A NOTE ON THE INTERACTION BETWEEN NUCLEON AND ANTI-NUCLEON

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ABSTRACT

The interaction potential between nucleon and anti-nucleon is derived by using the new Tamm-Dancoff formalism.

INTRODUCTION

WITH the discovery of anti-nucleons ($\overline{\mathrm{N}}$), many experiments on nucleon (N)—anti-nucleon ($\overline{\mathrm{N}}$) scattering have been performed in the energy range 100–400 Mev. In order to interpret the results of these experiments, we must know the forces between N and $\overline{\mathrm{N}}$. The interaction potential between N and $\overline{\mathrm{N}}$ has been recently studied by Chew and Ball (1958). As Chew (1959) pointed out, if we correctly understand charge conjugation, we can expect the interaction potential to have the general features of N-$\overline{\mathrm{N}}$ potential as given by Gartenhaus (1955) except that the N-N potential will have opposite sign to that of N-N. In this note we have derived an expression for N-$\overline{\mathrm{N}}$ potential due to an exchange of single pion using the new Tamm-Dancoff formalism of Dyson (1953).

As is well known in Dyson's new Tamm-Dancoff formalism, we employ the true or physical vacuum ψ_0 instead of the bare vacuum and the vacuum self-energy is thereby eliminated. Denoting by $A(\vec{p}, \vec{q})$ the amplitude for finding a nucleon of momentum $\vec{p}$ and anti-nucleon of momentum $\vec{q}$ in the state ψ, *i.e.*,

$$A(\vec{p}, \vec{q}) = [\psi_0, b(\vec{p})\, d(\vec{q})\, \psi]$$

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we obtain

$$(E_{p,\,q} - \epsilon)\, A(\vec{p}, \vec{q}) = \{\psi_0, [H_I, b(\vec{p})\, d(\vec{q})]\, \psi\} \tag{1}$$

where $E_{p,q}$ is the kinetic energy of free N and $\bar{N}$, ϵ is the total energy of the system and $b(\vec{p})$ and $d(\vec{q})$ are the annihilation operators for N and $\bar{N}$ with momentum $\vec{p}$ and $\vec{q}$ respectively. H_I is the Yukawa Hamiltonian in Schrodinger representation given by

$$H_I = g \int \bar{\psi}(\vec{x})\, \gamma_5\, \tau_i\, \psi(\vec{x})\, \phi_i(\vec{x})\, d^3x. \tag{2}$$

On evaluating the commutator on the right-hand side of (1), we find that the amplitude $A(\vec{p}, \vec{q})$ will be connected to various other amplitudes. But in order to obtain an integral equation for $A(\vec{p}, \vec{q})$ we will retain only those intermediate amplitudes which can be related back to $A(\vec{p}, \vec{q})$. Thus we consider only the following amplitudes $A(\vec{k}, \vec{q}, \vec{p} - \vec{k};)$, $A(\vec{k}, \vec{q};\ \vec{k} - \vec{p})$, $A(\vec{p} + \vec{q};)$, $A(;\, -\vec{p} - \vec{q})$, $A(\vec{p}, \vec{k}, \vec{q}, -\vec{k};)$ and $A(\vec{p}, \vec{k};\ \vec{k} - \vec{q})$ where the momentum occurring before the semicolon refer to Dyson's *plus* particles and the momentum occurring after the semicolon refer to Dyson's *minus* particles. The first two and the last two amplitudes stand for a $N\bar{N}$ and a meson while the middle two amplitudes refer to a meson alone.

In the centre of mass system of $N\bar{N}$ we have $\vec{p} = -\vec{q}$. Denoting by $A(\vec{p})$, the amplitude $A(\vec{p}, -\vec{p})$ the integral equation for $A(\vec{p})$ is given by

$$\begin{aligned}
&[2E(p) - \epsilon]\, A(\vec{p}) \\
&\quad = \frac{g^2}{(2\pi)^9} \frac{m}{E(p)} \Bigg[\bar{u}(\vec{p}) \int d^3k \, \frac{m}{E(k)} \frac{1}{2\omega(p-k)} \\
&\qquad \times \frac{\gamma_5 \tau_i u(\vec{k})\, \bar{u}(\vec{k})\, \gamma_5 \tau_i}{E(k) + E(p) \pm \omega(p-k) - \epsilon}\, u(\vec{p})\, A(\vec{p}) \\
&\quad - \bar{u}(\vec{p}) \int d^3k \, \frac{m}{E(k)} \frac{1}{2\omega(p-k)} \\
&\qquad \times \frac{\gamma_5 \tau_i u(\vec{k})\, \bar{v}(-\vec{k})\, \gamma_5 \tau_i\, v(-\vec{p})}{E(k) + E(p) \pm \omega(p-k) - \epsilon}\, A(\vec{k})
\end{aligned}$$

$$+\frac{1}{2\mu}\bar{u}(p)\int d^3k\frac{m}{\mathrm{E}(k)}\frac{\gamma_5\tau_i\bar{v}(\vec{k})\gamma_5\tau_i u(-\vec{k})v(\vec{p})}{\pm\mu-\epsilon}\mathrm{A}(\vec{k})$$

$$-\int d^3k\frac{m}{\mathrm{E}(k)}\frac{1}{2\omega(p+k)}$$

$$\times\frac{\bar{v}(\vec{k})\gamma_5\tau_i\bar{u}(\vec{p})\gamma_5\tau_i u(-\vec{k})v(-\vec{p})}{\mathrm{E}(p)+\mathrm{E}(k)\pm\omega(p+k)-\epsilon}\mathrm{A}(\vec{k})$$

$$+\int d^3k\frac{m}{\mathrm{E}(k)}\frac{1}{2\omega(p+k)}$$

$$\times\frac{\bar{v}(\vec{k})\gamma_5\tau_i\bar{v}(\vec{p})\gamma_5\tau_i v(\vec{k})v(\vec{p})}{\mathrm{E}(p)+\mathrm{E}(k)\pm\omega(p+k)-\epsilon}\mathrm{A}(\vec{p})\Big] \qquad (3)^*$$

We now neglect the nucleon recoil energies $\mathrm{E}(p)$ and $\mathrm{E}(k)$ when they occur together with meson energy ω. Also we omit the terms having $-\omega$ in the energy denominator. The first and last terms in (3) represent self-energy terms and thus they will be omitted. In these circumstances we can approximate Dirac spinors by their large components. Thus the 'potential' $\mathrm{V}(\vec{k})$ in the momentum space is identified to be

$$\mathrm{V}(\vec{k})=-\frac{g^2}{(2\pi)^9}\vec{\tau}_1\cdot\vec{\tau}_2\frac{(\vec{\sigma}_1\cdot\vec{k})(\vec{\sigma}_2\cdot\vec{k})}{[\omega(k)]^2}+\frac{g^2}{(2\pi)^9}\vec{\tau}_1\cdot\vec{\tau}_2\frac{1}{2\mu^2} \qquad (4)$$

The potential in configuration space will be given by, if γ denotes the relative distance,

$$\mathrm{V}(\gamma)=-\frac{g^2}{(2\pi)^8}\vec{\tau}_1\cdot\vec{\tau}_2\left\{\frac{\vec{\sigma}_1\cdot\vec{\sigma}_2}{\gamma}\int_1^\infty\frac{kdk}{[\omega(k)]^2}\sin k\gamma+3\mathrm{S}_{12}\int_0^\infty\frac{kdk}{[\omega(k)]^2}\right.$$

$$\left.\times\left[\left(\frac{k^2}{3\gamma}-\frac{1}{\gamma^3}\right)\sin k\gamma+\frac{k}{\gamma^2}\cos k\gamma\right]\right\}$$

$$+\frac{g^2}{(2\pi)^6}\frac{\vec{\tau}_1\cdot\vec{\tau}_2}{2\mu^2}\delta(\gamma) \qquad (5)$$

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* Here we have used the notation $+$ and $-$ in the denominator to indicate the sum of two terms in the integrand one involving a denominator with $+$ before ω and another having denominator with $-$ before ω. This is done purely for convenience in printing to avoid unnecessary duplication of terms.

where S_{12} is the familiar 'tensor force' operator, *i.e.*, $S_{12} = 3(\vec{\sigma}_1 \cdot \gamma)(\vec{\sigma}_2 \cdot \gamma)/\gamma^2$. An almost similar potential given below was obtained by Tarasov (1956) using old Tamm-Dancoff formalism

$$V(\gamma) = -\tfrac{1}{3}\vec{\tau}_1 \cdot \vec{\tau}_2 \frac{g^2}{4\pi}\left(\frac{\mu}{2m}\right)^2 \vec{\sigma}_1 \cdot \vec{\sigma}_2 + S_{12}\left[1 + \frac{3}{\mu\gamma} + \frac{3}{\mu\gamma^2}\right]\frac{e^{-\mu\gamma}}{\gamma}$$

$$- \tfrac{1}{3}(\vec{\sigma}_1 \cdot \vec{\sigma}_2)(\vec{\tau}_1 \cdot \vec{\tau}_2)(2m)^{-2}\,\delta(\gamma)$$

$$+ \frac{(\vec{\tau}_1 \cdot \vec{\tau}_2 + 3)(\vec{\sigma}_1 \cdot \vec{\sigma}_2 - 1)}{4\mu(2m - \mu)}\,\delta(\gamma) \qquad (6)$$

The first term in (5) should adequately describe the potential when V is of the order of Compton wavelength of pion. The second term denotes a 'contact' potential but as is well known we should not rely on it under the approximations made here. The first term agrees with that of Garterhaus potential for a single pion exchange except for the sign provided we replace g by the renormalised coupling constant. Of course, we are aware of the fact that in a Tamm-Dancoff formalism such as the one which we have employed in this note, renormalisation is not a consistent procedure.

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