A MAGNETO-OPTIC METHOD FOR THE DETERMINATION OF PIEZO-OPTIC COEFFICIENTS

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1. Introduction

When a cubic crystal or a glass is stressed, it becomes birefringent and the measurement of the relative retardation produced, gives some of the linear combinations of the piezo-optic constants q_{ij}. In particular, in glasses one gets the quantity $(q_{11}-q_{12})$. For the measurement of the relative retardation a Babinet Compensator is often used. While the measurements with a Babinet Compensator are very accurate the instrument is not too well suited for the measurement of the dispersion of the piezo-optic coefficients. From the recent studies on the magneto-optic rotation in birefringent media by the author (1949, 1951) a simple method has been developed for the measurement of the relative retardation produced on stressing an isotropic solid. The particular merit of the method is not so much its accuracy as its convenience for the measurement of the dispersion of the piezo-optic coefficient with wavelength.

This paper gives the experimental details and the theory of the method. Preliminary studies have been made of the piezo-optic coefficients of some glasses and they compare well with the determinations made with the Babinet Compensator (Vedam, 1950).

2. Theory of the Method

When plane polarised light is incident on a birefringent crystal (or a stressed isotropic solid) placed in a magnetic field, the emergent light is elliptically polarised. The major axis of the emergent ellipse is in general, inclined at an angle ψ to the plane of vibration of the incident light. This inclination is dependent on the angle which the incident electric vector makes with the principal planes of vibration of the crystal. In the particular case when the two are parallel, ψ is given by

$$\tan 2\psi = \frac{\sin 2\gamma \sin \Delta}{\cos^2 2\gamma + \sin^2 2\gamma \cos \Delta}, \tag{1}$$

where $\Delta = \Delta_0 t = t\sqrt{\delta_0^2 + (2\rho_0)^2}$ and $\tan 2\gamma = \frac{2\rho_0}{\delta_0}$

Δ_0 is the composite phase retardation per unit length, $\delta = t\delta_0$ the total phase retardation when there is no magnetic field and $\rho = t\rho_0$ the total rotation when there is no birefringence and t the thickness of the specimen.

It has been shown (Ramaseshan, 1951) that by suitable algebraic manipulation, when δ_0 and $2\rho_0$ are small, equation (1) could be written as

$$2\psi = 2\rho\left\{1 - \frac{\delta^2}{3!} + \frac{\delta^2}{5!}\left[(m^2+6)(2\rho)^2\right] - \frac{\delta^2}{7!}\left[(m^4+120\,m^2-90)(2\rho)^4\right] - - - \right\} \quad (2)$$

here $m = \dfrac{1}{\sin 2\gamma} = \dfrac{\Delta}{2\rho} = \dfrac{\sqrt{\delta^2+(2\rho)^2}}{2\rho}$

In the case when δ and 2ρ are very small the equation (2) reduces to

$$2\psi = 2\rho\left(1 - \frac{\delta^2}{6}\right) \quad (3)$$

As both ρ and ψ can be accurately measured with the aid of a half shade, these formulæ could be used in determining the stress-optic coefficients in isotropic solids. One measures the magnetic rotation when the solid is not stressed and again the apparent rotation with the same magnetic field and a known applied stress. Then the birefringence introduced δ, can be evaluated from equation (2) or (3). When δ and 2ρ are both less than 20° equation (3) gives an accuracy of about 1% in the value of δ. On the other hand for higher values of δ and 2ρ equation (2) must be used. In calculating δ by equation (2) the method of successive approximation must be resorted to in evaluating the value of m. By taking the four terms of the series given in equation (2) one could get the value of δ within 1% of its true value (when $\delta < 55°$ and $2\rho < 40°$).

In the earlier experiments one peculiarity was met with. When the solid was stressed the values of ψ determined for the magnetic field on and reversed, differed considerably. This was found to be due to the fact that the incident electric vector did not exactly coincide with the direction of the stress in the solid. Experimentally also, setting them exactly parallel was extremely difficult, particularly as the direction of stress in the experimental solid was never exactly the same at all points. Detailed mathematical investigation of the problem of *the dependence of ψ on the inclination of the electric vector to the principal axes of strain* has shown (Ramachandran and Ramaseshan, 1952) that the usual practice of taking the mean of the measurements for the two directions of the magnetic field practically eliminates the errors caused by the slight mis-setting of the polariser or specimen or as a result of the variation of the stress axis in the specimen. In fact when the setting was out by 3° (a case hardly to be met with in practice) while the individual readings for the two directions of the field varied by about 10% the mean value deviated from the correct value by less than 1%.

3. Experimental Procedure and Results

The usual Faraday effect arrangement with a half shade at the polarizer end was used in the measurement of ρ and ψ. A simple lever arrangement made of brass was employed to stress the specimen (in the present case a glass block 3 cm. × 3 cm. × 2 cm.). The light traversed the 2 cm. side of the glass block. Pressure was transmitted to the specimen by lead blocks suitably shaped to make the stress direction in the central portion almost vertical and the stress distribution fairly uniform. Some care was taken in setting the electric vector of the incident polarised beam parallel to the stress direction. It was found best to measure ρ and ψ for two or three different field strengths but for the same value of stress and later to repeat the measurements for different stresses. Table I gives a typical set of values obtained for one of the glasses (glass No. 10). δ in each case was calculated from formula (2) using 3 terms. It may be mentioned here that while the 4th term practically does not affect the values of δ, the omission of the 3rd term introduces an error of the order of 6% in the last three determinations.

TABLE I

Stress in Kgwt./mm.^{2}	Field in Oersteds	2ρ in degrees	2ψ in degrees	δ in degrees
0·091	6420	13·26	13·03	19·0
0·091	9490	19·62	19·27	19·0
0·179	6420	13·28	12·30	38·0
0·179	9490	19·60	18·21	39·1
0·268	6420	13·30	11·32	56·0
0·268	9490	19·60	16·71	56·5

Table II gives the results of the measurements made on 12 optical glasses. $[\delta_0]$ represents the path retardation introduced by a pressure of 1 dyne/cm.^{2} for λ 5893 when the thickness of the substance is 1 cm. and the values of $(p - q)$ are calculated from the formula

$$\delta = \frac{n^2}{2\,\mathrm{R}}(p - q)\,\mathrm{P}$$

where δ represents the birefringence introduced, R the rigidity modulus, n the refractive index for λ 5893, P the stress in dynes per sq. cm. and p and q the Neumann's strain-optical constants. The values of R obtained by Vedam (1950) for the same set of glasses have also been entered in the table. The last two columns give the values of $(p-q)$ obtained in the present studies as well as those obtained by Vedam (1950) who used the classical Babinet Compensator method.

One notices from Table II that the values obtained by the magneto-optic method are quite in agreement with those obtained by the Babinet Compensator method. The method could therefore be confidently used for the study of the variation of $(p-q)$ with wavelength. In doing so, it must be borne in mind that 2ρ must never be greater than 40°.

TABLE II

Glass* No.	n_D	$R \times 10^{-11}$ in dynes/cm.^{2}	$[\delta_0] \times 10^{13}$	$(p-q)$ (Author)	$(p-q)$ (Vedam)
1	1·4669	1·878	4·24	0·074	0·0762
2	1·4934	2·538	4·13	0·094	0·0918
3	1·5023	2·862	3·07	0·078	0·0774
4	1·5093	2·917	4·18	0·107	0·1049
5	1·5171	3·354	3·09	0·090	0·0909
6	1·5300	2·275	4·50	0·087	0·0875
7	1·5269	2·604	3·54	0·079	0·0771
9	1·5700	3·053	3·55	0·088	0·0855
10	1·5699	2·474	3·52	0·069	0·0698
11	1.5895	3·292	2·42	0·061	0·0633
12	1·5981	2·404	3·51	0·066	0·0642
13	1·6062	2·569	3·06	0·061	0·0630

* The numbering of these Jena glasses is the same as that adopted by Dr. R. S. Krishnan (*Proc. Ind. Acad. Sci.*, 1936, **4**, 211).

It is found most convenient to make the measurements from λ 7000 to λ 4500 visually and to use a spectrographic or a photomultiplier technique for lower wavelengths. Such techniques have been developed in this laboratory (V. Sivaramakrishnan) and the result of measurements for different wavelengths on glasses and other isotropic solids will be reported later. It is found that although the error in the absolute value of $(p - q)$ may be as great as 3 to 4% the relative values for different wavelengths would be of very much greater accuracy.

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4. SUMMARY

A simple magneto-optic method for the measurement of the relative retardation produced on stressing an isotropic solid is described. The method is based on the determination of the true and apparent magneto-optic rotation of the isotropic solid in the unstressed and the stressed states. The method is found to be useful in the measurement of the dispersion of the piezo-optic coefficients with wavelentgh.

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