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Bethe-Salpeter dynamics for two-photon processes*

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Abstract. Recent experimental data on single hadron production by two-photon beams in PETRA and PEP have provided a unique opportunity for testing specific models of confinement through a study of one of their cleanest predictions *viz* the $\gamma\gamma \to H$ amplitudes. Motivated by this new facility, a QCD-oriented Bethe-Salpeter model of harmonic confinement, which has already been found to describe rather well several classes of hadronic data (from mass spectra to electromagnetic and pionic couplings), is now employed for a detailed comparison of its predictions on $P \to \gamma\gamma$ and $T \to \gamma\gamma$ couplings with the data. The agreement is quite good for all cases except one ($\eta \to \gamma\gamma$).

Keywords. Bethe-Salpeter dynamics; two photon processes

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1. Introduction and summary

Two-photon processes, especially those involving $H\gamma\gamma$ ($H = q\bar{q}$ hadron) vertices, offer a unique probe into the mechanism of coupling of hadrons (as quark composites) to photons, and hence of the underlying dynamics for the quark structure of single hadrons, unmixed with unwanted hadronic interactions. The simplest processes of this kind are of course $H \to \gamma\gamma$ decays, but except for $\pi^0 \to \gamma\gamma$, these have not, until recently, been amenable to direct experimentation with any degree of reliability, in view of the huge hadronic background generally involved in the final state when vertices of order e^2 are sought to be extracted from standard hadron-hadron collision data. Theoretical tools for a deeper understanding of $H \to \gamma\gamma$ events have for a long time been limited to generalities like sum rules for $P \to \gamma\gamma$, based on the Adler-Bell-Jackiw anomaly (Adler 1969; Bell and Jackiw 1969), or at most VMD oriented models (Schwinger 1967), but with little prospects for more meaningful comparison with concrete dynamical models based on quark compositeness.

The spectacular developments of e^+e^- Collider techniques at PETRA and PEP in this very decade have let to powerful photon beams with high two-photon luminosities which compete with e^+e^- luminosities at high enough energies. Further the $\gamma\gamma$ cross-sections for hadron production which *increase* like $(\ln E)^2$, soon dominate over cross-sections (which *decrease* like E^{-2}), despite the disadvantage of the e.m. couplings (order α^4) compared to their e^+e^- counterparts (order α^2) (Kolanksi 1984). On the other hand, most of the γ-γ events have *low* invariant masses of the produced hadrons due to

*This paper is offered as a "Festschrift" in honour of Dr Raja Ramanna on the occasion of his sixtieth birthday. The subject is theoretical in content but seeks to exploit an entirely new window opened by the latest experimental technology (on two-photon physics). As such it is appropriately dedicated to one of the main architects of nuclear science in India.

the characteristic bremstrahlung spectrum ($\sim E_\gamma^{-1}$) of the radiated photons (Kolankski 1984). Thus the γ-γ collision mechanism facilitates, among other things, a clean measurement of $\gamma\gamma H$ vertex functions for relatively low mass hadrons (H), say up to a few GeV. This is mainly because of the stringent selection rules governing the $\gamma\gamma$ couplings of mesons, which greatly restrict the number and variety of final states involved in γ-γ inelastic processes, thus greatly simplifying the theoretical analysis of the process. In particular, $\gamma\gamma$ couplings of pseudoscalar (P), tensor (T) and scalar (S) mesons can be ideally probed by two photon production of these singlet and triplet $q\bar{q}$ states respectively. This way of studying $\gamma\gamma H$ coupling through the $\gamma\gamma \to H$ process merely amounts to interchanging the roles of the initial and final states compared to the old-fashioned way of looking at the same coupling through a decay process ($H \to \gamma\gamma$) but now with the advantage of a negligible hadronic background.

By this new method, the results of 'measurement' of several $H \to \gamma\gamma$ couplings ($H = T$, P, S) have already become available (Kolankski 1984) thus making it worthwhile to compare these relatively clean data with the predictions of specific (QCD-oriented) models of quark dynamics. Further, such processes, involving as they do invariant masses of modest magnitudes ($\lesssim$few GeV), bear not so much on the perturbative aspects of QCD (corresponding to the asymptotic freedom region), as on its (theoretically less tractable) confinement aspects. This twin feature of an emphasis on the confinement region on the one hand and a relative theoretical simplicity on the other makes a $\gamma\gamma \to H$ process an ideal laboratory for testing specific models of confinement.

In this paper we have been motivated by precisely such considerations to study $\gamma\gamma H$ couplmgs within the framework of a QCD-oriented Bethe-Salpeter (BS) model (Mitra and Santhanam 1981a) hadronic confinement which has been developed and tested in recent years through diverse applications from hadron mass spectra (Mitra and Santhanam 1981b; Kulshreshtha *et al* 1982; Mitra and Mittal 1984) to their e.m. and pionic form factors (Mitra and Kulshreshtha 1982; Kulshreshtha and Mitra 1983; Mitra and Mittal 1984). The only inputs are the (constituent) quark masses (m_q) and a universal reduced spring constant ($\tilde{\omega}$), which seem to describe a fairly large class of hadronic data (Mitra and Santhanam 1981a,b; Kulshreshtha *et al* 1982; Mitra and Mittal 1984; Mitra and Kulshreshtha 1982; Kulshreshtha and Mitra 1983; Mitra and Mittal 1984). The initial calculational techniques for the evaluation of matrix elements of e.m. and hadronic currents (Mitra and Kulshreshtha 1982; Kulshreshtha and Mitra 1982) by the method of null-plane variables (Bjorken *et al* 1971; Feldman *et al* 1973; Fishbane and Namyslowski 1980) have since been refined and found not only to simplify the calculations but also to lend a degree of elegance and transparency to the algebraic structures of the matrix elements.

The hadronic matrix elements can be classified into $H \to \gamma\gamma$, $H \to \gamma H'$ and $H \to H'H''$ types in ascending order of successively larger number of hadrons. The simplest process, *viz* $H \to \gamma\gamma$, involves only one hadron for which it is adequate to consider it to be in its *rest* frame. The other two classes of processes involve more hadrons than one, and for these it is necessary to take proper account of their motion in their respective wave functions. While the latter cases will be dealt with in a subsequent communication (Mitra and Mittal 1985), we concentrate in this paper on the specific process $H \to \gamma\gamma$ which is not only free from the complications due to hadronic motion but its predictions are also directly amenable to the new data on $\gamma\gamma \to H$ matrix elements (adequately summarized by Kolankski (1984)), thus hopefully providing a fairly clean (*albeit* limited) test of the dynamical content of a single $q\bar{q}$ wave function.

In §2 we summarize the essential features of the BS formalism for the wave functions of L-excited hadrons, and evaluation of $H \rightarrow \gamma\gamma$ matrix elements in terms of null-plane variables. In view of the relatively unfamiliar structure of the BS matrix elements in the null-plane variables, some attention is paid to the method of integration over-the corresponding time-like internal momentum (q_-) which is shown to exhibit the $P \rightarrow \gamma\gamma$ matrix elements in a simple and transparent form. Section 3 describes the corresponding algebraic structure of the $T \rightarrow \gamma\gamma$ matrix elements on closely parallel lines, except for the heavier algebra involved. Section 4 is devoted to a comparison of these predictions with the recent $\gamma\gamma \rightarrow H$ data (Kolankski 1984; PDG 1984; Bartel *et al* 1982). The theoretical implications of the agreement (which is rather good on the whole) are discussed.

2. BS formalism in null plane variables: The P $\rightarrow \gamma\gamma$ process

The necessary formalism for the calculations of various matrix elements in terms of Feynman rules was outlined earlier (Mitra and Kulshreshtha 1982; Kulshreshtha and Mitra 1983). In particular it was emphasised that the key role is played by the four-dimensional BS wave function which must first be reconstructed from its corresponding instantaneous (three dimensional) form. Now the three-dimensional wave functions have so far been worked out (Mitra and Kulshreshtha 1982; Kulshreshtha and Mitra 1983; Mitra and Mittal 1984) in the hadronic c.m. frame ($\mathbf{P} = 0$) which should be adequate for $H \rightarrow \gamma\gamma$ coupling, though *not* for $HH'\gamma$ and $HH'H''$ couplings involving more than one hadron. For the latter cases which were treated under the rest frame assumption for each hadron (Kulshreshtha and Mitra 1983; Mitra and Mittal 1984) defect was sought to be remedied through the compensating ansatz of a Lorentz-invariant adaptation of the corresponding scalar form factors (Santhanam *et al* 1979), *viz*

$$\mathscr{F}(\mathbf{P}^2, \mathbf{P}'^2, \mathbf{P}''^2) \rightarrow \mathscr{F}(P_\mu^2, P_\mu'^2, P_\mu''^2).$$

Such an adaptation is nevertheless *ad hoc*, and cannot be a substitute for a dynamical basis for wave functions of *moving* hadrons. The null-plane formulation of the hadronic wave functions seems to hold considerable promise in bridging the theoretical gap between rest-frame and moving frame wave functions, as will indeed be shown in a subsequent communication for $HH'\gamma$ and $HH'H''$ processes (Mitra and Mittal 1985). In the meantime, for the process $H \rightarrow \gamma\gamma$ on hand, it should be enough to summarize the essential details of the structure of the three-dimensional $q\bar{q}$ wave function ψ in the null-plane approximation (NPA) *vis-a-vis* its four-dimensional BS form Ψ. For any four-vector A_μ, the null-plane components are defined as (Bjorken *et al* 1971; Feldman *et al* 1973; Fishbane and Namyslowski 1980).

$$\mathbf{A}_\perp = A_1, A_2; \quad A_\pm = A_0 \pm A_3. \tag{1}$$

This decomposition holds for each of the 4-momenta of the $q\bar{q}$ hadron (P_μ) as well as those of the individual quark ($p_{1\mu}$) and antiquark ($-p_{2\mu}$) as shown in figure 1(a). The roles of the time-like momenta in NPA are played by $1/2\, p_{i-}$ ($i = 1, 2$), instead of by p_{i0} in the instantaneous approximation (IA), while the Z components in NPA are *effectively* p_{i+} instead of p_{i3} of IA. The relative (q) and total (P) 4-momenta are defined for equal mass quarks through

$$p_{1,2} = \tfrac{1}{2}P \pm q \tag{2}$$

and the null-plane assumption (NPA) consists in integrating out the BS four-dimensional $q\bar{q}$ wave function Ψ w.r.t. the variable $1/2\ q_-$ to obtain an effective three-dimensional entity ψ as (Kulshreshtha and Mitra 1983).

$$\psi(\mathbf{q}) = \psi(\mathbf{q}_\perp, q_+) = \int \tfrac{1}{2} \mathrm{d}q_- \Psi(p_1, p_2). \tag{3}$$

as a substitute for the instantaneous approximation (IA) which reads as (Mitra and Santhanam 1981a)

$$\psi(\mathbf{q}) = \psi(\mathbf{q}_\perp, q_3) = \int \mathrm{d}q_0 \Psi(p_1, p_2). \tag{4}$$

The inverse NPA relation between ψ and Ψ for a *moving* hadron, taking account of its spin-structure, is expressed by (Mitra and Kulshreshtha 1982; Kulshreshtha and Mitra 1982; Mitra and Mittal 1985)

$$\Psi(p_1, p_2) = S_F(-p_2)\Gamma(\mathbf{q})S_F(p_1), \tag{5}$$

$$\Gamma(\mathbf{q}) = N_H \Gamma \phi_L(\mathbf{q}) D(\mathbf{q})/2\pi i, \tag{6}$$

$$D(\mathbf{q}) = 2P_+(m_q^2 + q_\perp^2 - \tfrac{1}{4}M^2) + 2P_- q_+^2. \tag{7}$$

$\Gamma = \gamma_5$ or $i\gamma \cdot \varepsilon$ according as the $q\bar{q}$ hadron is a singlet or triplet structure respectively. $\phi(\mathbf{q})$ is the scalar part of the wave function which for an arbitrary L excited state has the form (Mitra and Mittal; Mitra and Sood 1977)

$$\phi_L = n_L \beta^{-L} q_{i_1} \ldots . q_{i_L} B^L_{i_1 \ldots . i_L} \phi_0, \tag{8}$$

$$n_L^2 = (M/P_+)^{\frac{1}{2}} (\pi\beta^2)^{-3/2} 2^L/L! \tag{9}$$

$$\phi_0 = \exp\left[-\tfrac{1}{2}\mathbf{q}_\perp^2 \beta^{-2} - \tfrac{1}{2}\frac{M}{P_+} q_+^2 \beta^{-2} \right], \tag{10}$$

$$\beta^2 = \tilde{\omega}(Mm_q)^{\frac{1}{2}}[1 + 2\tilde{\omega}^2/Mm_q]^{-\frac{1}{2}}. \tag{11}$$

$B^L_{(\mu)}$ is a symmetric traceless rank-L tensor normalised as (Mitra and Sood 1977)

$$\sum_M B^{L(M)}_{\mu_1 \ldots . \mu_L}(P) B^{*L(M)}_{\mu'_1 \ldots . \mu'_L}(P) = \theta^{\mu'_1 \ldots \mu'_L}_{\mu_1 \ldots . \mu_L}(P), \tag{12}$$

where the projection operator $\theta^{(\mu')}_{(\mu)}$ for a hadron of spin J and 4-momentum P_μ has the standard definition (Mitra and Sood 1977). The tensor B^L fully defines the relativistic structure of a singlet $q\bar{q}$ state of $J = L$. For a *triplet* $q\bar{q}$ state, on the other hand, the full spin-structure is expressed by the CG decomposition in three-dimensional form (Mitra and Sood 1977)

$$\varepsilon_i \otimes B^L_{(i)} = T^{L+1}_{i_1 \ldots i_L} + i\left(\frac{L}{L+1}\right)^{\frac{1}{2}} S_L \varepsilon_{ii_1 j} A^L_{ji_2 \ldots i_L} + \left(\frac{L(L+1)}{2(2L+1)}\right)^{\frac{1}{2}} S_{L+1} \delta_{ii_1} S^{L-1}_{i_2 \ldots i_L}, \tag{13}$$

of which only the first (tensor) state with unit coefficient corresponds to the highest J-value $(= L+1)$ while the satellites (A^L, S^{L-1}) have more involved coefficients which must be taken into account for determining the relative strengths of various couplings of the satellite states $(J < L+1)$ *vis-a-vis* those of the main state $(J = L+1)$. For formal covariance, each index i in (12) may be interpreted as a 4-vector as defined in the rest frame ($\mathbf{P}=0$) of the hadron through formal extensions such as

$$q_i \to \hat{q}_\mu(P); \quad \hat{q}_\mu(P) = q_\mu + q\cdot P P_\mu M^{-2}. \tag{14}$$

Substitution of (13) in (8) gives the following BS structures for $q\bar{q}$ singlet and triplet states respectively

$$\Gamma\phi_L = n_L \beta^{-L} \gamma_5 \hat{q}_{\mu_1}(P)\ldots \hat{q}_{\mu_L}(P) B^L_{\mu_1\ldots\mu_L}, \tag{15}$$

$$\Gamma\phi_L = n_L \beta^{-L} i\hat{\gamma}_\mu q_{\mu_1}\ldots q_{\mu_L} T^{L+1}_{\mu\mu_1\ldots\mu_L} + (ST), \tag{16}$$

where for $L=1$, the satellite terms (ST) are given by

$$(\mathrm{ST}) = n_1\beta^{-1}\left(i\varepsilon_{\lambda\mu\nu\mu_1}\frac{P_\lambda}{M} q_{\mu_1}\hat{\gamma}_\mu A^1_\nu + \frac{1}{\sqrt{3}} i\hat{\gamma}\cdot q S^0\right). \tag{17}$$

Here A^1 and S^0 are ($L=1$) axial vector and scalar mesons respectively of $C=+1$, while B^1 is an axial vector of $C=-1$, and B^0 ($\equiv P$) is simply a pseudoscalar meson.

The normaliser N_H, equation (6), associated with the BS vertex $\Gamma(\mathbf{q})$ is defined as (Mitra and Kulshreshtha 1982)

$$2iP_\mu = (2\pi)^4 \mathrm{Tr}\int d^4q\,[\bar{\Psi}\tfrac{1}{2}i\gamma_\mu \Psi(m_q - i\gamma\cdot p_2) - \bar{\Psi}(m_q + i\gamma\cdot p_1)\Psi\tfrac{1}{2}i\gamma_\mu]. \tag{18}$$

This equation, when simplified as in Mitra and Kulshreshtha (1982), yields the following results for N_H.

$$(2N_H)^{-2} = M(2\pi)^3\left\{m_q^2\begin{pmatrix}1\\1\end{pmatrix} + \beta^2\begin{pmatrix}J+\frac{3}{2}\\ \frac{1}{2}J+\frac{1}{2}\end{pmatrix}\right\}, \tag{19}$$

where the two entries refer to B^L-type ($J=L$) and T^{L+1}-type ($J=L+1$) states respectively.

In this paper we are interested only in the $\gamma\gamma$-couplings, figure 1 (b, c), for relevant meson states (T, B, S) at rest ($\mathbf{P}=0$), but with integrations carried out in the null-plane language. The cases of immediate physical interest are several tensor (f^0, f', A_2, χ_2) and pseudoscalar ($\pi^0, \eta, \eta', \eta_c$) mesons, while the corresponding data on scalar mesons (S^*,

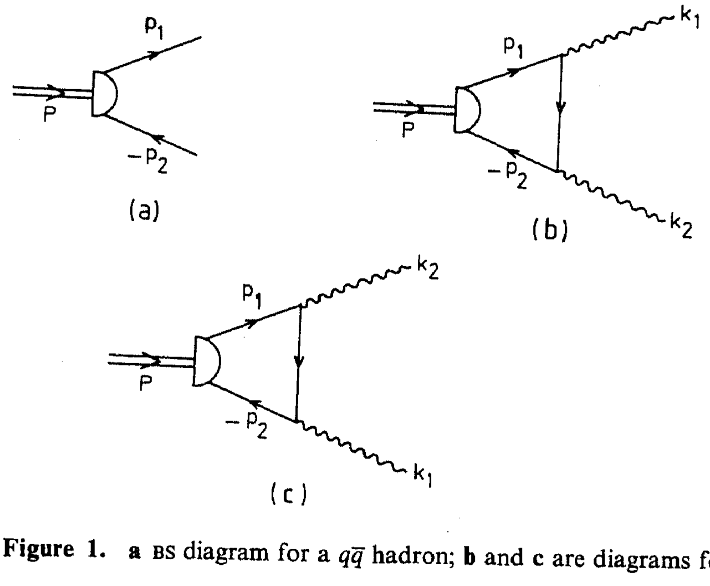

Figure 1. **a** BS diagram for a $q\bar{q}$ hadron; **b** and **c** are diagrams for $H\to\gamma\gamma$ process.

χ_0) are not yet available. To calculate the matrix elements it is good to separate out the colour-cum-charge factors (f_c) which are collectively given for the different cases (colour contributes $\sqrt{3}$ in all cases), all in units of $e^2/\sqrt{6}$, by the following table

$$\begin{array}{lccccc} & \pi^0(A_2) & f^0 & f' & \eta_c & \chi_2, \\ f_c = & 1 & 5/3 & \sqrt{2}/3 & 4\sqrt{2}/3 & 4\sqrt{2}/3. \end{array} \tag{20}$$

For the mixed states η, η', again in units of $e^2/\sqrt{6}$, the corresponding numbers must take account of the mass difference between ud- and s-quarks in the calculation of the matrix elements (Mitra and Mittal 1984). This requirement is however met by writing the effective $\gamma\gamma$ coupling constants (without the factor $e^2/\sqrt{6}$) in term of a $2\times 2 f_c$ matrix as;

$$\begin{pmatrix} g_\eta \\ g_{\eta'} \end{pmatrix} = \begin{pmatrix} 5/3\cos(\theta+\alpha) & -\frac{\sqrt{2}}{3}\sin(\theta+\alpha) \\ 5/3\sin(\theta+\alpha) & +\frac{\sqrt{2}}{3}\cos(\theta+\alpha) \end{pmatrix} \begin{pmatrix} g_{ud} \\ g_s \end{pmatrix}, \tag{21}$$

where the reduced $\gamma\gamma$ coupling constants g_{ud} and g_s are calculated from figure 1 (b,c) assuming only one type of quark (ud or s) emitting the photons. Here (Mitra and Mittal 1984)

$$\tan\alpha = \sqrt{2}; \quad \theta = -10{\cdot}8^\circ. \tag{22}$$

Such effective coupling constants may be formally defined through a corresponding Lagrangian which may be written for a $P\to\gamma\gamma$ process as (Kulshreshtha and Mitra 1983)

$$\mathscr{L}^P_{\text{eff}} = \frac{e^2}{\sqrt{6}} g_{\text{eff}}\,\varepsilon_{\mu\nu\rho\sigma}\,\varepsilon^{(1)}_\mu \varepsilon^{(2)}_\nu P_\rho Q_\sigma/4\pi^2 M, \tag{23}$$

where $2Q_\mu = k_{1\mu} - k_{2\mu}$. The quantity g_{eff} may be identified by comparison with the matrix element for $P\to\gamma\gamma$ *via* figure 1 (b,c), *viz*

$$M(P\to\gamma\gamma) = \frac{f_c e^2}{\sqrt{6}} \operatorname{Tr}\int d^4q[\Psi(p_1,p_2)i\gamma\cdot\varepsilon^{(1)}S_F(q-Q)i\gamma\cdot\varepsilon^{(2)} + (1\rightleftharpoons 2)], \tag{24}$$

where Ψ is the BS amplitude for the pseudoscalar meson. Substitution of (5) to (11) and (15) with $L=0$ in (24) yields:

$$\frac{g_{\text{eff}}}{4\pi^2} = (\pi\beta^2)^{-\frac{3}{4}} f_c N_H \int d^4q \frac{4Mm_q}{\Delta_1\Delta_2}\frac{D\phi_0}{2\pi i}\left[\frac{1}{\Delta_3^+} + \frac{1}{\Delta_3^-}\right], \tag{25}$$

where

$$\Delta_{1,2} = m_q^2 + p_{1,2}^2, \tag{26}$$

$$\Delta_3^\pm = m_q^2 + (q\mp Q)^2, \tag{27}$$

and f_c is the charge-colour factor as listed in (20) and (21).

The rest of the section is concerned with our procedure for integration of (25) w.r.t. the null-plane variables which will also hold in identical form for the $T\to\gamma\gamma$ matrix elements whose details are given in the next section. In view of (2) the q_- integration in (25) may be expressed in terms of either p_{1-} or p_{2-} whose pole position are governed by the three denominators $\Delta_1, \Delta_2, \Delta_3^\pm$. For definiteness we concentrate on the p_{2-}-variable which has a pole in Δ_2, *below* the real axis, *viz*

$$p_{2-} = (m_q^2 + q_\perp^2 - i\varepsilon)/p_{2+} \equiv (\omega_\perp^2 - i\varepsilon)/p_{2+}. \tag{28}$$

Since $p_{1\pm} = P_\pm - p_{2\pm}$ the corresponding p_{2-}-pole arising from Δ_1, *viz*

$$p_{2-} = P_- - (\omega_\perp^2 - i\varepsilon)/p_{1+}$$

lies *above* the real axis, and would not contribute if the semi-circle is closed from below. As for $\Delta^\pm$, their p_{2-}-poles are respectively:

$$p_{2-} = \tfrac{1}{2}P_- \mp Q_- - (\omega_\perp^2 - i\varepsilon)/(q_+ \mp Q_+), \tag{29}$$

where

$$k_{1,2} = \tfrac{1}{2}P \pm Q, \quad 2Q_\pm = k_{1\pm} - k_{2\pm}. \tag{30}$$

Thus the conditions for these poles to be *below* the real axis are $(q_+ \mp Q_+) < 0$ respectively. (Note that $P_\pm = M$ and $Q_+ = -Q_-$ in the rest frame of the hadron).

With these restrictions on the limits of the (subsequent) q_+-integration, the various residues from available poles can be evaluated in a straightforward manner and the result collected. Considerable simplification is achieved by noting the following result

$$\text{Res of } \left(\frac{1}{\Delta_1 \Delta_2}\right) \text{ at } \Delta_2\text{-pole} = \frac{1}{D}, \tag{31}$$

and using the following definitions

$$\begin{aligned}
\tfrac{1}{2}D_{1+} &= k_{1+}\omega_\perp^2 + p_{1+}k_{1-}(q-Q)_+,\\
\tfrac{1}{2}D_{2+} &= k_{2+}\omega_\perp^2 - p_{2+}k_{2-}(q-Q)_+,\\
\tfrac{1}{2}D_{1-} &= k_{2+}\omega_\perp^2 + p_{1+}k_{2-}(q+Q)_+,\\
\tfrac{1}{2}D_{2-} &= k_{1+}\omega_\perp^2 - p_{2+}k_{1-}(q+Q)_+.
\end{aligned} \tag{32}$$

The result for

$$\mathscr{I}_2 \equiv \int \tfrac{1}{2}\mathrm{d}p_{2-} \frac{D}{2\pi i}\frac{1}{\Delta_1\Delta_2}\left(\frac{1}{\Delta_3^+} + \frac{1}{\Delta_3^-}\right) \tag{33}$$

is

$$\left.\frac{2p_{2+}}{D_{2+}} + \frac{2(q_+ - Q_+)D}{D_{1+}D_{2+}}\right|_{q_+ - Q_+ < 0} + \left.\frac{2p_{2+}}{D_{2-}} + \frac{2(q_+ + Q_+)D}{D_{1-}D_{2-}}\right|_{q_+ + Q_+ < 0},$$

which simplifies to

$$\mathscr{I}_2 = \left.\frac{2p_{1+}}{D_{1+}}\right|_{q_+ < Q_+} + \left.\frac{2p_{2+}}{D_{2+}}\right|_{q_+ > Q_+} + \left.\frac{2p_{1+}}{D_{1-}}\right|_{q_+ < -Q_+} + \left.\frac{2p_{2+}}{D_{2-}}\right|_{q_+ > -Q_+}. \tag{34}$$

An identical result would have been obtained if the integration were performed over the

p_{1-}-variable. To further simplify the expression for $\mathscr{I}_2$, take $Q_+ > 0$ (without loss of generality). Since $Q_+ = \frac{1}{2}M$, one has $k_{1-} = k_{2+} = 0; k_{1+} = k_{2-} = M$. Then the second and third terms in (34) will make negligible contributions on q_+-integration, while the first and the fourth simplify just to:

$$\mathscr{I}_2 = \frac{p_{1+}}{M\omega_\perp^2} + \frac{p_{2+}}{M\omega_\perp^2} = \frac{1}{\omega_\perp^2}. \tag{35}$$

Substitution of (35) in (25) yields

$$g_{\text{eff}} = (\pi\beta^2)^{-\frac{3}{4}} 16\pi^2 M m_q f_c N_H \int \frac{d^2 q_\perp}{\omega_\perp^2} \int_{-|Q_+|}^{\infty} dq_+ \exp(-\tfrac{1}{2}\mathbf{q}^2 \beta^{-2}) \tag{36}$$

which simplifies to

$$g_{\text{eff}} = 4m_q (m_q^2 + \tfrac{3}{2}\beta^2)^{-\frac{1}{2}} \left(\frac{\pi M^2}{\beta^2}\right)^{\frac{3}{4}} f_c \mathscr{J}_0(y_0) \, Erf(x_0), \tag{37}$$

where

$$\mathscr{J}_n(y_0) = \pi \int_0^\infty x^n \, dx \exp(-x)/(x + y_0), \tag{38}$$

$$y_0 = \tfrac{1}{2} m_q^2 \beta^{-2}, \quad x_0 = |Q_+|/\sqrt{2}\beta. \tag{39}$$

The $P \to \gamma\gamma$ decay width is finally given by

$$\Gamma(P \to \gamma\gamma) = |g_{\text{eff}}|^2 \alpha^2 f_c^2 M (384\pi^3)^{-1}. \tag{40}$$

3. Structure of the $T \to \gamma\gamma$ amplitude

To evaluate the $\gamma\gamma$ couplings of tensor mesons, the tensor structure, (16), must be substituted in (6) for $\Gamma(\mathbf{q})$. The result is

$$\mathscr{F}(T \to \gamma\gamma) = \frac{e^2}{\sqrt{6}} \int d^4 q \frac{f_c N_H}{\Delta_1 \Delta_2} \frac{D\phi_0}{2\pi i} (\pi\beta^2)^{-\frac{3}{4}} \frac{4\sqrt{2}}{\beta} \left[\frac{T_+}{\Delta_3^+} + \frac{T_-}{\Delta_3^-}\right] \tag{41}$$

where $\Delta_{1,2}$ and $\Delta_3^\pm$ are given by (26), (27) and

$$T_\pm = \tfrac{1}{4} \text{Tr} (m_q + i\gamma\cdot p_2) i\gamma\cdot\hat{\varepsilon}(m_q - i\gamma\cdot p_1)\Gamma_\pm, \tag{42}$$

$$\Gamma_+ = i\gamma\cdot\varepsilon^{(1)} [m_q - i\gamma\cdot(q - Q)] i\gamma\cdot\varepsilon^{(2)}$$

$$\Gamma_- = i\gamma\cdot\varepsilon^{(2)} [m_q - i\gamma\cdot(q + Q)] i\gamma\cdot\varepsilon^{(1)}, \tag{43}$$

$$\hat{\varepsilon}_\mu = \hat{q}_{\mu 1}(P) T_{\mu\mu 1}(P). \tag{44}$$

The trace evaluation is considerably simplified by observing that each of the quantities T_+ can be effectively replaced by $1/2\,(T_+ + T_-)$, since the difference $(T_+ - T_-)$ vanishes on integration w.r.t. $\mathbf{q}$. The symmetric part is further simplified in *three-dimensional* tensor notation (in the rest frame of the hadron) as

$$\tfrac{1}{2}(T_+ + T_-) = -(\tfrac{1}{4}M^2 + \omega_\perp^2 + q_+^2) 2A_{ij} q_i q_l T_{jl} + 2q_i T_{lm} q_m (2A_{ij} q_i q_j - m_q^2 \varepsilon^{(1)}\cdot\varepsilon^{(2)}), \tag{45}$$

where

$$2A_{ij} = \varepsilon_i^{(1)}\varepsilon_j^{(2)} + \varepsilon_i^{(2)}\varepsilon_j^{(1)} - \theta_{ij}\varepsilon^{(1)}\cdot\varepsilon^{(2)} \tag{46}$$

$$\theta_{ij} = \delta_{ij} - \hat{Q}_i\hat{Q}_j, \tag{47}$$

θ_{ij} is a two-dimensional projection operator appropriate to the *transverse* character of each photon, and satisfies the relations

$$A_{ij} = \theta_{il}A_{lj}; \quad \theta_{lj}\theta_{jm} = \theta_{lm} \tag{48}$$

Equation (45) can be recast in terms of the two basic invariants

$$R \equiv 2A_{ij}T_{ij}; \quad S \equiv \hat{Q}_iT_{ij}\hat{Q}_j \tag{49}$$

representing the respective couplings of helicity *two* and *zero* states of the $\gamma\gamma$ system to the (tensor) hadron. This reduction is faciliated by an angular integration over the $\mathbf{q}_\perp$-vector, noting that the denominator after q_--integration in (41) will involve only $\omega_\perp^2$, vide (35). The main formulae are

$$\langle\, q_l q_m \,\rangle = \theta_{lm}\tfrac{1}{2}q_\perp^2 + q_+^2\hat{Q}_l\hat{Q}_m, \tag{50}$$

$$2A_{ij}f_{lm}\,\langle\, q_iq_jq_lq_m \,\rangle = (\tfrac{1}{2}q_\perp^2)^2 R. \tag{51}$$

After the q_--integration in (41) for which the result (35) for (33) can be directly taken over, the integrals over $\mathbf{q}_\perp$ and q_+ are again similar to those encountered in (36). The final result for (41), taking account of (42)–(51), is

$$\mathscr{F}(T\to\gamma\gamma) = \frac{8\alpha}{3}f_c\left(\frac{\pi\beta^2}{M^2}\right)^{\frac{1}{4}}(m_q^2 + \tfrac{3}{2}\beta^2)^{-\frac{1}{2}}$$

$$\times[-R\beta^2\tilde{J}_1(\beta^2 + m_q^2 + \tfrac{1}{4}M^2) + 2m_q^2\beta^2 S(\tilde{J}_1 - \tilde{J}_0)], \tag{52}$$

where $\tilde{J}_n(n = 0, 1)$ are as defined in (38) with the same argument as y_0 given by (39). (Since tensor mesons are all heavy, the q_+-integration has been taken as effectively extending to ∞, thus making $Erf(x_0) \approx 1$:). Expressing (52) in the form:

$$\mathscr{F}(T\to\gamma\gamma) \equiv AR + \mathrm{BS} \tag{53}$$

the $T\to\gamma\gamma$ width is given by

$$\Gamma_{\gamma\gamma} = \sum_{\text{all poll}} |\mathscr{F}(T\to\gamma\gamma)|^2|Q|/80\pi M^2 \tag{54}$$

$$= \frac{|Q|}{16\pi M^2}\left(\frac{28}{15}|A|^2 + \frac{4}{15}|B|^2\right) \tag{55}$$

4. Discussion and conclusions

With the help of the formulae (40) and (55) we are in a position to predict the $\gamma\gamma$ widths of P- and T- mesons respectively. The basic inputs for this purpose continue to be the same (Mitra and Mittal 1984) as those employed for several other successful predictions of the BS model, *viz*

$$m_{ud} = 0{\cdot}30,\ m_s = 0{\cdot}40, \quad m_c = 1{\cdot}66,\ \tilde{\omega} = 0{\cdot}14 \tag{56}$$

all in GeV units, while the masses of the concerned hadrons may be taken either from the predictions of the BS model (Mitra and Mittal 1984) or from the PDG (1984) tables, which are close enough to each other so as not to cause discrepancies in their $\gamma\gamma$-width predictions. For definiteness, the mass values have been taken from the PDG tables and the results for the $\gamma\gamma$ widths are listed in table 1, together with their recent data (PDG 1984; Althoff *et al* 1983; Edwards *et al* 1982; Weinstein *et al* 1983; Roussarie *et al* 1981; Jenni *et al* 1983; Behrend *et al* 1982, 1983; Bartel *et al* 1982).

$T \to \gamma\gamma$ couplings

We start by discussing the $\gamma\gamma$ couplings of tensor mesons which are believed to be free from theoretical problems (such as anomalies). As seen from table 1, the agreement with available data is remarkably close, except for the $\chi_2 \to \gamma\gamma$.

For the light quark mesons, the main question centres on what theoretical significance would be attached to this impressive agreement. Since the calculations have been made for the decay for a hadron into two (real) photons, and the corresponding "data" are extracted from the ($\gamma\gamma$) cross-section for the production of the hadron by two virtual photons, the theoretical and experimental conditions are not quite the same, thus tending to cast doubts on the genuineness of the agreement. While referring the interested reader to Kolankski (1984) for details of the analysis of $\gamma\gamma \to H$ data, we should nevertheless like to point out some salient features. Thus the nearer the experimental conditions are for *almost* real photons (k_1^2, $k_2^2 \approx 0$) the better is their overlap with our theoretical premises. For small k^2, it is quite reasonable to assume the predominance of the helicity-two ($\lambda = 2$) amplitude over the lower helicity ($\lambda = 0, 1$)

Table 1. $H \to \gamma\gamma$ widths (in keV) for different mesons ($H = P, T$). The experimental data listed are from the sources indicated in the last column.

Process	$\Gamma_{\gamma\gamma}$ (theor)	$\Gamma_{\gamma\gamma}$ (expt.)	Expt. references
$f^0 \to \gamma\gamma$	2·20	$2{\cdot}95 \pm 0{\cdot}30 \pm 0{\cdot}40$ ⟨ AV ⟩	Althoff *et al* (1983)
			Brandelik *et al* (1981)
			Edwards *et al* (1982)
			Weinstein *et al* (1983)
			Dainton (1983)
			Brighton (1983)
$A_2 \to \gamma\gamma$	0·74	$0{\cdot}79 \pm 0{\cdot}23 \pm 0{\cdot}25$ ⟨ AV ⟩	Edwards *et al* (1982)
			Weinstein *et al* (1983)
			Behrend *et al* (1982)
$f' \to \gamma\gamma$	0·134	$0{\cdot}11 \pm 0{\cdot}02 \pm 0{\cdot}04$	Althoff *et al* (1983)
			Brandelik *et al* (1981)
$\chi_2 \to \gamma\gamma$	0·178	$1{\cdot}8 \pm 1{\cdot}3$ (prelim)	Quoted in Kolankski (1984)
$\pi^0 \to \gamma\gamma$	$6{\cdot}61 \times 10^{-3}$	$(7{\cdot}85 \pm 0{\cdot}54) \times 10^{-3}$	PDG (1984)
$\eta \to \gamma\gamma$	0·060	$0{\cdot}324 \pm 0{\cdot}05$ AV; $0{\cdot}56 \pm 0{\cdot}1215$	PDG (1984)
			Edwards *et al* (1982)
			Weinstein *et al* (1983)
$\eta' \to \gamma\gamma$	4·59	$5{\cdot}5 \pm 0{\cdot}7$ AV; $3{\cdot}8 \pm 0{\cdot}6$	PDG (1984)
			Althoff *et al* (1983)
			Brandelik *et al* (1981)
$\eta_c \to \gamma\gamma$	2·18	$6{\cdot}2 \pm 0{\cdot}5$	Bloom *et al* (1983)
$\eta_l \to \gamma\gamma$	0·11	$0{\cdot}4 \pm ?$	Bloom *et al* (1983)

modes, an assumption which has generally been made for the extraction of $T \to \gamma\gamma$ couplings from the "raw" data. This assumption is fully borne out in the BS model where each photon is real ($k_i^2 = 0$), since an overwhelming contribution (90–95 %) comes from the R-term ($\lambda = 2$) and only a small amount ($< 10\%$) from the S-term ($\lambda = 0$). However, to the extent that the experimental conditions differ from $k^2 = 0$, uncertainties in the estimates cannot be ruled out. For example, the TASSO collaboration (Althoff *et al* 1983; Brandelik *et al* 1981) studied the k^2-dependence of the $f^0\ \gamma\gamma$ coupling by keeping one of the photon momenta fixed at $k^2 = 0{\cdot}35$ (the "tagged" photon) and found a suppression of as much as 50 % relative to the "no-tag" result. Our theoretical premises for $f^0 \to \gamma\gamma$ seem to be more in harmony with the "no-tag" experimental conditions, for which the $T \to \gamma\gamma$ results are listed in table 1.

Another source of uncertainty concerns the resonant *vs* non-resonant contribution to the specific $\gamma\gamma$ cross-section from which the $H \to \gamma\gamma$ couplings are extracted. For example, the $\pi\pi$ channel which is dominant for $f^0\gamma\gamma$ coupling has a large non-resonant background in the $\pi^+\pi^-$ case (TASSO) (Althoff *et al* 1983; Brandelik *et al* 1981), and a smaller one in the $\pi^0\pi^0$ state (Edwards *et al* 1982; Brandelik *et al* 1981), and these features require a modest model dependence of the corresponding data analysis. Similarly the $A_2 \to \gamma\gamma$ couplings have been measured through both the decay modes $\eta\pi^0$ (Edwards *et al* 1982; Weinstein *et al* 1983), and $\rho\pi$ (Behrend *et al* 1982, 1983), again with the helicity-two assumption, but with suitable parametrizations of the (modest) background effects. Similar remarks apply to $f' \to \gamma\gamma$.

$P \to \gamma\gamma$ couplings

The traditional example of the $P \to \gamma\gamma$ coupling is $\gamma\gamma$ decay of a π^0 which has not only played a central role in establishing the *colour* degree of freedom of quarks, but also acted as a laboratory for testing the twin concepts of PCAC and the axial vector anomaly associated with the so-called triangle diagram (Adler 1969; Bell 1969). In this respect we recall the original reasoning of Adler (1969) that the triangle (anomaly) diagram provides the "driving" matrix element for determining the $\pi^0 \to \gamma\gamma$ amplitude by interpreting it as the vacuum $\to \gamma\gamma$ transition of the PCAC term $m_\pi^2\phi_\pi$ in the anomaly-modified relation (Alder 1969; Bell 1969) for $\partial_\mu j_\mu^5$ and demanding that the latter should vanish in the limit $(k_1 + k_2)^2 = 0$. Thus the Alder reasoning provides a much-needed handle on the calculation of $\pi^0\gamma\gamma$ coupling at the physical π^0 mass $(k_1 + k_2)^2 = -m_\pi^2$, in terms of the Ward identity requirement that the $\pi^0 \to \gamma\gamma$ amplitude should vanish at $(k_1 + k_2)^2 = 0$, but such a handle presupposes that π^0 is an *elementary* field (Goldstone boson). On the other hand, in the standard quark-picture, the pion is just as much of a *$q\bar{q}$ composite* as any other meson, and figures 1 (b, c) tell precisely how the π^0 as a composite particle couples to $\gamma\gamma$, without any further need to invoke the anomaly-modified PCAC relation to define its $\gamma\gamma$ coupling at the physical pion mass (Adler 1969; Bell 1969). In other words figure 1 (b, c) not only play the roles of the anomaly diagrams themselves for the composite pion, but they also provide a complete description for the $\pi^0 \to \gamma\gamma$ amplitude without any further assumptions (Adler 1969; Bell 1969). Exactly similar considerations hold for the $\gamma\gamma$ couplings of the other pseudoscalar mesons.*

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* This point of view is somewhat at variance with the position adopted in our earlier paper (Kulshreshtha and Mitra 1983) which we no longer consider tenable. The integration technique presented here for $P \to \gamma\gamma$ is also numerically more accurate than the one employed earlier.

It is against this background that a comparison of the BS predictions with the data on $P \to \gamma\gamma$ couplings should be viewed. First of all we notice from (37) to (40) that the M-dependence of Γ is $M^{13/4}$, roughly in accord with the phenomenological analysis (Kolankski 1984) based on $\Gamma \sim M^3$ (the balance of M-dependence being a dynamical effect). The predicted $\pi^0 \to \gamma\gamma$ and $\eta' \to \gamma\gamma$ rates, the latter on the basis of the mixing angle (22), are seen to be in good accord with the data, but the η case seems, prima facie, to be a disaster. This is particularly so when it is remembered that the rate (0·56) estimated by the Crystal Ball group (Edwards *et al* 1982; Weinstein *et al* 1983) based on the analysis of the reaction

$$e^+e^- \to e^+e^-\eta \to e^+e^-\gamma\gamma \tag{57}$$

is even higher than the more conventional value (0·324) quoted by PDG (1984) based on the Primakoff effect. This is a non-trivial discrepancy but a major part of it must be attributed to the η-η' mixing effect. Indeed as seen from the mixing matrix, (21), the interference is constructive (and hence relatively insensitive) for $\eta' \to \gamma\gamma$. On the other hand, it is *destructive* for $\eta \to \gamma\gamma$, and hence highly sensitive to the θ-value which is far from 'ideal' in this pseudoscalar case. (For further discussion, see Kolankski 1984)

As for the heavy quarkonium states η_c, η_b, unfortunately the results of direct measurements of the type (57) are not yet available. Rather, the data (Bloom *et al* 1983) listed in table 1 are highly indirect, being based on the observation (Kolankski 1984) that the wave function at the origin should be the same for both ${}^1S_0\,(\eta_c)$ and ${}^3S_1\,(\psi/J)$ states, so that the accurately measured e^+e^- widths of the latter can be used to "determine" the corresponding $P \to \gamma\gamma$ widths through an essentially geometrical relationship. On the other hand it is possible to question the validity of this assumption, and indeed an independent estimate of $\eta_c \to \gamma\gamma$ based on the branching ratio for $\psi \to \gamma\eta_c$ yields a much smaller value, viz (Kolankski 1984)

$$\Gamma(\eta_c \to \gamma\gamma) = 1{\cdot}6 \pm 0{\cdot}8 \text{ keV}, \tag{58}$$

which seems to be consistent with our theoretical prediction. (The rate for $\eta_b \to \gamma\gamma$ has been listed only for illustration, since in any case the BS model for harmonic confinement cannot be expected to work for $b\bar{b}$ spectroscopy).

$\eta_c \to gg$ width

We have attempted another independent check on the η_c wave function through a prediction of the $\eta_c \to gg$ width as an OZI forbidden process. The $\eta_c \to gg$ coupling can be trivially calculated in our model by making the following replacements:

$$\textit{Color}: \sqrt{3} = \frac{1}{\sqrt{3}}\delta_{ii} \Rightarrow \frac{1}{\sqrt{3}}\mathrm{Tr}\left(\frac{1}{2}\lambda^a\tfrac{1}{2}\lambda^b\right)\frac{\delta_{ab}}{\sqrt{8}} = \sqrt{(2/3)}, \tag{59}$$

$$\textit{Charge}: \quad \frac{e^2}{3\sqrt{2}}f_c \Rightarrow \langle 4\pi\alpha_s \rangle, \tag{60}$$

where $\langle \alpha_s \rangle$ is the "strong" coupling constant. Thus the overall replacement in (40) for estimating $\Gamma(P \to gg)$ becomes

$$\frac{\alpha}{\sqrt{6}} \Rightarrow \sqrt{(2/3)}\,\langle \alpha_s \rangle. \tag{61}$$

For a realistic estimate of $\langle \alpha_s \rangle$ appropriate to the confinement region, the following replacement of the perturbative $Q-\bar{Q}$ interaction ($=\alpha_s/r$) by the harmonic interaction

$$\frac{\alpha_s(r)}{r} \Rightarrow \frac{3}{4}\omega_{Q\bar{Q}}^2 r^2 = \frac{3}{2} m_c \tilde{\omega}^2 r^2 \tag{62}$$

should be in order, thus leading to the determination

$$\langle \alpha_s \rangle = \frac{3}{2} m_c \tilde{\omega}^2 \langle r^3 \rangle \tag{63}$$

where $\langle r^3 \rangle$ must be calculated for the $Q\bar{Q}$ state under study.
This gives

$$\langle \alpha_s \rangle = 3 m_c \tilde{\omega}^2/\beta^3 \sqrt{\pi}. \tag{64}$$

Substitution of (61) and (64) in (40) yields the not unreasonable estimate

$$\Gamma(\eta_c \to gg) = 4{\cdot}66\ \text{MeV}, \tag{65}$$

which compares favourably with the values ($11{\cdot}5 \pm 5$; $8{\cdot}3 \pm {\cdot}5$) quoted by Bloom *et al* (1983). On the other hand, the same formula (62) yields for a typical OZI allowed process the value

$$\Gamma(f^0 \to gg) = 45{\cdot}95\ \text{MeV}. \tag{66}$$

which is only about 25% of the total f^0 width (PDG 1984).

To summarise, the overall consistency of the several $T \to \gamma\gamma$ data with the predictions should warrant a cautious optimism about the dynamical status of the $q\bar{q}$ structure of these hadrons in the BS model for harmonic confinement. Notwithstanding this conclusion, it need hardly be emphasized that such a limited comparison of the data at $H \to \gamma\gamma$ coupling level, requiring as it does a model-dependent data analysis, cannot be a substitute for a more direct comparison with the "raw" data available in the form of various two-photon inelastic cross-sections themselves. Such calculations are envisaged in the near future.

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