# ELASTIC CONSTANTS OF GARNETS

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## 1. Introduction

Elastic behaviour is an important aspect of crystal physics. In view of their availability in large and well-developed forms, garnets are chosen for study in this paper and examined for their elastic behaviour. The fact that they crystallise in the cubic system makes the investigation quite simple.

## 2. Structure and Composition of Garnets

It is a very well-known fact that garnets form a series of isomorphous crystals all belonging to the cubic system. The predominant habit is the dodecahedron. X-ray analysis has shown that they belong to the space group $O_h^{10}$ which is the last in the list of 230 space groups. There are 8 molecules per unit cell and the side of the cube varies from 11·50 Å and 12·02 Å. Chemical analysis of garnets has shown that they can be represented by the general formula $\dot{R}_3\ddot{R}_2(Si\,O_4)_3$ where $\dot{R}$ = Ca, Mg, Fe, Mn, $\ddot{R}$ = Al, Fe, Cr. They are of widely variable composition, the chief constituents of which are given below:

| | | | Density | Lattice constant in A° |
|---|---|---|---|---|
| Pyrope | .. | $Mg_3\,Al_2\,(Si\,O_4)_3$ | 3·51 | 11·51 |
| Grossularite | .. | $Ca_3\,Al_2\,(Si\,O_4)_3$ | 3·53 | 11·83 |
| Uvarovite | .. | $Ca_3\,Cr_2\,(Si\,O_4)_3$ | 3·77 | 11·95 |
| Andradite | .. | $Ca_3\,Fe_2\,(Si\,O_4)_3$ | 3·84 | 12·92 |
| Spessartite | .. | $Mn_3\,Al_2\,(Si\,O_4)_3$ | 4·18 | 11·60 |
| Almandite | .. | $Fe_3\,Al_2\,(Si\,O_4)_3$ | 4·33 | 11·50 |

All existing garnets are invariably isomorphous mixtures of the above in all possible proportions. On account of this fact colour, hardness, refractive index, specific gravity and other physical properties vary widely from specimen to specimen.

In all garnets quartz is invariably present as small inclusions. These inclusions are randomly distributed so that sections cut with different orientations but from the same mother crystal sometimes show slight variations in density. Hence in the following investigations the densities of the various sections are determined separately and used in the respective calculations.

## 3. THEORETICAL CONSIDERATIONS

The elastic constants for the cubic system are given by the following scheme:

$$\begin{array}{cccccc}
C_{11} & C_{12} & C_{12} & 0 & 0 & 0 \\
 & C_{11} & C_{12} & 0 & 0 & 0 \\
 & & C_{11} & 0 & 0 & 0 \\
 & & & C_{44} & 0 & 0 \\
 & & & & C_{44} & 0 \\
 & & & & & C_{44}
\end{array}$$

The velocities of propagation of sound waves in a direction $(l, m, n)$ in a cubic crystal are given by the roots of the equation[1] =

$$\begin{vmatrix}
C_{11}l^2 + C_{44}(m^2+n^2) - \rho c^2 & (C_{12}+C_{44})\,lm & (C_{12}+C_{44})\,nl \\
(C_{12}+C_{44})\,ml & C_{11}m^2 + C_{44}(l^2+n^2) - \rho c^2 & (C_{12}+C_{44})\,mn \\
(C_{12}+C_{44})\,nl & (C_{12}+C_{44})\,mn & C_{11}n^2 + C_{44}(l^2+m^2) - \rho c^2
\end{vmatrix} = 0$$

Specialising this equation for the specific cases of propagation of sound in directions perpendicular to (100), (110) and (111) faces, we have the solutions given below:

TABLE I

| Orientation of the plate | $v_1^2\rho$ | $v_2^2\rho$ | $v_3^2\rho$ |
|---|---|---|---|
| 100 .. .. | $C_{11}$ | $C_{44}$ | $C_{44}$ |
| 110 .. .. | $\frac{C_{11}+C_{12}+2C_{44}}{2}$ | $C_{44}$ | $\frac{C_{11}-C_{12}}{2}$ |
| 111 .. .. | $\frac{C_{11}+2C_{12}+4C_{44}}{3}$ | $\frac{C_{11}-C_{12}+C_{44}}{3}$ | $\frac{C_{11}-C_{12}+C_{44}}{3}$ |

where $v_1^2\rho$ corresponds to a longitudinal wave and $v_2^2\rho$ and $v_3^2\rho$ to shear waves.

The experimentally determined values of $v_i^2\rho$ in different cases are equated to the appropriate expressions given in Table I and hence the principal elastic constants calculated for each specimen.

## 4. EXPERIMENTAL DETERMINATIONS

In this series of investigations seven different specimens are used. Two of them, numbered six and seven in Table II are of the semi-precious variety. All the crystals used were of the dodecahedral habits with well-developed faces and free from twinning. From these crystals, sections parallel to the cube faces (100), the natural faces (110) and the octahedral faces (111) are cut and employed in the measurements. The (100) and (111) sections are obtained to the accuracy of a contact goniometer.

Ide[2] has recently measured the Young's modulus of a number of rock-forming minerals both by static and by dynamic methods and found that the dynamic values are always about 10% higher than the static ones. Theoretically the difference between the dynamic and static measurements cannot be expected to be more than about 0·30%.[3] This large difference is attributed by him to the non-homogeneity of the material and the possible existence of cracks which invariably give low values by static methods. Hence in studying the garnets it is always preferable to use dynamic methods.

Recently Bhagavantam and Bhimasenachar[4] have described a new method applicable to such substances. This new method has been employed in the present investigation. All the garnets studied showed cracks which precluded the application of static methods. Results of these determinations are given in Table II.

TABLE II

*Elastic Constants of Garnets and their Composition*

| Properties of Garnets No. : | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| Density in gm. per c.c. .. | 3·759 | 3·673 | 3·630 | 3·67 | 3·75 | 4·13 | 4·32 |
| Elastic constants in dynes/cm.$^2 \times 10^{-12}$ : | | | | | | | |
| $C_{11}$ .. | 1·97 | 1·92 | 2·10 | 2·22 | 2·26 | 2·73 | 3·27 |
| $C_{12}$ .. | 0·90 | 0·99 | 1·03 | 1·04 | 1·26 | 1·57 | 1·24 |
| $C_{44}$ .. | 0·57 | 0·59 | 0·67 | 0·70 | 0·62 | 0·68 | 0·89 |
| Bulk modulus K $\times 10^{-12}$ .. | 1·26 | 1·30 | 1·39 | 1·43 | 1·60 | 1·62 | 1·90 |
| Composition (percentage of FeO) .. | 21·8 | 22·7 | 23·6 | 23·0 | 26·2 | 28·7 | 33·5 |
| Compressibility in cm.$^2$/dynes $\times 10^{12}$: | 0·79 | 0·77 | 0·72 | 0·69 | 0·63 | 0·62 | 0·52 |

In the table, the bulk modulus K is calculated from the well-known relation $K = (C_{11} + 2C_{12})/3$. The garnets have been chemically analysed. Standard methods of analysis have been used and their ferrous content determined.

## 5. DISCUSSION OF RESULTS

From Table II we see that the elastic constants vary from specimen to specimen. In spite of the large variations in the principal constants, it is interesting to note that the bulk modulus K is directly proportional to the ferrous content of the specimen. This is clearly brought out by the curve of Fig. 1. It may be noted here that the points for the specimens 4 and 5 fall a little outside the curve. This may be due to the fact that the chemical method has not permitted us to estimate the ferric iron and it is possible that in these two specimens there is an excess of ferric iron. This is borne out by the fact that both these points fall on the left side of the curve.

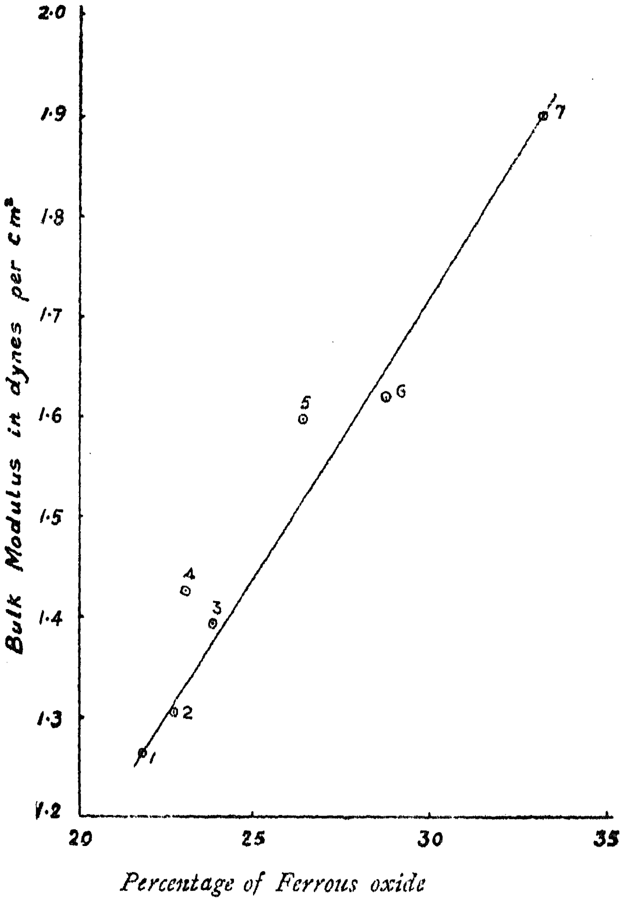

FIG. 1

We may mention here that Adams and Gibson[5] have determined the compressibilities of two varieties of garnets by using piezometric methods. They give a value of $0 \cdot 60 \times 10^{-12}$ cm.$^2$/dyne for the compressibility of almandite. In our measurements specimen No. 7 which contains about 80% of almandite shows a compressibility of $0 \cdot 52 \times 10^{-12}$ cm.$^2$/dyne. The agreement may be regarded as very satisfactory in view of the existing differences in composition and the well-known fact that piezometric values are usually high in consequence of the inherent difficulties of the method.

Finally it may be remarked that the curve given above can be used for an estimation of almandite in garnets provided the spessartite content is low.

## 6. SUMMARY

Employing the new method developed by Bhagavantam and Bhimasenachar, elastic constants of seven specimens of garnets have been determined. It has been found that the principal elastic constants show a large variation from specimen to specimen. Nevertheless the bulk modulus is found to vary linearly with the iron content.

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