ON A CLASS OF RELATIVISTIC WAVE-EQUATIONS OF SPIN 3/2

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§1. Introduction

It has been shown by the author[1] (1949) that on the basis of the general assumptions which underlie present relativistic quantum mechanics, the most general expression in the Lagrange-function, quadratic in the components of the wave-function ψ, can always be written in the canonical form

$$\psi^{\dagger} \mathrm{D} \, (\alpha^k p_k + \beta\chi) \, \psi, \tag{1 a}$$

where[1] $p_k = -i \, \partial/\partial x^k$, χ is an arbitrary constant of the same dimension as p_k, and D, $\mathrm{D}\alpha^k$, and $\mathrm{D}\beta$ are six hermitian matrices which have the transformation properties given below required to preserve the invariance of expression (1) under all Lorentz transformations. The variation of (1) leads to the wave-equation

$$(\alpha^k p_k + \beta\chi) \, \psi = 0 \tag{1 b}$$

since, the matrix D being non-singular by definition, we can divide by it from the left. It has also been shown (1949, 1951) that for an elementary particle-field the five matrices α^k, β must form an irreducible set and the nucleus of the representation $\mathscr{R}$, which determines the transformation properties of the wave-function ψ under all Lorentz transformations, must be expressible as a polynominal in them. The representation $\mathscr{R}$ is determined completely by the six infinitesimal transformations I^{kl} and the matrix R representing the improper Lorentz transformation r which reverses the directions of the three space axes.

It is well known that for the three relativistic wave-equations for particles of spin 0, ½ and 1 the I's and the α's are connected by the simplest possible relation between them, namely,

$$\mathrm{I}^{kl} = \alpha^k \alpha^l - \alpha^l \alpha^k, \tag{2}$$

and the I's and α's all satisfy minimal equations of the same degree, namely, 2 for the case of spin ½, and 3 for the cases of spin 0 and 1. On the other

[1] x^0, x^1, x^2, x^3 are the four co-ordinates of a point in space, and the metric tensor is taken as usual in the form $g^{00} = -g^{11} = -g^{22} = -g^{33} = 1$, $g^{kl} = 0$ for $k \neq l$.