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On the Penetrating Component of Cosmic Radiation

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Introduction

The position in cosmic radiation has changed considerably in the last year both from the experimental and the theoretical side, so that it is now possible to co-ordinate the various independent observations to a degree which was not hitherto possible and to draw some important conclusions from them. Since it has been shown by Rossi (1934*a*) and his co-workers, Auger and Ehrenfest (1934) and Street, Woodward and Stevenson (1935) that there are single ionizing particles in cosmic radiation which penetrate more than a metre of lead, and further, according to the theory, no electron of any reasonable energy can make its effects felt through more than about 15 cm. of lead, it has become clear that the behaviour of the penetrating component of cosmic radiation faces us with at least one of the two following conclusions:

a—The theoretical formulae for the energy loss of fast electrons break down for energies above some critical energy, where this critical energy may or may not depend on the material.

b—The penetrating component does not consist of electrons.

Moreover, since Blackett (1937*a*) has found that there is a rough equality in the number of positive and negative particles up to the highest measurable energies, it follows that the second alternative already demands the existence of a hitherto unknown particle, since even if we assume, as was supposed at first, that these penetrating particles are protons, for which the radiation loss is small due to their larger mass, resulting in a corresponding increase of penetrating power, the presence in the penetrating group of negatively charged particles forces one to admit the existence of negative protons. The assumption that these particles are protons has, however, met with the difficulty noticed by several investigators that far fewer protons, identifiable at the end of their range by a heavy track in a Wilson Chamber, are observed than there should be, a difficulty which can only be removed by certain plausible but *ad hoc* assumptions about processes (such as nuclear collisions and explosions) which would remove protons from the beam before they became slow enough to show a noticeably greater ionization.

Street and Stevenson (1937) have, however, shown that among those particles which have already traversed 10 cm. of lead, there are some with curvatures which would correspond to energies of less than $7 \cdot 5 \times 10^8$ e-volts if they were electrons, whose behaviour is not that to be expected theoretically for electrons, and which are certainly not protons. The latest experiments of Blackett and Wilson (1937) also show that at energies as low as 4×10^8 e-volts in lead, cosmic-ray particles have an energy loss much less than the theoretical loss for electrons, and they further claim that it is possible to exclude the hypothesis of protons since they would show a noticeably greater ionization along their tracks at these energies. For the further discussion, therefore, we will only consider the conclusion (*b*) stated above in the form:

b—There are in the penetrating component of cosmic radiation new particles of electronic charge of both signs, and mass or masses intermediate between those of the electron and proton. For brevity, in the further discussions we shall describe such particles simply as heavy electrons.

The two alternatives (*a*) and (*b*) are not mutually exclusive.

As has been shown by Rossi (1934*b*) and especially stressed by Auger and Leprince-Ringuet (1934), the radiation at and above sea-level definitely consists of two groups distinguishable by their penetrating power. If we measure the absorption in lead of the vertically incident particles at sea-level, we find that the first 10 cm. of lead absorb 25–30 % of the total number of particles. At about this point a distinct change in the slope of the absorption curve occurs, and a metre of lead only serves to decrease further the number of particles by about 30 %. We shall call the group of particles absorbed in about 10 cm. of lead the soft group, and the other the hard or penetrating group. It should be emphasized that in this paper the words soft and hard are used to denote only the penetrating power of the particles and have nothing whatsoever to do with their energy.

As we shall see in the next section the soft component is not in equilibrium with the hard component above sea-level. Further, the hard component obeys the mass absorption law, whereas the soft component shows an absorption per atom which varies as the square of the atomic number as would be the case for electrons obeying the theory.

In recent papers it has been shown independently by Carlson and Oppenheimer (1937) and Bhabha and Heitler (1937) that a consistent application of the formulae of Bethe and Heitler (1934) to the successive emission of radiation by electrons and the creation of electron pairs by

γ-rays cannot only explain the showers and bursts observed in cosmic-ray phenomena, but is capable of describing the typical features of these phenomena, namely, the transition curves investigated by Rossi (1933*b*) and the absorption curve in the atmosphere. It is therefore certain that the soft group consists of electrons and positrons and their accompanying γ-radiation, and its behaviour is described correctly by the theory.

In view of this fact it is of interest to discuss the experimental material, particularly with regard to the hard component, and to see whether it supports the first or the second of the alternatives mentioned above. We carry through this discussion in § 1, where it will appear that although assumption (*a*) may also be true *for extremely high energies*, it is *insufficient* to explain all the facts, and that there are sufficient grounds to justify us in considering the presence of new particles of electronic charge and mass between those of the electron and proton at least as a possibility. Under these circumstances we think it not unprofitable to consider the behaviour of such particles theoretically, in order that a comparison may then be made with experiment. Of course only that part of the behaviour of such a particle can be calculated which depends essentially only on its charge and mass, namely, the magnitudes of the various collision processes and the ionization and radiation losses, as also the creation of pairs of such particles by γ-rays. This has been done in § 2, assuming that the particle obeys the Dirac equation. We shall see that the radiation loss is not just inversely proportional to the square of the mass of the particle, since for particles of different masses the effect of screening is different. Moreover, since we do not know what mass to attribute to such a particle if it exists, we have carried through the calculations for particles of rest energy equal to 5×10^6 e-volts and 5×10^7 e-volts, i.e. 10 and 100 times the electron mass respectively, so that together with the already known results for the electron and proton, we obtain curves from which the energy loss can be read off for particles of any given rest mass and energy. It will appear in the course of this paper that if the existence of new particles is to be assumed as a solution of the various difficulties which still remain in the cosmic-ray phenomena, it seems probable that particles of various different rest masses will have to be assumed with perhaps the possibility of particles making transitions from one rest mass to another.

In § 4 we discuss the production of showers by heavy particles, that is, particles with any mass greater than that of the electron, either by the emission of a sufficiently large quantum of radiation, or by the production of a sufficiently fast secondary electron by collision. In § 3 we calculate the average number of positive and negative electrons accompanying the penetrating heavy particles as a result of these processes, thus forming a soft component in equilibrium with the penetrating particles. In § 5 we have briefly considered the effect of the creation of pairs of heavy particles in decreasing the rate at which cascade processes die out. This may find a possible application in the penetration of particles to sea-level.

1—Discussion of the experimental material

A—Latitude effect

We have already stated in the introduction that the soft group (defined as those particles absorbed in about the first 10 cm. of lead) is not in equilibrium with the hard group. This statement is established by the experiments of Auger, Ehrenfest and Leprince-Ringuet (1936), who have measured the absorption curve in lead at sea-level and at Jungfraujoch. Table I gives their results.

Table I

	Jung-fraujoch	Sea-level	Absorption coefficient in air in cm.²/g.	Absorption coefficient in lead in cm.²/g.
Hard group	190	120	$0{\cdot}70 \times 10^{-3}$	$0{\cdot}70 \times 10^{-3}$
Soft group	170	25–30	6×10^{-3}	$32 \pm 2 \times 10^{-3}$

The first two columns give in arbitrary units the number of particles of each group found at the two heights, the third column then gives the absorption coefficient deduced from these figures assuming an exponential absorption. The last column gives the absorption coefficients deduced from the absorption curve in lead. These results again confirm those of Rossi, Alocco (1935), and of Clay (1936), and show that the hard component obeys a mass absorption law. The soft group, on the other hand, is seen to show an absorption per atom which is roughly proportional to the square of the atomic number.

The above figures show that the soft group increases with height much more rapidly than the hard group. Ionization measurement by Compton and Stevenson (1934) and Bowen, Millikan and Neher (1934) at great heights with an ionization chamber shielded by 6 and 12 cm. of lead confirm this, and show that at these heights the hard component contributes less than 30 % to the total ionization. The shower intensity as measured by Woodward (1936) and Braddick and Gilbert (1936) increases with height much more rapidly than the total radiation and runs more parallel with the intensity of the soft group. Heitler (1937) has shown that the variation of showers with altitude and latitude can be understood if they are due predominantly to the soft component. Further, Montgomery and Montgomery (1935) find that the intensity of bursts increases with height even more rapidly than the shower intensity, and recently Young (1937) has shown for small bursts containing ten particles and more that the increase with height of bursts of a given size is greater, the larger the size of the bursts. Thus we regard this as evidence that bursts, which have been shown by Ehrenberg (1936) to be very large showers, are also predominantly due to the soft component. But such a view is incompatible with the assumption (*a*). For on this assumption, the soft component consisting of particles which are absorbed in about 10 cm. of lead must be electrons of energy below the critical energy in *lead*, which is less than 4×10^8 e-volts. Such electrons would be incapable of producing bursts, merely because they have less energy than the bursts themselves. There is no difficulty of this sort on assumption (*b*), since if the soft component consists of electrons obeying the theory, they would be absorbed in about 10 or 15 cm. of lead even for energies of the order of 10^{11} e-volts.

We finally come to consider the latitude effect at sea-level. Bhabha and Heitler (1937) have shown that the shape of the absorption curve in the atmosphere is a proof that electrons of energy near 3×10^9 e-volts (the lowest energy which can reach a magnetic latitude 50° due to the magnetic field of the earth) multiply according to the cascade theory. Heitler (1937) has further demonstrated by a more detailed analysis of the same curve that if a break-down of the theoretical energy loss formulae takes place, the breakdown energy must be at least as high as 5×10^9 e-volts. The latest measurements of Bowen, Millikan and Neher (1937) show that particles of energy at least as high as 10^{10} e-volts produce a rapid multiplication in the upper layers of the atmosphere which is at least in qualitative agreement with the cascade theory, thus proving that the radiation loss of electrons demanded by quantum mechanics is correct in air for energies up to 10^{10} e-volts.

Let us consider these facts on assumption (*a*). Now the theory shows (Bhabha and Heitler 1937, p. 454) that the chance of an electron of energy less than 10^{10} e-volts making its effect felt at sea-level is negligible. Further, particles of energy greater than 10^{10} e-volts will not show a latitude effect at latitudes greater than about 35°. Thus there should be no variation of intensity at sea-level at latitudes greater than 35°.

Bhabha and Heitler have already shown that if the theory of energy loss for electrons be right for all energies, then no latitude effect at sea-level could be due to the electrons. Our present considerations show that even if the theory of energy loss break down above some critical energy, no latitude effect beyond 35° could be due to electrons, provided the breakdown energy is greater than 10^{10} e-volts. Since at sea-level the latitude effect starts at about 50°, and is quite considerable at 35°, we regard this as a very strong argument against the hypothesis (*a*) and in favour of the existence of new particles.

Indeed, in our opinion, the very discrepancy between the theoretical absorption curve in the atmosphere and the experimental difference curve of Bowen, Millikan and Neher for primaries of an average energy of 10^{10} e-volts is evidence of the existence of new particles. For the first part of the experimental curve shows that an enormous multiplication takes place in the atmosphere, thus establishing the existence of large radiative losses, from which it follows merely from arguments of self-consistency that the rest of the curve should agree with the theoretical curve at least as regards the order of magnitude, whereas in fact some thirty times as many particles are found at sea-level as there should be. We believe that a large fraction of these sea-level particles are heavy electrons, either of primary origin, or secondaries created in the atmosphere, together with the electron component in equilibrium with them which they produce.

B—Bursts and transitions curves

It has been shown by Bhabha and Heitler, and Carlson and Oppenheimer in the papers quoted above that not only does the theory show that electrons and γ-rays will create showers in their passage through various substances, but that it also gives the shape of the curve connecting the number of showers containing a given number of particles as a function of the thickness of the material in which they are produced. These curves are like the curves found by Rossi (1933) for the number of coincidences between a number of counters placed below a plate of some heavy substance, usually lead, plotted as a function of the thickness of the plate. Further, Bhabha and Heitler have shown that the maximum of the Rossi curve for large showers should lie at greater thicknesses than the maximum of the Rossi curve for smaller showers, a prediction which has been confirmed recently by Auger, Ehrenfest, Freon and Grivet (1937) for small showers containing a few particles. Moreover, Bøggild (1936) has also observed that for bursts of different size there are indications of the same shift of the maximum.

The questions we have to answer then are: (1) Do electrons of sufficiently high energy produce bursts by cascade multiplication only, as the quantum theory predicts? (2) Do electrons and other particles produce showers in *one elementary process*? The two alternatives are not mutually exclusive.

Before we discuss the bursts, we will analyse the Rossi curve for showers in some detail. The Rossi curve for showers at and above sea-level as measured by Auger and Meyer (1933) has the shape shown by curve 1, fig. 1. It is important to notice that after the maximum the curve falls away rapidly till about 10 cm. of lead, after which the decrease would be much more gradual, being then comparable to the decrease of the penetrating component (cf. 4). The same curve, measured by these authors at a depth below the ground equivalent to 30 m. of water, where only the hard component is found, has the shape shown by curve 2, and below 75 m. by curve 3. The decrease of both these curves after the maximum has an absorption coefficient ten times less

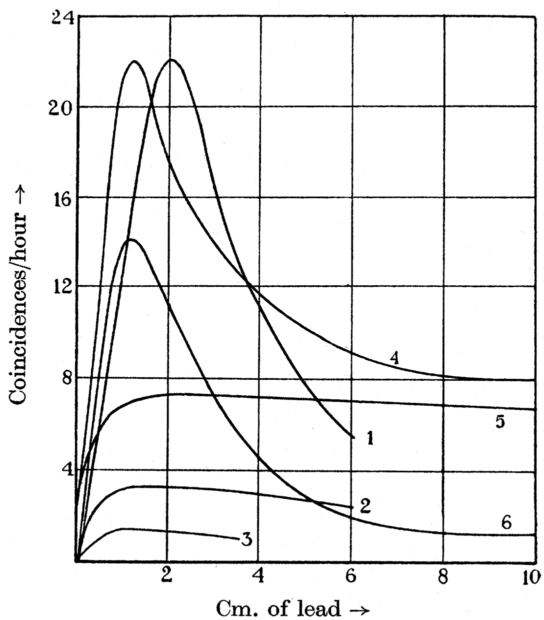

Fig. 1—Rossi curves for showers. Curves 1, 2, 3, Auger and Meyer; curves 4, 5, 6, Schwegler. The ordinates of curves 1, 2, and 3 have been reduced so that the maximum heights of 1 and 4 shall be the same.

than that of (1) and is parallel to the decrease of the vertical intensity. Moreover, the observation of Clay, Gemert, and Wiersma (1936) that after thicknesses of 200 g./cm.^{2} the decrease of the number of showers is parallel to the decrease of the primary intensity also shows that at least some showers are to be associated with the penetrating component. It is therefore reasonable to suppose that the sea-level curve 1 is made up of the superposition of two effects, those due to the soft and hard components respectively. This supposition has been confirmed directly by Schwegler (1935). By placing a 10 cm. lead block between the three counters used for recording the triple coincidences and thus eliminating the effect of the soft component he gets curve 5, being the number of coincidences plotted as function of the lead above the counters. Without the 10 cm. block he gets the usual curve 4. The difference between the two is shown by the curve 6, and gives then the showers produced by the soft component alone. This is just what we should expect theoretically, for 10 or 15 cm. of lead should suffice to absorb more or less completely all electrons or γ-rays of energy below 10^{11} e-volts.

The Rossi curve for bursts measured by Bøggild (1936) in iron and Carmichael (1936) in lead have the same shape as curve 1, except that the maximum lies at greater thicknesses. We wish to emphasize particularly the fact that after the maximum the *decrease is much more rapid than that of the total intensity*. Moreover, Bøggild has found that there are definite indications that the maximum shifts to greater thicknesses for larger bursts. We now wish to consider these facts in the light of the assumptions (*a*) and (*b*).

We will first discuss them on the assumption that for energies above a certain critical energy the theory of energy loss fails, and that above these energies the radiation loss is much less than the theoretical, and goes more or less gradually to zero with increasing energy. According to Blackett and Wilson's experiments this critical energy must be below 4×10^8 e-volts in lead. The energy of the particle starting the burst must be at least as great as the total energy of the burst, which in the case of a burst of 100 particles of 5×10^6 e-volts each amounts to 5×10^8 e-volts and is higher than the critical breakdown energy. Since, according to the cascade theory, bursts of a hundred particles or more require an energy in lead which is at least $1{\cdot}5 \times 10^9$ e-volts, it means that the cascade theory of bursts will fail. A burst would then have to be started by some elementary process in which a number of very energetic particles or quanta are created at once. Of course, those electrons or quanta so formed, whose energy was less than the critical energy, or became less than it on penetrating the lead plate, would then have the theoretically correct energy loss, and multiply according to the cascade theory.

But the Rossi curve for such bursts will show a shape characteristically different from curve 1. As the thickness of the lead plate above the ionization chamber is increased from zero, bursts started anywhere in the plate will be registered, and so the number of bursts will increase. The effect of adding more lead at the top is merely to add those bursts which are started in this new layer of lead, although it may be that these bursts do not add to the number of registered bursts because the particles are absorbed by the intervening lead. *A decrease of the number of recorded bursts can only take place if less bursts are started in the lower layers of material*, which can only happen if the intensity of the electrons starting these bursts has been decreased by the superposed lead. Since these electrons have energies above the critical breakdown energy they belong to the penetrating group, and their decrease will be proportional to the decrease of the *penetrating* component. Thus, after the maximum, *the curve for bursts should decrease at roughly the same rate as the penetrating component*, which is contrary to observation. It should be noticed that his conclusion is independent of the exact mechanism of the bursts and holds whether or not there are highly absorbable intermediate links. In fact, with this mechanism the transition curve would look like curves 2, 3 and 5 for the showers which we have seen to be associated directly or indirectly with the penetrating component. We conclude then that the observed shape of *the transition curve for bursts makes it highly improbable that the theory of energy loss fails for electrons in lead at energies of* 4×10^8 e-volts, and it allows us to answer the first question stated at the beginning of this section in the affirmative, i.e. *that electrons of sufficiently high energy do produce bursts entirely by cascade multiplication.* We are still not in a position to answer the second question, for those showers and bursts which are associated with the hard component may be produced by a penetrating particle emitting a sufficiently hard light quantum, or producing a very high energy electron by collision, either of which would then produce showers according to the cascade theory.

It would be possible to decide whether or not particles can produce showers in one elementary process by investigating the showers produced in sheets of material so thin that it would be impossible for sufficient multiplication of the number of particles to take place on the cascade theory in such sheets. According to Bhabha and Heitler, in a thickness corresponding to 2 in the units characteristic of the material (Bhabha and Heitler 1937, eq. (15)) the mean number of particles with energy greater than 10^7 e-volts produced by an electron of 2×10^{11} e-volts is 30, and since roughly an equal number of particles have an energy below 10^7 e-volts, this shower will contain on the average about 60 particles. Electrons of less energy produce still smaller showers. Messerschmidt (1936) has observed behind 9 cm. of aluminium or 20 cm. of coal in a chamber whose walls were of 0·7 cm. iron, bursts of more than 200 particles with a quite comparable frequency.* Since these thicknesses of absorber correspond to thicknesses less than 2, one would have to suppose that these bursts represent fluctuations in the number of particles in a shower from the mean number. The chance that a shower of more than 200 particles should appear as a fluctuation when the expected average number is 60, is (Bhabha and Heilter 1937, eq. (31)) of the order 10^{-45}. It would thus appear as if these experiments proved without doubt that high-

* I wish to express my thanks to Professor Heisenberg for drawing my attention to these results in the course of an exchange of letters.

energy particles can in fact also create a shower of particles in one elementary process. The force of these results is, however, weakened by the fact that there was a low wall at a distance of 1·5 m. from the ionization chamber, and the surrounding walls were only 4 m. removed from it. A number of particles of a shower occurring in the walls might then quite easily hit the apparatus, thus upsetting our calculations. We also do not know to what extent the particles of a shower produced, say, in the top of the chamber produce subsidiary showers in the walls by cascade multiplications. A similar difficulty is met in interpreting the results of Carmichael (1936) who, in a chamber made of iron $\frac{1}{8}$ in. thick, and with no heavy material above the chamber, found bursts corresponding to more than 200 particles. One cannot be sure that the burst was not started in some iron girder in the roof.* We nevertheless hold it for not improbable that cases do occur in which a number of particles are created in one elementary process. Indeed, that such processes are to be expected on Fermi's β-ray theory has been shown by Heisenberg, although the theory has to be modified before it will give results of the right order of magnitude.

C—Wilson chamber experiments

Blackett and Wilson (1937) have recently measured the energy loss of cosmic-ray particles passing through lead and aluminium plates put across their Wilson chamber. Their results may be summarized as follows. For energies up to 2×10^8 e-volts the energy loss of cosmic-ray particles in lead is in agreement with that to be expected theoretically for electrons, confirming the earlier results of Anderson and Neddermeyer (1936). For energies greater than 2×10^8 e-volts the ratio of the experimental to the theoretical energy loss in lead decreases rapidly, reaching a value of about a quarter at energies of about 4×10^8 e-volts. After this the decrease is more slow, the ratio being less than about a twentieth for energies near 4×10^9 e-volts.

In aluminium the relative energy loss seems to be about one-fifth of that in lead for the energy range from 5×10^8 e-volts to 2×10^9 e-volts, although the accuracy of the aluminium measurements is admittedly not as high as that in lead.

It is clear from these experiments that there is already a marked discrepancy with the theory for curvatures which correspond to an energy of 4×10^8 e-volts if the particles be electrons. Since for such curvatures the ionization of a proton would be about two and a half times that on an electron, and it is claimed by Blackett and Wilson that it is possible to notice differences in the ionization of this amount, the possibility of these particles being

* Mr Carmichael in a conversation himself drew my attention to this possibility.

protons may be excluded. Thus these experiments compel one directly to accept either hypothesis (*a*) or (*b*).

If the explanation of these experiments is to be found in a breakdown above some critical energy of the theory for radiative energy loss, then it is quite clear that this critical energy must depend on the atomic number of the material, for the critical energy in lead must be put between 2 and 4×10^8 e-volts, whereas we have seen that in air it cannot be below 10^{10} e-volts. Indeed, the rough energy loss measurements in aluminium seem to support this view.

To explain the observed energy loss in lead on hypothesis (*b*) it would be necessary to assume that at sea-level the radiation consisted of a mixture of electrons and heavy electrons, possibly having several different rest masses. Particles below 4×10^8 e-volts would be mostly electrons, those above this energy mostly heavy electrons. If the energy losses of all these particles be due entirely to the ionization and the ordinary Bremsstrahlung, then we would have to conclude that the majority of particles of energy round about 4×10^9 e-volts should have a rest mass of about five times the electron mass or more, since the observed radiation loss is less than about one-twentieth of that for electrons. This sets a lower limit to the mass of heavy electrons with energies in the neighbourhood of 4×10^9 e-volts. We cannot give an upper limit from such considerations, since the observed loss may be due to an average of the energy loss of heavy particles and a small number of electrons. The fact that in the atmosphere the particles seem to show the theoretical loss for electrons up to energies as high as 10^{10} e-volts presents no difficulty, for in the upper atmosphere the soft electron component would predominate and control the shape of the absorption curve, whereas, by the time the radiation has reached sea-level, most of these electrons would have already been absorbed.

But an explanation along these lines meets with the difficulty that in lighter elements, for example, aluminium, the observed energy loss would also be less than about a twentieth of the theoretical at about the same energies, whereas in fact it seems to be comparable with the theoretical energy loss. Thus, if the findings of Blackett and Wilson are correct, namely, that the actual energy loss deviates from the theoretical in lead for energies above 4×10^8 e-volts whereas in aluminium no considerable deviation occurs for energies up to some much higher value, say 2×10^9 e-volts, then we are forced to the following conclusion: *If the explanation of these experiments is to be sought by assuming the existence of new particles, then we must conclude that* for high energies *part of the loss is not due to the ordinary Bremsstrahlung* (which varies as Z^2), *but to some other process which allows the* *possibility of large losses besides the ionization loss, such losses varying in different substances not as Z^2 but rather as Z.* One possibility for such losses is discussed in the preceding section.

Recently Neddermeyer and Anderson (1937) have reported energy-loss measurements in a platinum plate put across the chamber in which they divide the tracks entering the chamber into two groups, shower particles and single tracks. The tracks of the first group show the energy loss to be expected theoretically for electrons, while the particles of the second group show a much lower energy loss in the same range. Since the lowest energy tracks of the second group have a curvature corresponding to an energy of about $1 \cdot 4 \times 10^8$ e-volts if they were electrons, one may, as before, exclude the possibility of these particles being protons, since protons of this curvature would exhibit a much larger ionization. Under these circumstances Neddermeyer and Anderson conclude that their experiments indicate the existence of a new particle of electronic charge and a mass intermediate between those of the electron and proton.*

Lastly, Anderson and Neddermeyer have found that in certain photographs of showers, particles are seen which show an ionization definitely heavier than that of an electron. On the other hand, the range of these particles is much longer than it would be for protons having the observed curvature. If, therefore, no error has occurred in the estimate of the curvature or range, these photographs would supply additional evidence of a new particle of the type we are considering, as the authors themselves point out. The mass of these particles would be of the order of a few hundred times the electron mass.

To sum up then, we may say that while the energy-loss measurements of Blackett and Wilson would find a more simple explanation on the hypothesis (a) that the theory of radiation loss fails for electrons, *this hypothesis would seem to be in contradiction with other definitely established*

* It must be remarked however that these results are not quite in harmony with those of Blackett and Wilson. Since the first group shows a normal energy loss, and the second group a lower energy loss, it follows that the energy loss averaged over particles of both groups is less than that to be expected theoretically, whereas Blackett and Wilson find that the energy loss up to 2×10^8 e-volts is in reasonable agreement with the theory. On the other hand, apart from the actual value of the energy losses for the two groups, the fact that the two groups show a markedly different energy loss seems to be clear, in accordance with the observation often made by various investigators that the particles in showers seem to show a higher energy loss and are more absorbable than those not in showers. In passing we may remark that the suggestion that has been made, that the shower particles have a mass smaller than that of the electron, may be rejected on the ground that a gamma ray would have a greater chance of producing a pair of such particles than an electron pair, and the threshold frequency for pair creation would also be lower than 10^6 e-volts.

phenomena connected with the latitude effect and the transition curves for bursts. In these circumstances we seem to be compelled to accept the hypothesis (*b*) of the existence of new particles.

2—Energy loss

Free collision and ionization

We now proceed to investigate as far as possible the behaviour of a particle, of charge e equal to the electronic charge, and of some arbitrary mass M between those of the electron and proton. We shall assume that the particle obeys the Dirac equation, and that its interaction with other charged particles is that given by quantum electrodynamics for the interaction of point charges. The correctness of our results will then be limited by two possibilities. First, the particle may have a direct interaction with other particles like itself or even with electrons other than that operating through the electromagnetic field, as is indeed the case for protons. We, however, regard it as unlikely that such interactions, if they exist, would affect the ionization loss appreciably, since the important contribution to the loss comes from processes which take place at large distances, and direct interactions between particles are usually short-range forces. Secondly, the interaction of the particle with the radiation field may differ from that due to a point charge owing to the particle possessing something corresponding in the classical picture to its charge being spread over a region of finite extension. This will again not affect the ionization loss appreciably, but, as we shall see when we come to consider radiation loss, it may impose restrictions on the validity of the radiation formulae which are more stringent than those for electrons since the "Compton wave-length" $\hbar/Mc$ of such a particle is less than the Compton wave-length $\hbar/mc$ of electrons.

We shall first consider the cross-sections for the collision of a "heavy electron" with an ordinary electron at rest in the material. The general expression for this cross-section taking retardation into account has been given by Møller (1932), and it can be evaluated exactly as has been done in a previous paper (Bhabha 1936). We give only the result here. The differential effective cross-section $Q\,dq$ for the production by a "heavy electron" of total energy E_0 of a secondary electron of kinetic energy W lying in the interval corresponding to dq, where

$$q \equiv W/(E_0 - Mc^2)$$

and

$$\gamma \equiv E_0/Mc^2,$$

is

$$Q(E_0, W)\,dW \equiv Q(q)\,dq = 2\pi r_0^2 \frac{m}{M(\gamma-1)}\left[\frac{\gamma^2}{\gamma^2-1} - \frac{q}{q_m} + \frac{1}{2}\left(\frac{\gamma-1}{\gamma+1}\right)q^2\right]\frac{dq}{q^2}. \quad (2\cdot1)$$

Here m is the mass of the electron, $r_0 \equiv e^2/mc^2$, and

$$\frac{W_m}{E_0 - Mc^2} \equiv q_m = \frac{2mM(\gamma+1)}{m^2+M^2+2mM\gamma}. \quad (2\cdot2)$$

q may take on all values consistent with the conservation of energy and momentum, namely, from 0 to q_m. Then $W_m \equiv q_m(E_0 - Mc^2)$ is the maximum energy which can be transferred to an initially stationary electron in a free collision, and it plays a certain role in our later calculations. It is clear that q_m tends to unity for sufficiently large γ however great M may be.

We must emphasize that just the cross-sections for hard collisions given by expression (2·1) might be considerably altered by the existence of close-range forces. In the absence of any knowledge about such forces and a relativistic formulation of them, we cannot estimate the magnitude of this correction. We would mention in passing that the spin of the particle also plays an important role for just these hard collisions, and since the spin of the proton, for example, is not described completely by the Dirac equation, it is of interest to estimate the order of this correction. If we had described the heavy particles by the Klein-Gordon equation instead of the Dirac equation, thus ascribing no spin to them, the last term in square brackets in (2·1) would have been replaced by

$$\frac{1}{\gamma+1}\frac{m}{M}q.$$

The difference due to the spin therefore bears a ratio to the total cross-section of the order $(\gamma^2-1)\,m^2/M^2$, which for protons is small compared with unity except for energies of the order of 10^{12} e-volts.

The total ionization loss per centimetre is given by the formula of Bloch

$$-\left(\frac{dE_0}{dx}\right)_{\text{coll}} = 2\pi\sigma Z r_0^2 \frac{mc^2}{\beta^2}\left[\log\frac{mc^2\beta^2 W_m}{(1-\beta^2)\,I^2Z^2} + (1-\beta^2)\right], \quad (2\cdot3)$$

where σ is the number of atoms per cubic centimetre of the substance, Z the atomic number, $c\beta$ the velocity of the heavy electron given by $\beta = \sqrt{(1-1/\gamma^2)}$ and IZ the mean ionization potential of the atom, where we put, with Bloch, $I = 13\cdot5$ e-volts. W_m has been defined above. Our results are given in Tables II and III.

Radiation loss and creation of pairs

We mentioned in the preceding section that the quantum-mechanical formulae for the radiation loss of heavy electrons are not on as sound a footing as for ordinary electrons. To see this we consider the derivation of the formulae by the method of Weizsäcker and Williams. We consider the whole process in a Lorentz system in which the radiating particle is at rest. In this system the nucleus moves along a straight line with extremely high velocity, and its field may then be considered as a superposition of quanta of different frequencies. The radiation process corresponds in this system to the scattering of one of these quanta by the stationary particle. Weizsäcker has shown that the important contribution to the radiation comes from the scattering of all quanta whose energy is equal to or less than the rest energy of the particle. In other words, the validity of the radiation formulae depends on the validity of the Klein-Nishina formula for wave-lengths as small as the "Compton wave-length" of the particle. Now in the case of electrons, the Compton wave-length $\hbar/mc$ is large compared to the classical radius of the electron e^2/mc^2, so that the validity of the Klein-Nishina formula is not in question. The classical radius e^2/Mc^2 of a heavy electron would also be small compared to its Compton wave-length $\hbar/Mc$, so that in this case, too, no difficulty would arise. We, however, think that it may be a property of charge in general that it may not be possible to localize it in a region smaller than e^2/mc^2, whatever the mass of the particle with which it is associated. In this case the Klein-Nishina formula may no longer be valid for quanta of energy equal to the rest energy of "heavy electrons" if

$$\frac{\hbar}{Mc} \lesssim \frac{e^2}{mc^2},$$

i.e. if

$$\frac{m}{M} \lesssim \frac{e^2}{\hbar c}. \tag{2·4}$$

Thus we must be prepared to find that for particles of mass greater than $137m$ the radiation formulae may not be valid.

It is quite easy to calculate the radiation loss for heavy electrons, taking screening into account to the same degree of accuracy as in the original calculations of Bethe and Heitler. The calculations follow those of Bethe (1934) closely. It can be shown exactly, as has been done there, that the differential effective cross-section for the emission of a quantum k in the energy interval dk by a heavy electron of energy E_0 is

$$\phi(k)\,dk = \frac{Z^2}{137}\left(\frac{e^2}{Mc^2}\right)^2 \frac{4dk}{E_0^2 k}\Bigg[(E_0^2+E^2)\left\{\int_\delta^{Mc^2}(1-F)^2(q-\delta)^2\frac{dq}{q^3}+1\right\}$$
$$-\tfrac{2}{3}E_0E\left\{\int_\delta^{Mc^2}(1-F)^2\left(q^3-6\delta^2 q\log\frac{q}{\delta}+3\delta^2 q-4\delta^3\right)\frac{dq}{q^4}+\frac{5}{6}\right\}\Bigg], \tag{2·5}$$

where $\delta = \dfrac{(Mc^2)^2 k}{2E_0 E}$. We shall assume throughout that

$$E_0,\ E,\ k \gg Mc^2,$$

for it is only then that screening is important, so that

$$\delta \ll Mc^2.$$

The form factor F which represents the effect of screening is given by

$$F\left(\frac{q\mu Z^{-\frac{1}{3}}}{c\hbar}\right) \equiv \frac{1}{4\pi}\int e^{\frac{i}{c\hbar}\mu Z^{-\frac{1}{3}}(q,\,x)} \frac{\phi^{\frac{3}{2}}(r)}{r^{\frac{3}{2}}}\,dx \tag{2·6}$$

and is a function of $q\mu Z^{-\frac{1}{3}}/c\hbar$ only. The integration in (2·6) extends over the whole of space. ϕ is the function tabulated by Fermi, and determines the density of electrons at a distance r from the nucleus in the units of length defined by μ, where

$$\mu = \left(\frac{3\pi}{8\sqrt{2}}\right)^{\frac{2}{3}} \frac{\hbar^2}{me^2} = 0{\cdot}876\frac{\hbar^2}{me^2}. \tag{2·7}$$

Now ϕ is only considerable when r is less than or of the order unity, so that $F \ll 1$ when

$$q \gg \frac{c\hbar}{\mu} Z^{\frac{1}{3}} \sim \frac{e^2}{\hbar c} Z^{\frac{1}{3}} mc^2. \tag{2·8}$$

Screening is therefore only effective when δ, the minimum value of q, satisfies

$$\delta \lesssim \frac{e^2}{\hbar c} Z^{\frac{1}{3}} mc^2 \ll mc^2. \tag{2·8a}$$

Hence if the inequality (2·8a) is not satisfied, i.e. if

$$\frac{Mc^2 k}{2E_0 E}\frac{M}{m} \gg Z^{\frac{1}{3}}\frac{e^2}{\hbar c}, \tag{2·8b}$$

we may neglect F in (2·5) altogether, and the integrations can then be carried out exactly. We now consider the first q integral in (2·5) when (2·8a) is satisfied. We may write it

$$\int_{\delta}^{mc^2} (1-F)^2 (q-\delta)^2 \frac{dq}{q^3} + \int_{mc^2}^{Mc^2} (1-F)^2 (q-\delta)^2 \frac{dq}{q^3}. \tag{2·9}$$

In virtue of (2·8) and (2·8a) both F and δ may be neglected in the second integral in (2·9) and it becomes $\log M/m$. The first integral has been evaluated numerically by Bethe and may be written in the form

$$\tfrac{1}{4}\phi_1(\xi)+\log Z^{-\frac{1}{3}},$$

where
$$\xi = \frac{200}{mc^2}Z^{-\frac{1}{3}}\delta = \frac{100}{Z^{\frac{1}{3}}}\frac{Mc^2k}{E_0E}\cdot\frac{M}{m}. \qquad (2\cdot10)$$

(2·9) therefore reduces to

$$\tfrac{1}{4}\phi_1(\xi)+\log\frac{M}{m}Z^{-\frac{1}{3}}.$$

The second q integral in (2·5) can be treated similarly. We get finally

$$\phi(k)\,dk = \frac{Z^2}{137}\left(\frac{e^2}{Mc^2}\right)^2\frac{dk}{E_0^2k}\left[(E_0^2+E^2)\left\{\phi_1(\xi)+4\log\frac{M}{m}Z^{-\frac{1}{3}}\right\}\right.$$
$$\left. -\tfrac{2}{3}E_0E\left\{\phi_2(\xi)+4\log\frac{M}{m}Z^{-\frac{1}{3}}\right\}\right]. \qquad (2\cdot11)$$

Here ξ is defined by (2·10), and the functions ϕ_1 and ϕ_2 have been given by Bethe and Heitler (1934, fig. 1). Moreover, as Bethe (1934, eq. (63)) has shown, for the case of complete screening $\delta = 0$

$$\phi_1(0) = \phi_2(0)+\tfrac{2}{3} = 4\log 183,$$

so that (11) reduces to

$$\phi(k)\,dk = 4\frac{Z^2}{137}\left(\frac{e^2}{Mc^2}\right)^2\frac{dk}{E_0^2k}\left\{(E_0^2+E^2-\tfrac{2}{3}E_0E)\log 183\frac{M}{m}Z^{-\frac{1}{3}}+\tfrac{1}{9}E_0E\right\}. \qquad (2\cdot12)$$

For low energies when the inequality (2·8 b) is satisfied, i.e. when $\xi \gg 1$,

$$\phi_1(\xi) = \phi_2(\xi) = 4\left(\log\frac{200}{\xi}-\frac{1}{2}\right),$$

so that (2·11) reduces to

$$\phi(k)\,dk = 4\frac{Z^2}{137}\left(\frac{e^2}{Mc^2}\right)^2\frac{dk}{E_0^2k}\left\{(E_0^2+E^2-\tfrac{2}{3}E_0E)\left(\log\frac{2E_0E}{kMc^2}-\frac{1}{2}\right)\right\}, \qquad (2\cdot13)$$

as indeed a direct integration of (2·5) neglecting the factors F would have given.

The average energy loss by radiation per centimetre is given by

$$-\left(\frac{dE_0}{dx}\right)_{\text{rad}} = \sigma\int_0^{E_0}k\phi(k)\,dk, \qquad (2\cdot14)$$

where σ is the number of atoms per c.c. and $\phi(k)$ is given by (2·11). Introducing the variable $\epsilon = k/E_0$, it may be written in the form

$$\sigma \frac{Z^2}{137}\left(\frac{e^2}{Mc^2}\right)^2 E_0 \int_0^1 d\epsilon \left[\{1+(1-\epsilon)^2\}\phi_1\left(\frac{50}{\xi'}\frac{\epsilon}{1-\epsilon}\right) - \tfrac{2}{3}(1-\epsilon)\,\phi_2\left(\frac{50}{\xi'}\frac{\epsilon}{1-\epsilon}\right) + 4\log\frac{M}{m}Z^{-\frac{1}{3}}\{\tfrac{4}{3}(1-\epsilon)+\epsilon^2\}\right], \quad (2\cdot 14a)$$

where
$$\xi' = Z^{\frac{1}{3}}\left(\frac{m}{M}\right)^2\left(\frac{E_0}{2mc^2}\right). \quad (2\cdot 15)$$

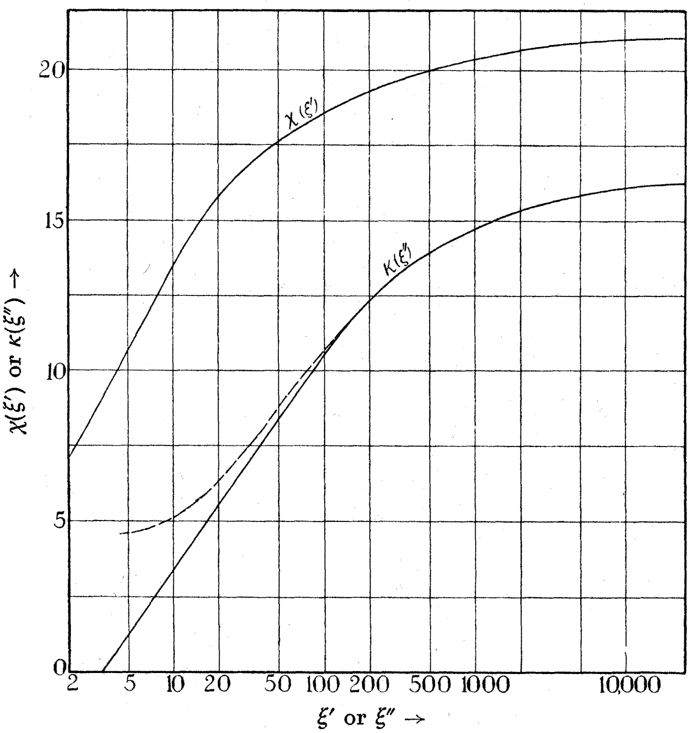

FIG. 2—The above curves are accurate only as long as $E_0 \gg Mc^2$. For electrons this condition ceases to hold for ξ'' less than about 20, and the dotted line gives the more accurate curve for this case.

The integrals containing ϕ_1 and ϕ_2 can only be evaluated numerically, and are functions of ξ' only. Equation (2·14a) may therefore be written in the form

$$-\left(\frac{dE_0}{dx}\right)_{\text{rad}} = \sigma \frac{Z^2}{137}\left(\frac{e^2}{Mc^2}\right)^2 E_0\left\{\chi(\xi') + 4\log\frac{M}{m}Z^{-\frac{1}{3}}\right\}. \quad (2\cdot 16)$$

We have plotted χ as a function of ξ' in fig. 2, where ξ' defined by (2·15) is the energy measured in millions of volts multiplied by the factor $Z^{\frac{1}{3}}(m/M)^2$. Since for the case of electrons, $M = m$, the expression in curly brackets must reduce to ϕ_{rad} already calculated by Bethe and Heitler (1934, eq. (48)), the values of χ can be deduced from those of ϕ_{rad}. For $\xi' \ll 137$ (negligible screening)

$$\chi(\xi') \to 4 \log 4\xi - \tfrac{4}{3},$$

and for $\xi' \gg 137$ (complete screening)

$$\chi(\xi') \to 4 \log 183 + \tfrac{2}{9},$$

so that in the first case

$$-\left(\frac{dE_0}{dx}\right)_{\text{rad}} = \sigma \frac{Z^2}{137}\left(\frac{e^2}{Mc^2}\right)^2 E_0 \left\{4 \log \frac{2E_0}{Mc^2} - \frac{4}{3}\right\}, \tag{2·16a}$$

and in the second case

$$-\left(\frac{dE_0}{dx}\right)_{\text{rad}} = \sigma \frac{Z^2}{137}\left(\frac{e^2}{Mc^2}\right)^2 E_0 \left\{4 \log 183 \frac{M}{m} Z^{-\frac{1}{3}} + \frac{2}{9}\right\}. \tag{2·16b}$$

We therefore see that the larger the mass of the particle, the later screening becomes effective and the less its effect.

In Tables II and III we give the collision and radiation losses in lead and water of particles of masses 10 and 100 times the electron mass for various energies. The total energy loss is shown in fig. 3.

TABLE II—ENERGY LOSS IN LEAD

Millions of electron-volts per centimetre

$E_0 - Mc^2$		10^6	10^7	10^8	10^9	10^{10}	10^{11}	10^{12}
Electron	Coll.	11·4	13·9	18·6	23·4	28·2	33·0	37·7
	Rad.	—	14·4	177	1900	19,400	$1·94 \times 10^5$	$1·94 \times 10^6$
$Mc^2 = 5 \times 10^6$ e-volts	Coll.	26·4	12·9	16·0	20·8	25·4	30·3	35·0
$\approx 10mc^2$.	Rad.	—	—	1·72	26·4	296	3080	31,100
$Mc^2 = 5 \times 10^7$ e-volts	Coll.	132	26·8	13·2	16·9	22·2	27·1	31·8
$\approx 100mc^2$.	Rad.	—	—	—	0·17	2·9	38·1	413
Proton	Coll.	(537)	203	40·3	13·9	15·7	21·7	27·4
$M = 1840m$.	Rad.	—	—	—	—	—	0·07	1·01

Using (2·3) and (2·16) we find that the ratio of the radiation to the collision loss is given by

$$\frac{-(dE_0/dx)_{\text{rad}}}{-(dE_0/dx)_{\text{coll}}} = \frac{ZE_0}{1300mc^2}\left(\frac{m}{M}\right)^2. \tag{2·17}$$

TABLE III—ENERGY LOSS IN WATER

Millions of electron volts per centimetre

$E_0 - Mc^2$		10^6	10^7	10^8	10^9	10^{10}	10^{11}	10^{12}
Electron	Coll.	1·93	2·15	2·72	3·29	3·94	4·53	5·10
	Rad.	—	0·16	2·07	22·5	233	$2 \cdot 33 \times 10^3$	$2 \cdot 33 \times 1$
$Mc^2 = 5 \times 10^6$ e-volts	Coll.	4·76	2·10	2·44	3·02	3·61	4·19	4·77
$\approx 10mc^2$	Rad.	—	—	0·02	0·28	3·26	34·5	350
$Mc^2 = 5 \times 10^7$ e-volts	Coll.	28·1	4·82	2·14	2·54	3·20	3·80	4·39
$\approx 100mc^2$	Rad.	—	—	—	—	0·03	0·40	4·45
Proton	Coll.	(284)	47·1	7·48	2·32	2·40	3·13	3·84
$M = 1840m$				Radiation loss negligible				

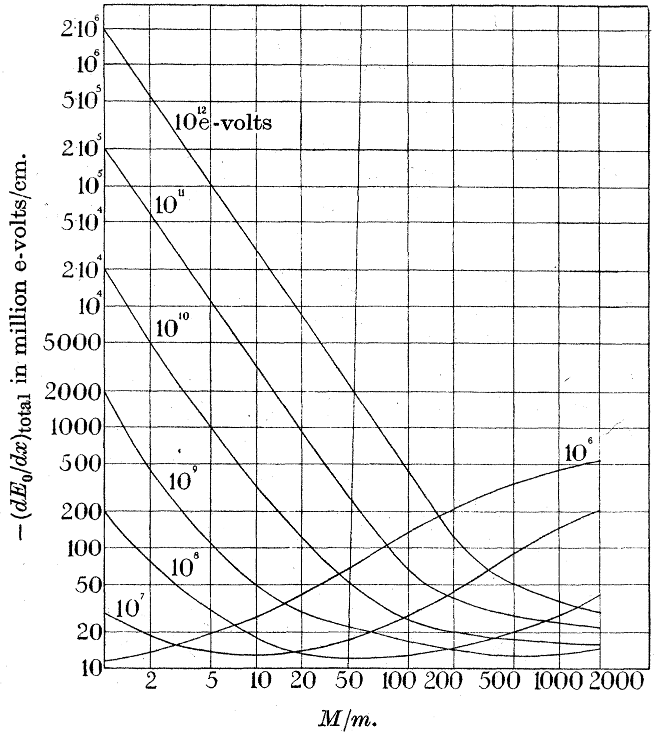

FIG. 3—Energy loss in lead.

By putting (2·17) equal to unity, we may define a critical energy at which the rates of radiation and collision losses are equal, namely

$$E_M = \frac{1300}{Z}\left(\frac{M}{m}\right)^2 mc^2. \tag{2·18}$$

For energies greater than this the loss is predominantly due to radiation, for smaller energies to collision. This energy is correctly given by (2·18) to within 20 %. More accurately, the numerical constant in (2·18) should be 1600 for $M = m$, 1300 for $M = 10m$ and 1100 for $M = 100m$.

The total rate of energy loss of the particle may be roughly written in the form

$$-\left(\frac{dE_0}{dx}\right)_{\text{total}} = \frac{Z^2}{137} r_0^2 \sigma \left(\frac{m}{M}\right)^2 C_M [E_0 + E_m], \qquad (2\cdot19a)$$

where C_M is a numerical constant;

$$C_M = 23 \text{ for } M = 10m, \quad \text{and} \quad C_M = 30 \text{ for } M = 100m.$$

The mean range R is then roughly given by

$$R = \frac{137}{Z r_0^2 \sigma \left(\frac{m}{M}\right)^2 C_M} \log\left(1 + \frac{E_0 - Mc^2}{E_M + Mc^2}\right), \qquad (2\cdot19b)$$

which is accurate to within 30 % except for kinetic energies small compared to the rest mass of the particle. (2·19*b*) would give a better fit with the numerically calculated ranges if we chose slightly different values for E_M, putting the numerical constant in (2·18) equal to 1100 for $M = 10m$ and 920 for $M = 100m$ respectively.

The cross-sections for the creation of pairs by γ-rays are obtained from the above formulae if we change the sign of the energy E_0 of the initial state, writing in its place $-E_+$, E_+ being the energy of the positron, and replacing the factor $dk/E_0^2 k$ by dE/k^3. We thus get for the cross-section for the creation by a quantum $k = h\nu$ of a pair, the electron of which has an energy in dE, the expression

$$\phi(E)\,dE = \frac{Z^2}{137}\left(\frac{e^2}{Mc^2}\right)^2 \frac{dE}{(h\nu)^3}\left[(E^2 + E_+^2)\left\{\phi_1(\xi) + 4\log\frac{M}{m} Z^{-\frac{1}{3}}\right\} + \tfrac{2}{3} E E_+ \left\{\phi_2(\xi) + 4\log\frac{M}{m} Z^{-\frac{1}{3}}\right\}\right]. \qquad (2\cdot20)$$

The total cross-section for pair creation is given by

$$\phi_{\text{pair}} = \frac{Z^2}{137}\left(\frac{e^2}{Mc^2}\right)^2 \left[\kappa(\xi'') + \frac{28}{9}\log\frac{M}{m} Z^{-\frac{1}{3}}\right]. \qquad (2\cdot21)$$

The function $\kappa(\xi'')$ is plotted in fig. 2, where

$$\xi'' = Z^{\frac{1}{3}}\left(\frac{m}{M}\right)^2\left(\frac{h\nu}{2mc^2}\right). \qquad (2\cdot22)$$

When $\xi'' \ll 137$ (negligible screening)

$$\kappa(\xi'') \to \tfrac{28}{9}\log 4\xi'' - \tfrac{218}{27},$$

and for complete screening $\xi'' \gg 137$

$$\kappa(\xi'') \to \tfrac{28}{9}\log 183 - \tfrac{2}{27},$$

so that (2·21) becomes

$$\phi_{\text{pair}} = \frac{Z^2}{137}\left(\frac{e^2}{Mc^2}\right)^2 \begin{cases} \left\{\dfrac{28}{9}\log\dfrac{2h\nu}{Mc^2} - \dfrac{218}{27}\right\}, \\ \left\{\dfrac{28}{9}\log 183\dfrac{M}{m}Z^{-\frac{1}{3}} - \dfrac{2}{27}\right\}, \end{cases} \tag{2·23}$$

in the two cases respectively.

3—The secondaries accompanying heavy particles

A heavy particle in its passage through matter will directly or indirectly produce secondary electrons which on account of their lower energy and lighter mass will behave like a soft component accompanying the penetrating heavy particle. It is therefore of interest to calculate the number of electrons or positrons of energy greater than some value E in equilibrium with a homogeneous beam of heavy particles of mass M and energy E_0.

The number of secondaries produced by direct hard encounters is given by (2·1). But this is not the quantity, which is of direct interest in cosmic-ray experiments. These direct secondaries will produce further positive and negative electrons by cascade multiplication in the subsequent layers of material. In addition to this, the heavy particle may emit a quantum of radiation which will also produce electrons by cascade multiplication. The average number of electrons which emerge from the bottom of some layer of substance through which the heavy particle has passed is therefore different from that given by a simple application of (2·1). We now proceed to estimate this number.

We will first calculate the average number of positrons and electrons which accompany the passage of a heavy particle of energy E_0, and which are due only to the direct production of fast secondaries by collision as given by (2·1) and the ensuing cascade processes. We will suppose that the layer already traversed by the heavy particle is so thick that a cascade process started at the beginning of the layer does not reach the point we are considering. Such a layer may be described as "infinitely thick". A layer satisfying this condition has a thickness of about 30 in the units λ_0 characteristic of the material in the cascade theory. λ_0 is given by (Bhabha and Heitler 1937 eq. (15))

$$\lambda_0 = \frac{137}{aZ^2 r_0^2 \sigma}, \tag{3·1}$$

with a equal to 20 for lead and 23 for air or water, so that an infinitely thick layer is about 10 m. of water or 12 cm. of lead. Under these circumstances it can be shown (Arley and Bhabha 1937) that the number $N_{s>}$ of electrons (+ and −) with energy greater than E is given by

$$N_{s>} = \frac{2\pi \,.\, 137}{aZ}\frac{mc^2}{E}\left[\frac{\gamma^2}{\gamma^2-1}\left\{1-\frac{1}{r}(\log r+1)+2\alpha\left[\frac{1}{\beta-1}(r^{\beta-1}-1)+\frac{1}{r}-1\right]\right\}\right.$$
$$-\frac{(\log r)^2}{2r}-\frac{2\alpha}{r}\left\{\frac{1}{\beta}(r^{\beta}-1)-\log r\right\}$$
$$\left.+\frac{1}{2}\frac{\gamma-1}{\gamma+1}q_m^2\left\{\frac{\log r-1}{r}+\frac{1}{r^2}+\frac{2\alpha}{r}\left(\frac{1}{\beta+1}[r^{\beta+1}-1]-r+1\right)\right\}\right], \tag{3·2}$$

where $r = W_m/E$, and $\alpha = 0{\cdot}224$ and $\beta = 1{\cdot}029$ are numerical constants. W_m is defined in (2·2). This is valid for $E > E_c$, where E_c is the "critical energy" for the substance, being the energy at which the ionization loss of an electron is equal to the radiation loss. This limit is (Bhabha and Heitler 1937, § 1)

$$10^7 \text{ e-volts in lead; } 1{\cdot}5\times 10^8 \text{ e-volts in air or water.} \tag{3·3}$$

(3·2) is accurate to the same degree as the calculations of the cascade theory, i.e. to within about 30%, except when $W_m \sim E_c$.

To estimate roughly the number of electrons below E_c we use the result of the cascade theory discussed in § 4, namely, that in a shower the number of particles with energy below E_c is very roughly equal to the number above E_c. We may thus deduce the following expression for the number $N_{s<}$ of positrons and electrons with energy below E_c accompanying the heavy particle,

$$N_{s<} = \frac{2\pi \,.\, 137}{aZ}\frac{mc^2}{E_c}\left[\left\{1-\frac{1}{r}+\frac{2\alpha\beta}{\beta-1}(r^{\beta-1}-1)\right\}\frac{\gamma^2}{\gamma^2-1}-\frac{1}{r}\log r-\frac{2\alpha}{r}(r^{\beta}-1)\right.$$
$$\left.+\frac{1}{2}\frac{\gamma-1}{\gamma+1}\frac{q_m{}^2}{r^2}\left\{r-1+\frac{2\alpha\beta}{\beta+1}(r^{\beta+1}-1)\right\}\right]. \tag{3·4}$$

We give the values of (3·2) and (3·4) in Table IV.

We see that for extremely high energies (10^{12} e-volts) the number of secondaries *due to this process alone* is practically independent of the mass of the particle, being about 33 % of the hard component in lead. For lower energies it depends on the mass to a greater extent. For particles with a kinetic energy of 10^{10} e-volts passing through lead the secondary electron component would constitute about 20 % for particles of ten to a hundred times the electron mass, whereas it is about 7 % for protons. In water the secondary electron component varies from about 7 % for particles with kinetic energy of 10^{10} e-volts to about 15 % for particles of 10^{12} e-volts. According to Auger and his co-workers, the soft component in water which is in equilibrium with the penetrating component is of the order of 5 % or less,* which allows one to deduce from Table IV, that the mean energy of the penetrating particles must be of the order of 10^{10} e-volts *or less*, as is in fact otherwise known. It must be emphasized that in estimating the equilibrium intensity, the number of particles appearing in showers must be *included* in calculating the average. We see that the amount of the secondary electron component in equilibrium with the penetrating component as given by the theory is in qualitative agreement with experiment, although both the theoretical and experimental results are too inaccurate to allow of a quantitative comparison.

TABLE IV—THE MEAN NUMBER PER CENT OF POSITRONS AND ELECTRONS WITH ENERGY GREATER THAN E_c ($N_{s>}$) AND ENERGY LESS THAN E_c ($N_{s<}$) ACCOMPANYING THE HEAVY PARTICLE

The addition of secondaries from radiative losses would roughly double the fiigures for $M = 10m$ (figures in brackets).

		Lead			Air or water		
$E_0 - Mc^2$ in mV		10^8	10^{10}	10^{12}	10^8	10^{10}	10^{12}
$Mc^2 = 5 \times 10^6$ e-volts	$N_{s>}$	1·8	9·4	16	—	3·0	7·2
$\approx 10mc^2$	$N_{s<}$	3·0	11·0	18	—	3·9	8·1
	Total	4·8	20·4	34	—	6·9	15·3
		(10)	(41)	(70)	—	(14)	(31)
$Mc^2 = 5 \cdot 10^7$ e-volts	$N_{s>}$	—	9·0	16	—	2·9	7·2
$\approx 100mc^2$	$N_{s<}$	—	10·0	18	—	3·9	8·1
	Total	—	19·0	34	—	6·8	15·3
Proton	$N_{s>}$	—	2·5	15·1	—	—	6·5
$M = 1840m$	$N_{s<}$	—	4·1	16·7	—	—	7·4
	Total	—	6·6	31·8	—	—	13·9

We must add to the figures of Table IV the number of electrons which result from cascade processes caused by quanta emitted by the heavy particle. It will appear in § 4, eq. (4·1), that the ratio of the relative prob-

* This equilibrium intensity must be taken from measurements under water, for as we have pointed out in § 1, the two components are not in equilibrium above sea-level.

abilities of the emission of a quantum and of the production of an electron by collision both with energy greater than E_c is of the order unity for $M = 10m$ and is small compared to one for $M \gg 10m$. A quantum, moreover, produces roughly the same number of particles by cascade multiplication as an electron, so that in Table IV the figures for $M = 100m$ and for protons will not be appreciably altered, *whereas the figures for $M = 10m$ will be somewhat more than doubled by the addition of this process.* A comparison with the measurements of Auger and others therefore allows one to conclude that if the hypothesis of new particles is right, *the majority of the penetrating particles must have masses nearer to a hundred times the electron mass rather than ten times the same.*

4—Number and size of showers

We now proceed to investigate the production of showers by heavy particles. There are three distinct ways in which a heavy particle may cause a shower. (1) It may produce a very fast secondary electron by direct collision which in its turn may produce a shower by cascade multiplication if it has sufficient energy. (2) It may emit a quantum of radiation of sufficient energy to produce a shower by cascade multiplication.* (3) It may produce a shower directly by a multiple process. The second process may itself occur in two distinct ways. The emitted quantum may be just the ordinary Bremsstrahlung, which is emitted during a change in the motion of the particle as a whole in a given external field. The probability of this process has been calculated in § 2. But in addition to this we must be prepared to find that under certain circumstances a particle may change its rest mass, the difference in energy being liberated as a quantum of radiation. Present quantum mechanics of course does not enable one to calculate the probability of such a process. We shall not concern ourselves here with processes of the type (3). Heisenberg has shown how they may result from a modification of Fermi's β-ray theory.

As has been shown by Bhabha and Heitler, cascade multiplication of the number of particles is only effective provided the particles or quanta have energies above the critical energy E_c for the material under consideration, although a certain amount of multiplication may also take place below this

* Since this paper was sent to press, a note has appeared by Landau and Rumer (1937) in which they estimate the probability of showers due to process (2). Our calculations below show that for $M > 10m$ the process (1) becomes much more important than the process (2), so that for $M \approx 100m$, we get shower probabilities which are some hundred times larger than those of Landau and Rumer.

energy. We will begin by estimating the relative probability of a shower being produced by processes (1) and (2), which amounts to estimating the ratio of the chance of a heavy particle emitting a quantum of energy greater than E to the chance of its producing by collision an electron of energy greater than the same amount. The cross-sections for these two processes are given by formulae (2·11) and (2·1) respectively. It can be shown by an easy calculation that this ratio is

$$\frac{\int_E^{E_0} \phi(k)\,dk}{\int_{E/E_0}^{q_m} Q(q)\,dq} = \frac{2}{\pi}\cdot\frac{Z}{137}\left(\frac{m}{M}\right)^2\left(\frac{E}{mc^2}\right)\log q\,.\log X \tag{4·1}$$

to a fair approximation provided $\gamma^2 \gg 1$ and $q \equiv \dfrac{E}{E_0 - Mc^2} \ll q_m$, i.e. provided $E \ll W_m$, the maximum energy which can be communicated to an electron is a free collision. Here

$$X \equiv \begin{cases} 183Z^{-\frac{1}{3}} & \text{when } \dfrac{2E_0}{Mc^2}\dfrac{1-q}{q} > 183Z^{-\frac{1}{3}}, \\ \dfrac{2E_0}{Mc^2}\cdot\dfrac{1-q}{q} & \text{when } \dfrac{2E_0}{Mc^2}\dfrac{1-q}{q} < 183Z^{-\frac{1}{3}}. \end{cases} \tag{4·1 a}$$

Even when E is not very small compared to W_m, (4·1) still gives the correct order of magnitude. Since cascade multiplication of the number of particles is only important provided the electrons or quanta have energies above the critical energy E_c for the material under consideration, although a certain amount of multiplication may also take place below this energy, the above formula is of interest when $E \geqslant E_c$. Putting $E = E_c \approx 1600mc^2/Z$ in (4·1) we find that the ratio (4·1) is less than unity if

$$\frac{M}{m} > 2{\cdot}73\sqrt{(\log q_c\,.\log X)}, \tag{4·2}$$

where $q_c = E_c/(E_0 - Mc^2)$. The right-hand side of (4·2) is practically independent of the atomic number of the material, and is roughly equal to 9 for $E_0 \sim 10^9$ e-volts and 14 for $E_0 \sim 10^{11}$ e-volts. Therefore if the mass of the heavy particle $M \gg 10m$ the effect of radiative processes is small compared with the effect of direct collisions in producing showers and electronic secondaries. For $M \lesssim 10m$ the radiative process is important, and may even predominate over the other.

We will now consider the following question. When a particle of mass M emerges after its passage through a plate of some substance of atomic number

Z and thickness l in the units (3·1) with energy E_0, what is the probability of its being accompanied by N electrons and positrons? We will first consider only the process (1), namely, the production of a fast secondary electron by collision and the subsequent cascade process. The chance of an electron of energy E in the interval dE being provided in the thin layer dl', l' being measured from the lower surface of the material from which the particle emerges is just

$$\lambda_0 \sigma Z Q(E_0'(l'), E')\, dE'\, dl',$$

using (2·1). As a result of this electron, $F(l', y')$ electrons and positrons will emerge *on the average* by cascade multiplication from the lower surface of the material ($l' = 0$) with an energy greater than E, where $y' = \log E'/E$. The function F is given by

$$F(l', y') \equiv W(l', y') + 2f_-(l', y'). \tag{4·3}$$

Here $W(l', y')$ is the incomplete γ-function and $f_-(l', y')$ is the function calculated by Bhabha and Heitler (1937, eqq. (22) and (23) and Table I). The probability of N particles appearing instead of F by a fluctuation is then (Bhabha and Heitler, eq. (31))

$$e^{-F}\frac{F^N}{N!}. \tag{4·4}$$

The total probability of the heavy particle appearing accompanied by N positrons and electrons of energy greater than E is therefore

$$P(N) = \lambda_0 \sigma Z \int_0^l dl' \int_E dE'\, e^{-F(l', y')} \frac{F(l', y')^N}{N!} Q(E_0'(l'), E'). \tag{4·5}$$

We may neglect the small variation in the energy of the heavy particle in thicknesses of the plate in which F is considerable and put $E_0'(l') = E_0$. We may then write

$$P(N) = \lambda_0 \sigma Z \int_E^{W_m} dE'\, Q(E_0, E')\, J_N(y'), \tag{4·6}$$

where
$$J_N(y) = \int_0^l dl'\, e^{-F(l', y)} \frac{F(l', y)^N}{N!}. \tag{4·7}$$

As before, we are interested mainly in large thickness of material, so that we may put $l = \infty$ in (4·7), since for $l > 30$ no significant contribution comes to the integral. It does not seem possible to proceed further analytically.

We have worked out numerically the values of J for several values of N and y and they are given in Table V.

TABLE V—VALUES OF $J_N(y)$

$N \backslash y$	0	1	3	5	7	10
1	0	(1·2)	3·03	2·40	2·60	3·05
2	0	(0·43)	2·0	1·78	1·40	1·85
5	0	0	0·48	1·18	0·65	0·70
10	0	0	0	0·95	0·43	0·38
50	0	0	0	0	0·23	0·11

The accuracy of the figures is not high, since the function F is itself only known to within about 30 %. It is easy to see the general form of the dependence of J on N and y. The function $F(l, y)$ for a given y has the value of 1 for $l = 0$, increases rapidly to a maximum for an l between 4 and 12, and then falls away somewhat more slowly to zero. The actual value at the maximum depends on the value of y and increases rapidly with increasing y being determined by the equation

$$F_m = 0{\cdot}062e^{0{\cdot}93y} \tag{4·8}$$

(Bhabha and Heitler, eq. (28)) for y larger than about 3. Moreover, the expression (4·4) has a maximum when $F = N$ and is small when F differs considerably from this value. Thus, for a given N, if y be so small that $F_m \ll N$ the integrand of (4·7) will always remain small, and hence the value of J will be small. As y increases the value of F_m increases rapidly, but the value of J will remain small until a y is reached such that $F_m \simeq N$. For all values of y equal to and less than this, the integrand in (4·7) has but one maximum, namely, at the point at which $F = F_m$. As y increases still further, however, F_m becomes larger than N, and the integrand shows two maxima. For still larger y such that $F_m \gg N$ the contribution to the integral (4·7) comes from two quite separate regions, one for small l' when F is increasing and is in the neighbourhood of N, and the other for large l' when F is decreasing and is also in the neighbourhood of N. Thus as y increases beyond the point for which $F_m \gtrsim N$ the positions of the two maxima in the integrand of (4·7) change, but the value of the integral does not alter appreciably. Due to the form (4·4) in which F occurs, the contribution to the integral (4·7) of two regions of l' in which $F \sim N$ is not the same when the two regions overlap as when they are separate. The value of J is largest when the two regions overlap for then the integrand of (4·7) is considerable for the largest domain of l'. Hence it follows that the largest value of J occurs for some y such that $F_m \sim CN$, where C is a numerical constant somewhat greater than unity. Table V clearly displays this behaviour of J. For a given N and y small, J is negligible. It rises rapidly to its maximum for values of y near some value y_N, say, depending on N, after which it decreases a little, but nevertheless remains of the same order of magnitude. Further, for large N all values of J are smaller than for small N since then the maximum of the expression (4·4) is itself smaller.

For a rough estimate, therefore, we may put $J = 0$ in (4·6) for $y' < y_N$, i.e./or $E' < E_N$, and equal to a constant value $J_N(y_N)$ for $y' \geqslant y_N$ or $E' \geqslant E_N$, where y_N and E_N are the values of y and E respectively at which the maximum of J occurs. We may determine them approximately by the condition that the maximum number of particles F_m which can be created for this y as given by (4·8) shall be equal to CN, C being a numerical constant somewhat larger than unity. This gives

$$\left(\frac{E_N}{E}\right)^{0\cdot93} \equiv e^{0\cdot93 y_N} = \frac{CN}{0\cdot062},$$

or

$$E_N = 1\cdot67(CN)^{1\cdot075}\,E. \tag{4·9}$$

Using the expression (2·1) for $Q(E_0, E')$ and (3·1) for λ_0 we get

$$P(N) \approx \frac{2\pi\,.\,137}{aZ}\cdot\frac{mc^2}{1\cdot67(CN)^{1\cdot075}\,E}\,J_N(y_N)\left[\frac{\gamma^2}{\gamma^2-1}(1-u)+u\log u+\frac{1}{2}\frac{\gamma-1}{\gamma+1}q_m^2u(1-u)\right],$$

where

$$u \equiv \frac{1\cdot67(CN)^{1\cdot075}\,E}{W_m}. \tag{4·10}$$

This expression is valid only provided $u \leqslant 1$. The constant a is equal to 20 for lead and 23 for air or water. We should regard (4·10) merely as a rather careful determination of the order of magnitude of $P(N)$. The expression (4·6) is of course much more accurate.

We see at once from (4·10) that for a given E, $P(N)$ varies inversely as Z, i.e. the chance of a shower containing N electrons and positrons *with energy greater than E* being produced by a heavy particle is *inversely* proportional to the atomic number. The reason for this is obvious, for the production of fast secondary electrons by collision is just proportional to the number of electrons, i.e. to Z, whereas the distances within which this production has to take place are determined by λ_0, the unit of length in the cascade theory for the material concerned which is inversely proportional to Z^2. The above result is, however, not of direct experimental interest.

In the usual cosmic ray experiments we are interested in the relative frequency of showers containing different numbers of particles irrespective of their energy. The lowest value we can give to E in (4·10) is the critical energy E_c. Substituting this in (4·10) we would get the probability of a shower containing N particles above the critical energy of the substance. To a rough approximation E_c may be put equal to $1600mc^2/Z$, so that in this case the factor outside the square brackets in (4·10) ceases to depend explicitly on Z. In other words, the frequency of *small showers for which* $u \ll 1$ *is independent of the atomic number of the material to a first approximation.** *The average energy of the particles in a shower is however higher in lighter elements since this is roughly proportional to* E_c. More accurately, $E_c = 10^7$ e-volts in lead and $1{\cdot}5 \times 10^8$ e-volts in air or water, and the value of a is also somewhat larger in lighter elements, so that the frequency of showers in lighter elements is also somewhat less than in heavier ones. *Large showers* will, however, be markedly less frequent in light elements due to the operation of the expression in square brackets in (4·10).

The size of the largest shower which occurs with any probability is determined by the fact that when $u = 1$ (34) vanishes. This gives

$$N_{\max} = 0{\cdot}062\left(\frac{W_m}{E_c}\right)^{0{\cdot}93}. \tag{4·11}$$

E_c being roughly inversely proportional to Z, it follows that $N_{\max}$ is crudely proportional to it. The frequency of small showers is very roughly inversely proportional to N.

All we have said above refers to the number of particles with energies above the critical energy; the total number of particles in a shower is roughly double this, as we shall deduce from the following general results of the cascade theory (Bhabha and Heitler 1937, fig. 4 and § 7, A). The number of particles in a shower with energy $> E$ is proportioned to $1/E$ provided $E \geqslant E_c$. The number in any energy range dE therefore varies as

* *Note added in proof.*—The recent experiments of Morgan and Nielsen (1937) show that the intensity of secondaries and showers in equilibrium after large thicknesses of absorber with penetrating particles is roughly the same in lead and iron, in agreement with our theory. This is not the case on the theory of Landau and Rumer, where only the process (2) is considered. (More exactly, Morgan and Nielsen find that the intensity in iron is 20 % higher. We believe that this discrepancy can be attributed to geometrical differences in the two cases, since the dimensions of the counter system are the same in both, whereas the characteristic units of length λ_0 are quite different in iron and lead.) They also observe typical transition effects on adding lead or iron after large thicknesses (274 g./cm².) of iron or lead respectively, the addition of lead causing a rapid increase followed by a decrease to the air-lead curve, and the addition of iron to lead causing a rapid decrease followed by an increase to the air-iron curve. These results are completely in accordance with our theory and result from the fact that although *the secondary intensities are roughly the same in all elements, the mean energy of the secondaries is much higher in lighter elements*, as we have already pointed out in the text.

dE/E^2. Multiplication does not continue below the critical energy E_c, so the number above E does not continue to increase as $1/E$. But the number of particles in a given energy range dE is roughly the same as at the critical energy, namely dE/E_c^2. Hence the total number of particles below E_c is just proportional to $1/E_c$ whereas the number above E_c is also proportional to E_c. We thus get the following rough result of the cascade theory. The number of particles in a shower with energies below the critical energy is roughly equal to the number above the critical energy, which is what we wished to deduce.

In Table VI we give the values of $P(N)$ for lead and water assuming the constant C to be unity.* The figures give the probability for a shower containing N particles above the critical energy. The total number of particles in the shower is roughly twice this as we have already stated. The figures show what we have already said, that while the probability of shower production is less in lighter elements, it is nothing like as small as would follow from a Z^2 law. This result seems to be in accordance with what little is known about shower production by the penetrating component. We also see that the chance of a shower of some ten electrons and positrons accompanying the heavy particle is not very small, being of the order of 0·5 %. This seems to be of the right order of magnitude to agree with experiment. Further, except for large showers, the figures are not very sensitive to the mass of the heavy particle.

From these figures we may calculate the average number of particles accompanying the heavy particle. This is what we have calculated fairly accurately in § 3. We thus get figures which are on the average about twice those of Table IV. This is due to the fact that in calculating Table VI, we have put the constant C equal to unity whereas in fact we know that it is larger. The comparison shows that C is nearly 2 and that hence the figures in Table VI are roughly too large by a factor 2. The agreement is as good as could be expected and confirms our statement that the formula (4·10) is a fairly accurate determination of the order of magnitude of the shower probabilities.

Lastly we would remark that Table VI gives the shower probabilities due only to the first process mentioned at the beginning of this section. The

* *Note added in proof.*—We might have proceeded more accurately as follows. Putting $E = E_c$ in (4·10), the mean number of particles of energy greater than E_c accompanying the heavy particle is just $\sum_N NP(N)$, which is a function of C. This is just $N_{s>}$ of (3·2). By equating the two we can determine the value of C. This is to be inserted in (4·10) in evaluating $P(N)$. We find that C is in the neighbourhood of 2. Due to the occurrence of C in u, this method would have given slightly smaller relative probabilities for larger showers than those of Table VI, in addition to reducing all the probabilities by roughly a half, as is stated in the text below.

probabilities for the second process, i.e. for showers caused by quanta emitted by the heavy particles can be calculated similarly. We shall not do this explicitly. Indeed the expressions (4·2) and (4·3) show that for particles of mass $M \gg 10m$ the chance of emitting a hard quantum is negligible compared to the chance of producing an electron of the same energy. Thus for particles of $M = 100m$ and protons the second process is negligible. For particles of mass 5×10^6 e-volts the two probabilities are roughly equal, so that if we took the second process into account as well, all figures in Table VI for particles of 5×10^6 e-volts would be roughly doubled. Moreover the second process would make larger showers relatively somewhat more probable since the emission of light quanta does not favour the lower energies as much as the production of fast secondaries.

TABLE VI

$E_0 - Mc^2$	Mc^2	N	1	2	4	5	10	50
10^8 e-volts	5×10^6 e-volts	Lead	2·4	0·4	0·014	—	—	—
		Water	—	—	—	—	—	—
10^{10} e-volts	5×10^6 e-volts	Lead	4·7	1·5	0·58	0·40	0·12	$0·37 \times 10^{-2}$
		Water	2·5	0·72	0·25	0·16	0·034	—
	5×10^7 e-volts	Lead	4·6	1·4	0·57	0·38	0·12	$0·34 \times 10^{-2}$
		Water	2·5	0·69	0·22	0·11	0·026	—
	Protons	Lead	2·9	0·6	0·08	0·025	—	—
		Water	—	—	—	—	—	—
10^{12} e-volts	5×10^6 e-volts	Lead	4·7	1·5	0·60	0·42	0·13	$0·54 \times 10^{-2}$
		Water	2·8	0·88	0·36	0·24	0·077	$0·29 \times 10^{-2}$
	5×10^7 e-volts	Lead	4·7	1·52	0·60	0·42	0·13	$0·54 \times 10^{-2}$
		Water	2·8	0·88	0·36	0·24	0·077	$0·29 \times 10^{-2}$
	Protons	Lead	4·7	1·5	0·60	0·42	0·13	$0·53 \times 10^{-2}$
		Water	2·8	0·86	0·35	0·24	0·074	$0·26 \times 10^{-2}$

The figures give the probabilities per cent of the heavy particle being accompanied by a shower containing N particles above the critical energy. The total number of particles in the shower is roughly twice this. The upper figures in each row refer to lead, the lower figures to air or water. *If showers started by emitted quanta be also taken into account, then the figures for particles of $M = 10m$, would be somewhat more than doubled, and the others would be unaffected.*

5—THE CREATION OF HEAVY PARTICLES

In this section we will just discuss very briefly what effect the possibility of pair-creation of heavy particles would have on cosmic ray phenomena. Since the pair-creation cross-sections vary roughly inversely as the square of the mass of the particle it follows that it is much more likely that an electron rather than a heavy particle should be created by a γ-ray. Thus, if a heavy particle emit a quantum, the subsequent range of the quantum is almost entirely controlled by the probability of its creating electrons, and the number of quanta and electrons at any point is determined by the cascade process which follows. The chance of a pair of heavy particles being created by one of the quanta emitted by a heavy particle is therefore a process of a higher order.

The creation of heavy particles however has an effect on questions concerned with penetrating power. Suppose a very high energy quantum or electron enters some material. The number of particles and the ionization up to distances of 20 or 30 in the characteristic units λ_0 is determined by the usual cascade process. In this process many quanta take part at some stage, and there is a finite though small chance of some heavy particles being created. These, due to their low radiation loss would continue to penetrate to distances much larger than $30\lambda_0$ while the cascade electrons do not do so with any comparable probability. The following question is therefore of interest. Supposing a quantum of energy $h\nu$ enters a sheet of material, what is the number of heavy particles of mass M with energy greater than some arbitrary value E found at distances so large that the cascade process following on the original quantum has died out? The distances however have to be small compared to the range of the heavy particles. Both these conditions can be fulfilled if M and E are large enough. We have solved this problem very crudely, and give only the answer here. The number of heavy particles with energy greater than E is of the order

$$\tau\left(1-\frac{E}{h\nu}\right)+\alpha\tau\frac{h\nu}{E}\left(1-\frac{E}{h\nu}\right)^2, \tag{5·1}$$

where
$$\tau = 0{\cdot}6\left(\frac{m}{M}\right)^2.$$

The order of magnitude of the expression (5·1) is determined by τ, and it is only valid when $\tau \ll 1$ for then the creation of heavy particles does not appreciably influence the cascade process.

It is premature to draw any definite conclusions, but we wish to point out in passing that the above considerations may have some bearing on the following discrepancy between theory and experiment. While the ordinary cascade theory predicts that an electron of 10^{10} e-volts at the top of the atmosphere will produce an entirely negligible number of electrons at sea level, Bowen, Millikan and Neher find a number of particles which is an order of magnitude larger, being about 1/500th of the number at the maximum. A considerable fraction of these particles at sea level could be attributed to pair-creation in the atmosphere of heavy electrons with rest mass of the order of $10m$, if such pair-creation is possible at all. This does not exclude the possibility that some heavy particles at sea level may have come in from outer space.

In this connection we would remark that the well-known second maximum of the Rossi curve which occurs at 17 cm. of lead shows that penetrating particles are also produced in the lead (cf. Schmeiser and Bothe 1937).

6—General discussion and conclusions

In discussing the experimental evidence in the light of assumption (b) we have not limited ourselves to the hypothesis that only one new particle is concerned in cosmic radiation. Indeed in the experimental evidence itself there are definite hints that one new particle alone may not suffice to explain all the facts. We must therefore be prepared for the eventuality that a later and more complete theory may allow particles to exist whose rest masses may take on one of an infinite number of possible values of which only a few may turn out to be stable. With this idea is connected the possibility of a particle changing its rest mass, the difference in the energy being radiated or communicated to some other particle in the immediate neighbourhood. This change in the rest mass may be spontaneous, or caused by an external agency. The former possibility is not very interesting as far as cosmic radiation is concerned, for if the probability is large, the change will take place before the particle reaches the earth, if small, then the chance of its taking place in the very short time taken by the particle in penetrating the earth's surface is also negligible. We will therefore only consider the second possibility, and in particular the chance of a heavy electron changing its rest mass while moving very rapidly near a nucleus. We consider the process in the system in which the heavy electron is at rest, so that the nuclear field acts like a superposition of quanta. The change of rest mass with the emission of radiation then corresponds, as it were, to radiation induced by the presence of one of these quanta, the particle passing into a state of lower rest mass. We of course do not ascribe any internal structure to the particle. The emitted quantum would therefore be sent out in the direction in which the nucleus is moving, and hence in the system in which the nucleus is at rest would have very little energy indeed due to the action of the Lorentz transformation. The particle would nevertheless have changed into a particle of smaller rest mass, and its subsequent behaviour would, therefore, be different. The only chance of a large quantum being emitted in the system in which the nucleus is at rest is therefore if a direct collision takes place. The cross-section for this is therefore at most the nuclear cross-section, i.e.

of the order $(e^2/mc^2)^2$ and it may be very considerably smaller. Since the process however depends on a direct collision, we might expect it to vary for different nuclei as Z rather than Z^2. This behaviour might be connected with the fact observed by Blackett and Wilson that for very high energy particles the energy losses in lead and aluminium are comparable. Of course, the possibility of such radiation losses does not affect the ordinary Bremsstrahlung loss we have calculated above, but will be in addition to it.

Finally, we wish to draw attention to a discrepancy of which not sufficient notice has been taken. A comparison of the results of several investigators has shown, as has been put into direct evidence by Auger, Ehrenfest, Freon and Fournier (1937) that measurements of the absorption of cosmic radiation by change in the zenith angle of three counters in a plane are not in agreement with measurements in which the depth of the counters below the top of the atmosphere is changed. *It directly follows that the absorption of the radiation is not a unique function of the thickness of atmosphere traversed.* The discrepancy is not attributable to differences in intensity at the top of the atmosphere due to the earth's magnetic field since the results do not depend on the direction in which the zenith angle measurements are made. Nor is this attributable to a transition effect as Blackett (1937*b*) suggests, since air and water have roughly the same atomic number, and further, geometry does not play a role when single track coincidences are measured. These discrepancies however could be reconciled by the assumption that for some unknown reason the uppermost layer of the atmosphere absorbs more strongly than we should expect it to. This would also be in agreement with the observation of Bowen, Millikan and Neher (1937) that the maximum of the absorption curve in the atmosphere occurs at about two-thirds of the distance from the top that the theory predicts.

We may then sum up the results of the discussion of this paper as follows:

1—The measurements of energy loss by Blackett and Wilson in lighter elements, if they are correct, are easily reconcilable with a "breakdown" theory in which the breakdown energy depends on the atomic number. They can only be explained on a "new particle" theory by attributing a behaviour to the new particles which is not described entirely by present quantum mechanics. Processes giving rise to large energy losses must be assumed which however vary as Z rather than Z^2, as for example a possible change in the rest mass of the particle (§ 1 C).

2—An analysis of the transition curves for large showers and bursts shows that a breakdown theory is unable to explain the shape of these curves, and allows one to conclude that at least some large bursts and showers must be entirely due to cascade processes in accordance with the theory (§ 1 B).

3—The latitude effect at sea level which extends to 50°, in conjunction with the direct proof by Bowen, Millikan and Neher that the cascade theory is correct in air up to 10^{10} e-volts demands the existence of new particles. A breakdown theory cannot explain this sea level latitude effect since a breakdown below 10^{10} e-volts cannot be in question (§ 1 A).

4—A soft component consisting of electrons must accompany any energetic penetrating particle and *for heavy electrons of mass* $M \sim 100m$ is in air or water of the order of 7 % for particles of 10^{10} e-volts and of the order of 15 % for particles of 10^{12} e-volts. The former figure is of the same order as the experimental findings of Auger and others. In lead the soft component varies from about 15 to 30 %. For $M \sim 10m$ the figures are roughly double these (§ 3).

5—The frequency of production of showers by a heavy particle does not depend on the atomic number for the smaller showers, though it is somewhat less in lighter elements. The size of the largest shower which is produced with any reasonable probability by penetrating particles of a given energy is, however, proportional to Z. The mean energy of the shower particles is higher in lighter elements for showers of the same size (§ 4).

6—The probability of a shower of N^{γ} particles being produced by a very energetic heavy particle is roughly inversely proportional to N^{γ} provided N is not too large, where γ is somewhat larger than 1 (§ 4).

7—The large number of particles observed at sea level by Bowen, Millikan and Neher may partly be heavy electrons of mass almost ten times the electron mass created in the atmosphere by the quanta emitted by electrons, though some of them may be heavy electrons which entered the atmosphere from outside (§ 5).

8—The discrepancy between the zenith angle and direct absorption measurements, and also the observation of Bowen, Millikan and Neher that the maximum of the atmospheric absorption curve occurs at two thirds of the theoretical thickness from the top, may both be explained by assuming that for some unknown reason the uppermost layer of the atmosphere absorbs more than we should expect it to, i.e. more like a substance of larger atomic number. This may be due to the fact that the atoms in these layers are largely ionized (§ 6).

9—A breakdown theory based on limiting the acceleration of the electron in the rest system would carry with it not only a modification of the radiation formulae but also a diminution in the ionization loss. According to Cosyns the ionization of cosmic ray particles seems to be less than the theoretical.

Summary

An analysis of the experimental data is carried through to show that a "breakdown" theory for radiation loss of electrons cannot explain (1) the latitude effect at sea level from latitudes of 35–50°, (2) the large number of particles found at sea level in the difference curves of Bowen, Millikan and Neher for charged particles of 10^{10} e-volts, (3) the shape of the transition curve for large bursts. All these facts can be explained by assuming that the penetrating component consists of new particles with masses between those of the electron and proton. But in order to explain the energy loss measurements of Blackett and Wilson, one must then assume that these particles suffer large energy losses in addition to the ordinary Bremsstrahlung which must vary in different substances as Z rather than Z^2, as for example a change in the rest mass of the particles.

The radiation loss and the pair-creation cross-sections taking screening into account accurately for "heavy" electrons are calculated. The frequency of the production of showers of different sizes by such heavy electrons as also the intensity of electrons in equilibrium with such particles forming a soft component are also calculated, and it is shown that though both these are somewhat larger in heavier elements, the variation is much less than a Z^2 law would give. A comparison with experiment gives the mass of the penetrating particles as of the order $100m$.

If such heavy electrons can be created as usual in pairs, then a part of the hard component at sea level could consist of heavy electrons of mass $10m$ created by the soft component in the upper atmosphere.

It is shown that there are reasons to suppose that the uppermost layers of the atmosphere are more absorbing than one should expect from their mass. This may be due to the fact that the atoms in these layers are largely ionized thus increasing the effective radiation and pair-creation cross-sections.

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