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Hypotheses for earthquake occurrences

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Abstract. Very little work has been done in generating alternatives to the Poisson process model. The work reported here deals with alternatives to the Poisson process model for the earthquakes and checks them using empirical data and the statistical hypothesis testing apparatus. The strategy used here for generating hypotheses is to compound the Poisson process. The parameter of the Poisson process is replaced by a random variable having prescribed density function. The density functions used are gamma, chi and extended (gamma/chi). The original distribution is then averaged out with respect to these density functions. For the compound Poisson processes the waiting time distributions for the future events are derived. As the parameters for the various statistical models for earthquake occurrences are not known, the problem is basically of composite hypothesis testing. One way of designing a test is to estimate these parameters and use them as true values. Moment-matching is used here to estimate the parameters. The results of hypothesis testing using data from Hindukush and North East India are presented.

Keywords. Compound Poisson processes; hypothesis testing; parameter estimation; waiting time distributions; Polya process; statistical models for earthquake occurrences.

1. Introduction

The debate whether the earthquake occurrences follow a Poisson process model is still inconclusive (Benioff 1951; Aki 1956; Shalanger 1960; Knopoff 1964; Lomnitz 1966; Ferraes 1967; Vere Jones 1970; Schlien and Toksoz 1970; Utsu 1972; Udias and Rice 1975) though a predominant view is that the Poisson process model is not particularly appropriate. One reason why the debate is still alive is that the Poisson process model has great conceptual appeal and those who rejected the Poisson process model have tried to restore it by removing the aftershocks and foreshocks from the sequence of earthquakes (Vere Jones 1970; Schlien and Toksoz 1970; Udias and Rice 1975). These attempts have lacked internal consistency because the definitions of the main event and the cluster of the main event and its aftershocks and foreshocks have been ad hoc. These have been in terms of the arbitrarily defined 'cluster length' which violates the assumption of independent events in a Poisson process model for the cluster centres, because two or more cluster centres are then forbidden within a cluster length. The second reason is that not much work is done in generating alternatives to the Poisson process model. The work of Vere Jones (1970) in which a contagious Poisson process model of the Neyman type has been suggested for the number of earthquakes per unit time is almost exceptional. No model can be effectively rejected unless more successful models are obtained. The third reason is that only some consequences of a Poisson process model have earlier been studied. The present paper points out that different random variables derived from the stochastic process models may lead to different conclusions unless a really superior model is generated. Towards this end three different alternatives to the Poisson process model are generated here *viz.* Polya compound, χ-Poisson and compound (γ-χ)-Poisson. Under all the four hypotheses, distributions are obtained for the discrete random variable, number of earthquakes per unit time and density functions for continuous random variables such as waiting times for the next, second, third, etc. event.

The strategy used to generate alternatives to Poisson process model is compounding. Let

$$P(x = k \mid \lambda) = \frac{\lambda^k}{k!} e^{-\lambda}, \quad \lambda > 0, \quad k = 0, 1, 2 \tag{1}$$

be the probability of k earthquakes per unit time under Poisson process model where λ is a parameter, called the rate of the process. Let λ be awarded a status of a random variable with density function $p(\lambda)$ over the range $[0, \infty)$. Then

$$\begin{aligned} P(x = k) &= \int_0^\infty P(x = k \mid \lambda) p(\lambda)\, \mathrm{d}\lambda \\ &= E_\lambda[P(x = k \mid \lambda)], \end{aligned} \tag{2}$$

gives the probability of k earthquakes per unit time under a compound Poisson process where $E_\lambda[\cdot]$ is expectation with respect to the density function of λ (Fisz 1963; Johnson and Kotz 1969).

If the density function of λ is chosen to be γ, that is, if

$$p(\lambda) = \frac{a^\nu}{\Gamma(\nu)} \lambda^{\nu-1} \exp(-a\lambda)\, u(\lambda), \tag{3}$$

where $u(\lambda)$ is unit step function, and

$$\Gamma(\nu) = \int_0^\infty \lambda^{\nu-1} \exp(-\lambda)\, \mathrm{d}\lambda \tag{4}$$

is the gamma function and where $a > 0$ and $\nu > 0$ are parameters, it has been shown that (Fisz 1963; Johnson and Kotz 1969) (2) becomes

$$P(x = k) = (-1)^k \binom{-\nu}{k} p^k q^\nu, \quad k = 0, 1, 2, \tag{5}$$

where

$$p = \frac{1}{1+a}, \quad q = 1 - p = \frac{a}{1+a}, \quad 0 < p < 1, \tag{6}$$

and

$$\binom{-\nu}{k} = \frac{(-\nu)(-\nu-1)\ldots(-\nu-k+1)}{k!} \tag{7}$$

Equation (7) gives a negative binomial distribution. If ν is a positive integer, it is sometimes called Pascal distribution (Johnson and Kotz 1969). When the postulates of

Poisson distribution, particularly independence, are suspect, the negative binomial distribution is a frequently suggested alternative (Johnson and Kotz 1969).

Compounding of Poisson distribution of (1) by rectangular, truncated normal and log-normal density functions has also been considered in literature (Johnson and Kotz 1969). Two new compound Poisson distributions are obtained in the next section.

2. Compound Poisson distributions

2.1 *Compound Chi-Poisson distribution*:

Let λ have a χ density function given by (Papoulis 1965)

$$p(\lambda) = \frac{2}{2^{n/2}\Gamma(n/2)\sigma^n}\lambda^{n-1}\exp(-\lambda^2/2\sigma^2)\,u(\lambda), \quad n > 1, \quad \sigma > 0. \tag{8}$$

Then (2) becomes

$$P(x = k) = \frac{2}{2^{n/2}\Gamma(n/2)\sigma^n k!}\int_0^\infty \lambda^{n+k-1}\exp[-(\lambda^2/2\sigma^2 + \lambda)]\,d\lambda \tag{9}$$

which is a compound Poisson distribution which will be named χ-Poisson. The problem is to evaluate an integral

$$C(m, a, b) = \int_0^\infty \lambda^m \exp[-(a\lambda^2 + b\lambda)]\,d\lambda. \tag{10}$$

We have (Spiegel 1968)

$$\int_0^\infty \exp[-(a\lambda^2 + b\lambda + c)d\lambda = \frac{1}{2}(\pi/a)^{1/2}\exp[(b^2 - 4ac)/4a\,\mathrm{erfc}(b/2\sqrt{a}), \tag{11}$$

where

$$\mathrm{erfc}(p) = \frac{2}{\sqrt{\pi}}\int_p^\infty \exp(-\lambda^2)\,d\lambda, \tag{12}$$

is the complementary error function. By putting $c = 0$ in (11) we get

$$\int_0^\infty \exp\{-(a\lambda^2 + b\lambda)\}\,d\lambda = \frac{1}{\sqrt{a}}\exp(b^2/4a)\int_{b/2\sqrt{a}}^\infty \exp(-\lambda^2)\,d\lambda. \tag{13}$$

Differentiating both sides of (13) with respect to b and using Leibnitz's theorem (Spiegel 1968) that

$$D^m(uv) = \sum_{r=0}^{m} \binom{m}{r} D^{m-r}u\,D^r v, \tag{14}$$

where D is a differential operator which is $\mathrm{d}/\mathrm{d}b$ in our case, we get

$$\int_0^\infty \lambda^m \exp[-(a\lambda^2 + b\lambda)]\,\mathrm{d}\lambda$$

$$= \frac{(-1)^m}{\sqrt{a}} \left\{ \sum_{r=0}^{m} \binom{m}{r} \frac{\mathrm{d}^r}{\mathrm{d}b^r} [\exp(b^2/4a)] \frac{\mathrm{d}^{m-r}}{\mathrm{d}b^{m-r}} \left(\int_{b/2\sqrt{a}}^{\infty} \exp(-\lambda^2)\,\mathrm{d}\lambda \right) \right\}$$

$$= \frac{(-1)^m}{\sqrt{a}} \left\{ \sum_{r=0}^{m-1} \binom{m}{r} \frac{\mathrm{d}^{m-r-1}}{\mathrm{d}b^{m-r-1}} \left[-\frac{1}{2\sqrt{a}} \exp(-b^2/4a) \right] \right.$$

$$\left. \frac{\mathrm{d}^r}{\mathrm{d}b^r} [\exp(b^2/4a)] + \left[\int_{b/2\sqrt{a}}^{\infty} \exp(-\lambda^2)\,\mathrm{d}\lambda \right] \frac{\mathrm{d}^m}{\mathrm{d}b^m} [\exp(b^2/4a)] \right\} \tag{15}$$

Now, let

$$\phi(x) = \exp(-x^2/2), \qquad \psi(x) = \exp(x^2/2), \tag{16}$$

and let

$$\phi^{(n)}(x) = (-1)^n \phi(x) H_n(x), \qquad \psi^{(n)}(x) = \psi(x)\bar{H}_n(x), \tag{17}$$

where $H_n(x)$ and $\bar{H}_n(x)$ are polynomials and $\phi^{(n)}(x)$ and $\psi^{(n)}(x)$ are nth derivatives of $\phi(x)$ and $\psi(x)$ respectively. $H_n(x)$ are known as Hermite polynomials (Abramowitz and Stegun 1966) of degree n. They satisfy recurrence relation (Whalen 1971)

$$H_{n+1}(x) = xH_n(x) - nH_{n-1}(x), \quad H_0(x) = 1, \quad H_1(x) = x. \tag{18}$$

It can be easily shown that $\bar{H}_n(x)$ must satisfy the recurrence relation

$$\bar{H}_{n+1}(x) = x\bar{H}_n(x) + \bar{H}_n^{(1)}(x), \quad \bar{H}_0(x) = 1, \quad \bar{H}_1(x) = x. \tag{19}$$

Using (18) and (19) to tabulate the polynomials $H_n(x)$ and $\bar{H}_n(x)$, it is easily seen that the two sets of polynomials are closely related. In $H_n(x)$ and $\bar{H}_n(x)$ the absolute values of the coefficients of various powers of x are the same. However, whereas in $\bar{H}_n(x)$ all the coefficients are positive, in $H_n(x)$ the coefficients of x^{n-2}, x^{n-6}, x^{n-10}, etc. are negative, where $n-2$, $n-6$, $n-10$, etc are non-negative integers. This relation between $H_n(x)$ and $\bar{H}_n(x)$ can be summarized by

$$\bar{H}_n(x) = (-j)^n H_n(jx). \tag{20}$$

Equation (20) could also be obtained more directly by using (16) and (17) after writing

$$\psi(x) = \exp[-(jx)^2]/2. \tag{21}$$

The polynomials $\bar{H}_n(x)$ can be named modified-Hermite polynomials because (20) is similar to

$$I_n(x) = (-j)^n J_n(jx), \tag{22}$$

for Bessel and modified Bessel functions $J_n(x)$ and $I_n(x)$, respectively (Bateman 1953). $H_n(x)$ and $\bar{H}_n(x)$ must now be regarded as known polynomials. Using (16) and (17), (15) becomes

$$C(m, a, b) = \int_0^\infty \lambda^m \exp[-(a\lambda^2 + b\lambda)]\,d\lambda$$

$$= \frac{1}{(2a)^{(m+1)/2}}\Big\{\sum_{r=0}^{m-1}\binom{m}{r}(-1)^r H_{m-r-1}[b/(2a)^{1/2}]\,\bar{H}_r[b/(2a)^{1/2}]$$

$$+ (-1)^m\sqrt{\pi/2}\,\mathrm{erfc}\,[b/(2a)^{1/2}]\,\bar{H}_m\,[b/(2a)^{1/2}]\exp(b^2/4a)\Big\} \tag{23}$$

which now becomes a known function. Using (9), (10) and (23) we can write

$$P(x=k) = \frac{2}{2^{n/2}\Gamma(n/2)\;k!\,\sigma^n}\;C(n+k-1,\;1/2\sigma^2,\;1)$$

$$= \frac{2\sigma^k}{2^{n/2}\Gamma(n/2)k!}\Big\{\sum_{r=0}^{n+k-2}\binom{n+k-2}{r}(-1)^r H_{n+k-r-2}(\sigma)\,\bar{H}_r(\sigma)$$

$$+ \sqrt{\pi/2}\,\mathrm{erfc}\,(\sigma/\sqrt{2})\,(-1)^{n+k-1}\,\bar{H}_{n+k-1}(\sigma)\exp(\sigma^2/2)\Big\}, \tag{24}$$

which is the compound χ-Poisson distribution. Of course, the modified Hermite polynomials in (24) can be expressed in terms of Hermite polynomials with imaginary arguments using (20). A computer program to evaluate probabilities in (24) on DEC-20 has been written (Sharma 1982).

2.2 *Compound (gamma/chi) — Poisson Distribution*:

Let the density function of λ be

$$p(\lambda) = \frac{1}{C(n,a,b)}\,\lambda^n\exp[-(a\lambda^2 + b\lambda)]\;u(\lambda), \qquad a > 0,\; b > 0,\; n = 0, 1, 2, \ldots \tag{25}$$

where $C(n, a, b)$ is given in (23). This density function can be named extended (gamma/chi) because, if $a = 0$,

$$C(n, a, b) = C(n, 0, b) = \int_0^\infty x^n \exp(-bx)\,dx = \frac{\Gamma(n+1)}{b^{n+1}} \tag{26}$$

and $p(\lambda)$ of (25) reduces to the form

$$p(\lambda) = \frac{\lambda^n e^{-b\lambda} b^{n+1}}{\Gamma(n+1)}\;u(\lambda), \tag{27}$$

which can be seen to be a gamma density function from (3) and if $b = 0$

$$C(n, a, b) = C(n, a, 0) = \int_0^\infty x^n \exp(-ax^2)\,dx$$
$$= \frac{\Gamma\{(n+1)/2\}}{2a^{(n+1)/2}} \tag{28}$$

and $p(\lambda)$ of (25) reduces to the form

$$p(\lambda) = \frac{2a^{(n+1)/2}}{\Gamma\{(n+1)/2\}} \lambda^n \exp(-a\lambda^2) u(\lambda), \quad n > 0,\ a > 0$$

which can be seen to be a χ density function from (8). Substituting (25) in (2) and using (10)

$$P(x = k) = \int_0^\infty \frac{\lambda^k}{k!} e^{-\lambda} \frac{1}{C(n, a, b)} \lambda^n \exp[-(a\lambda^2 + b\lambda)]\,d\lambda$$
$$= \frac{C(n + k, a, b + 1)}{C(n, a, b)k!} \tag{29}$$

where (23) can be used to get an explicit expression. This can be named compound (gamma/chi)-Poisson distribution. It subsumes gamma-Poisson and chi-Poisson distributions as special cases. A computer program to evaluate probabilities in (29) on DEC-20 has been written (Sharma 1982).

Probabilities under the Poisson, negative binomial, compound chi-Poisson and (gamma/chi)-Poisson distributions are shown in table 1 with a few chosen values of the parameters.

3. Compound Poisson process models

3.1 Poisson process model

Let X_t be the number of earthquakes during the time interval $[0, t)$ where $0 \leq t < \infty$. Then $\{X_t, 0 \leqq t < \infty\}$ is a stochastic process, where for every t the random variable can take on integer values $k = 0, 1, 2, \ldots$. Under certain ideal assumptions (Feller 1956; Fisz 1963) the earthquakes follow a Poisson process model given by

$$P\{X_t = k \mid \lambda\} = \frac{(\lambda t)^k}{k!} \exp(-\lambda t), \quad k = 0, 1, 2, \ldots, \lambda > 0. \tag{30}$$

If the parameter λ is awarded a status of a random variable having density function $p(\lambda)$, we get

$$P(X_t = k) = \int_0^\infty \frac{(\lambda t)^k}{k!} \exp(-\lambda t) p(\lambda)\,d\lambda \tag{31}$$

as a compound Poisson process model (Fisz 1963).

Table 1. Probabilities for each distribution.

k	Poisson distribution $\lambda=3.2808219$	Negative binomial distribution $a=1.0118489$ $\nu=3.3196956$	Compound chi-Poisson distribution $n=2$, $\sigma=2.65$	Compound (gamma/chi)-Poisson distribution $n=3$, $a=0.1$, $b=0.01$
0	0.0375973	0.1021264	0.1046892	0.0390846
1	0.1233501	0.1685158	0.1601306	0.0975718
2	0.2023448	0.1809124	0.1729216	0,1444864
3	0.2212858	0.1594551	9.1574826	0.1633476
4	0.1814998	0.1252216	0.1283219	0.1550099
5	0.1190937	0.0911185	0.0962408	0.1294238
6	0.0651208	0.0628011	0.0676109	0.0978149
7	0.0305214	0.0415600	0.0453229	0.0679416
8	0.0125169	0.0266475	0.0279501	0.0450210
9	0.0045629	0.0016659	0.0171359	
10	0.0014970	0.0102013	0.0107734	
11	0.0004465	0.0061399		
12	0.0001221	0.0036418		
13	0.0000308	0.0021332		

3.2 *Polya process model*

If λ has a gamma density function of (3), (31) becomes (Fisz 1963)

$$P(X_t = k) = \int_0^\infty \frac{(\lambda t)^k}{k!} \exp(-\lambda t) \frac{a^\nu}{\Gamma\nu} \lambda^{\nu-1} \exp(-a\lambda)\, d\lambda$$

$$\frac{\nu(\nu+1)\ldots(\nu+k-1)}{k!}\left(\frac{t}{a+t}\right)^k \left(\frac{a}{a+t}\right)^\nu, \quad k = 1, 2, 3, \ldots$$

$$= \left(\frac{a}{a+t}\right)^\nu, \quad k = 0 \qquad (32)$$

using the definition of (4) and the properties of the γ functions.

3.3 *Compound chi-Poisson process model*

If λ has a chi density function of (8) (31) becomes

$$P(X_t = k) \int_0^\infty \frac{(\lambda t)^k}{k!} \exp(-\lambda t) \frac{2}{2^{n/2}\Gamma(n/2)\sigma^n} \lambda^{n-1} \exp(-\lambda^2/2\sigma^2)\, d\lambda$$

$$= \frac{2t^k}{2^{n/2}\Gamma(n/2)\sigma^n k!} \int_0^\infty \lambda^{n+k-1} \exp[-(\lambda^2/2\sigma^2 + \lambda t)]\, d\lambda$$

$$= \frac{2t^k}{2^{n/2}\Gamma(n/2)\sigma^n k!} C(n+k-1, (2\sigma^2)^{-1}, t)$$

$$= \frac{2t^k\sigma^k}{2^{n/2}\Gamma(n/2)k!} \left\{ \sum_{r=0}^{n+k-2} \binom{n+k-1}{r} (-1)^r \right.$$

$$H_{n+k-r-2}(\sigma t)\bar{H}_r(\sigma t) + (\pi/2)^{1/2}(-1)^{n+k-1}$$

$$\left. \operatorname{erfc}\left(\frac{\sigma t}{\sqrt{2}}\right) \bar{H}_{n+k-1}(\sigma t)\exp(\sigma^2 t^2/2) \right\} \qquad (33)$$

where (10) and (23) have been used. Equation (33) represents compound chi-Poisson process.

3.4 *Compound (gamma/chi)-Poisson process model*

If λ has an extended (gamma/chi) density function of (25), (31) becomes

$$_tP(X_t = k) \int_0^\infty \frac{(\lambda t)^k}{k!} \exp(-\lambda t) \qquad \frac{\lambda^n \exp[-(a\lambda^2 + b\lambda)]}{C(n, a, b)} d\lambda$$

$$= \frac{t^k C(n+k, a, b+t)}{k!\, C(n, a, b)} \qquad (34)$$

where (10) has been used. An explicit expression can be obtained if substitutions are made for $C(n+k, a, b+t)$ and $C(n, a, b)$ from (23). Equation (33) represents compound (gamma/chi)-Poisson process.

4. Waiting time distributions for earthquakes

4.1 *General theory*

Let T_1 be the waiting time for the next earthquake, and let t_1 be a specific value which the random variable T_1 takes. Then

$$\text{Prob}\,(T_1 < t_1) = \text{Prob. (one or more earthquakes occur during time } t_1)$$

$$= \sum_{k=1}^{\infty} P(X_{t_1} = k) = 1 - P(X_{t_1} = 0). \qquad (35)$$

Let T_p be the waiting time for the pth earthquake, and let t_p be a specific value the random variable T_p takes.

Then

$$\text{Prob}\,(T_p < t_p) = \text{Prob (}p\text{ or more earthquakes occur during time } t_p)$$

$$= \sum_{k=p}^{\infty} P(X_{t_p} = k) = 1 - \sum_{k=0}^{p-1} P(X_{t_p} = k). \qquad (36)$$

Differentiating (35) with respect to t_1, the density function for T_1 is obtained as (Massey 1971):

$$p_{T_1}(t_1) = -\frac{d}{dt_1} P(X_{t_1} = 0). \tag{37}$$

Similarly, differentiating (36) with respect to t_p, the density function for T_p is obtained as

$$p_{T_p}(t_p) = -\sum_{k=0}^{p-1} \frac{d}{dt_p} P(X_{t_p} = k). \quad p = 1, 2\ldots \tag{38}$$

of which (37) is a particular case.

4.2 *Poisson process model*

Substituting for $P(X_{t_1} = 0)$ in (37) from (30), we get (Massey 1971)

$$p_{T_1}(t_1) = \lambda \exp(-\lambda t_1) u(t_1). \tag{39}$$

which is a negative exponential density function. T_1 can also be considered the inter arrival time of the earthquakes. Thus, the interarrival times for the Poisson process are negative — exponentially distributed.

Substituting for $P(X_{t_p} = k)$ in (38) from (30), we get, after mundane adjustments

$$p_T(t_p) = \frac{\lambda^p t_p^{p-1}}{(p-1)!} \exp(-\lambda t_p) u(t_p). \tag{40}$$

From (3) it can be seen that the waiting time for the pth earthquake is γ-distributed if the earthquakes follow a Poisson process model.

4.3 *Polya process model*

Substituting for $P(X_{t_p} = k)$ in (38) from (32) and simplifying, we get

$$p_{T_p}(t_p) = \frac{\nu(\nu+1)\ldots(\nu+p-1)a^{\nu}}{(a+t_p)^{\nu+p}} \frac{t_p^{p-1}}{(p-1)!} u(t_p). \tag{41}$$

which is the density function for waiting time for the pth earthquake under the Polya process model for earthquakes. This density function can also be obtained by compounding the gamma density function of (40) for T_p by assuming a gamma density function of (3) for λ (Sharma 1982). Hence, the density function of (41) can be named compound gamma-gamma. This also proves that (a) compounding a Poisson process by gamma-density function for λ to get the Polya process and then to obtain the waiting time density functions under Polya process model and (b) obtaining the waiting time density functions under Poisson process model and then compounding them by a gamma density function for λ, both give the same results. This is certainly reassuring and similar results would be expected for other compound processes.

4.4 *Compound chi-Poisson process model*

Substituting for $P(X_{t_p} = k)$ in (37) from (24), obtaining a relation (Sharma 1982)

$$H_n^{(1)}(x) = xH_n(x) - H_{n+1}(x), \tag{42}$$

for Hermite polynomials and using it together with

$$\bar{H}_n^{(1)}(x) = \bar{H}_{n+1}(x) - x\bar{H}_n(x), \tag{43}$$

which is obtained from (19) and simplifying, we get (Sharma 1982)

$$p_{T_1}(t_1) = -\frac{2}{2^{n/2}\Gamma(n/2)}\sum_{r=0}^{n-2}\Big\{(-1)^r\binom{n-1}{r}\sigma H_{n-r-2}(\sigma t_1)\bar{H}_{r+1}(\sigma t_1)$$
$$-(-1)^r\binom{n-1}{r}\sigma H_{n-r-1}(\sigma t_1)\bar{H}_r(\sigma t_1)$$
$$+(-1)^n\sigma\bar{H}_{n-1}(\sigma t_1)+(-1)^{n-1}\sigma(\pi/2)^{1/2}\,\mathrm{erfc}(\sigma t_1/\sqrt{2})\bar{H}_n(\sigma t_1)\exp(\sigma^2 t_1^2/2)\Big\} \tag{44}$$

As the same result can be obtained by compounding the negative exponential density function of (39) under the Poisson process model by assuming λ to be chi-distributed according to (8), the density function of (44) can be named compound chi-negative exponential.

Substituting for $P(x_{t_p} = k)$ in (38) from (24) we get

$$p_{T_p}(t_p) = -\frac{2}{2^{n/2}\Gamma(n/2)}\sum_{k=1}^{p-1}\Big\{\frac{t_p^k}{k!}\frac{\mathrm{d}}{\mathrm{d}t_p}C(n+k-1,\frac{1}{2\sigma^2}, t_p)$$
$$+\frac{t_p^{k-1}}{(k-1)!}C(n+k-1,\frac{1}{2\sigma^2}, t_p)\Big\}. \tag{45}$$

For reasons alluded to above this density function can be named compound chi-gamma density function. Using (23) an explicit expression for it can be obtained.

4.5 *Compound (gamma/chi)-Poisson process model*

Substituting for $P(X_{t_p} = k)$ in (37) from (34) and using (23), (42) and (43) to simplify, we get (Sharma 1982)

$$p_{T_1}(t_1) = -\Big[\sum_{r=0}^{n-1}\Big\{(-1)^r\binom{n}{r}H_{n-r-1}[b+t_1/(2a)^{1/2}]\,H_{r+1}[b+t_1/(2a)^{1/2}]$$
$$(2a)^{1/2} - (-1)^r\binom{n}{r}\frac{1}{(2a)^{1/2}}H_{n-r}[b+t_1/(2a)^{1/2}]\bar{H}_r[b+t_1/(2a)^{1/2}]$$
$$+(-1)^{n+1}\frac{1}{(2a)^{1/2}}\bar{H}_n[b+t_1/(2a)^{1/2}] +$$

$$+(-1)^n(\pi/2)^{1/2}\,\mathrm{erfc}\;\;(b+t_1/2\sqrt{a})\;\;\frac{1}{(2a)^{1/2}}\;\bar{H}_{n+1}\,[b+t_1/(2a)^{1/2}]$$

$$\exp(b+t_1)^2/4a\Big\}\Big]\Big/$$

$$\Big[\sum_{r=0}^{n-1}\binom{n}{r}(-1)^r H_{n-r-1}[b/(2a)^{1/2}]\,H_r[b/(2a)^{1/2}] + (-1)^n(\pi/2)^{1/2}$$

$$\mathrm{erfc}\,(b/2\sqrt{a})\,\bar{H}_n\;\;[b/(2a)^{1/2}]\exp(b^2/4a)\Big]. \tag{46}$$

For reasons alluded to above this density function can be named compound (gamma/chi)-negative exponential density function.

Substituting for $P(X_{t_p} = k)$ in (38) from (34), we get

$$p_{T_p}(t_p) = -\frac{1}{C(n,a,b)}\sum_{k=1}^{P-1}\Big\{\frac{t_p^k}{k!}\;\frac{\mathrm{d}}{\mathrm{d}t_p}\;C(n+k,\,a,\,b+t_p)$$

$$\frac{t_p^{k-1}}{(k-1)!}\;\;C(n+k,a,b+t_p)\Big\} \tag{47}$$

which can be named compound (gamma/chi)-gamma density function for which explicit expression can be obtained by using (23).

5. Estimation of the parameters of various distributions

5.1 *Poisson distribution*

The moments of the Poisson distribution of (1) are (Johnson and Kotz 1969)

$$E(x) = \lambda,\quad E(x^2) = \lambda + \lambda^2,$$
$$E(x^3) = \lambda + 3\lambda^2 + \lambda^3,\quad E(x^4) = \lambda + 7\lambda^2 + 6\lambda^3 + \lambda^4. \tag{48}$$

If expression for any population moment is equated with the corresponding sample moment, estimate $\hat{\lambda}$ for λ can be obtained. Thus

$$\hat{\lambda} = \frac{1}{N}\sum_{i=1}^{N} x_i, \tag{49}$$

could be the simplest estimate of λ where N is the number of samples. As the second and third central moments of the Poisson distribution are also equal to λ, other estimators for λ can be obtained.

5.2 *Negative binomial distribution*

The moments of the negative binomial distribution of (5) are (Fisz 1963):

$$\text{mean} = m_1 = vp/q,$$

$$\text{variance} = \mu_2 = (v/qp)\Big(1+\frac{p}{q}\Big). \tag{50}$$

Using (6), we can get

$$a = [(\mu_2/m_1) - 1)]^{-1}, \quad v = m_1 a. \tag{51}$$

Therefore,

$$\hat{a} = [(\hat{\mu}_2/\hat{m}_1) - 1]^{-1}, \quad \hat{v} = \hat{m}_1\hat{a}, \tag{52}$$

can be taken as the estimates of a and v, where $\hat{\mu}_2$ and $\hat{m}_1$ are sample variance and sample mean respectively. If v is assigned some value and only a is considered a parameter, we get

$$\hat{a} = v/\hat{m}_1. \tag{53}$$

5.3 *Compound chi-Poisson and (gamma/chi)-Poisson distributions*

For these distributions given by (24) and (29) respectively, expressions for the first two and three moments respectively can be written and equated with the corresponding sample moments. However, the resulting equations cannot be analytically solved. Therefore the parameters (a) n and σ (b) n, a and b respectively of the two distributions have to be evaluated by computing these distributions and their moments for various parameter values till the population moments match the sample moments.

Here also the value of n can be fixed and (a) σ and (b) a and b respectively can be obtained by matching the first and first two moments.

5.4 *Gamma distribution*

The mth moment for the gamma density function for the waiting time for the pth earthquake under Poisson process model for earthquake occurrences of (40) can be shown to be (Sharma 1982):

$$E(t_p^m) = \frac{\lambda^p}{(p-1)!}\int_0^\infty t_p^{m+p-1}\exp(-\lambda t_p)\,dt_p$$

$$= \frac{p(p+1)\ldots(p+m-1)}{\lambda^m} \tag{54}$$

Therefore, by putting $m = 1$, we can get

$$\hat{\lambda} = p/\hat{E}(t_p), \tag{55}$$

where $\hat{E}(t_p)$ is the sample mean of t_p. We can, of course, get estimates as

$$\hat{\lambda} = \left\{\frac{p(p+1)\ldots(p+m-1)}{\hat{E}(t_p^m)}\right\}^{1/m}, \tag{56}$$

where $\hat{E}(t_p^m)$ is the mth sample moment of t_p.

5.5 *Compound gamma-gamma distribution*

The mth moment for the compound gamma-gamma density function of (41) for the waiting time for the pth earthquake under Polya process model for earthquake occurrences can be shown to be (Sharma 1982):

$$E(t_p^m) = \frac{\nu(\nu+1)\ldots(\nu+p-1)a^\nu}{(p-1)!} \sum_{j=0}^{m+p-1} \binom{m+p-1}{j} (-a)^{m+p-j-1} \int_0^\infty y^{j-\nu-p}\,dy. \quad (57)$$

In particular, by putting $p = 1, 2, 3$ and $m = 1, 2, 3$ in (57), we get

$$E(t_1) = \frac{a}{\nu - 1}, \quad \nu > 1, \quad (58)$$

$$E(t_1^2) = \frac{2a^2}{(\nu-1)(\nu-2)}, \quad \nu > 2, \quad (59)$$

$$E(t_1^3) = \frac{6a^3}{(\nu-1)(\nu-2)(\nu-3)}, \quad \nu > 3, \quad (60)$$

$$E(t_2) = \frac{2a}{(\nu-1)}, \quad \nu > 1, \quad (61)$$

$$E(t_2^2) = \frac{6a^2}{(\nu-1)(\nu-2)}, \quad \nu > 2, \quad (62)$$

$$E(t_2^3) = \frac{24a^3}{(\nu-1)(\nu-2)(\nu-3)}, \quad \nu > 3, \quad (63)$$

$$E(t_3) = 3a/(\nu - 1), \quad \nu > 1 \quad (64)$$

$$E(t_3^2) = \frac{12a^2}{(\nu-1)(\nu-2)}, \quad \nu > 2, \quad (65)$$

$$E(t_3^3) = \frac{60a^3}{(\nu-1)(\nu-2)(\nu-3)}, \quad \nu > 3. \quad (66)$$

It is clear from (57) that the mth moment for t_p exists only if $\nu > m$. That is, all the moments do not exist for the density functions of (41). Solving the pairs of equations (58) and (59), (61) and (62) and (64) and (65), respectively, we get

$$\hat{a} = \frac{\hat{E}(t_1)\,\hat{E}(t_1^2)}{\hat{E}(t_1^2) - 2\hat{E}^2(t_1)}, \quad \hat{\nu} = 2\,\frac{\hat{E}(t_1^2) - \hat{E}^2(t_1)}{\hat{E}(t_1^2) - 2\hat{E}^2(t_1)} \quad (67)$$

$$\hat{a} = \frac{\hat{E}(t_2)\,\hat{E}(t_2^2)}{2\hat{E}(t_2^2) - 3\hat{E}^2(t_2)}, \quad \hat{v} = \frac{4\hat{E}(t_2^2) - 3\hat{E}(t_2)}{2\hat{E}(t_2^2) - 3\hat{E}^2(t_2)} \tag{68}$$

and

$$\hat{a} = \frac{\hat{E}(t_3)\,\hat{E}(t_3^2)}{3\hat{E}(t_3^2) - 4\hat{E}^2(t_3)}, \quad \hat{v} = 2\,\frac{3\hat{E}(t_3^2) - 2\hat{E}^2(t_3)}{3\hat{E}(t_3^2) - 4\hat{E}^2(t_3)} \tag{69}$$

as estimates for parameters in the density functions $p_{T_1}(t_1)$, $p_{T_2}(t_2)$ and $p_{T_3}(t_3)$ respectively of (41). If the conditions on v in (58) to (66) are not satisfied, the estimates of a and v may not be valid. These estimates may or may not be consistent with (60), (63) and (66) respectively.

5.6 *Compound chi-gamma and (gamma/chi)-gamma distributions*

The moments of the compound chi-gamma and (gamma/chi)-gamma density functions of (45) and (47) for the waiting time for the pth earthquake under the compound chi-Poisson and (gamma-chi)-Poisson process models for earthquake occurrences can in principle be obtained, and equated with the corresponding sample moments to obtain the estimates of the parameters (a) n and σ and (b) n, a and b, respectively. However, the expression for $p_{T_p}(t_p)$ themselves are complicated and, therefore, this procedure is clearly impractical. The only hope is to tabulate the density functions and their moments on a computer for various values of the parameters till the sample moments match the computed population moments.

6. Hypothesis testing

6.1 *The regions and data*

The data of earthquakes in the Hindukush region are taken from the catalogue of epicentral locations of earthquakes published by the Indian Meteorological Department. For computing the number of earthquakes per unit time, 2395 earthquakes of magnitude greater than 3.5, for the period from January 1963 to December 1974, having focal depth less than 250 km in an area bounded by 62 to 76° E longitude and 30 to 39° N latitude are used. In tests for waiting time density functions, 1935 earthquakes were considered from a region restricted to 69 to 72° E longitude and 35 to 38° N latitude during the period from January 1970 to December 1976.

Another set of data of microearthquakes from the North-Eastern region of India is also used. These data were collected by the University of Roorkee and the Geological Survey of India under a joint project, for a period of 5.5 months from May 1979 to October 1979. There were seismic stations at Raliang (25.47° N, 92.43° E), Borjori (26.40° N, 92.94° E), Burnihat (26.06° N, 91.89° E) and Shillong (25.57° N, 91.88° E). The Shillong station was run by the Indian Meteorological Department. Magnitudes were between 2.2 to 5.1. Earthquake sources having distances more than about 300 km, *i.e.* those with S-P times greater than 40 seconds, were excluded. 235 events were used for the analysis.

6.2 *Choice of a test*

The likelihood ratio test was used. That is, if $P_1(x)$ and $P_0(x)$ were probabilities under the alternative and the null hypotheses, the alternative hypothesis was accepted if (Whalen 1971)

$$\frac{P_1(x)}{P_0(x)} \geq 1. \tag{70}$$

Similarly, for the continuous random variable t_p, the alternative hypothesis was accepted if (Whalen 1971)

$$\frac{p_1(t_p)}{p_0(t_p)} \geq 1 \tag{71}$$

where $p_1(t_p)$ and $p_0(t_p)$ are the density functions under the alternative and the null hypotheses.

Under multiple observations, the observations could be considered to be independent (though not always a correct assumption) and the alternative hypothesis was accepted if (Whalen 1971)

$$\prod_{i=1}^{N} \frac{P_1(x_i)}{P_0(x_i)} \geq 1 \quad \text{or} \quad \prod_{i=1}^{N} \frac{p_1(t_{pi})}{p_0(t_{pi})} \geq 1, \tag{72}$$

where x_i is the ith sample for number of earthquakes per unit time or t_{pi} is the ith sample for the waiting time for the pth earthquake, and N is the number of samples.

Alternatively, whenever the inequality (70) or (71) was satisfied for a particular sample, it was considered as one vote for an alternative hypothesis, and an alternative hypothesis was accepted if it collected a majority of votes. Thus, in the two decision rules, the integration of samples can be said to be before the decision and after the decision, respectively.

The tests discussed above assume that the parameters (λ, n, σ, a or b) are known. However, if the parameters are not known and have to be regarded as random variables, the hypotheses are called composite. One of the strategies in such a case is to obtain estimates of the unknown parameters and use these estimates as the operative values of the parameters to obtain the likelihood ratio (Whalen 1971). Normally, maximum likelihood estimates are used, but here we have used moment-matching estimates discussed in §5.

6.3 *Tests for number of earthquakes*

6.3a *Poisson versus negative binomial distribution:* For number of earthquakes per week occurring in Hindukush region as random variable, we have

$$\begin{aligned} &\text{Sample mean} = 3.281, \ \text{second sample moment} = 17.287, \\ &\text{third sample moment} = 113.64, \ \text{sample variance} = 6.523 \end{aligned} \tag{73}$$

Therefore, from (49) and (52)

$$\hat{\lambda} = 3.281, \ \hat{a} = 1.012, \ \hat{\nu} = 3.32. \tag{74}$$

Using (72), (1) (5) and (6), the negative binomial distribution is to be accepted if (Sharma 1982)

$$- N\hat{\lambda} + N \ln[(\hat{\nu} - 1)!] - N\hat{\nu} \ln \hat{q} + \\ + \sum_{i=1}^{N} k_i \ln(\hat{\lambda}/\hat{p}) - \sum_{i=1}^{N} \ln[(\hat{\nu} + k_i - 1)!] \leq 0 \tag{75}$$

The value of the left side in (75) comes out to be − 1768.89. Thus, the negative binomial distribution is to be accepted.

Taking the number of earthquakes per fortnight as a random variable, we get

$$\text{Sample mean} = 7.011; \text{ second sample moment} = 70.038; \\ \text{sample variance} = 20.419 \tag{76}$$

Therefore, from (49) and (52)

$$\hat{\lambda} = 7.011, \quad \hat{a} = 0.527, \quad \hat{\nu} = 3.710 \tag{77}$$

and the left side of (75) turns out to be − 6513 suggesting again that the negative binomial distribution can be accepted.

Taking the number of micro-earthquakes per day in the NE India as a random variable, we get

$$\text{Sample mean} = 9.198; \text{ second sample moment} = 110.51; \\ \text{sample variance} = 25.899 \tag{78}$$

so that from (49) and (52) we get

$$\hat{\lambda} = 9.198, \quad \hat{a} = 0.558, \quad \hat{\nu} = 5.135, \tag{79}$$

and the left side of (75) turns out to be − 11841.8 which once again suggests that the negative binomial distribution can be accepted.

Using the number of earthquakes per fortnight in the Hindukush region as a random variable and the voting scheme for decision-making, the votes in favour of the Poisson and negative binomial distributions are 153 and 187 respectively. Thus, the negative binomial distribution is favoured. Using the number of earthquakes per week in Hindukush region as a random variable, the votes in favour of the Poisson and negative binomial distributions are 448 and 282 respectively! In this case the Poisson distribution is favoured.

6.3b *Poisson versus compound chi-Poisson distribution:* Taking the number of earthquakes per week in the Hindukush region as a random variable, the parameters of the compound chi-Poisson distribution were estimated to be

$$\hat{n} = 2, \quad \hat{\sigma} = 2.65, \tag{80}$$

With these parameters the population moments are

$$\text{First moment} = 3.316 \text{ and second moment} = 17.121. \tag{81}$$

These values are reasonably close to the sample moments of (73).

The votes in favour of the Poisson and compound chi-Poisson distributions were 404 and 326 respectively, thereby favouring the Poisson distribution.

6.3c *Poisson versus compound (gamma/chi)-Poisson distribution:* Taking the number of earthquakes per week in the Hindukush region as a random variable, the parameters of the compound (gamma/chi)-Poisson distribution were estimated as

$$\hat{n} = 3, \quad \hat{a} = 0.1, \quad \hat{b} = 0.01 \tag{82}$$

With these parameters the population moments are

$$\text{First moment} = 3.566, \quad \text{second moment} = 17.593 \text{ and}$$
$$\text{third moment} = 99.245 \tag{83}$$

These values are reasonably close to the sample moments of (73).

The votes in favour of the Poisson and compound (gamma/chi)-Poisson distributions were 432 and 298 respectively, thereby favouring the Poisson distribution.

6.4 *Tests for waiting time distributions*

6.4a *Gamma-versus compound gamma-gamma distribution:* From (71), (40) and (41), the decision rule is to prefer compound gamma-gamma density function over the gamma density function if

$$g(t_p) = \hat{\lambda} t_p - (\hat{\nu} + p) \ln (\hat{a} + t_p) \geq - \hat{\nu} \ln \hat{a} + p \ln \hat{\lambda}$$
$$- \sum_{j=0}^{p-1} \ln (\hat{\nu} + j) = \Lambda_p \tag{84}$$

where t_p is the waiting time for the pth earthquake, and the test is resolved by comparing a statistic $g(t_p)$ against the threshold Λ_p.

With inter-arrival time t_1 as the random variable and using (55) and (67), we get

$$\hat{\lambda} = 6.983 \times 10^{-6}, \quad \hat{a} = 1.411 \times 10^6 \text{ and } \hat{\nu} = 10.851 \tag{85}$$

With waiting time t_2 as the random variable and using (55) and (68), we get

$$\hat{\lambda} = 6.979 \times 10^{-6}, \quad \hat{a} = 1.996 \times 10^6 \text{ and } \hat{\nu} = 14.930 \tag{86}$$

With waiting time t_3 as the random variable and using (55) and (69), we get

$$\hat{\lambda} = 6.978 \times 10^{-6}, \quad \hat{a} = 2.555 \times 10^6 \text{ and } \hat{\nu} = 18.830. \tag{87}$$

Using (85) to (87) for $p = 1, 2, 3$ and (84), the thresholds Λ_p turn out to be

$$\Lambda_1 = -167.896, \quad \Lambda_2 = -245.808 \text{ and } \Lambda_3 = -322.392. \tag{88}$$

The data used are from the Hindukush region and the time is measured in seconds.

The votes in favour of gamma and compound gamma-gamma density functions were (a) 864 and 670, (b) 906 and 627, (c) 870 and 662 respectively for waiting times t_1, t_2 and t_3, thus the Poisson process model is preferred to Polya process model.

6.4b *Gamma versus compound chi-gamma and (gamma/chi)-gamma distributions*

The expressions for compound chi-gamma and (gamma/chi)-gamma density functions are complicated and these density functions have not been computed yet. Therefore, they have not been tested against the gamma distribution yet.

7. Discussion

7.1 *Analysis of results*

(a) While comparing various stochastic process models for earthquake occurrences the decision of hypothesis test may depend on the definition of the random variable chosen for the test. If the test is based on the number of earthquakes in a fortnight the negative binomial distribution is favoured. The Poisson distribution is accepted if the unit of time is a week (for the data from Hindukush region using voting scheme).

(b) The decision to accept or to reject a particular hypothesis may depend on how the test is set up. When the random variable was the number of earthquakes per week, the Poisson distribution was accepted on the basis of a voting scheme whereas the negative binomial distribution was preferred when various samples were treated as independent samples.

(c) When stochastic process models for earthquake occurrences are being tested, different results may be obtained on the basis of different implications of the stochastic process models. If the number of earthquakes per unit time is used for the hypothesis test, the Polya process model may be accepted, but on the basis of waiting times for the first, second, third, etc. earthquakes Poisson process model may be preferred.

(d) There are situations wherein the Poisson process model is not the best and the Polya process model seems superior. But, also in situations where the Poisson process model is preferred the difference in the number of votes gathered by the two hypotheses under comparison is not very large. That is, the alternatives to the Poisson process model proposed here are not very poor. In fact, if instead of deciding the issue by simple majority and taking a hard decision, if the soft decision were entertained, that is, if a particular hypothesis was to be accepted only if it got $(50 + \Delta)\%$ votes and if the decision was to be deferred if the votes gathered by the winning hypothesis were less than $(50 + \Delta)\%$, it is conceivable that the decisions in this paper would have been deferred until larger data sets are available for $\Delta = 12\%$.

7.2 *Limitations of the analysis*

(a) It is known that the mean, variance and the third central moment are all equal to λ for the Poisson distribution. It can be seen that this equality does not hold for the data from (73), (76) and (78). Similarly, values of $\hat{a}$ and $\hat{\nu}$ estimated from the different waiting time statistics are significantly different under Polya process model, as can be seen from (85)–(87). This might be interpreted to mean that neither the Poisson nor the Polya process model is particularly good for the data considered here. Nevertheless it can also be used as an argument that the parameters of a stochastic process model should not be estimated by matching just the requisite number of moments for a chosen random variable. But an attempt may be made to match more than the requisite number of moments for a moderate number of random variables simultaneously under a suitable matching criterion. Alternatively, estimators such as maximum likelihood or maximum entropy estimators may be sought. The results of the hypothesis test may depend on how the parameters are estimated.

(b) After the parameters are estimated by a method, the estimate of the density function or the probability is not obtained by substituting the estimate instead of parameter. That is, if $\hat{\lambda}$ is obtained from (49), $\hat{\lambda}$ is a maximum likelihood estimator of λ (Johnson and Kotz 1969), but then $\hat{\lambda}^k \exp(-\hat{\lambda})/k!$ is not h maximum likelihood estimator of $P(x = k)$. In fact, the minimum variance unbiased estimator of $P(x = k)$ is shown to be (Barton 1961 and Glasser 1962)

$$\hat{P}(x = k) = \binom{N\hat{\lambda}}{k}\left(\frac{N-1}{N}\right)^{N\hat{\lambda}} \frac{1}{(N-1)^k} \tag{89}$$

A similar situation would arise for other distributions. As an example, let $P(x = k; n, a, b)$ be a distribution of (29) where parameters n, a, b are explicitly shown. Once the estimates $\hat{n}, \hat{a}, \hat{b}$ are obtained, for likelihood ratio test we have used

$$\hat{P}(x = k; n, a, b) = P(x = k; \hat{n}, \hat{a}, \hat{b}) \tag{90}$$

as an estimate of $P(x = k; n, a, b)$. That this is erroneous can be easily shown by expanding $P(x = k; n, a, b)$ as a function of n, a, b as Taylor's series around $\hat{n}, \hat{a}, \hat{b}$. Then

$$\begin{aligned}
P(x = k; n, a, b) &= P(x = k; \hat{n}, \hat{a}, \hat{b}) + \\
&(n - \hat{n}) \frac{\partial P(x = k; \hat{n}, \hat{a}, \hat{b})}{\partial n} + (a - \hat{a}) \frac{\partial P(x = k; \hat{n}, \hat{a}, \hat{b})}{\partial a} \\
&+ (b - \hat{b}) \frac{\partial P(x = k; \hat{n}, \hat{a}, \hat{b})}{\partial b} + \frac{(n - \hat{n})^2}{2} \frac{\partial^2 P(x = k; \hat{n}, \hat{a}, \hat{b})}{\partial n^2} \\
&+ \frac{(a - \hat{a})^2}{2} \frac{\partial^2 P(x = k; \hat{n}, \hat{a}, \hat{b})}{\partial a^2} + \frac{(b - \hat{b})^2}{2} \frac{\partial^2 P(x = k; \hat{n}, \hat{a}, \hat{b})}{\partial b^2} \\
&+ (n - \hat{n})(a - \hat{a}) \frac{\partial^2 P(x = k; \hat{n}, \hat{a}, \hat{b})}{\partial n\, \partial a} \\
&+ (n - \hat{n})(b - \hat{b}) \frac{\partial^2 P(x = k; \hat{n}, \hat{a}, \hat{b})}{\partial n\, \partial b} \\
&+ (a - \hat{a})(b - \hat{b}) \frac{\partial^2 P(x = k; \hat{n}, \hat{a}, \hat{b})}{\partial a\, \partial b}
\end{aligned} \tag{91}$$

wherein as $n \neq \hat{n}$, $a \neq \hat{a}$, $b \neq \hat{b}$, the terms other than the first on the right side, do not vanish. Taking expectation with respect to the joint density function of $\hat{n}$, $\hat{a}$, $\hat{b}$, (91) becomes

$$P(x = k; n, a, b) = E[P(x = k; \hat{n}, \hat{a}, \hat{b})]$$

$$+ \frac{\operatorname{var}(\hat{n})}{2} \frac{\partial^2 P(x = k; \hat{n}, \hat{a}, \hat{b})}{\partial n^2} + \frac{\operatorname{var}(\hat{a})}{2} \frac{\partial^2 P(x = k; \hat{n}, \hat{a}, \hat{b})}{\partial a^2}$$

$$+ \frac{\operatorname{var}(\hat{b})}{2} \frac{\partial^2 P(x = k; \hat{n}, \hat{a}, \hat{b})}{\partial b^2} + \operatorname{covar}(\hat{n}, \hat{a}) \frac{\partial^2 P(x = k; \hat{n}, \hat{a}, \hat{b})}{\partial n\, \partial a}$$

$$+ \operatorname{covar}(\hat{n}, \hat{b}) \frac{\partial^2 P(x = k; \hat{n}, \hat{a}, \hat{b})}{\partial n\, \partial b} + \operatorname{covar}(\hat{a}, \hat{b}) \frac{\partial^2 P(x = k; \hat{n}, \hat{a}, \hat{b})}{\partial a\, \partial b} \tag{92}$$

assuming that the series in (92) converges, that higher order terms can be neglected, that $\hat{n}$, $\hat{a}$, $\hat{b}$ are unbiased estimators of n, a, b and that the derivatives of $P(x = k; \hat{n}, \hat{a}, \hat{b})$ are independent of $(a - \hat{a})$ $(a - \hat{a})$ $(b - \hat{b})$, etc. Thus, in general

$$E[P(x = k; \hat{n}, \hat{a}, \hat{b})] \neq P(x = k; n, a, b) \tag{93}$$

that is $P(x = k; \hat{n}, \hat{a}, \hat{b})$ is not an unbiased estimator of $P(x = k; n, a, b)$ in whatever way $\hat{n}$, $\hat{a}$, $\hat{b}$ are obtained. To estimate an error, low order moments of the estimators of parameters must be estimated. There are many methods of estimating parameters. For example, four methods of estimating parameters of the negative binomial distribution are described (Johnson and Kotz 1969). Such an analysis should be done for all the methods. As this exercise has not been conducted in this paper, the results need not be reliable.

(c) Compounding has been used here to generate alternative hypotheses for earthquake occurrences in a heuristic way. An attempt has not been made to attach physical meaning to the parameters of the compounding distribution or to the compounding distribution as a whole. Thus the work reported here needs further investigation.

(d) The alternative hypotheses generated here relate only to earthquake occurrences and do not take into account the location of the epicentres, the magnitudes of the earthquakes, etc. A more comprehensive and utilitarian statistical model for earthquakes must include these aspects.

(e) The computed probabilities for the compound chi-Poisson and (gamma/chi)-Poisson distributions violated the necessary constraint that the probabilites must always be positive, for large values of k. Thus, the formulae for these distributions are not numerically appropriate. Numerically appropriate formulae for them should be obtained. These may be in terms of the recurrence relations between $P(x = k)$, $P(x = k - 1)$, ..., $P(x = k - m)$. The choice of time period of one week was dictated by this numerical difficulty.

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