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Understanding fields using strings: A review

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Abstract. In addition to being a prime candidate for a fundamental unified theory of all interactions in nature, string theory provides a natural setting to understand gauge field theories. This is linked to the concept of 'D-branes': extended, solitonic excitations of string theory which can be studied using techniques of string theory and which support gauge fields localized along their world-volumes. It follows that the techniques of string theory can be very useful even for those particle physicists who are not specifically interested in unification and/or quantum gravity. In this talk I attempt to review how strings help us to understand fields. The discussion is restricted to 3+1 spacetime dimensions.

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1. Introduction

A long-standing goal in theoretical high-energy physics is to understand the dynamics of gauge theory beyond perturbation theory. This is particularly important for QCD where non-perturbative effects are responsible for many, if not most, of the physical behaviour of the theory. While the lattice offers one hope to address this problem in a very explicit way, it is often the case that a continuum picture, even qualitative, can be a rich source of insight.

Such a continuum picture — of confinement and other non-perturbative effects in gauge theory — has been conjectured by many outstanding physicists since the early 1970's. While beautiful and reasonably convincing physical pictures of QCD emerged from this analysis, it proved very hard to substantiate much of this thinking by evidence, even 'theoretical evidence'.

What is theoretical evidence? In the last five years, we understand this term much better. Conjectures about strongly-coupled gauge theory cannot be proved without having a definite and effective computational procedure in mind, and this is still lacking at present. But there is a more realistic goal: having formulated a conjecture, one can make a large list of its consequences, and then hope to isolate, from this list, a few consequences that can actually be theoretically tested. This then constitutes a body of theoretical evidence for the conjecture.

Supersymmetry and string theory have turned out to be the twin planks on which a large body of theoretical evidence, embodied in 'duality symmetries', has accumulated over the last few years. Since not all high-energy physicists are interested in the goal of string theory (to unify all four fundamental interactions, including gravity, into a consistent quantum theory), I have chosen to focus this talk on the areas in which a more or less conventional particle physicist can gain insight from string theory.

Along with string theory, supersymmetry will turn out to be a key ingredient in our story. Supersymmetry is accepted by most high-energy physicists as a plausible proposal for what the world is like above a TeV or so. Even if this proposal turns out not to be correct, string theory might still be a helpful way to understand the correct field theory. This is because many of the 'miraculous' symmetries of string theory that we will use, might well be present even in the absence of supersymmetry. It is our knowledge of non-supersymmetric string theory that is still insufficient to put it to the service of gauge theory.

This review should be fairly accessible to readers who are not knowledgeable about string theory. Such readers may, however, wish to consult refs [1,2] to learn more about the subject.

2. Classical SUSY gauge theory

To set the stage for our discussions, it is useful to review the structure of supersymmetric gauge fields in 4 spacetime dimensions [3,4]. Supersymmetry requires different bosonic and fermionic fields to fall into multiplets.

2.1 *Multiplets and Lagrangians*

$\mathcal{N}=1$ *supersymmetry*: With $\mathcal{N}=1$ supersymmetry we have two possible multiplets. The first is a vector multiplet:

$$\text{vector multiplet}: \qquad A^a_\mu, \lambda^a, \quad a=1,\ldots,\text{dim}G \tag{1}$$

consisting of a gauge field and a Majorana spinor ('gaugino') in the adjoint of the gauge group G.

The second multiplet, called the 'chiral multiplet', contains no gauge fields but only scalars and fermions:

$$\text{chiral multiplet}: \qquad \phi^i_I, \psi^i_I, \quad i=1,\ldots,\text{dim}R, \;\; I=1,\ldots,N_f, \tag{2}$$

where R is a representation of the gauge group G, and N_f denotes the number of flavours. The scalars in this multiplet are usually called 'squarks' since that is what they would be if we were writing a supersymmetric version of the standard model.

Together, supersymmetry and gauge symmetry constrain the most general renormalizable classical action one can write with these fields. The action is made up of three terms:

$$S = S_{\text{kinetic}} + S_{D-\text{term}} + S_{\text{superpotential}}, \tag{3}$$

where S_{kinetic} contains the usual kinetic terms for all the fields, and

$$\begin{aligned} S_{D-\text{term}} &= \int \mathrm{d}^4x \sum_{a=1}^{\dim G} \left({\phi^i_I}^\dagger T^a_{ij} \phi^j_I \right)^2 + \text{fermions}, \\ S_{\text{superpotential}} &= \int \mathrm{d}^4x \sum_{i,I} \left| \frac{\partial W}{\partial \phi^i_I} \right|^2 + \text{fermions}, \end{aligned} \tag{4}$$

where $W(\phi_i^I)$ is an analytic function of the complex field on which it depends, and is called the 'superpotential'. Supersymmetry allows this to be arbitrary, but for renormalizability it should be at most cubic in its argument.

$\mathcal{N}=2$ *Supersymmetry:* With $\mathcal{N}=2$ supersymmetry we again have two possible multiplets. The first is again called a vector multiplet but its content is different from the $\mathcal{N}=1$ vector multiplet:

$$\text{vector multiplet}: \qquad A_\mu^a, \Phi^a, \lambda^a, \quad a = 1\ldots\dim G. \tag{5}$$

Here λ^a is a Dirac gaugino, while Φ^a is a complex scalar field in the adjoint. This multiplet is actually the combination of a vector and a chiral multiplet of $\mathcal{N}=1$ supersymmetry.

The second multiplet of $\mathcal{N}=2$ supersymmetry is called a 'hypermultiplet' and has the following content:

$$\text{hypermultiplet}: \qquad Q_I^i, \tilde{Q}_I^i, \Psi_I^i, \tilde{\Psi}_I^i, \qquad i = 1\ldots\dim \mathrm{R},\ \ I = 1\ldots N_f, \tag{6}$$

where Q_I^i and $\tilde{Q}_I^i$ are complex scalars in the representation N_c and $\bar{N}_c$ respectively of the gauge group, and $\Psi_I^i, \tilde{\Psi}_I^i$ are Weyl fermions in the same representations. Thus the hypermultiplet is a combination of two chiral multiplets of $\mathcal{N}=1$ supersymmetry, in conjugate representations.

The most general renormalizable action compatible with $\mathcal{N}=2$ supersymmetry is:

$$S = S_{\text{kinetic}} + S_2 + S_{\text{superpotential}}, \tag{7}$$

where

$$S_2 = \int \mathrm{d}^4x \sum_a |f^{abc}\bar{\Phi}^b\Phi^c|^2 + \text{fermions}$$

$$S_{\text{superpotential}} = \int \mathrm{d}^4x |W'|^2 + \text{fermions}$$

$$W(\Phi^a, Q_I^i, \tilde{Q}_I^i) = \tilde{Q}_I^i\Phi^a T_{ij}^a Q_I^j + m_{IJ}\tilde{Q}_I^i Q_j^i. \tag{8}$$

There is an SU(2) $\times$ U(1) R-symmetry under which λ^a decomposes into a doublet. The squarks $(Q, \tilde{Q}^+)$ also form an SU(2)$_R$ doublet.

$\mathcal{N}=4$ *supersymmetry*: With $\mathcal{N}=4$ supersymmetry there is only a single multiplet, called the vector multiplet:

$$\begin{aligned}\text{vector multiplet}: \quad & A_\mu^a, \Phi_r^a, \lambda_R^a, \quad a = 1\ldots\dim G,\\ & r = 1\ldots 6,\ R = 1\cdots 4.\end{aligned} \tag{9}$$

In terms of $\mathcal{N}=2$ super-multiplets, this is a combination of a vector multiplet and an adjoint hypermultiplet, while in $\mathcal{N}=1$ language this is a combination of a vector multiplet and three adjoint chiral multiplets.

With such a high degree of supersymmetry, the action is completely determined if we allow only renormalizable (dimension 4) interactions. It takes the form:

$$S = S_{\rm kinetic} + S_2,$$
$$S_2 = \int {\rm d}^4x \left(\sum_{r,s=1}^{6} |f^{abc}\Phi_r^b \Phi_s^c|^2 + {\rm fermions} \right). \quad (10)$$

This is the maximally supersymmetric situation if we restrict ourselves to field theories in 3+1 dimensions without gravity. In components, there are 16 supersymmetry charges (4 Majorana spinors of 4 components each).

2.2 *Classical parameter space ('moduli space')*

The parameter space, or 'moduli space', of a field theory is the space of degenerate vacuum configurations. This amounts to the space of energy-minimising vacuum expectation values of various scalar fields. Classically, this is easy to determine by examining the Lagrangian and looking for flat directions in field space along which the potential does not vary. Quantum mechanically, one has to replace the Lagrangian by the effective Lagrangian incorporating quantum corrections.

Without supersymmetry, there is often no moduli space since the potential will have a unique minimum. Even if we choose a potential with flat directions, quantum corrections will generically lift this degeneracy. However, with supersymmetry, the classical moduli space is already constrained and moreover, quantum corrections can fail to lift degeneracies because of cancellations between fermion and boson loops. We will denote the classical moduli space by $\mathcal{M}_c$ and the quantum moduli space by $\mathcal{M}_q$.

The moduli spaces are most constrained when there is the greatest degree of supersymmetry. Hence in this discussion we start with the maximally supersymmetric case.

$\mathcal{N} = 4$ *Supersymmetry*: In this case, the classical moduli space consists of those vacuum expectation values of the 6 scalars which together minimise the potential energy. The result is simple but interesting. We require:

$$\sum_{r,s} (f^{abc}\phi_r^b \phi_s^c)^2 = 0, \quad (11)$$

where the scalar fields are understood to represent the VEV's. Positivity implies

$$f^{abc}\phi_r^b \phi_s^c = 0 \text{ for all } a, r, s. \quad (12)$$

This condition will be satisfied if and only if the VEV's all lie in the Cartan subalgebra of the gauge group:

$$\phi_r^\alpha \ \text{ arbitrary},\ \alpha = 1 \cdots {\rm rank}\, G$$
$$\phi_r^a = 0,\ a = ({\rm rank}\, G) + 1, \ldots, \dim G. \quad (13)$$

Recall that r takes values from 1 to 6, labelling the 6 scalar fields in the vector multiplet.

As a simple example, with gauge group SU(2), we have

$$\phi_r^3 \ \text{ arbitrary}, \quad \phi_r^{1,2} = 0 \ \ (r = 1, \ldots, 6). \quad (14)$$

It is convenient to label the VEV's by a collection of 6-vectors:

$$\phi_r^\alpha = v_r^\alpha = (v_1^\alpha, v_2^\alpha, \ldots, v_6^\alpha) = \vec{v}^\alpha \tag{15}$$

Then, the classical moduli space is the space of all 6-vectors $\vec{v}^{\,\alpha}$. However, there are global identifications by the Weyl group of G, a discrete subgroup which must still be imposed as a gauge symmetry. Thus the true classical moduli space is really the quotient of the naive one by this group.

The Weyl group of SU(2) is just Z_2, while for general SU(N_c) it is the permutation group S_{N_c}. Thus the classical moduli space in these cases is:

$$\begin{aligned} &\mathrm{SU}(2) : \mathcal{M}_c = \mathbf{R}^6/Z_2 \\ &\mathrm{SU}(N_c) : \mathcal{M}_c = \mathbf{R}^{6(N_c-1)}/S_{N_c}. \end{aligned} \tag{16}$$

Let us consider the SU(2) case in more detail. $\mathbf{R}^6$ has coordinates $(v_1, \ldots, v_6) = \vec{v}$. The action of the Weyl group is:

$$Z_2 : \quad \vec{v} \to -\vec{v}. \tag{17}$$

There is a fixed point of this action at $\vec{v} = \vec{0}$. This is the point where SU(2) gauge symmetry is restored, since the adjoint scalar VEV's all vanish. Elsewhere, $\phi_r^3 = v_r \neq 0$ breaks SU(2) to U(1).

Geometrically, a fixed point of the quotienting group corresponds to a singularity of the space. The space becomes an *orbifold*, so while it is flat everywhere else, it has infinite curvature at the origin.

Far away from the origin ($\vec{v} \neq \vec{0}$), the off-diagonal SU(2) gauge particles, which we may denote $W^\pm$, are massive, with a mass $g_{\mathrm{YM}}|\vec{v}|$. As $\vec{v} \to \vec{0}$, these gauge particles become massless. We see that singularities of the moduli space $\mathcal{M}$ are associated to the presence of new massless particles in the spectrum.

For SU(N_c), at a generic point of $\mathcal{M}_c$ we have the symmetry-breaking pattern:

$$\mathrm{SU}(N_c) \to (\mathrm{U}(1))^{N_c-1}$$

Note that the Cartan subgroup $(\mathrm{U}(1))^{N_c-1}$ of SU(N_c) can never be broken by the VEV of an adjoint scalar (since adjoint scalars are uncharged under this subgroup). Hence at such generic points we always have a number (rank G) of massless photons, and the theory is in the Coulomb phase.

However, there are special points where the breaking pattern is different:

$$\begin{aligned} \mathrm{SU}(N_c) &\to \mathrm{SU}(2) \times (\mathrm{U}(1))^{N_c-2} \\ &\to \mathrm{SU}(3) \times (\mathrm{U}(1))^{N_c-3} \\ &\to \mathrm{SU}(2) \times \mathrm{SU}(3) \times \mathrm{U}(1) \times \cdots \end{aligned} \tag{18}$$

and so on. All such points have 'enhanced nonabelian symmetry', hence extra massless particles. These points are fixed under the action of some element of S_{N_c}, hence they are singularities of the moduli space.

In addition to the above moduli space, there is the parameter space for the gauge coupling g_{YM} and the θ-angle, which combine into a complex parameter:

$$\tau_{\rm YM} = \frac{\theta}{2\pi} + \frac{4\pi i}{g_{\rm YM}^2}. \tag{19}$$

In field theory these parameters are fixed by hand and are quite distinct from VEV's of scalar fields. However, in string theory they arise as VEV's of some appropriate scalar fields, hence string theorists usually consider this parameter space to be part of the moduli space.

$\mathcal{N} = 2$ *Supersymmetry*: In this case, there are other phases besides the Coulomb phase. Thus the classical moduli space $\mathcal{M}_c$ splits into branches. One branch is characterised by the following:

$$\begin{aligned} &Q_I^i = \tilde{Q}_I^i = 0 \\ &\phi^\alpha = v^\alpha, \alpha = 1, \ldots, \mathrm{rank}\, G. \end{aligned} \tag{20}$$

Note that with $\mathcal{N} = 2$ supersymmetry, the field ϕ^α and its VEV v^α are *complex* numbers.

The above equation defines the 'Coulomb branch', on which as before, the generic breaking pattern is:

$$\mathrm{SU}(N_c) \to (\mathrm{U}(1))^{N_c - 1}. \tag{21}$$

In particular, for SU(2) we have the Coulomb branch:

$$\mathcal{M}_c^{\rm Coulomb} = \mathbf{R}^2 / Z_2, \tag{22}$$

where $v = v^3$ is the (complex) coordinate on $\mathbf{R}^2$.

At generic points of $\mathcal{M}_c^{\rm Coulomb}$ we cannot give a VEV to $Q_I^i, \tilde{Q}_I^i$, since their couplings to the adjoint scalars would increase the potential energy. But it is not hard to see that if v^α takes some special values, then we can turn on VEV's for $Q_I^i, \tilde{Q}_I^i$ at no cost in energy. Since the hypermultiplets are usually in the fundamental representation, they are charged under the Cartan subgroup of the gauge group. Hence such VEV's break even $U(1)$ factors. This branch of the moduli space is therefore called the Higgs branch.

$\mathcal{N} = 1$ *supersymmetry*: In this case the vector multiplet contains no scalars, hence there is no moduli space unless we couple some chiral (matter) multiplets. With matter, we have to minimize

$$S_{D-\rm term} + S_{\rm superpotential}.$$

The result for $\mathcal{M}_c$ depends on the details of the fields, representations and choice of superpotential. Not much can be said about it without going into a detailed classification of cases.

We see that the classical moduli space $\mathcal{M}_c$ is relatively simple for $\mathcal{N} = 4$ and consists of a Coulomb phase, while for $\mathcal{N} = 2$ supersymmetry, it consists of intersecting Coulomb and Higgs branches. With $\mathcal{N} = 1$ supersymmetry, the moduli space depends largely on one's choice of field content and superpotential in the theory.

3. Quantum SUSY gauge theory

We now turn to the question of how quantum corrections modify the classical moduli space of a supersymmetric gauge theory. In general, the quantum effective action will be different from the classical one and will incorporate non-renormalizable terms, including more general kinetic terms than the usual ones.

$\mathcal{N}=4$ *supersymmetry*: Because of the high degree of supersymmetry, the quantum moduli space $\mathcal{M}_q$ is identical to the classical one $\mathcal{M}_c$. At the origin of $\mathcal{M}_q$, the theory has unbroken SU(N_c) gauge symmetry and vanishing β-function. Thus, it is a conformal field theory (CFT). Note that as a consequence there is no asymptotic freedom, and hence also no confinement, in this theory. Away from the origin, conformal invariance is broken by the scalar VEV and we have massive theory coupled to U(1) gauge fields.

$\mathcal{N}=2$ *supersymmetry*: Consider SU(2) gauge theory with no hypermultiplets. It was shown non-perturbatively, by Seiberg and Witten [5,6], that the structure of $\mathcal{M}_q$ is rather different from that of $\mathcal{M}_c$. Whereas in $\mathcal{M}_c$ the Coulomb branch is singular at the origin and SU(2) gauge symmetry is restored there, in $\mathcal{M}_q$ the Coulomb branch has no singularity at the origin. Moreover, in this theory SU(2) gauge symmetry is *never* restored at any point of the moduli space!

Instead, it is found that there are two other singular points in $\mathcal{M}_q$. At these points, some particles do become massless – but not the gauge bosons. The massless particles at these points are monopoles and dyons. It becomes useful to make an electric–magnetic duality transformation near these points and study the magnetic theory instead.

This $N=2$ theory has a nontrivial β-function and is asymptotically free, so the coupling $\tau=(\theta/2\pi)+(4\pi i/g_{\rm YM}^2)$ depends on the scale. This coupling was shown to vary complex analytically ('holomorphically') as a function of the complex VEV $\phi^3=v$: so we can write $\tau=\tau(v)$. This dependence is known exactly as a certain non-trivial 'fibre bundle'. Since τ is valued in the upper half plane, it can naturally be interpreted as the 'shape' parameter (technically, 'complex structure parameter') of a torus, thus the moduli space looks like a torus varying over a plane.

The above holds for SU(2) gauge group and $N_f=0$ (no matter). Analogous exact results for $\mathcal{N}=2$ supersymmetry are also known for SU(N_c) gauge groups and for $N_f\leq 2N_c$ flavours, for which the theories are always asymptotically free. For $N_f=2N_c$ these theories are finite (the β-function vanishes) and hence they are conformal field theories. For $N_f>N_c$ the β-function is positive and the theory becomes ill-defined.

Note that the interesting results about quantum corrections always concern the Coulomb branch. The Higgs branch is protected from quantum corrections.

$\mathcal{N}=1$ *supersymmetry*: A complex array of results have been found for the quantum moduli space of $N=1$ supersymmetric gauge theories. However, just as the classical moduli space in this case depends on the detailed choice of matter fields, representations and couplings, the structure of $\mathcal{M}_q$ too will depend on these choices. The interested reader is referred to appropriate review articles on this topic, such as ref. [7].

4. D-branes and $\mathcal{N}=4$ SUSY

In this section we show how supersymmetric gauge theories in 3+1 spacetime dimensions naturally arise as a subsector of superstring theory. For a more detailed review of the relevant material on D-branes, see ref. [2].

Introducing fundamental extended objects such as strings leads to a variety of interesting new physical consequences. For one thing, closed string excitations produce gravity, so string theories are theories of quantum gravity. But we will be more interested in the sector of string theory that contains open strings.

Open strings have a pair of ends. This requires the specification of boundary conditions at the endpoints. While it is most natural to allow these to lie anywhere in space, one can consistently choose to restrict the endpoints onto a p-dimensional spatial hypersurface in the 9-dimensional space where strings propagate. In fact, one can show that such choices must necessarily be consistent: starting with unconstrained endpoints and applying known symmetries of string theory, we end up with endpoints constrained on a hypersurface.

What is the physical interpretation of these constrained endpoints? They define a spatial region on which the strings can end. Suppose we choose $p = 0$ and constrain our open strings to end on a fixed point in space. Then, that point breaks translation invariance exactly as an elementary particle would do. (For example, applying a Lorentz boost to the theory would cause the point to start moving with a fixed velocity). Fluctuations of the string give rise to motions and oscillations of this fixed endpoint. Hence in all respects this string endpoint can be treated as a particle with a definite mass. Because constrained endpoints satisfy Dirichlet boundary conditions, we call the associated particle a 'D-particle'.

D-particles can also be understood as solitonic excitations in the string theory. Hence we have two different mental pictures of the same object: as a soliton, and as a string endpoint. Now suppose we choose $p = 2$ instead of 0. Then the string endpoint sweeps out a 2-dimensional space. The associated object looks like a membrane. Indeed, it is called a 'D-brane'. It too has a complementary description as an extended solitonic excitation in string theory, much like the cosmic strings and domain walls that can be found as classical solutions of more physically relevant field theories. For arbitrary p, we say that the string endpoint describes a Dp-brane.

For suitable values of p, Dp-branes are stable objects in type II superstring theory. They are charged under some generalised gauge field and hence, in the solitonic picture, they correspond to stable solitons.

A key property of open superstrings is that their lowest excitations are massless gauge fields. These gauge fields propagate only on the locus where the endpoints are free to move, namely on the Dp-brane. Thus, the low-energy field theory coming from the dynamics of open strings is a gauge theory in $p+1$ spacetime dimensions. This is the central observation that links string theory and gauge field theory. For our purposes we will select the value $p = 3$, so we intend to realise the supersymmetric field theories discussed in the preceding sections as modes of open strings ending on $D3$-branes. The underlying string theory which has stable $D3$-branes is called type IIB string theory.

Because of supersymmetry, the gauge fields arising from open string endpoints lie in supermultiplets containing scalars and fermions. The basic $D3$-brane of type IIB string theory can be shown to inherit $\mathcal{N} = 4$ supersymmetry from the underlying spacetime supersymmetry of the 10-dimensional string theory. Hence the theory on the world-volume of a single $D3$-brane is an $\mathcal{N} = 4$ supersymmetric gauge theory. A single $D3$-brane gives rise to abelian gauge theory. We will argue below that to get higher gauge groups one must stack several identical $D3$-branes together. We will also see that lower supersymmetry can be obtained by combining $D3$-branes with other D-branes and 'orientifolds'.

Before doing this, let us note one amusing fact. A soliton has 'collective coordinates' for the symmetries that it breaks. In particular, extended solitons (branes) break translational invariance in the directions transverse to their own world-volume. For example, suppose a $D3$-brane is arranged to lie along (x^1, x^2, x^3). It breaks the remaining 6 translational

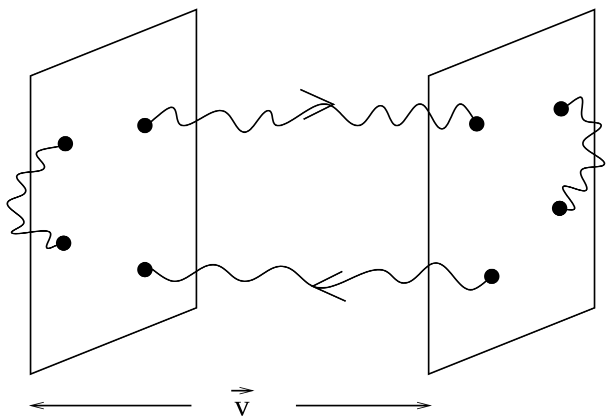

Figure 1. Two parallel $D3$-branes.

symmetries in the $9+1$ dimensional string theory, along $(x^4, x^5, \cdots, x^9)$. So it should have 6 massless scalar fields on its world-volume. And it does, because $\mathcal{N} = 4$ supersymmetry requires precisely 6 scalar fields in a vector multiplet!

We see that the 6 scalar fields in the $\mathcal{N} = 4$ supersymmetry multiplet, whose presence was deduced from the supersymmetry algebra long before superstrings and D-branes were understood, are most naturally interpreted as translational collective coordinates of a $D3$-brane. Moreover, the SO(6) R-symmetry comes from transverse rotational invariance: the 10-dimensional Lorentz group SO(9,1) is broken by the $D3$-brane into SO(3,1) $\times$ SO(6). Thus R-symmetry (a key property of field theories with extended supersymmetry) gets re-interpreted as a spacetime symmetry!

Now consider two parallel $D3$-branes (figure 1). Both are aligned along (x^1, x^2, x^3) but they can be at arbitrary locations in the other six directions. We let the vector $\vec{v}$ denote the relative location of one brane with respect to the other along these directions.

From the previous discussion we should expect that together, these $D3$-branes support a U(1) $\times$ U(1) $\mathcal{N} = 4$ supersymmetric gauge theory. The two vector multiplets arise from open strings having both ends on the first brane or both ends on the second brane. But now we also have two more types of open strings: those beginning on the first brane and ending on the second, and those beginning on the second brane and ending on the first. The corresponding states are charged as $(1, -1)$ and $(-1, 1)$ under U(1) $\times$ U(1). Under the diagonal U(1) they are neutral. With respect to the other U(1), they have exactly the charges of massive W-bosons! In fact their mass is

$$m_W \sim T|\vec{v}|, \tag{23}$$

where T is the string tension. In suitable units, this is related to the Yang–Mills coupling constant for the $D3$-brane gauge theory by $T \sim 1/g^2_{\rm YM}$. (Note that $\vec{v}$ in this section is a distance, while in the previous sections it was the VEV of a scalar field. The translation between these two involves a change of units and some rescaling.)

Particles obeying a mass-charge relationship like the one above correspond to quantum states in the gauge theory that do not break all the underlying supersymmetry (as a generic state would do) but preserve a fraction of supersymmetry. Such states are known as ‘BPS

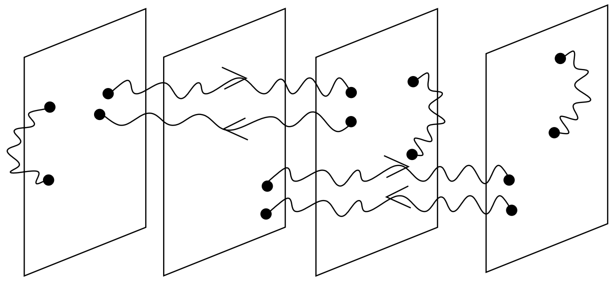

Figure 2. N_c parallel $D3$-branes.

states', and the corresponding particles are necessarily stable by virtue of the supersymmetry that they preserve.

Thus, two parallel $D3$-branes realize the Coulomb branch of $N = 4$ SU(2) gauge theory (apart from a decoupled centre-of-mass U(1)) [8]. When the parallel $D3$-branes coincide, the stretched open strings shrink to zero length, and $|\vec{v}| = 0$. There, SU(2) is restored. This is the origin of the Coulomb branch.

Since D-branes are indistinguishable objects, the parameter space is $\mathbf{R}^6/Z_2$, as we predicted from purely field-theoretic considerations. Thus we see that in string theory, the Weyl group factor in the gauge group comes from D-brane statistics!

For N_c parallel, separated $D3$-branes we have the following picture. The total number of stretched strings between pairs of $D3$-branes is $N_c(N_c - 1)$. Add N_c strings that begin and end on the same brane, and we end up with N_c^2 fields altogether. This is the dimension of the group $\mathrm{U}(N_c) \sim \mathrm{SU}(N_c) \times \mathrm{U}(1)$. So, N_c parallel $D3$-branes describe the moduli space of $\mathrm{U}(N_c) \sim \mathrm{SU}(N_c) \times \mathrm{U}(1)$ $\mathcal{N} = 4$ supersymmetric gauge theories (figure 2).

We have already identified some stable BPS states ($W^{\pm}$ bosons) in these theories. These carry electric charge under the U(1) factors. Now let us use string duality to extract more information. The type IIB string in 10 dimensions has a pair of massless scalar particles: the dilaton φ, and the axion $\tilde{\varphi}$. These appear naturally in the complex combination

$$\tau_s = \frac{\tilde{\varphi}}{2\pi} + 4\pi i \mathrm{e}^{-\varphi} = \frac{\tilde{\varphi}}{2\pi} + \frac{4\pi i}{g_s}. \tag{24}$$

We have used the fact, well-known to string theorists, that the string coupling is determined by the expectation value of the dilaton field: $g_s = \mathrm{e}^{\varphi}$.

Since the modes propagating on the $D3$-brane are excitations of open strings, they 'inherit' this coupling. In fact, the complex combination τ_{YM} of Yang–Mills coupling and theta-angle which we encountered in eq. (19) is equal to the complex combination τ_s above:

$$\tau_{\mathrm{YM}} = \frac{\varphi}{2\pi} + \frac{4\pi i}{g_{\mathrm{YM}}^2} = \tau_s = \frac{\tilde{\varphi}}{2\pi} + \frac{4\pi i}{g_s}. \tag{25}$$

Hence, in particular, $g_{\mathrm{YM}}^2 = g_s$.

Now, it is believed that type IIB string theory has a group of duality symmetries, SL(2, **Z**), under which

$$\tau_s \to \frac{a\tau_s + b}{c\tau_s + d}, \qquad \begin{pmatrix} a & b \\ c & d \end{pmatrix} \ \epsilon \ \mathrm{SL}(2, \mathbf{Z}). \tag{26}$$

This group of transformations includes, as a special case, a simple integer shift of the axion,

$$\tilde{\varphi} \to \tilde{\varphi} + 1$$

which tells us that it is an angle-valued field. It also includes the more nontrivial strong–weak coupling duality (‘S-duality’)

$$\tau_s \to -\frac{1}{\tau_s}$$

which for zero axion acts as $g_s \to 1/g_s$, inverting strong and weak coupling in the string theory.

Under S-duality, we know how all the massless fields of type IIB string transform. Since D-branes carry charges under specific massless fields, this also tells us how the branes transform. In particular one finds that S-duality converts the fundamental type II string into a $D1$-brane or ‘D-string’, but leaves the $D3$-brane invariant.

It follows that $\mathcal{N} = 4$ supersymmetric gauge theory must have a symmetry under

$$\tau_{\rm YM} \to \frac{a\tau_{\rm YM} + b}{c\tau_{\rm YM} + d},$$

For vanishing θ-angle, this includes a transformation $g_{\rm YM} \to -(1/g_{\rm YM})$, which interchanges a weakly coupled gauge theory with a strongly coupled one.

In addition, we saw that this duality acts on the string theory to interchange a fundamental type IIB string with a D-string. But we know that the end-point of a fundamental string when it terminates on a brane behaves like an electrically charged particle of the brane world-volume theory. It is also known that the endpoint of a D-string when it terminates on a brane, behaves like a magnetically charged particle [9,10]. Thus when acting on a $D3$-brane, S-duality must interchange electric with magnetic fields.

Thus, stringy S-duality implies strong–weak, electric–magnetic duality of $\mathcal{N} = 4$ supersymmetric gauge theory. This in turn implies the existence of a definite spectrum of monopoles and dyons as a consequence of the existence of electrically charged W-bosons, which can be identified as perturbative states. While this result was originally argued from field-theoretic considerations [11], this way of understanding it through string theory is very powerful and conceptually illuminating. (It is not as rigorous, though, since the string duality that we invoke remains a conjecture, which is in some ways harder to prove or justify than the field-theoretic duality.)

Here we have seen perhaps the simplest example wherein, by realising a field theory in terms of world-volume excitations on a brane, one can derive properties of this field theory using known (or conjectured) properties of the underlying string theory. These results are nonperturbative, since the duality acts non-perturbatively.

For general gauge groups SU(N_c), one re-discovers in this way a rich and complex spectrum of BPS monopoles and dyons, which field theorists had been slowly discovering over the last two decades.

For $N_c \geq 3$, SU(N_c) gauge theory also admits exotic BPS dyons whose existence had been conjectured (but not demonstrated) by field theorists. Such dyons have electric and magnetic charge vectors that are not proportional. String theory can be used to show that they must exist. One starts with the fact that type IIB string theory admits BPS junctions where a fundamental string meets a D-string and a bound-state of the two comes out from the junction point (figure 3). More general junctions also exist.

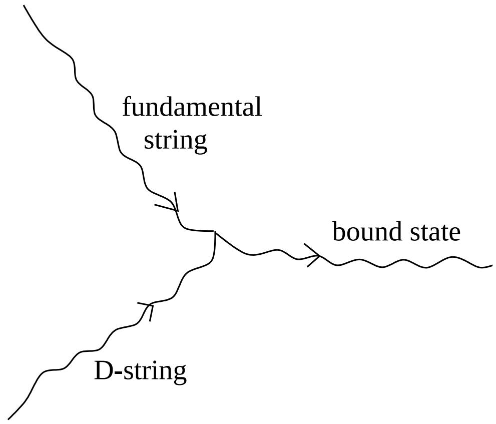

Figure 3. A three-string junction.

Following the observation that string junctions are stable, BPS objects, [12], it was argued [13] that an exotic dyon is obtained by suspending such a string junction between $D3$-branes. Such dyons exist in all SU(N_c) $\mathcal{N} = 4$ supersymmetric theories, for $N_c \geq 3$, and are stable.

With this impetus, field theorists began to generate the appropriate classical solutions for such field-theoretic solitons, and quite a lot is known about them by now.

5. Brane probes and $\mathcal{N} = 2$

Type IIB symmetric is invariant under orientation-reversal of the closed string. This symmetry, denoted by Ω, generates a Z_2 group and has a definite action on the fields of the theory (and therefore, as we have seen, also on the branes). Let us compactify IIB string theory on a 2-torus, with coordinates (x^8, x^9), and take the quotient by the symmetry $\Omega\,\mathcal{I}_{89}$, where $\mathcal{I}_{89}$ denotes reflection of the two toroidal directions:

$$\mathcal{I} : (x^8, x^9) \rightarrow (-x^8, -x^9)$$

As we might expect, Ω creates unoriented closed strings out of oriented ones, and $\mathcal{I}_{89}$ makes the two toroidal space dimensions into the orbifoldT^2/Z_2 (details about orientifolds can be found in ref. [2]).

The reflection symmetry has 4 fixed points on T^2. Let us focus on one of them, say the one at the origin. This is a point on the 2-torus, but it is independent of the other 7 spatial directions and is therefore a 7-dimensional hyperplane that extends along those directions. We call it an ‘orientifold 7-plane’.

This object is like a mirror: the spatial regions on the two opposite sides of it get identified. If we bring a $D3$-brane near it, we get new light states coming from open strings joining the $D3$-brane to its mirror image (figure 4). These become massless precisely when the $D3$-brane meets the orientifold 7-plane. This leads to two effects. The 7-plane breaks the supersymmetry on the $D3$-brane (which was originally $\mathcal{N} = 4$) down to $\mathcal{N} = 2$.

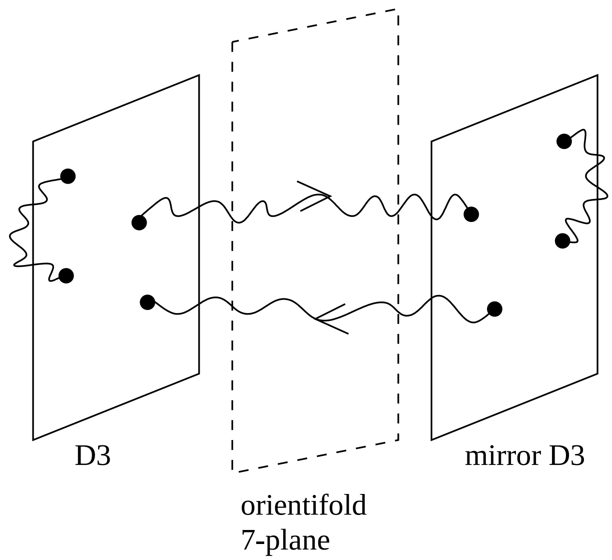

Figure 4. $D3$-branes at an orientifold 7-plane.

The other effect is that out of four open string sectors on a pair of $D3$-branes, one is projected out, leading to an SU(2) gauge group rather than U(2).

The result is pure $\mathcal{N} = 2$ supersymmetric gauge theory with SU(2) gauge group. This means that the moduli space of that theory must be given by the geometric space encountered by the $D3$-brane. In fact, we have recovered the classical moduli space R^2/Z_2 of this theory!

What about quantum effects? In the presence of this orientifold plane, the type IIB theory becomes a 'type-I' string theory with reduced supersymmetry. It was shown [14], following the construction of 'F-theory' [15], that quantum effects split the orientifold 7-plane into two dynamical 7-branes.

These 7-branes do not allow a type IIB string to end on them. So there are no massless 'W-bosons' when the $D3$-brane touches them. However, they allow dyonic (p, q) strings (bound states of p fundamental strings and q D-strings) to end on them. Since the end point of a fundamental string on a $D3$-brane is an electric charge, and the endpoint of a D-string on a $D3$-brane is a magnetic charge, it must be true that the endpoint of a (p, q) string is a dyon of electric charge p and magnetic charge q. Hence when the $D3$-brane touches either of the $D7$-branes, we get corresponding massless (p, q) dyons.

We have recovered an essential part of the Seiberg–Witten picture! The origin of the Coulomb branch has split into two singularities where dyons become massless. There is no point where W-bosons become massless.

To complete the picture, we use the existence of 'F-theory' [15], which is a novel way of compactifying the type IIB string where its coupling τ_s is allowed to vary over the compact manifold. Since the $D3$-brane inherits this coupling, the gauge coupling $\tau_{\rm YM}$ too varies over the v-plane (where $v = x^8 + ix^9$)) exactly as predicted by Seiberg and Witten. The Seiberg–Witten torus, which was a mathematical artifact in their solution, is realised geometrically: it turns out to be the torus whose shape is parametrised by $\tilde{\varphi}$ (the axion) and φ (the dilaton).

We can also introduce $D7$-branes parallel to the orientifold plane, this gives rise to (massive) hypermultiplets coupled to the pure $\mathcal{N} = 2$ gauge theory. In this way one recovers the more general Seiberg–Witten theories incorporating $\mathcal{N} = 2$ matter multiplets, and the Higgs branch appears as well.

One can use this stringy setup to predict new field-theoretic phenomena. The usual Seiberg–Witten theories have a maximal flavour symmetry group SO(8), which is realised in the case of four massless flavours. However, it was argued [16] that some configurations of 7-branes give rise to gauge theories on the 3-brane with E_6, E_7, E_8 global symmetry. This phenomenon (unlike the familiar SO(8) case) cannot occur at weak coupling. It is a new non-perturbative field-theoretic effect predicted by string theory!

6. Large-N_c gauge theories and the AdS/CFT correspondence

$D3$-branes have some features that we have not yet explored. Complementary to their description as D-objects (loci of open-string endpoints), they can also be understood as solitonic classical solutions of type IIB string theory – more specifically, of its low-energy limit, type IIB supergravity. Hence there is a spacetime metric describing the gravitational field around a collection of N_c $D3$-branes:

$$\begin{aligned} \mathrm{d}s^2 = {} & f(r)^{-\frac{1}{2}}(-\mathrm{d}t^2 + (\mathrm{d}x^1)^2 + (\mathrm{d}x^2)^2 + (\mathrm{d}x^3)^2) \\ & + f(r)^{\frac{1}{2}}((\mathrm{d}x^4)^2 + \cdots + (\mathrm{d}x^9)^2), \end{aligned} \tag{27}$$

where

$$f(r) = 1 + \frac{R^4}{r^4}, \qquad r = \left((x^4)^2 + \ldots + (x^9)^2\right)^{\frac{1}{2}} \tag{28}$$

and

$$R \sim (g_s(\alpha')^2 N_c)^{\frac{1}{4}}. \tag{29}$$

This metric describes a massive object localised along three spatial directions. Some generalised gauge fields of the low-energy supergravity theory must also be excited to make this a genuine classical solution. As a result, the solution describes a charged object. In fact, it is supersymmetric (BPS), and has a mass–charge relationship analogous to that in eq. (23), except that mass is replaced by mass per unit 3-volume or ‘brane tension’.

Something remarkable happens in the limit of large R (which, from eq. (29) is the same as large $g_s N_c = g^2_{\mathrm{YM}} N_c$). From the form of $f(r)$ above, this limit is equivalent to the ‘near-horizon’ limit $r \ll R$ in which we probe the metric very close to the brane. In this limit, we can make the replacement

$$f(r) = 1 + \frac{R^4}{r^4} \;\to\; \frac{R^4}{r^4} \tag{30}$$

and as a result the spacetime metric around N_c $D3$-branes becomes:

$$\begin{aligned}\mathrm{d}s^2 &= \frac{r^2}{R^2}(-\mathrm{d}t^2 + (\mathrm{d}x^1)^2 + (\mathrm{d}x^2)^2 + (\mathrm{d}x^3)^2) \\ &\quad + \frac{R^2}{r^2}(\mathrm{d}r^2 + r^2(\mathrm{d}\Omega_5)^2) \\ &= \left\{ \frac{r^2}{R^2}(-\mathrm{d}t^2 + (\mathrm{d}x^1)^2 + (\mathrm{d}x^2)^2 + (\mathrm{d}x^3)^2) + R^2\frac{\mathrm{d}r^2}{r^2} \right\} \\ &\quad + R^2(\mathrm{d}\Omega_5)^2. \end{aligned} \tag{31}$$

The factor in large braces is the metric of a $(4+1)$-dimensional space-time called 'anti-de Sitter', and denoted AdS_5, while the last term is the metric of a 5-sphere. Thus we have shown that the near-horizon metric of N_c $D3$-branes is the space-time Ad$S_5 \times S^5$.

As N_c grows, the near-horizon region expands. In the limit of infinite N_c, the entire spacetime (not just near the branes) is Ad$S_5 \times S^5$. Based on these facts, Maldacena [17] made a novel conjecture. According to this, the following two descriptions of $D3$-branes for large N_c are equivalent:

(i) The description as the limit of $\mathcal{N} = 4$ supersymmetric gauge theory as $(g^2_{\mathrm{YM}} N_c)$ becomes large,

(ii) The description as the nontrivial spacetime background Ad$S_5 \times S^5$ of type IIB string theory.

This is a duality between a gauge theory, and a theory of gravity and strings. It is remarkable how the symmetries of the problem match up in the two descriptions. In the gravity description, we have the symmetry groups SO(4,2) and SO(6), making up the isometries of the maximally symmetric spaces AdS_5 and S^5 respectively. In the gauge theory description, SO$(4,2)$ is realised as the conformal symmetry group of $3+1$-dimensional gauge theory, which includes the Poincaré group. On the other hand, SO(6) $\sim$ SU(4) is the R-symmetry group of $\mathcal{N} = 4$ supersymmetric Yang–Mills theory.

If we are only interested in the leading behaviour in the limit of large $g^2_{\mathrm{YM}} N_c$, we can really ignore string theory in favour of its low energy limit, type IIB supergravity. This is because the massive stringy modes decouple in this limit.

Precise prescriptions have been given [18,19] to relate correlation functions in $\mathcal{N} = 4$ gauge theory to computations in supergravity. This opens up the possibility of solving the quantum gauge theory completely in the large-N_c limit just using the classical Lagrangian of supergravity!

Some of the remarkable results obtained in this direction concern the computation of expectation values of Wilson loops [20,21], properties of baryons and domain walls [22], and thermal properties and phase transitions in gauge theory [23]. The correspondence was also extended to the case of lower supersymmetry: $\mathcal{N} = 2$, $\mathcal{N} = 1$, and even $\mathcal{N} = 0$ (no supersymmetry) [24–26].

An interesting example of lower supersymmetry is a case with $\mathcal{N} = 1$ supersymmetry in four dimensions. This arises by placing N_c $D3$-branes at the singular tip of a singular noncompact manifold called a 'conifold'. One finds in this case an interesting $\mathcal{N} = 1$ supersymmetric field theory on the $D3$-branes, which exhibits a nontrivial flow in the infrared to a superconformal field theory [26]. A dual brane description of this was found [27–29] which leads to a description of the field theory and its symmetries using strongly coupled string theory or 'M-theory'.

7. Conclusions

String theory has found a new role: to help in ‘solving’ gauge theories non-perturbatively. Such solutions range from a qualitative understanding of the theories, including their symmetries, to a detailed description of the moduli space in the same sense that Seiberg and Witten initially achieved using only field theoretic techniques.

Due to a shortage of time, I could not discuss a fascinating approach to realising field theories in terms of intersecting branes, the so-called ‘brane constructions’ [31,30]. These provide much more general examples of the utility of string theory in understanding quantum field theory.

Though such constructions exist for various different amounts of supersymmetry up to the maximal case of $\mathcal{N} = 4$, it remains true that at present our understanding is best for the most highly supersymmetric, and hence less interesting, gauge theories. It is important to improve our understanding of theories with $\mathcal{N} = 1$ supersymmetry, which is the amount of supersymmetry in the MSSM (such theories are dynamically quite similar to non-supersymmetric theories). Some partial progress has also been made towards directly studying non-supersymmetric gauge theories using string theory. The day may not be far off when the Standard Model will be most easily understood by representing it as a sector of string theory.

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