SPECTROPHOTOMETRIC STUDY OF SUNSPOTS

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Received December 23, 1952

INTRODUCTION

THE well-known paper by Prof. H. N. Russell (1921) entitled "Cooling by Expansion in Sunspots" is the first attempt towards a quantitative explanation of the observed fact that sunspots are some 1500° to 2000° K. cooler than the surrounding photosphere. Russell considers the sunspot as a region whose equilibrium is governed by the equations of adiabatic change of state while the surrounding photosphere is assumed to be in radiative equilibrium. The observed difference of temperature between the spot and the photosphere is then made use of to calculate the pressure, temperature and density at the depth from which the spot is supposed to originate. Retaining the basic idea of Russell but modifying his simple theory in some important respects Petrie (1930) and Milne (1930) made calculations of the temperature and pressure at the base of sunspots and the depth below the photosphere of the base of the umbral column. From the horizontal pressure gradients between the umbra and the adjacent photosphere, Milne also made estimates of the velocity of horizontal flow of gases in sunspots first observed by Evershed (1909) at Kodaikanal.

In two important papers entitled "Konvektion in der Sternatmosphäre," Unsöld (1930, 1931) showed that in a stellar atmosphere composed mainly of hydrogen, there should be a zone just below the surface where convection currents can be sustained because of the super-adiabatic lapse rate prevailing in this zone consequent on the rapid increase with depth in the ionisation of hydrogen. Under plausible assumptions Unsöld found that in the solar atmosphere convective instability would set in at a depth of about 100 km. below the surface and continue downwards for the next 100 km. He identified this zone as the source of sunspots and granulations and also calculated the minimum possible cooling in sunspots.

In 1930 Pettit and Nicholson of the Mt. Wilson Observatory published the results of measurements of the ratio of the energy in the umbræ of spots to that in the adjacent photosphere over the wavelength interval $0{\cdot}4\,\mu$ to $2{\cdot}2\,\mu$. From this they derived the spectral energy curve of sunspots and deduced a mean black body temperature of 4750° K. for the sunspot as against 5955° K. for the centre of the solar disc. They also measured the total energy in the umbræ of spots in terms of that for a corresponding area of the adjacent photosphere and found a value of 0·471 for this ratio.

The experimental results of Pettit and Nicholson formed the starting point of an important theoretical investigation by Minnaert and Wanders (1932) on the theory of sunspots. They investigated the wavelength dependance of the intensity ratio umbra/photosphere and also the variation of this ratio with the distance of the spot from the centre of the solar disc for the two cases in which the thermal equilibrium in the optically accessible regions of the sunspots is: (*a*) convective, and (*b*) radiative. The main results brought out were:—

(*a*) Sunspot in Convective Equilibrium:

(i) The contrast between the spot and the photosphere should increase (or the intensity ratio umbra/photosphere should decrease) *rapidly* towards the solar limb;

(ii) The intensity ratio umbra/photosphere should increase *gradually* with increasing wavelength.

(*b*) Sunspot in Radiative Equilibrium:

(i) The contrast between the spot and the photosphere (or the intensity ratio umbra/photosphere) should remain practically constant over the entire solar disc.

(ii) The intensity ratio umbra/photosphere should increase more rapidly with wavelength than in the case of convective equilibrium.

Minnaert and Wanders found that the experimental results were incompatible with adiabatic equilibrium, but were in good agreement with the assumption that sunspots are in radiative equilibrium at an effective temperature of 4300° K. They concluded that the levels at which cooling by ascending motion takes place in sunspots are below the depth accessible for optical observation, a result in conformity with the fact that displacements of the spectral lines towards the violet have not been observed in the umbræ of spots near the centre of the solar disc.

Following this investigation, Wanders (1935) made an experimental study of the intensity ratio umbra/photosphere for a large number of spots in the λ 4000 A region. He found that the ratio remains practically constant over the solar disc, and that larger spots show smaller central intensities. The latter conclusion was confirmed by Wormell (1936) who made measurements of the intensity of total radiation from sunspots. During the sunspot maximum epoch of 1938, Richardson (1939) made measurements of the centre to limb variation of the intensities for a number of large stable spots at four different wavelengths ($\lambda\lambda$ 4100, 5100, 5800 and 6600 A). His results appeared to support the conclusion that sunspots behave as though they are in radiative rather than in adiabatic equilibrium. He deduced an effective temperature of 4300° K. for the umbra.

Abetti and Colacevich (1939) made a spectrophotometric study of a large sunspot in the blue (λ 4227 A) and red (λ 6568 A) regions of the spectrum. They found the values 3·7% and 12% respectively for the intensity ratio umbra/photosphere corresponding to an umbral temperature of 3690° K.—the lowest recorded temperature of a spot.* Ten Bruggencate and Von Klüber (1939) studied the total absorption in the Fraunhofer lines in the spectra of some large sunspots from which they deduced an excitation temperature of 3800° K. for the spots.

Thackeray (1940) showed that if the absorption coefficient of the solar atmosphere is assumed to increase linearly with pressure, (Minnaert and Wanders considered the two cases of k constant and $k \propto pT^{-3/2}$) then the observed "angle variation" and "wavelength variation" of the intensity ratio spot/photosphere can be explained equally well either as the assumption that both disc and spot are in convective equilibrium or that both are in radiative equilibrium.

On the assumption that sunspots are in radiative equilibrium, Waldmeier (1941, 1942) showed that it is possible to account for the Evershed Effect based on a model solar atmosphere.

2. Object of the Present Work

Experimental data on the "angle dependance" and "wavelength dependance" of the intensity ratio umbra/photosphere which are obviously of great importance for the theory of sunspots are comparatively scanty. The conclusions of Minnaert and Wanders as well as of Thackeray are mainly based on the experimental results of Pettit and Nicholson. Although the subsequent work of Wanders and Richardson has given support to the idea of radiative equilibrium for sunspots there appears to be no other mechanism excepting convective ascent of gases from below which can account for the cooling observed in sunspots. The collection of more quantitative experimental data on the spectral energy distribution for a larger number of sunspots appears desirable and hence it was decided to undertake a systematic programme of spectrophotometry of sunspots extending over the entire photographic region of the spectrum whenever comparatively large

* *Transactions of the International Astronomical Union*, Vol. VII, p. 381 (1950).

stable spots could be studied under good sky and seeing conditions. This work was taken up in the beginning of 1951. Unfortunately the number of favourable occasions has not been many. The results of measurements made on some spots during 1951 are reported in this paper.

3. Experimental Arrangements

The optical system employed in the present work for forming the solar image was the same as that used for taking the daily spectroheliograms at the Kodaikanal Observatory. It consists of an 18-inch siderostat and a 12-inch photovisual lens of 21 ft. focal length giving a solar image of about 60 mm. diameter. The spectra were photographed with an auto-collimating spectrograph of 20 ft. focal length employing a 6¼ inch Michelson grating. All photographs were taken in the first order, the dispersion being practically uniform and very nearly 3 A/mm. between λ 6600 A and λ 5000 A. In place of the grating a set of 3½ prisms can be employed as the dispersing system in this spectrograph. Some spectra in the H-α region were taken in this manner, the dispersion being about 4 A/mm.

By means of the slow motion controls of the siderostat the solar image was so maintained that the vertical slit of the spectrograph passed through the centre of the umbra of the spot which in turn bisected the length of the slit. A slit width of ·04 mm. was employed for photographing the spectra. Since the spectrograph was not capable of rotation and since the optical field of the siderostat gradually rotates, the orientation of the spectrograph slit with reference to the spot was computed in each case from the times at which the spectra were taken.

Ilford Special Rapid Panchromatic plates specially sensitised for H-α were used for the spot spectra in that region; ordinary Ilford Special Rapid Panchromatic plates were used for the Na-D and Mg-b regions. Five spectra were taken on each plate in rapid succession, the exposure times varying from 1 to 5 seconds for the H-α and Na-D regions and from 2 to 10 seconds for the Mg-b region. The width of each spectrum was about 6 mm., the central spot band covering a width of 1 mm. in the case of the larger spots. A set of intensity marks was also impressed on each plate using a photographic step wedge which was prepared in the manner described in the next paragraph. The centre of the solar disc was used as the source of light in this case, the slit width of the spectrograph being so adjusted that the time of exposure required to get intensity marks with suitable density with the wedge in front of the slit being about the mean of the exposure times for the set of spot spectra. All plates were developed for 5 minutes in M-Q developer at room temperature, taking the usual precautions.

Since the solar image was only about 60 mm. diameter, the step wedge had to be correspondingly small to avoid the error due to limb darkening. The wedges employed were prepared in the following manner. A master wedge containing 18 steps with suitable density gradation was prepared on an Ilford Process plate by exposing the plate in the blue region of the second order spectrum in a high dispersion grating spectrograph the slit of which was illumined by a Philips tungsten ribbon lamp. The different density steps were obtained by appropriately varying the exposure times from 10 seconds to 150 seconds. The 18 steps and the intervening gaps together occupied 28 mm. on the plate. Because of the narrow spectral range involved, there was little variation of intensity along the length of any one of the steps. This wedge was next placed in an enlarger-*cum*-reducer and a large number of *reduced negatives* of it were made on fine-grained Ilford Lantern and Thin Film Half Tone plates. In this process, special care was taken to see that the master wedge was uniformly illumined. By suitably controlling the exposure time and development, negatives having various gradations of density could be easily prepared. The total width occupied by the 18 steps and the intervening spaces in the reduced wedges was 9 mm. While the gap between successive steps is transparent in the master wedge, it is opaque in the reduced wedges, the density gradation of the steps being in the opposite direction to that on the master wedge. Since the steps in the reduced wedges employed in the work extended only 4·5 mm. on either side of the centre of the solar disc, the error due to limb darkening was negligible compared with the other errors of photographic photometry (For $\sin\theta = 0{\cdot}2$ which corresponds to a distance of 6 mm. from the centre of the disc on a 60 mm. image, the intensity of the solar disc varies from 98% of the central intensity at λ 3700 A to 99% at λ 6700 A).

The step wedges employed in the photometric work were calibrated in the manner described by Shane (1932). The relative transmissions of two wedges employed in the present work are given in Table I.

TABLE I

Relative Transmission Factors of the Step Wedges

Step No. / Wedge No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
A	100	75·1	58·9	43·6	34·1	28·4	19·0	13·8	11·8	10·0	8·5	7·0	6·2	5·4	4·8	4·2	3·7	3·4
B	..	100	85·8	69·3	58·9	49·2	38·9	30·6	26·8	23·7	20·6	18·0	16·0	14·3	12·9	11·8	10·8	10·1

4. Results

A good series of spectra were obtained on a single large spot in the great sunspot group of 1951 May 9–22. This group whose mean heliographic latitude was 14° N, crossed the central solar meridian on May 16 and attained a maximum area of 4850 millionths of the sun's visible hemisphere. According to H. W. Newton (1951) it is the third if not the second largest sunspot group on record. The successive appearances of this group during its passage across the disc are illustrated in Pl. X, Fig. 1.

At Kodaikanal atmospheric conditions were good on May 12 and 13 and a large number of spectrograms were taken on these two days in the H_α, Na-D and Mg-b regions of the spectrum. All spectra showed excellent definition and fine details. A typical set of spectra in the Mg-b region is reproduced in Pl. XI, Fig. 2 (*a*).

Spectrograms in the H_α region were also taken on three other spots during May 1951, while the spectra of a single large spot were photographed on three days in October 1951.

Particulars regarding the spots whose spectra were studied are given in Table II.

The appearances of the spots on the respective dates are illustrated in Pl. XII, Fig. 2 (*b*).

It is well known that apart from the umbra and the penumbra which are the most conspicuous features of a sunspot, there is a bright ring surrounding the umbra which is slightly more intense than the photosphere. This is quite distinct from the faculæ which are generally noticed around spots especially when they are near the limb. The bright ring around the penumbra can be seen on all good photoheliograms particularly for spots near the central meridian when the faculæ are quite inconspicuous. According to Waldmeier (1939) this bright ring is some 2 to 3% brighter than the photosphere on photoheliograms taken in violet light ($\sim$4000 A). Just as the transition from the penumbra to the photosphere is discontinuous, the transition from the umbra to the penumbra is also discontinuous. It was noticed by Secchi (1875) long ago that the inner edge of the penumbra—the edge towards the periphery of the umbra—is brighter than its outer edge. This observation was doubted by Pettit and Nicholson but has been confirmed by later workers (Strebel, 1932; Ananthakrishnan, 1951). Visual and microphotometric examination of regular spots on good photoheliograms show that the inner edge of the penumbra is brighter in many cases, the intensity maximum at the outer edge of the umbra being quite conspicuous in some cases. The microphotometer

TABLE II

Co-ordinates and Areas of Sunspots (From Kodaikanal Photoheliograms)

Date and Time	Spot Group Number	Lat.	Date of C.M. Passage	d/R = sin θ	θ	U′	S′	U = $\frac{U'}{2 \cos \theta}$	S = $\frac{S'}{2 \cos \theta}$	U*	S*	U/S
1951 May—												
13·101 ..	KKL 9620 Mt. W 10662	14° N	16·0	·578	35° 18′	167	850	102	521	475	2420	·195
19·104 ..	KKL 9687 Mt. W 10671	15° N	21·7	·599	36° 48′	92	455	57	326	262	1296	·175
	KKL 9688 Mt. W 10673	21° N	22·2	·705	44° 50′	131	628	92	443	373	1790	·212
24·088 ..	KKL 9692 Mt. W 10678	21° S	27·9	·790	52° 11′	146	728	117	590	416	2074	·198
1951 Oct.—												
14·106 ..	KKL 9768 Mt. W 10796	9° S	14·2	·256	14° 50′	212	1180	111	610	604	3360	·182
16·099 ..	do			·480	28° 42′	176	911	100	520	501	2600	·193
18·087 ..	do			·804	53° 30′	121	607	103	515	344	1730	·200

U′, S′ — Projected area of umbra and whole spot in millionths of sun's disc.

U, S — Area of umbra and whole spot corrected for foreshortening in millionths of sun's visible hemisphere.

U*, S* — Projected area of umbra and whole spot in square seconds of arc.

records of two sunspots illustrating the discontinuous nature of the transition from the umbra to the penumbra are reproduced in Pl. XIII, Fig. 3. Thus, starting from the undisturbed photosphere we have first the outer bright ring, then the penumbra, then the inner bright ring and finally the umbra of the spot. In conformity with this structure of sunspots it is found that on many of the good spectrograms of spots the umbral and penumbral spectra are bordered by bright margins along their entire length.

Each of the spectrum plates covered a range of about 300 A. They were microphotometered perpendicular to the direction of dispersion at

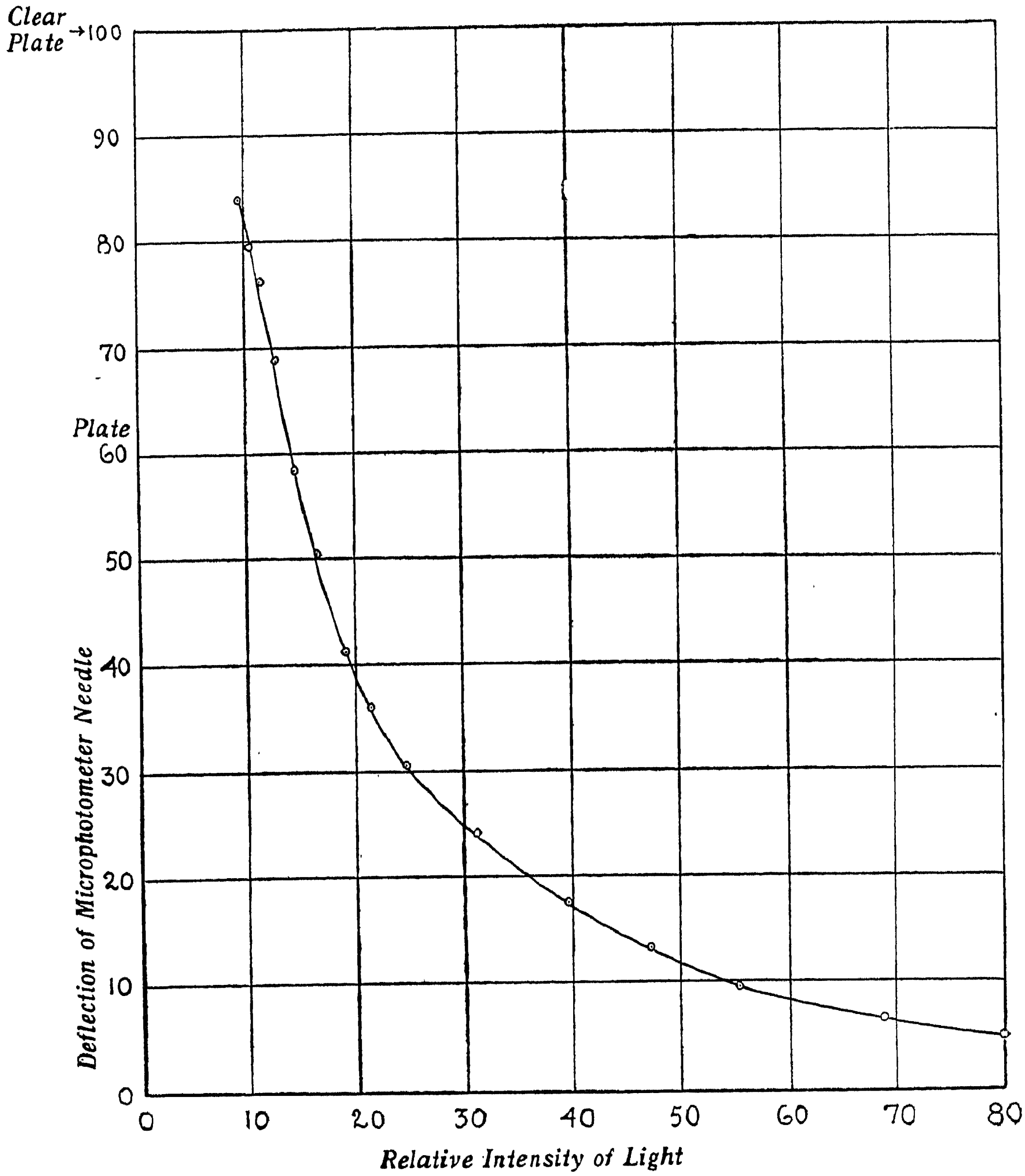

FIG. 4 *b*. Typical Standardising Curve

selected wavelengths in the continuous spectrum using the Cambridge photo-electric microphotometer of the observatory. All records were taken under a magnification of ×50, care being taken to avoid the absorption lines. A typical record is reproduced in Pl. XIV, Fig. 4 (*a*). The density variations were converted into intensity variations using the characteristic curves worked out from the intensity marks Fig. 4 (*b*). Figs. 5 (*a*), (*b*), (*c*) and (*d*)

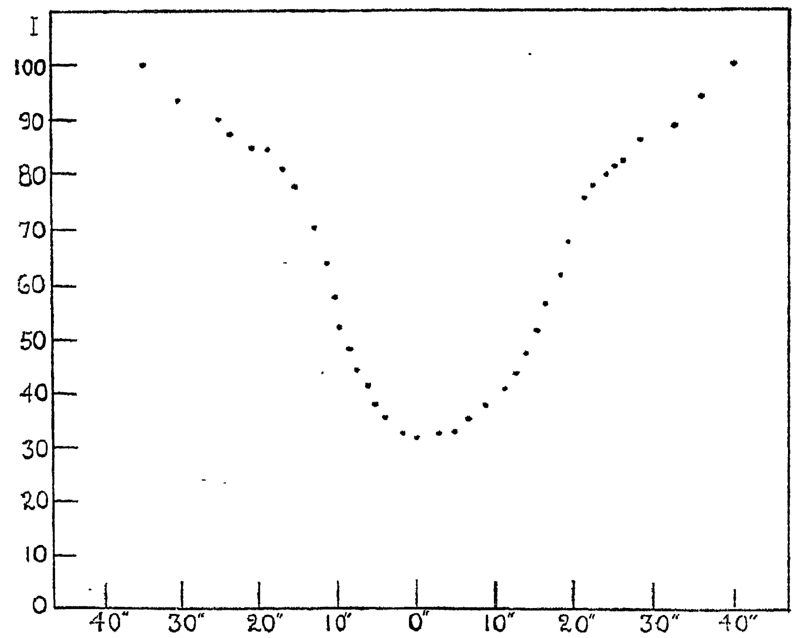

FIG. 5 *a*. Intensity Variation across Leading Spot of KKL 9620 (λ 6620 A).

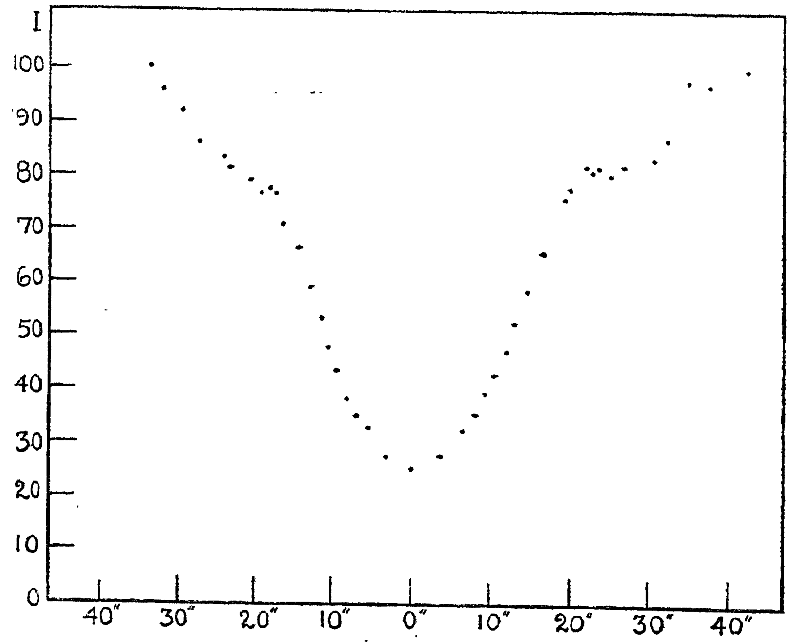

FIG. 5 *b*. Intensity Variation across Leading Spot of KKL 9620 (λ 6002 A)

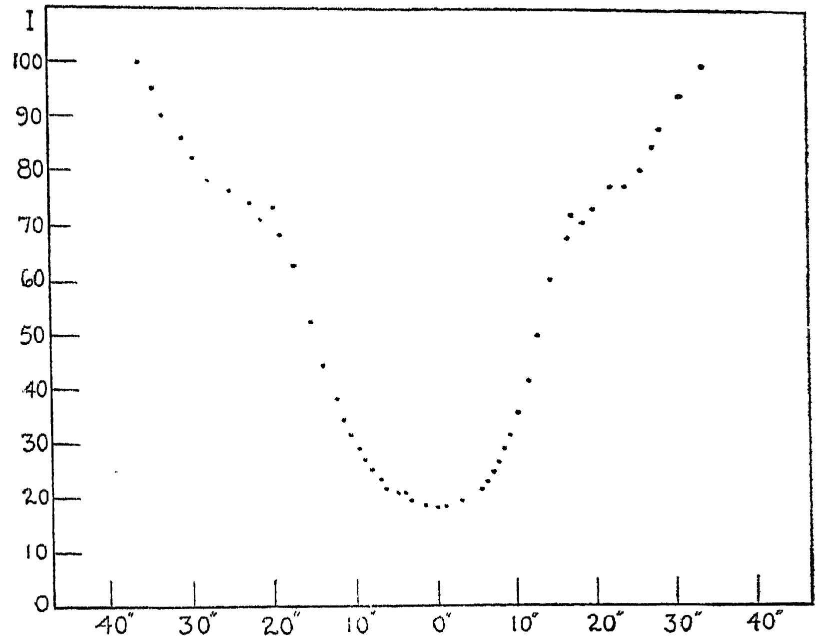

FIG. 5 *c*. Intensity Variation across Leading Spot of KKL 9620 (λ 5290 A)

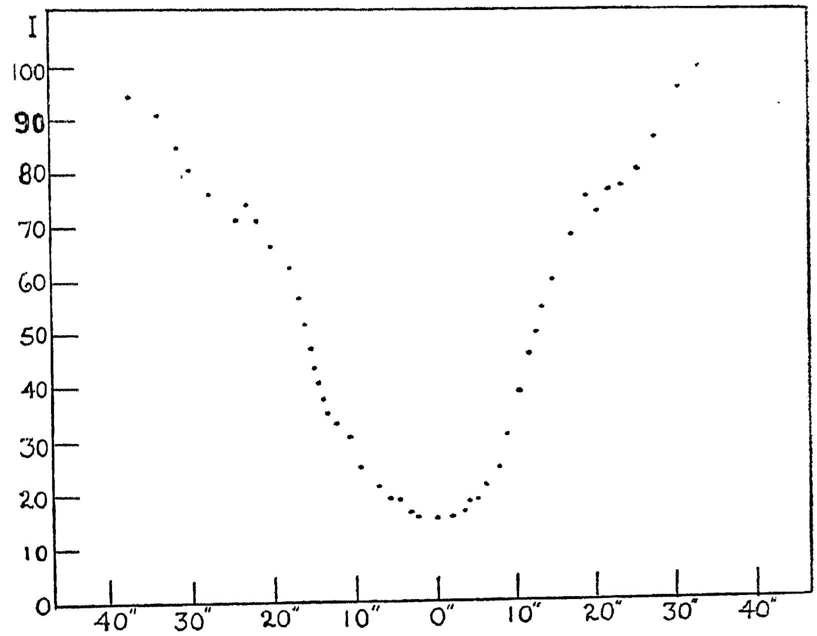

FIG. 5 *d*. Intensity Variation across Leading Spot of KKL 9620 (λ 5078 A)

illustrate typical curves of the intensity variations across one of the sunspots studied. The abscissa for these curves is seconds of arc (geocentric) and the ordinate relative intensities. (The intensity of the photosphere sufficiently far away from the spot is taken as 100.)

Table III gives the relative intensities of the umbra and the penumbra for the various spots studied. The penumbral intensities are the average of the values on either side of the umbra.

TABLE III

Spot No.	λ (Å)	Rel. Int. (%)		Eff. Temp. (° K.)	
		U	P	U	P
KKL 9620 (13–5–1951)	6620	31·8	82·5	4420	5460
	6540	27·2	81·2	4300	5440
	6375	27·5	81·3	4330	5450
	6002	25·2	79·7	4310	5450
	5871	23·8	80·4	4300	5460
	5768	22·6	79·2	4280	5430
	5290	18·1	73·9	4220	5390
	5170	14·4	70·2	4100	5350
	5078	15·3	73·2	4160	5390
KKL 9687 (19–5–1951)	6620	27·6	76·5	4270	5360
	6342	27·0	77·4	4310	5400
KKL 9688 (19–5–1951)	6620	28·2	77·0	4320	5360
	6342	25·2	77·8	4170	5400
KKL 9692 (24–5–1951)	6620	21·8	76·8	4130	5360
	6540	16·6	66·1	3930	5160
	6375	15·7	66·3	3910	5220
KKL 9768 (14–10–1951)	6620	30·3	85·3	4380	5510
	6540	29·0	75·5	4350	5360
	6375	25·7	75·6	4260	5360
Do (16–10–1951)	6620	28·4	78·0	4320	5400
	6540	26·2	75·3	4270	5350
Do (18–10–1951)	6375	22·0	81·3	4150	5490
	6620	29·2	79·0	4340	5420

U = umbra; P = penumbra

Because of the rapid fall in the sensitivity of Ilford special rapid panchromatic H-α plates on the longer wavelength side of H_α it was found that the same spectrum could not be utilised for photometry for wavelengths on the red and violet sides the H_α line. Spectra with longer exposures (about 3 to 4 seconds) had to be used for the red side of H_α. This fact would, to some extent, account for the sudden change in the intensity ratio for a comparatively small change of wavelength shown by some spots listed in Table III. Nevertheless, it is clear that the spot of May 24 had a smaller umbral intensity as compared with the other spots.

The major sources of error in the spectrophotometry of sunspots arise from the scattering by the earth's atmosphere and by the instrumental optics as well as from " poor seeing ". The result of all these is to throw photospheric light into the spot and thereby enhance the measured intensity ratio umbra/photosphere. The manner in which the measured values may be corrected has been theoretically discussed at some length by Wanders (1934). The method is based on the study of the observed curve of intensity fall just outside the solar limb *photographed simultaneously with the sunspot spectrum under identical sky and instrumental conditions.* However, the practical application of the formula worked out by Wanders is far from simple and he himself has made use of a procedure originally employed by Pettit and Nicholson to correct his measurements. These authors measured the ratio of the radiation from the sky and instrument 15″ of arc outside the solar limb in terms of the radiation 15″ of arc within the limb. If this ratio is x then they assumed that the measured energy ratio umbra/photosphere has to be diminished by $2x$. Wormell corrected his measurements by making some simplifications in the formula given by Wanders to render its practical applicability more easy.

In the present work the spectra of the limb and adjacent sky were photographed on some days almost simultaneously with the spot spectra to find out the magnitude of the correction for the measured umbral intensities. An attempt was made to evaluate this correction on the lines of Wormell. However, the method was found to be rather uncertain and in some respects arbitrary. Hence the attempt was given up. All the umbral and penumbral intensities given in Table III are only the values as obtained by direct measurements on the plates. Almost all the spot spectra were photographed between 8 and 10 A.M. when the definition of the solar image is comparatively good at Kodaikanal; sky conditions were generally good on all the days on which the spectra were photographed. Hence the corrections required for the measured umbral intensities are expected to be small.

5. Effective Temperatures of Spots

According to the measurements of Wanders and Wormell, the intensity ratio umbra/photosphere varies within wide limits depending upon the size of the spot. Wanders found values of umbral intensities ranging from 5% to 40% of the photospheric intensity in the λ 4000 A region, while in integrated light Wormell found values ranging from 28% to over 70%. The values in Table III also show some variations, but the range of variation is smaller. This is probably because all the spots were comparatively large and the sky and seeing conditions generally good on all the days.

TABLE IV

Intensity Ratio $I(\lambda, T^*)/I(\lambda, T)$ *according to Planck's Law* ($T = 5740°$ K.)

T^* °K \ λ(A)	3000	3500	4000	4500	5000	5500	6000	6500	7000	7500	8000	8500	9000	9500	10000
8000 ..	1052·0	754·8	587·3	486·4	418·1	370·4	335·5	308·9	288·0	271·8	258·1	246·8	237·6	229·8	223·3
7500 ..	706·2	535·3	434·6	371·2	327·3	297·0	272·6	254·1	239·8	228·1	218·9	210·6	204·3	198·5	193·4
7000 ..	447·8	362·1	308·6	273·4	247·9	229·5	215·3	204·0	194·9	187·8	181·6	176·7	172·2	168·5	165·0
6500 ..	264·9	231·2	207·7	191·9	180·2	171·2	164·4	158·6	153·8	150·1	146·9	144·2	142·1	140·0	138·1
6000 ..	143·6	136·6	131·2	127·4	124·3	121·9	120·1	118·7	117·1	116·1	115·1	114·5	113·8	113·2	112·7
5740 ..	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
5500 ..	69·66	73·35	76·07	78·52	80·20	81·85	83·24	84·33	85·29	86·42	86·78	87·40	87·96	88·67	88·96
5000 ..	29·25	34·84	39·64	43·98	47·58	50·78	53·67	56·14	58·28	60·22	61·94	63·52	64·91	66·18	67·26
4500 ..	10·19	14·07	17·92	21·69	25·19	28·43	31·49	34·24	36·79	39·10	41·26	43·22	45·04	46·62	48·14
4000 ..	2·70	4·50	6·62	8·95	11·33	13·76	16·20	18·51	20·74	22·88	24·89	26·82	28·64	30·32	31·88
3500 ..	0·488	1·04	1·84	2·87	4·08	5·43	6·90	8·43	9·95	11·52	13·08	14·61	16·10	17·51	18·91
3000 ..	0·050	0·149	0·336	0·635	1·04	1·57	2·21	2·94	3·75	4·63	5·56	6·52	7·50	8·49	9·48

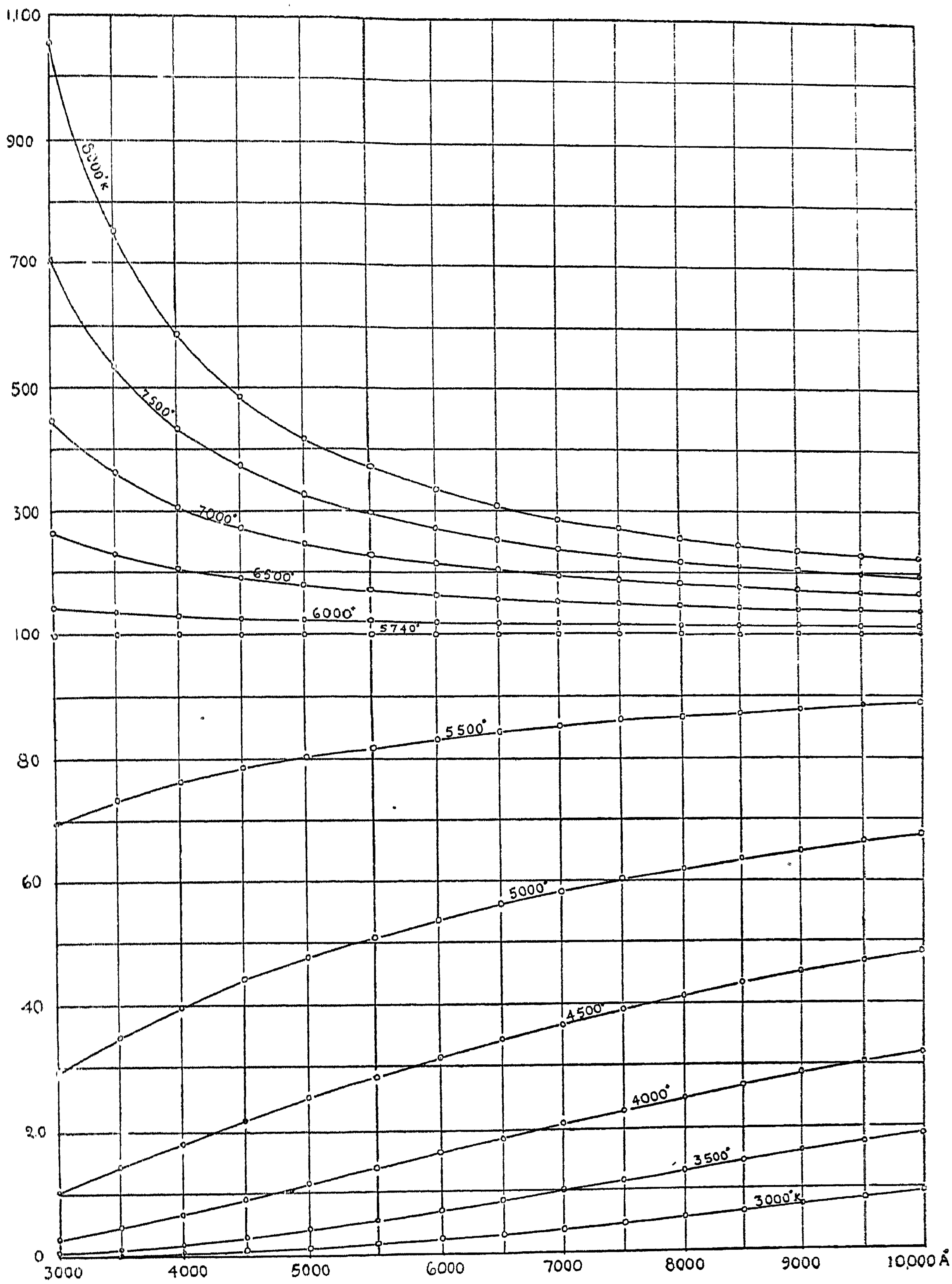

FIG. 6. Curves of I (λ, T*)/I (λ, T)

Assuming T = 5740° K. for the effective temperature of the photosphere, the values of the ratio I (λ, T*)/I (λ, T) for various values of λ and T* computed according to Planck's radiation formula are given in Table IV and represented graphically in Fig. 6, for the temperature range 3000° K. to 8000° K. and for the wavelength range λ 3000 A to λ 10000 A.

The range of values for the intensity ratio umbra/photosphere found by Wanders corresponds to effective spot temperatures from a little under 4000° K. to a little over 5000° K. For total radiation the values of Wormell give effective spot temperatures ranging from 4175° K. to 5250° K. The effective temperatures for the spots studied in the present work are given in Table III.

6. ORIGIN OF THE "BRIGHT RINGS" IN SUNSPOTS

Odgers (1946) has given a theory of sunspots which indicates the probable origin of the bright rings seen around the umbra and the penumbra. He regards the sunspot as a region where the product of the mass absorption coefficient and the density is higher than that in the photosphere. It is well known that this product is a measure of the "obstructive power" of the material of the stellar atmosphere to the passage of radiation through it. Hence the assumption of Odgers implies that radiation encounters greater obstruction to its passage through the sunspot region than through the surrounding photosphere. The mathematical problem of evaluating the energy density and the flux of radiant energy inside and outside the sunspot region assuming radiative equilibrium in both cases becomes analogous to the classical electrostatic problem of the effect of introducing a dielectric in a uniform electric field.* Three cases have been considered by Odgers, *viz.*, (i) when the sunspot can be idealised by a sphere of radius a and absorption coefficient k_1/ρ_1 embedded in the homogeneous radiation field of the surrounding photosphere whose absorption coefficient is k/ρ; (ii) when the sunspot has the shape of a prolate ellipsoid; (iii) when the sunspot has the shape of an oblate ellipsoid. In all cases the problem reduces to the solution of Laplace's equation with the appropriate boundary conditions.

For the first case referred to above, the solutions are:—

$$\mathrm{E}_0 = \mathrm{F}x\left[1 - \frac{k-k_1}{k+2k_1}\left(\frac{a}{r}\right)^3\right] + \mathrm{A} \tag{1}$$

$$\mathrm{E}_i = \mathrm{F}x\left[\frac{3k_1}{k+2k_1}\right] + \mathrm{A}, \tag{2}$$

* Sir James Jeans, *The Mathematical Theory of Electricity and Magnetism*, Fifth Edn., p. 228 (1951).

Assuming T = 5740° K. for the effective temperature of the photosphere, the values of the ratio I (λ, T*)/I (λ, T) for various values of λ and T* computed according to Planck's radiation formula are given in Table IV and represented graphically in Fig. 6, for the temperature range 3000° K. to 8000° K. and for the wavelength range λ 3000 A to λ 10000 A.

The range of values for the intensity ratio umbra/photosphere found by Wanders corresponds to effective spot temperatures from a little under 4000° K. to a little over 5000° K. For total radiation the values of Wormell give effective spot temperatures ranging from 4175° K. to 5250° K. The effective temperatures for the spots studied in the present work are given in Table III.

6. ORIGIN OF THE "BRIGHT RINGS" IN SUNSPOTS

Odgers (1946) has given a theory of sunspots which indicates the probable origin of the bright rings seen around the umbra and the penumbra. He regards the sunspot as a region where the product of the mass absorption coefficient and the density is higher than that in the photosphere. It is well known that this product is a measure of the "obstructive power" of the material of the stellar atmosphere to the passage of radiation through it. Hence the assumption of Odgers implies that radiation encounters greater obstruction to its passage through the sunspot region than through the surrounding photosphere. The mathematical problem of evaluating the energy density and the flux of radiant energy inside and outside the sunspot region assuming radiative equilibrium in both cases becomes analogous to the classical electrostatic problem of the effect of introducing a dielectric in a uniform electric field.* Three cases have been considered by Odgers, *viz.*, (i) when the sunspot can be idealised by a sphere of radius a and absorption coefficient k_1/ρ_1 embedded in the homogeneous radiation field of the surrounding photosphere whose absorption coefficient is k/ρ; (ii) when the sunspot has the shape of a prolate ellipsoid; (iii) when the sunspot has the shape of an oblate ellipsoid. In all cases the problem reduces to the solution of Laplace's equation with the appropriate boundary conditions.

For the first case referred to above, the solutions are:—

$$E_0 = Fx\left[1 - \frac{k - k_1}{k + 2k_1}\left(\frac{a}{r}\right)^3\right] + A \tag{1}$$

$$E_i = Fx\left[\frac{3k_1}{k + 2k_1}\right] + A, \tag{2}$$

* Sir James Jeans, *The Mathematical Theory of Electricity and Magnetism*, Fifth Edn., p. 228 (1951).

where E_0 and E_i are the energy densities outside and inside the spot, r is the distance from the centre of the sunspot sphere which is taken as the origin of co-ordinates, x is the distance measured in the direction of the flux, and A is a constant.

Sufficiently away from spot ($r = \infty$) the energy density in the photosphere is given by:

$$E_\infty = Fx + A$$

If F_i is the energy flux inside the spot we have:

$$F_i \propto \frac{1}{k_1}\left(\frac{\partial E_i}{\partial x}\right) = F\left[\frac{3}{k + 2k_1}\right] = \text{constant.}$$

In the yz plane the energy flux outside the spot is given by:

$$F_0 \propto \frac{1}{k}\left(\frac{\partial E_0}{\partial x}\right)_{x=0} = \frac{F}{k}\left[1 - \frac{k - k_1}{k + 2k_1} \cdot \left(\frac{a}{r}\right)^3\right]$$

$$F_\infty \propto F/k.$$

Hence:

$$F_i/F_\infty = \frac{3k}{k + 2k_1} \tag{3}$$

$$F_0/F_\infty = \left[1 - \frac{k - k_1}{k + 2k_1}\left(\frac{a}{r}\right)^3\right]. \tag{4}$$

From (3) and (4) we see that the ratio of the flux just outside the spot ($r=a$) to that inside is given by k_1/k. From the observed intensity ratio umbra/photosphere we can calculate k_1/k according to equation (3). The manner in which the energy flux outside the spot falls off with distance from the centre of the spot for various values of the intensity ratio umbra/photosphere is shown in Table V.

The flux is a maximum just outside the spot and falls off at first rapidly and then slowly till it merges with the value for the undisturbed photosphere. Physically this means that the flux held back by the increased obstructive power of the spot region reappears as an excess of flux immediately surrounding the spot analogous to the flow round a sphere immersed in a fluid moving with uniform velocity. The increased energy flux around the spot appears as the " bright ring ".

Odgers has also discussed the case of a spherical spot with umbra and penumbra. In this case three different values of k corresponding to the umbra, penumbra and the photosphere have to be considered. The analysis shows that there should be two bright rings one around the umbra and

TABLE V

Decrease of Energy Flux Outside a Spherical Spot

F_i/F_∞	k_1/k	Values of F_0/F_∞ for various values of r										
		$r=a$	$r=1\cdot 1a$	$r=1\cdot 2a$	$r=1\cdot 3a$	$r=1\cdot 5a$	$v=1\cdot 7a$	$r=1\cdot 9a$	$r=2\cdot 2a$	$r=2\cdot 5a$	$r=3\cdot 0a$	$r=\infty$
0 ..	∞		1·338	1·260	1·205	1·133	1·091	1·066	1·042	1·029	1·017	1·0
0·1 ..	14·5	1·45	1·301	1·232	1·182	1·119	1·081	1·058	1·038	1·026	1·015	1·0
0·2 ..	7·0	1·40	1·263	1·203	1·159	1·104	1·071	1·051	1·033	1·022	1·013	1·0
0·3 ..	4·5	1·35	1·225	1·174	1·137	1·089	1·061	1·044	1·028	1·019	1·011	1·0
0·4 ..	3·25	1·30	1·188	1·145	1·114	1·074	1·051	1·036	1·023	1·016	1·009	1·0
0·5 ..	2·5	1·25	1·150	1·116	1·091	1·059	1·041	1·029	1·019	1·013	1·007	1·0
0·6 ..	2·0	1·20	1·113	1·087	1·068	1·044	1·031	1·022	1·014	1·010	1·006	1·0
0·7 ..	1·643	1·15	1·075	1·058	1·046	1·030	1·020	1·015	1·009	1·006	1·004	1·0
0·8 ..	1·375	1·10	1·038	1·029	1·023	1·015	1·010	1·007	1·005	1·003	1·002	1·0
0·9 ..	1·167	1·05	1·0	1·0	1·0	1·0	1·0	1·0	1·0	1·0	1·0	1·0
1·0 ..	1·0	1·0										

the other around the penumbra. The consideration of spots which resemble prolate or oblate ellipsoids shows that the numerical value of the sudden increase in flux just outside the spot depends on the shape of the spot. For a spot whose depth is great compared with the aperture the increase of flux just outside the spot is smaller than for a shallow spot.

According to Odgers' theory the variation in the intensity ratio umbra/photosphere as well as the variation in the depth/aperture ratio of spots can apparently account, at least qualitatively, for the observed variation in the intensity of the bright rings. There is no other theory which attempts to explain this almost universal phenomenon associated with sunspots. There is however an important point which has to be remembered in this connection. It is generally believed that the depth of a sunspot is much smaller compared with its aperture. In the case of medium and large spots the aperture is of the order of 10^4 km, while according to Unsöld's theory, for instance, the depth will only be of the order of 10^2 km. In such a case, the intensity of the bright ring should be quite appreciable according to Odgers' theory. However, the observed intensities are generally small judged from visual and microphotometric examination of sunspots. For the spots studied in the present work the inner bright ring was about 2% more intense than the penumbra in one case and less than this in the other cases. The brightness of the outer ring was also estimated to be the same order. Interpreted in terms of Odgers' theory this would imply that spots extend to appreciable depths below the photosphere.

Summary

After a brief review of some of the earlier work on sunspots the experimental technique employed for the spectrophotometry of sunspots and the results obtained in the case of 5 spots are described. In the case of one of the spots studied on 1951 May 13, the intensity ratio umbra/photosphere decreased from about 32% at λ 6620 A to 15% at λ 5078 A. The lowest value of 22% at λ 6620 A was recorded for the umbral intensity of a spot on May 24. Microphotometer records of two spots are reproduced showing the discontinuous nature of the transition from the umbra to the penumbra first observed visually by Secchi. Tables and curves of $I(\lambda, T^*)/I(\lambda, T)$ based on Planck's radiation law are given and the effective temperatures of the spots calculated. Odgers' theory of the "bright rings" in sunspots is considered and it is pointed out that the observed small values of excess intensity of the bright rings probably imply that sunspots extend to appreciable depths below the photosphere.

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