Soliton collisions with shape change by intensity redistribution in mixed coupled nonlinear Schrödinger equations

T. Kanna,[1,*] M. Lakshmanan,[2] P. Tchofo Dinda,[1] and Nail Akhmediev[3]

[1]*Laboratoire de Physique de l'Université de Bourgogne, UMR CNRS No 5027, Av. A Savary, BP47 870, 21078 Dijon Cédex, France*

[2]*Centre for Nonlinear Dynamics, Department of Physics, Bharathidasan University, Tiruchirapalli-620 024, India*

[3]*Optical Sciences Group, Research School of Physical Sciences and Engineering, The Australian National University, Canberra, ACT 0200, Australia*

(Received 7 July 2005; published 7 February 2006)

A different kind of shape changing (intensity redistribution) collision with potential application to signal amplification is identified in the integrable N-coupled nonlinear Schrödinger (CNLS) equations with mixed signs of focusing- and defocusing-type nonlinearity coefficients. The corresponding soliton solutions for the $N=2$ case are obtained by using Hirota's bilinearization method. The distinguishing feature of the mixed sign CNLS equations is that the soliton solutions can both be singular and regular. Although the general soliton solution admits singularities we present parametric conditions for which nonsingular soliton propagation can occur. The multisoliton solutions and a generalization of the results to the multicomponent case with arbitrary N are also presented. An appealing feature of soliton collision in the present case is that all the components of a soliton can simultaneously enhance their amplitudes, which can lead to a different kind of amplification process without induced noise.

DOI: 10.1103/PhysRevE.73.026604 PACS number(s): 42.65.Tg, 42.81.Dp, 02.30.Ik

I. INTRODUCTION

It was suggested a long time ago that solitons could be used to carry data at a very high bit rate in optical communication systems, because of their ability to overcome the dispersion limitation through a balance between the self-phase modulation and dispersion effects [1]. In fact soliton pulses are known to have many other desirable properties, such as their robustness against small changes in the pulse shape or amplitude around the exact soliton profile leads to treat such changes only as small perturbations on soliton propagation [2–4]. Strictly speaking, the soliton properties can exit only in an ideal fiber. Indeed, in a standard telecommunication fiber, the propagation of light pulses gives rise to a host of perturbing effects which inhibit the desirable properties of solitons [5]. One of the strongly perturbing effects that comes inevitably into play is the linear attenuation of light along the fiber (which is of the order of 0.2 dB/km at carrier wavelength 1.55 μm), which does not permit us to keep a constant balance between the self-phase modulation and the group-velocity dispersion [5]. Although the fundamental soliton propagation cannot be obtained in standard fibers, pulse propagation over relatively long distances (and even transoceanic distances) can still be obtained through an appropriate combination of dispersion management and optical amplification (now mostly based on Er-doped fiber amplifiers and Raman amplifiers) [6–8].

All the existing amplification processes involve three major ingredients: The first one is a *pump wave*, which serves as a photon reservoir. The second one is an *amplification medium*, that is, a special material in which the pump wave is mixed with the signal. The third ingredient is a *physical mechanism* that can cause a transfer of photons from the pump to the signal. Only three types of physical mechanisms have been exploited so far in optical amplifiers, namely the *laser process* used in laser optical amplifiers (e.g., Er-doped fiber amplifiers, semiconductor optical amplifiers) [9], the *stimulated Raman scattering* (used in Raman amplifiers) [5,8] and *parametric wave mixing* (used in parametric amplifiers) [5,8]. Such optical amplifiers do permit us to fully compensate the fiber losses, but the amplification process is unavoidably accompanied by an undesirable effect of noise generation which is commonly referred to as the "amplified spontaneous emission" (ASE) [10–12]. Hence one of the most important characteristic parameters of the optical amplifiers developed so far is the so-called "noise figure," which serves as a measure of the amount of noise generated during the amplification process [13]. The ASE increases with the amplifier gain, and there exists an unavoidable amount of noise, known as the amplifier noise figure limit of 3 dB [13–15]. The ASE is one of the major effects that severely degrades the transmission quality of ultrashort light pulses over long distances [5,7,16]. To radically resolve the problem of ASE limitation in high-speed long-distance transmission systems, it is clear that the conceptual approach of optical amplification based on the three ingredients mentioned above needs to be partially or totally reformulated.

In the present work, we examine shape changing (intensity redistribution) collisions of vector solitons in mixed coupled nonlinear Schrödinger (CNLS) equations, and report some results that suggest the possibility of constructing a different approach of signal amplification. The difference lies in viewing the collision process of solitons as a fundamental physical mechanism for transferring energy from the pump to the signal. The collision involves two vector solitons. One of the two solitons, say S_1, is chosen to be the signal, while

*Corresponding author. Email address: Thambithurai.Kanna@u-bourgogne.fr

the other soliton (S_2) serves as the energy reservoir (pump wave). The major virtue of this type of collision-based amplification process is that it does not induce any noise, as it does not make use of any external amplification medium.

On the other hand, the study of physical and mathematical aspects of CNLS equations is of considerable current interest as these equations arise in diverse areas of science like nonlinear optics, optical communication, biophysics, Bose-Einstein condensates, and plasma physics [3,4,17–19]. The fundamental integrable N-CNLS system is given by the following set of equations:

$$iq_{j,z} + q_{j,tt} + 2\mu\left(\sum_{l=1}^{N} \sigma_l |q_l|^2\right) q_j = 0, \quad j = 1,2, \ldots, N, \tag{1a}$$

where q_j, $j=1,2,\ldots,N$, is the complex amplitude of the jth component, the subscripts z and t denote the partial derivatives with respect to normalized distance and retarded time, respectively, μ represents the strength of nonlinearity ($\mu > 0$) and the coefficients σ_l's define the sign of the nonlinearity. System (1a) can be classified into three classes as focusing, defocusing, and mixed types depending on the signs of the nonlinearity coefficients σ_l's. The focusing case arises where all σ_l's are equal to 1 and the corresponding system admits bright soliton solutions [20–25]. These bright solitons are found to undergo fascinating shape changing (intensity redistribution) collisions [21,23,24] (for other details see, for example, Refs. [26–28]) and such collision properties are not observed in systems with defocusing nonlinearity which arises for all $\sigma_l=-1$ in Eq. (1a). The latter system possesses either dark solitons in all the components or dark-bright solitons which undergo standard elastic collision [25,29,30]. Also special analytic solutions for the focusing and defocusing types are given in Refs. [31,32]. The third case arises for mixed signs of σ_l's (that is, +1 or −1). For convenience, we define σ_l's for this mixed case as

$$\begin{aligned} \sigma_l &= 1 \quad \text{for } l = 1,2, \ldots ,n, \\ &= -1 \quad \text{for } l = n+1, n+2, \ldots ,N. \end{aligned} \tag{1b}$$

Here onwards we refer to Eq. (1) with the above choice of σ_l's as *mixed* CNLS equations.

From a physical point of view, system (1) with $N=2$ corresponds to the modified Hubbard model in one dimension [33]. A similar equation, for $N=2$, is observed in the context of electromagnetic pulse propagation in left handed materials [34]. The above set of equations (1) is found to be completely integrable [33,35,36] and the corresponding Lax pair was obtained in Ref. [33]. In their pioneering works Makhankov *et al.* [33,35] have shown that Eq. (1), for $N=2$, admits particular bright-bright, bright-dark, dark-dark type one soliton solutions depending upon the asymptotic behavior of the complex amplitudes q_j, $j=1,2$. Since then very few works have appeared in the literature to analyze the problem further [25,29,37–39] (for a detailed review of existing results one can refer to Ref. [38]). Particularly, in a recent work [38], Kanna *et al.* have obtained stationary solutions of mixed CNLS equations with singularities by following an algebraic approach [22,40,41]. It was observed that despite the points of singularities the solutions behave smoothly in a finite region of the temporal domain. Then the natural question arises as to whether multisoliton solutions exhibiting regular behavior over the entire space-time regions exist and, if so, what is the nature of soliton interactions?

Being motivated by the above fundamental and intriguing aspects, in the present paper we perform a detailed study on the bright soliton collision dynamics arising in the mixed CNLS system. In particular, we point out that bright solitons of regular type do exist, provided the soliton parameters satisfy certain conditions and that the underlying solitons undergo interesting shape changing–intensity redistribution collisions. The singular solutions turn out to be special cases (with specific parametric choices) of the general soliton solutions. An important feature which we identify in the collision process of regular solitons in the mixed CNLS case is that after collision a soliton can gain energy in all its components, while the opposite takes place in the other soliton.

This paper is organized as follows. Section II contains the details of Hirota's bilinearization procedure [42] for the CNLS equations to obtain soliton solutions. Though the solutions obtained in this paper admit both singular and nonsingular behaviors, we call them soliton solutions ascribing to their soliton nature in some specific region. In Sec. III, we obtain the one and two soliton solutions. Section IV is devoted to a detailed analysis of shape changing (intensity redistribution) collisions exhibited by these soliton solutions. The procedure to obtain one and two soliton solutions is extended to multisoliton solutions in Sec. V. The results of two component case are generalized in a systematic way to the multicomponent case with arbitrary number of components following the lines of Ref. [24]. The final section is allotted for a conclusion. In Appendix A we present the singular stationary three soliton solution for mixed 3-CNLS equations. The multicomponent multisoliton solutions of mixed N-CNLS equations, for arbitrary N, is given in Appendix B.

II. BILINEARIZATION OF MIXED CNLS EQUATIONS

The set of equations (1) has been shown to be completely integrable [33,36], admitting certain types of single soliton solutions [33,35], for the $N=2$ case, as mentioned in the Introduction. Here we are concerned with bright-bright multisoliton solutions whose intensity profiles vanish asymptotically and with the nature of soliton interactions.

Let us apply the bilinearizing transformation [42]

$$q_j = \frac{g^{(j)}}{f}, \quad j = 1,2, \ldots ,N \tag{2}$$

to Eq. (1) similar to the focusing case $\sigma_l=1$, $l=1,2,\ldots,N$ [24]. This results in the following set of bilinear equations:

$$(iD_z + D_t^2) g^{(j)} \cdot f = 0, \quad j = 1,2, \ldots ,N, \tag{3a}$$

$$D_t^2(f\cdot f)=2\mu\sum_{l=1}^{N}\sigma_l g^{(l)}g^{(l)*}, \tag{3b}$$

where σ_l is given by Eq. (1b), $*$ denotes the complex conjugate, $g^{(j)}$'s are complex functions, while $f(z,t)$ is a real function, and the Hirota's bilinear operators D_z and D_t are defined by

$$D_z^n D_t^m(a\cdot b)=\left(\frac{\partial}{\partial z}-\frac{\partial}{\partial z'}\right)^n\left(\frac{\partial}{\partial t}-\frac{\partial}{\partial t'}\right)^m a(z,t)b\,(z',t')|_{(z=z',t=t')}. \tag{3c}$$

The above set of equations can be solved by introducing the following power series expansions for $g^{(j)}$'s and f:

$$g^{(j)}=\chi g_1^{(j)}+\chi^3 g_3^{(j)}+\cdots,\quad j=1,2,\ldots,N, \tag{4a}$$

$$f=1+\chi^2 f_2+\chi^4 f_4+\cdots, \tag{4b}$$

where χ is the formal expansion parameter. The resulting set of equations, after collecting the terms with the same power in χ, can be solved recursively to obtain the forms of $g^{(j)}$'s and f.

III. SOLITON SOLUTIONS FOR $N=2$ CASE

The mixed system (1) with $N=2$ and $\sigma_1=1$, $\sigma_2=-1$ is of special physical interest. To start with, we consider this particular case.

A. One soliton solution

In order to write down the one soliton solution we restrict the power series (4) to the lowest order,

$$g^{(j)}=\chi g_1^{(j)},\quad j=1,2,\quad f=1+\chi^2 f_2. \tag{5}$$

Then by solving the resulting set of linear partial differential equations recursively, one can write down the explicit one soliton solution as

$$\begin{pmatrix}q_1\\ q_2\end{pmatrix}=\begin{pmatrix}\alpha_1^{(1)}\\ \alpha_1^{(2)}\end{pmatrix}\frac{e^{\eta_1}}{1+e^{\eta_1+\eta_1^*+R}} \tag{6a}$$

$$=\begin{pmatrix}A_1\\ A_2\end{pmatrix}k_{1R}\,\mathrm{sech}\left(\eta_{1R}+\frac{R}{2}\right)e^{i\eta_{1I}}, \tag{6b}$$

where

$$\eta_1=k_1(t+ik_1 z)=\eta_{1R}+i\eta_{1I}, \tag{6c}$$

$$A_j=\frac{\alpha_1^{(j)}}{[\mu(\sigma_1|\alpha_1^{(1)}|^2+\sigma_2|\alpha_1^{(2)}|^2)]^{1/2}},\quad j=1,2,$$

$$e^R=\frac{\mu(\sigma_1|\alpha_1^{(1)}|^2+\sigma_2|\alpha_1^{(2)}|^2)}{(k_1+k_1^*)^2},\quad \sigma_1=-\sigma_2=1. \tag{6d}$$

Note that this one soliton solution is characterized by three arbitrary complex parameters $\alpha_1^{(1)}$, $\alpha_1^{(2)}$, and $k_1=k_{1R}+ik_{1I}$, where the suffixes R and I represent the real and imaginary parts, respectively. The quantities $k_{1R}A_1$ and $k_{1R}A_2$, give the amplitude of the soliton in components q_1 and q_2, respectively, subject to the condition

$$\sigma_1|A_1|^2+\sigma_2|A_2|^2=\frac{1}{\mu}, \tag{6e}$$

and the soliton velocity in each component is given by $2k_{1I}$. The position of the soliton is found to be

$$\frac{R}{2k_{1R}}=\frac{1}{2k_{1R}}\ln\left[\frac{\mu(\sigma_1|\alpha_1^{(1)}|^2+\sigma_2|\alpha_1^{(2)}|^2)}{(k_1+k_1^*)^2}\right]. \tag{6f}$$

From Eq. (6b), it is clear that singular solutions start occurring when $|\alpha_1^{(1)}|=|\alpha_1^{(2)}|$. In this case, one can easily observe from Eq. (6d) that the quantity e^R becomes 0, and one gets the solution

$$\begin{pmatrix}q_1\\ q_2\end{pmatrix}=\begin{pmatrix}\alpha_1^{(1)}\\ \alpha_1^{(2)}\end{pmatrix}e^{\eta_1} \tag{7}$$

which is unbounded. Such an unbounded solution is depicted in Fig. 1(a) for $k_1=1+i$, $\alpha_1^{(1)}=\alpha_1^{(2)}=1$, and $\mu=1$.

When $|\alpha_1^{(1)}|<|\alpha_1^{(2)}|$, e^R becomes negative (so R becomes complex). In this case, singularity occurs, whenever

$$1-|e^R|e^{2\eta_{1R}}=0, \tag{8a}$$

or

$$\eta_{1R}=\frac{1}{2}\ln\left(\frac{1}{|e^R|}\right). \tag{8b}$$

Again a singular solution in this case is plotted in Fig. 1(b) for $k_1=1+i$, $\alpha_1^{(1)}=0.8$, $\alpha_1^{(2)}=1$, and $\mu=1$.

However the bright soliton solution is always regular as long as the condition $|\alpha_1^{(1)}|>|\alpha_1^{(2)}|$ is valid in which case e^R is always real and positive, as the denominator $(1+e^{\eta_1+\eta_1^*+R})$ in

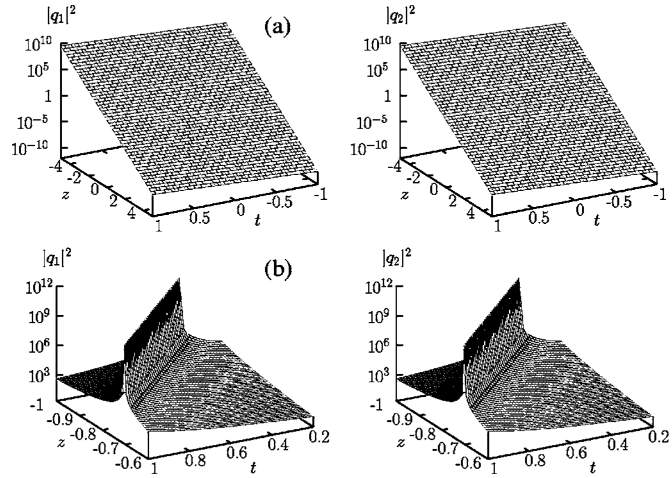

FIG. 1. Intensity plots of singular one soliton solution of Eq. (1) for $N=2$: (a) for the case $|\alpha_1^{(1)}|=|\alpha_1^{(2)}|$; (b) for the case $|\alpha_1^{(1)}|<|\alpha_1^{(2)}|$.

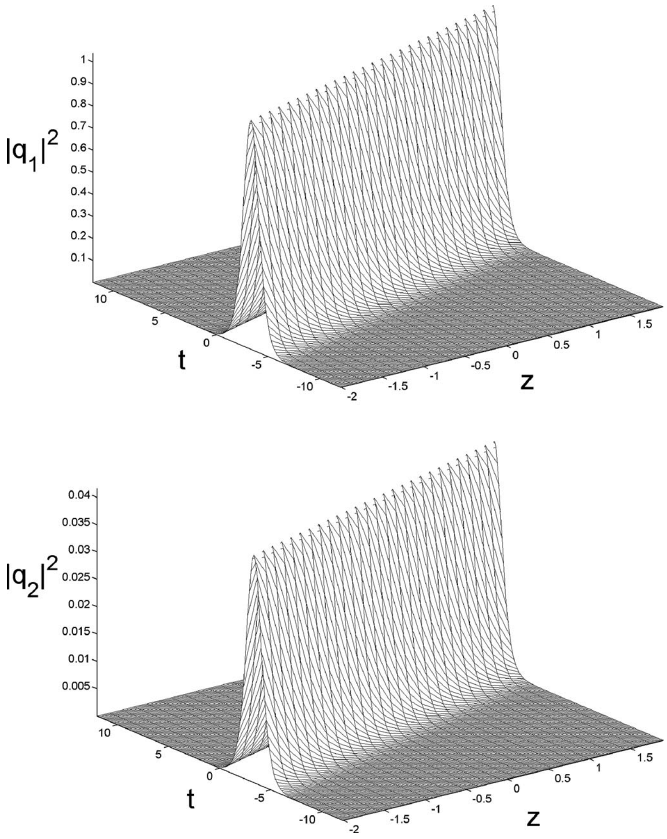

FIG. 2. Intensity plots of regular one soliton solution of Eq. (1) for the $N=2$ case.

Eq. (6a) is always positive definite (as η_{1R} is real) for this choice. This regular one soliton solution is shown in Fig. 2 for $k_1=1+i$, $\alpha_1^{(1)}=1$, $\alpha_1^{(2)}=0.2$, and $\mu=1$.

It is also interesting to note here that the polarization vector evolves in a hyperboloid defined by the surface $|A_1|^2-|A_2|^2=1/\mu$ [33], whereas in the Manakov case it is a sphere (that is $|A_1|^2+|A_2|^2=1/\mu$) [24]. This allows Eq. (1) to admit a rich variety of singular and nonsingular solutions and makes significant difference in the collision scenario of bright solitons arising in the two systems as we will see in the following sections.

B. Two soliton solution

To obtain the two soliton solution the power series expansion (4) is terminated at the higher-order terms

$$g^{(j)}=\chi g_1^{(j)}+\chi^3 g_3^{(j)}, \quad j=1,2, \tag{9a}$$

$$f=1+\chi^2 f_2+\chi^4 f_4. \tag{9b}$$

Then by solving the resultant linear partial differential equations recursively, we can write the explicit form of the solution as

$$q_j=\frac{\alpha_1^{(j)}e^{\eta_1}+\alpha_2^{(j)}e^{\eta_2}+e^{\eta_1+\eta_1^*+\eta_2+\delta_{1j}}+e^{\eta_1+\eta_2+\eta_2^*+\delta_{2j}}}{D}, \quad j=1,2, \tag{10a}$$

where

$$D=1+e^{\eta_1+\eta_1^*+R_1}+e^{\eta_1+\eta_2^*+\delta_0}+e^{\eta_1^*+\eta_2+\delta_0^*}+e^{\eta_2+\eta_2^*+R_2} + e^{\eta_1+\eta_1^*+\eta_2+\eta_2^*+R_3}. \tag{10b}$$

Various quantities found in Eq. (10), are defined as below:

$$\eta_i=k_i(t+ik_iz), \quad e^{\delta_0}=\frac{\kappa_{12}}{k_1+k_2^*}, \quad e^{R_1}=\frac{\kappa_{11}}{k_1+k_1^*},$$

$$e^{R_2}=\frac{\kappa_{22}}{k_2+k_2^*},$$

$$e^{\delta_{1j}}=\frac{(k_1-k_2)(\alpha_1^{(j)}\kappa_{21}-\alpha_2^{(j)}\kappa_{11})}{(k_1+k_1^*)(k_1^*+k_2)},$$

$$e^{\delta_{2j}}=(k_2-k_1)(\alpha_2^{(j)}\kappa_{12}-\alpha_1^{(j)}\kappa_{22})\Big/(k_2+k_2^*)(k_1+k_2^*),$$

$$e^{R_3}=\frac{|k_1-k_2|^2}{(k_1+k_1^*)(k_2+k_2^*)|k_1+k_2^*|^2}(\kappa_{11}\kappa_{22}-\kappa_{12}\kappa_{21}), \tag{10c}$$

and

$$\kappa_{ij}=\frac{\mu(\sigma_1\alpha_i^{(1)}\alpha_j^{(1)*}+\sigma_2\alpha_i^{(2)}\alpha_j^{(2)*})}{(k_i+k_j^*)}, \quad i,j=1,2, \tag{10d}$$

where $\sigma_1=1$ and $\sigma_2=-1$. This solution is characterized by six arbitrary complex parameters $\alpha_1^{(1)}$, $\alpha_1^{(2)}$, $\alpha_2^{(1)}$, $\alpha_2^{(2)}$, k_1, and k_2. Note that the form of the above two soliton solution remains the same as that of the Manakov case (where $\sigma_1=+1$, $\sigma_2=+1$) [21,24], except for the crucial difference that in the expressions for the parameters κ_{ij} in Eq. (10d), $\sigma_1=+1$ and $\sigma_2=-1$.

It can also be easily verified that the singular stationary solution for the $N=2$ case given by Eq. (17) in Ref. [38] can be obtained for the specific parametric choice

$$\alpha_1^{(1)}=-e^{\eta_{10}}, \quad \alpha_2^{(2)}=e^{\eta_{20}}, \quad \alpha_1^{(2)}=0, \quad \alpha_2^{(1)}=0, \tag{11}$$

$$k_{1I}=k_{2I}=0, \quad \mu=1,$$

where η_{10} and η_{20} are two arbitrary real parameters. For this choice of parameters, Eq. (10) reduces to the form

$$q_1=\frac{1}{\widetilde{D}}\left(-e^{\eta_1}+\frac{(k_{1R}-k_{2R})e^{\eta_1+\eta_2+\eta_2^*}}{4k_{2R}^2(k_{1R}+k_{2R})}\right), \tag{12a}$$

$$q_2=\frac{1}{\widetilde{D}}\left(e^{\eta_2}-\frac{(k_{1R}-k_{2R})e^{\eta_1+\eta_1^*+\eta_2}}{4k_{1R}^2(k_{1R}+k_{2R})}\right), \tag{12b}$$

where

$$\widetilde{D}=1+\left[\frac{e^{\eta_1+\eta_1^*}}{4k_{1R}^2}-\frac{e^{\eta_2+\eta_2^*}}{4k_{2R}^2}\right]-\frac{(k_{1R}-k_{2R})^2e^{\eta_1+\eta_1^*+\eta_2+\eta_2^*}}{16k_{1R}^2k_{2R}^2(k_{1R}+k_{2R})^2}, \tag{12c}$$

and η_j is redefined as

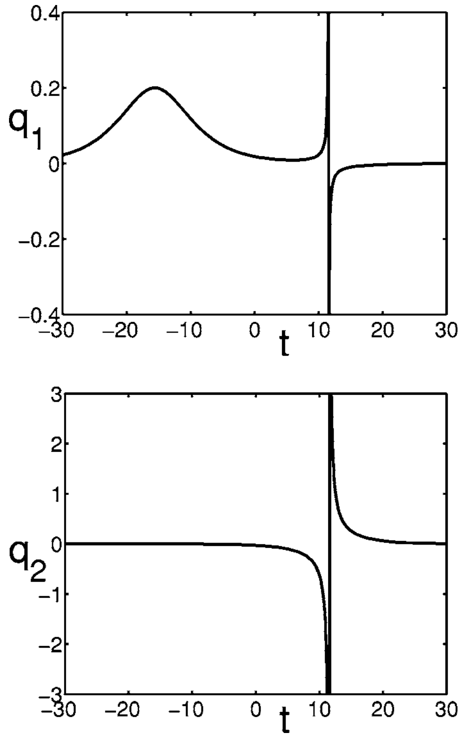

FIG. 3. Stationary singular two soliton solution for the $N=2$ case.

$$\eta_j = k_{jR}(t + ik_{jR}z) + \eta_{j0}, \quad j = 1,2, \tag{12d}$$

where η_{j0}'s are arbitrary real parameters. The above equation (12) can be expressed in terms of hyperbolic functions as

$$q_1 = \frac{2k_{1R}}{\hat{D}} \sqrt{\frac{k_{1R}+k_{2R}}{k_{1R}-k_{2R}}} \sinh\left[k_{2R}t + \eta_{20} + \frac{1}{2}\ln\left(\frac{k_{1R}-k_{2R}}{4k_{2R}^2(k_{1R}+k_{2R})}\right)\right] e^{ik_{1R}^2 z}, \tag{13a}$$

$$q_2 = -\frac{2k_{2R}}{\hat{D}} \sqrt{\frac{k_{1R}+k_{2R}}{k_{1R}-k_{2R}}} \sinh\left[k_{1R}t + \eta_{10} + \frac{1}{2}\ln\left(\frac{k_{1R}-k_{2R}}{4k_{1R}^2(k_{1R}+k_{2R})}\right)\right] e^{ik_{2R}^2 z}, \tag{13b}$$

where

$$\hat{D} = -\sinh\left[k_{1R}t + k_{2R}t + \eta_{10} + \eta_{20} + \ln\left(\frac{k_{1R}-k_{2R}}{2k_{1R}k_{2R}(k_{1R}+k_{2R})}\right)\right] + \left(\frac{k_{1R}+k_{2R}}{k_{1R}-k_{2R}}\right) \times\sinh\left[k_{1R}t - k_{2R}t + \eta_{10} - \eta_{20} + \ln\left(\frac{k_{2R}}{k_{1R}}\right)\right]. \tag{13c}$$

One can check that Eq. (17) given in Ref. [38] can be re-expressed in terms of hyperbolic functions in a form similar to Eq. (13). Figure 3 represents the stationary singular two soliton solution at $z=0$ for $k_{1R}=0.2$, $k_{2R}=-0.25$, $\alpha_1^{(1)}=-\alpha_2^{(2)}=-1$, $\alpha_1^{(2)}=\alpha_2^{(1)}=0$, and $\mu=1$.

Now from the expression (10) it can be observed that the denominator can become zero for finite values of z and t leading to singular solutions. However, in the case of the general two soliton solution (10), it is possible to make the denominator [D in Eq. (10b)] to be nonzero for any value of t and z for suitable choice of k_j and $\alpha_j^{(l)}$'s, $j,l=1,2$. In order to do so we rewrite the denominator D [see Eq. (10b)] as

$$D = 2e^{\eta_{1R}+\eta_{2R}}\{e^{(R_1+R_2)/2}\cosh[\eta_{1R}-\eta_{2R}+(R_1-R_2)/2] + e^{\delta_{0R}}\cos(\eta_{1I}-\eta_{2I}+\delta_{0I}) + e^{R_3/2}\cosh(\eta_{1R}+\eta_{2R}+R_3/2)\}, \tag{14a}$$

where the suffixes R and I denote the real and imaginary parts, respectively. Then the solution is regular if the above expression is positive for all values of z and t. For this purpose, a definite set of criteria can be identified as follows. As in the case of one soliton solution in Sec. III A, if we choose the parameters $\alpha_i^{(j)}$, $i,j=1,2$, such that $|\alpha_i^{(1)}|^2 > |\alpha_i^{(2)}|^2$, $i=1,2$, $k_{1R}>0$ and $k_{2R}>0$ then

$$\kappa_{11} > 0, \quad \kappa_{22} > 0. \tag{14b}$$

Correspondingly, from Eqs. (10c) we note that $e^{R_1}>0$ and $e^{R_2}>0$, so that $e^{R_1+R_2}>0$. Then, $e^{(R_1+R_2)/2}\cosh[\eta_{1R}-\eta_{2R}+(R_1-R_2)/2]>0$. There is also the other possibility $\kappa_{11}<0$, $\kappa_{22}<0$. But it will not lead to regular solution as in this case e^{R_1} and e^{R_2} become negative thereby making R_1 and R_2 complex.

The term $e^{R_3/2}$ becomes greater than zero if

$$\kappa_{11}\kappa_{22} - |\kappa_{12}|^2 > 0. \tag{14c}$$

Then for this choice $e^{R_3/2}\cosh(\eta_{1R}+\eta_{2R}+R_3/2)$ is always greater than zero.

However, the term $\cos(\eta_{1I}-\eta_{2I}+\delta_{0I})$ oscillates between -1 and 1. So in order that the middle term does not compensate the other two terms at any point in space or time resulting in D being equal to zero, we should have

$$e^{(R_1+R_2)/2} + e^{R_3/2} > e^{\delta_{0R}}. \tag{14d}$$

Consequently using the expressions (10c) in (14d) one may deduce the condition

$$\frac{1}{2}\sqrt{\frac{\kappa_{11}\kappa_{22}}{k_{1R}k_{2R}}} + \frac{|k_1-k_2|}{2|k_1+k_2^*|}\sqrt{\frac{\kappa_{11}\kappa_{22}-|\kappa_{12}|^2}{k_{1R}k_{2R}}} > \frac{|\kappa_{12}|}{|k_1+k_2^*|}. \tag{14e}$$

Note that the conditions (14b) and (14c) are necessary conditions to obtain regular solution as their falsity will always result in singular solution. Condition (14e) is a sufficient one as its validity confirms that the solution is always regular. We are unable to prove whether condition (14e) is also necessary or not due to the complicated form of the function D as a function of the variables t and z given by Eqs. (10b) and (14a). It appears that the latter can only be checked numerically for given soliton parameter values. In terms of soliton parameters the conditions (14b) and (14c) read as

$$|\alpha_1^{(1)}|^2 - |\alpha_1^{(2)}|^2 > 0, \tag{15a}$$

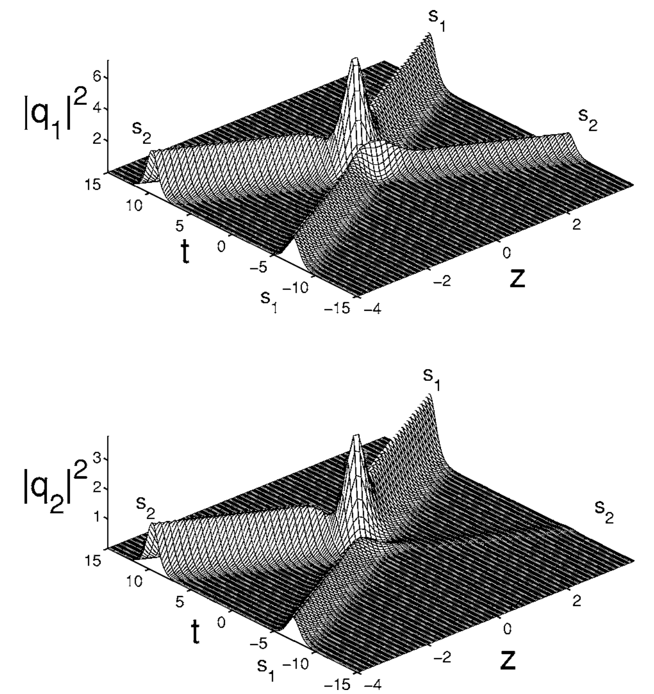

FIG. 4. Shape changing (intensity redistribution) collision of two solitons in the mixed CNLS system for the $N=2$ case.

$$|\alpha_2^{(1)}|^2-|\alpha_2^{(2)}|^2>0, \tag{15b}$$

while Eq. (14e) becomes

$$\frac{(|\alpha_1^{(1)}|^2-|\alpha_1^{(2)}|^2)(|\alpha_2^{(1)}|^2-|\alpha_2^{(2)}|^2)}{|\alpha_1^{(1)}\alpha_2^{(1)*}-\alpha_1^{(2)}\alpha_2^{(2)*}|^2}>\frac{16k_{1R}^2k_{2R}^2}{(k_{1R}+k_{2R})^2+(k_{1I}-k_{2I})^2}. \tag{15c}$$

Thus the two soliton solution satisfying these conditions represent the interaction of two finite amplitude bright solitons with definite velocities and their collision behavior is analyzed in the following section.

For illustrative purpose we consider the case $k_{1R}>0$, $k_{2R}>0$, $\mu=1$, $\alpha_1^{(1)}=\cosh(\theta_1)e^{i\phi_1}$, $\alpha_2^{(1)}=\cosh(\theta_2)e^{i\phi_1}$, $\alpha_1^{(2)}=\sinh(\theta_1)e^{i\phi_2}$, and $\alpha_2^{(2)}=\sinh(\theta_2)e^{i\phi_2}$, for some arbitrary θ_1,θ_2,ϕ_1, and ϕ_2. Then, the conditions (14b), (14c), and (14e) become

$$\kappa_{11}=\frac{1}{2k_{1R}},\quad \kappa_{22}=\frac{1}{2k_{2R}},$$

$$|k_1+k_2^*|^2-4k_{1R}k_{2R}\cosh^2(\theta_{12})>0,$$

$$\frac{1}{4k_{1R}k_{2R}}+\frac{|k_1-k_2|\sqrt{|k_1+k_2^*|^2-4k_{1R}k_{2R}\cosh^2(\theta_{12})}}{4k_{1R}k_{2R}|k_1+k_2^*|^2}>\frac{\cosh(\theta_{12})}{|k_1+k_2^*|}, \tag{16}$$

where $\theta_{12}=\theta_1-\theta_2$. A two soliton collision process corresponding to the condition (16) is shown in Fig. 4 for the parameter choice $k_1=1.0+i$, $k_2=1.1-i$, $\theta_1=0.8$, $\theta_2=0.2$, $\phi_1=1$, and $\phi_2=0.3$. This collision behavior is analyzed in detail in the following section.

IV. SHAPE CHANGING (INTENSITY REDISTRIBUTION) COLLISIONS OF SOLITONS

Now it is of interest to understand the collision behavior, shown in Fig. 4, of the regular two soliton solution. Figure 4 shows the interaction of two solitons S_1 and S_2 which are well separated before and after collision, in the q_1 and q_2 components. This figure shows that after collision, the first soliton S_1 in the component q_1 gets enhanced in its amplitude while the soliton S_2 is suppressed. Interestingly, the same kind of changes are observed in the second component q_2 as well. This collision scenario is entirely different from the one observed in the Manakov system where one soliton gets suppressed in one component and is enhanced in the other component with commensurate changes in the other soliton.

On the other hand, conceptually, the collision scenario shown in Fig. 4 may be viewed as an amplification process in which the soliton S_1 represents a signal (or data carrier) while the soliton S_2 represents an energy reservoir (pump). The main virtue of this amplification process is that it does not require any external amplification medium and therefore the amplification of S_1 does not induce any noise.

The understanding of this fascinating collision process can be facilitated by making an asymptotic analysis of the two soliton solution as in the Manakov case [21,24,43]. We perform the analysis for the choice $k_{1R},k_{2R}>0$ and $k_{1I}>k_{2I}$. For any other choice the analysis is similar. The study shows that due to collision, the amplitudes of the colliding solitons S_1 and S_2 change from $(A_1^{1-}k_{1R},A_2^{1-}k_{1R})$ and $(A_1^{2-}k_{2R},A_2^{2-}k_{2R})$ to $(A_1^{1+}k_{1R},A_2^{1+}k_{1R})$ and $(A_1^{2+}k_{2R},A_2^{2+}k_{2R})$, respectively. Here the superscripts in A_i^j's denote the solitons [number(1,2)], the subscripts represent the components [number(1,2)] and "±" signs stand for "$z\to\pm\infty$." They are defined as

$$\begin{pmatrix}A_1^{1-}\\ A_2^{1-}\end{pmatrix}=\begin{pmatrix}\alpha_1^{(1)}\\ \alpha_1^{(2)}\end{pmatrix}\frac{e^{-R_1/2}}{(k_1+k_1^*)}, \tag{17a}$$

$$\begin{pmatrix}A_1^{2-}\\ A_2^{2-}\end{pmatrix}=\begin{pmatrix}e^{\delta_{11}}\\ e^{\delta_{12}}\end{pmatrix}\frac{e^{-(R_1+R_3)/2}}{(k_2+k_2^*)}, \tag{17b}$$

$$\begin{pmatrix}A_1^{1+}\\ A_2^{1+}\end{pmatrix}=\begin{pmatrix}e^{\delta_{21}}\\ e^{\delta_{22}}\end{pmatrix}\frac{e^{-(R_2+R_3)/2}}{(k_1+k_1^*)}, \tag{17c}$$

$$\begin{pmatrix}A_1^{2+}\\ A_2^{2+}\end{pmatrix}=\begin{pmatrix}\alpha_2^{(1)}\\ \alpha_2^{(2)}\end{pmatrix}\frac{e^{-R_2/2}}{(k_2+k_2^*)}. \tag{17d}$$

All the quantities in the above expressions are given in Eq. (10) [21,24,43]. The analysis reveals the fact that, for the nonsingular two soliton solution, the colliding solitons change their amplitudes in each component according to the conservation equation

$$|A_1^{j-}|^2-|A_2^{j-}|^2=|A_1^{j+}|^2-|A_2^{j+}|^2=\frac{1}{\mu},\quad j=1,2. \tag{18}$$

This can be easily verified from the actual expressions given in Eq. (17).

This condition allows the given soliton to experience the same effect in each component during collision, which may find potential applications in some physical situations like noiseless amplification of a pulse. It can be easily observed from the conservation relation (18) that each component of a given soliton experiences the same kind of energy switching during collision process. The other soliton (say S_2) experiences an opposite kind of energy switching due to the conservation law

$$\int_{-\infty}^{\infty} |q_j|^2 dt = \text{const}, \quad j=1,2, \tag{19}$$

as required from Eq. (1).

The asymptotic analysis also results in the following expression relating the intensities of solitons S_1 and S_2 in q_1 and q_2 components before and after interaction [see Eq. (17)]:

$$|A_j^{l+}|^2 = |T_j^l|^2 |A_j^{l-}|^2, \quad j,l=1,2, \tag{20}$$

where the superscripts $l\pm$ represent the solitons designated as S_1 and S_2 at $z \to \pm\infty$. The transition intensities are defined as

$$|T_j^1|^2 = \frac{|1-\lambda_2(\alpha_2^{(j)}/\alpha_1^{(j)})|^2}{|1-\lambda_1\lambda_2|}, \tag{21a}$$

$$|T_j^2|^2 = \frac{|1-\lambda_1\lambda_2|}{|1-\lambda_1(\alpha_1^{(j)}/\alpha_2^{(j)})|^2}, \quad j=1,2, \tag{21b}$$

$$\lambda_1 = \frac{\kappa_{21}}{\kappa_{11}}, \quad \lambda_2 = \frac{\kappa_{12}}{\kappa_{22}}. \tag{21c}$$

In fact, this way of energy (amplitude) redistribution can also be expressed in terms of linear fractional transformations (LFTs) as in the CNLS system with focusing nonlinearities [24,44,45]. For example, one can identify from the asymptotic expressions (17) that the state of S_1 after interaction (say $\rho_{1,2}^{1+}=A_1^{1+}/A_2^{1+}$) is related to its state before interaction (say $\rho_{1,2}^{1-}=A_1^{1-}/A_2^{1-}$) through the following LFT:

$$\rho_{1,2}^{1+} = \frac{A_1^{1+}}{A_2^{1+}} = \frac{C_{11}^{(1)}\rho_{1,2}^{1-} + C_{12}^{(1)}}{C_{21}^{(1)}\rho_{1,2}^{1-} + C_{22}^{(1)}}, \tag{22a}$$

where

$$C_{11}^{(1)} = \alpha_2^{(1)}\alpha_2^{(1)*}(k_2-k_1) + \alpha_2^{(2)}\alpha_2^{(2)*}(k_1+k_2^*),$$

$$C_{12}^{(1)} = -\alpha_2^{(1)}\alpha_2^{(2)*}(k_2+k_2^*),$$

$$C_{21}^{(1)} = \alpha_2^{(2)}\alpha_2^{(1)*}(k_2+k_2^*),$$

$$C_{22}^{(1)} = \alpha_2^{(2)}\alpha_2^{(2)*}(k_1-k_2) - \alpha_2^{(1)}\alpha_2^{(1)*}(k_1+k_2^*). \tag{22b}$$

A similar expression can be obtained for soliton S_2 also. The analysis of such state transformations preserving the difference of intensities among the components, during collision, in the context of optical computing and their advantage in constructing logic gates is kept for future study.

For the standard elastic collision property ascribed to the scalar solitons to occur here we need the magnitudes of the transition intensities to be unity which is possible for the specific choice

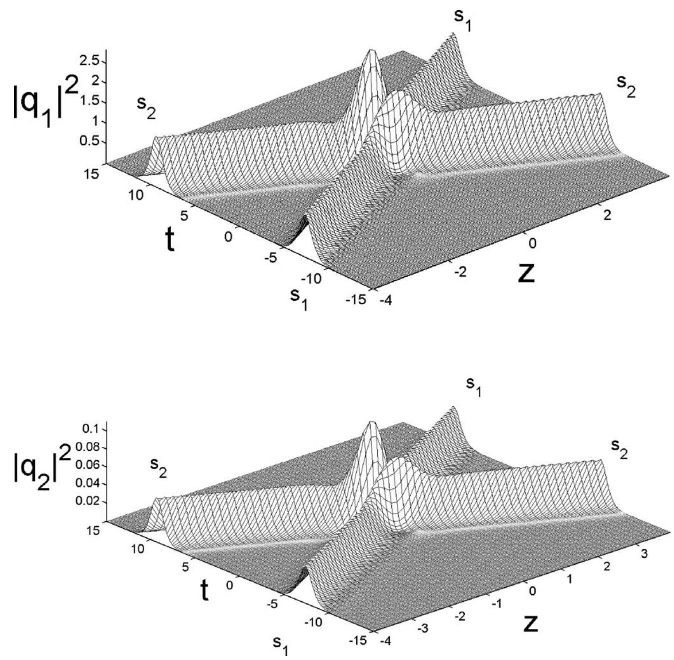

FIG. 5. Elastic collision of two solitons in the mixed CNLS system for the $N=2$ case.

$$\frac{\alpha_1^{(1)}}{\alpha_2^{(1)}} = \frac{\alpha_1^{(2)}}{\alpha_2^{(2)}}. \tag{23}$$

As an example, in Fig. 5 we present the elastic collision for $\theta_1=\theta_2=0.2$, $\phi_1=\phi_2=0.3$ [see Eq. (16)], with k_j's unaltered, $j=1,2$ [Note that this choice satisfies the above condition (23)]. For all other values of $\alpha_i^{(j)}$'s, the soliton energies get exchanged between the solitons in both the components as in Fig. 4.

The other quantities characterizing this collision process, along with this energy redistribution, are the amplitude dependent phase shifts and change in relative separation distances. Their explicit forms can be obtained as in the case of the Manakov model [21,24]. Explicit expressions for the phase shifts Φ_1 and Φ_2 of solitons S_1 and S_2, respectively, during the collision are obtained from the asymptotic analysis as

$$\Phi_1 = -\Phi_2 = \frac{(R_3-R_1-R_2)}{2}, \tag{24}$$

where R_1, R_2, and R_3 are defined in Eq. (10).

Then, the change in relative separation distance between the solitons can be expressed as

$$\Delta t_{12} = t_{12}^- - t_{12}^+ = \frac{(k_{1R}+k_{2R})}{k_{1R}k_{2R}}\Phi_1, \tag{25}$$

where $t_{12}^{\pm}$= the position of S_2 (at $z\to\pm\infty$) minus position of S_1 (at $z\to\pm\infty$).

V. GENERALIZATION OF THE RESULTS TO MULTISOLITON SOLUTIONS AND MULTICOMPONENT CASE

Having discussed the nature of two soliton collision in the two component case ($N=2$), we now wish to study multisoliton collisions for the $N=2$ as well as $N>2$ cases. For this purpose, we will consider first the three soliton collision scenario for the $N=2$ case and then extend the analysis to more general cases.

A. Multisoliton solutions

It is straightforward to extend the bilinearization procedure of obtaining one and two soliton solutions to multisoliton solutions as was done in Ref. [24] for the integrable CNLS equations with focusing nonlinearity coefficients. Below, we present the form of the three soliton solution for the mixed CNLS equations (1) as

$$q_j = \frac{\alpha_1^{(j)}e^{\eta_1} + \alpha_2^{(j)}e^{\eta_2} + \alpha_3^{(j)}e^{\eta_3} + e^{\eta_1+\eta_1^*+\eta_2+\delta_{1j}} + e^{\eta_1+\eta_1^*+\eta_3+\delta_{2j}} + e^{\eta_2+\eta_2^*+\eta_1+\delta_{3j}}}{D} + \frac{e^{\eta_2+\eta_2^*+\eta_3+\delta_{4j}} + e^{\eta_3+\eta_3^*+\eta_1+\delta_{5j}} + e^{\eta_3+\eta_3^*+\eta_2+\delta_{6j}} + e^{\eta_1^*+\eta_2+\eta_3+\delta_{7j}} + e^{\eta_1+\eta_2^*+\eta_3+\delta_{8j}}}{D} + \frac{e^{\eta_1+\eta_2+\eta_3^*+\delta_{9j}} + e^{\eta_1+\eta_1^*+\eta_2+\eta_2^*+\eta_3+\tau_{1j}} + e^{\eta_1+\eta_1^*+\eta_3+\eta_3^*+\eta_2+\tau_{2j}} + e^{\eta_2+\eta_2^*+\eta_3+\eta_3^*+\eta_1+\tau_{3j}}}{D}, \quad j=1,2, \tag{26a}$$

where

$$\begin{aligned} D = {} & 1 + e^{\eta_1+\eta_1^*+R_1} + e^{\eta_2+\eta_2^*+R_2} + e^{\eta_3+\eta_3^*+R_3} + e^{\eta_1+\eta_2^*+\delta_{10}} \\ & + e^{\eta_1^*+\eta_2+\delta_{10}^*} + e^{\eta_1+\eta_3^*+\delta_{20}} + e^{\eta_1^*+\eta_3+\delta_{20}^*} + e^{\eta_2+\eta_3^*+\delta_{30}} \\ & + e^{\eta_2^*+\eta_3+\delta_{30}^*} + e^{\eta_1+\eta_1^*+\eta_2+\eta_2^*+R_4} + e^{\eta_1+\eta_1^*+\eta_3+\eta_3^*+R_5} \\ & + e^{\eta_2+\eta_2^*+\eta_3+\eta_3^*+R_6} + e^{\eta_1+\eta_1^*+\eta_2+\eta_3^*+\tau_{10}} + e^{\eta_1+\eta_1^*+\eta_3+\eta_2^*+\tau_{10}^*} \\ & + e^{\eta_2+\eta_2^*+\eta_1+\eta_3^*+\tau_{20}} + e^{\eta_2+\eta_2^*+\eta_1^*+\eta_3+\tau_{20}^*} + e^{\eta_3+\eta_3^*+\eta_1+\eta_2^*+\tau_{30}} \\ & + e^{\eta_3+\eta_3^*+\eta_1^*+\eta_2+\tau_{30}^*} + e^{\eta_1+\eta_1^*+\eta_2+\eta_2^*+\eta_3+\eta_3^*+R_7}. \end{aligned} \tag{26b}$$

Expressions for various quantities given in Eq. (26) have the following forms:

$$\eta_i = k_i(t+ik_iz), \quad i=1,2,3, \tag{27a}$$

$$e^{\delta_{1j}} = \frac{(k_1-k_2)(\alpha_1^{(j)}\kappa_{21}-\alpha_2^{(j)}\kappa_{11})}{(k_1+k_1^*)(k_1^*+k_2)},$$

$$e^{\delta_{2j}} = \frac{(k_1-k_3)(\alpha_1^{(j)}\kappa_{31}-\alpha_3^{(j)}\kappa_{11})}{(k_1+k_1^*)(k_1^*+k_3)},$$

$$e^{\delta_{3j}} = \frac{(k_1-k_2)(\alpha_1^{(j)}\kappa_{22}-\alpha_2^{(j)}\kappa_{12})}{(k_1+k_2^*)(k_2+k_2^*)},$$

$$e^{\delta_{4j}} = \frac{(k_2-k_3)(\alpha_2^{(j)}\kappa_{32}-\alpha_3^{(j)}\kappa_{22})}{(k_2+k_2^*)(k_2^*+k_3)},$$

$$e^{\delta_{5j}} = \frac{(k_1-k_3)(\alpha_1^{(j)}\kappa_{33}-\alpha_3^{(j)}\kappa_{13})}{(k_3+k_3^*)(k_3^*+k_1)},$$

$$e^{\delta_{6j}} = \frac{(k_2-k_3)(\alpha_2^{(j)}\kappa_{33}-\alpha_3^{(j)}\kappa_{23})}{(k_3^*+k_2)(k_3^*+k_3)},$$

$$e^{\delta_{7j}} = \frac{(k_2-k_3)(\alpha_2^{(j)}\kappa_{31}-\alpha_3^{(j)}\kappa_{21})}{(k_1^*+k_2)(k_1^*+k_3)},$$

$$e^{\delta_{8j}} = \frac{(k_1-k_3)(\alpha_1^{(j)}\kappa_{32}-\alpha_3^{(j)}\kappa_{12})}{(k_1+k_2^*)(k_2^*+k_3)},$$

$$e^{\delta_{9j}} = \frac{(k_1-k_2)(\alpha_1^{(j)}\kappa_{23}-\alpha_2^{(j)}\kappa_{13})}{(k_1+k_3^*)(k_2+k_3^*)},$$

$$e^{\tau_{1j}} = \frac{(k_2-k_1)(k_3-k_1)(k_3-k_2)(k_2^*-k_1^*)}{(k_1^*+k_1)(k_1^*+k_2)(k_1^*+k_3)(k_2^*+k_1)(k_2^*+k_2)(k_2^*+k_3)} \times [\alpha_1^{(j)}(\kappa_{21}\kappa_{32}-\kappa_{22}\kappa_{31}) + \alpha_2^{(j)}(\kappa_{12}\kappa_{31}-\kappa_{32}\kappa_{11}) + \alpha_3^{(j)}(\kappa_{11}\kappa_{22}-\kappa_{12}\kappa_{21})],$$

$$e^{\tau_{2j}} = \frac{(k_2-k_1)(k_3-k_1)(k_3-k_2)(k_3^*-k_1^*)}{(k_1^*+k_1)(k_1^*+k_2)(k_1^*+k_3)(k_3^*+k_1)(k_3^*+k_2)(k_3^*+k_3)} \times [\alpha_1^{(j)}(\kappa_{33}\kappa_{21}-\kappa_{31}\kappa_{23}) + \alpha_2^{(j)}(\kappa_{31}\kappa_{13}-\kappa_{11}\kappa_{33}) + \alpha_3^{(j)}(\kappa_{23}\kappa_{11}-\kappa_{13}\kappa_{21})],$$

$$e^{\tau_{3j}} = \frac{(k_2-k_1)(k_3-k_1)(k_3-k_2)(k_3^*-k_2^*)}{(k_2^*+k_1)(k_2^*+k_2)(k_2^*+k_3)(k_3^*+k_1)(k_3^*+k_2)(k_3^*+k_3)} \times [\alpha_1^{(j)}(\kappa_{22}\kappa_{33}-\kappa_{23}\kappa_{32}) + \alpha_2^{(j)}(\kappa_{13}\kappa_{32}-\kappa_{33}\kappa_{12}) + \alpha_3^{(j)} \times(\kappa_{12}\kappa_{23}-\kappa_{22}\kappa_{13})],$$

$$e^{R_m} = \frac{\kappa_{mm}}{k_m + k_m^*}, \quad m = 1,2,3, \quad e^{\delta_{10}} = \frac{\kappa_{12}}{k_1 + k_2^*},$$

$$e^{\delta_{20}} = \frac{\kappa_{13}}{k_1 + k_3^*}, \quad e^{\delta_{30}} = \frac{\kappa_{23}}{k_2 + k_3^*},$$

$$e^{R_4} = \frac{(k_2 - k_1)(k_2^* - k_1^*)}{(k_1^* + k_1)(k_1^* + k_2)(k_1 + k_2^*)(k_2^* + k_2)}[\kappa_{11}\kappa_{22} - \kappa_{12}\kappa_{21}],$$

$$e^{R_5} = \frac{(k_3 - k_1)(k_3^* - k_1^*)}{(k_1^* + k_1)(k_1^* + k_3)(k_3^* + k_1)(k_3^* + k_3)}[\kappa_{33}\kappa_{11} - \kappa_{13}\kappa_{31}],$$

$$e^{R_6} = \frac{(k_3 - k_2)(k_3^* - k_2^*)}{(k_2^* + k_2)(k_2^* + k_3)(k_3^* + k_2)(k_3 + k_3^*)}[\kappa_{22}\kappa_{33} - \kappa_{23}\kappa_{32}],$$

$$e^{\tau_{10}} = \frac{(k_2 - k_1)(k_3^* - k_1^*)}{(k_1^* + k_1)(k_1^* + k_2)(k_3^* + k_1)(k_3^* + k_2)}[\kappa_{11}\kappa_{23} - \kappa_{21}\kappa_{13}],$$

$$e^{\tau_{20}} = \frac{(k_1 - k_2)(k_3^* - k_2^*)}{(k_2^* + k_1)(k_2^* + k_2)(k_3^* + k_1)(k_3^* + k_2)}[\kappa_{22}\kappa_{13} - \kappa_{12}\kappa_{23}],$$

$$e^{\tau_{30}} = \frac{(k_3 - k_1)(k_3^* - k_2^*)}{(k_2^* + k_1)(k_2^* + k_3)(k_3^* + k_1)(k_3^* + k_3)}[\kappa_{33}\kappa_{12} - \kappa_{13}\kappa_{32}], \tag{27b}$$

$$e^{R_7} = \frac{|k_1 - k_2|^2|k_2 - k_3|^2|k_3 - k_1|^2}{(k_1 + k_1^*)(k_2 + k_2^*)(k_3 + k_3^*)|k_1 + k_2^*|^2|k_2 + k_3^*|^2|k_3 + k_1^*|^2} \times [(\kappa_{11}\kappa_{22}\kappa_{33} - \kappa_{11}\kappa_{23}\kappa_{32}) + (\kappa_{12}\kappa_{23}\kappa_{31} - \kappa_{12}\kappa_{21}\kappa_{33}) + (\kappa_{21}\kappa_{13}\kappa_{32} - \kappa_{22}\kappa_{13}\kappa_{31})], \tag{27c}$$

and

$$\kappa_{ij} = \frac{\mu\sum_{l=1}^{2} \sigma_l \alpha_i^{(l)} \alpha_j^{(l)*}}{(k_i + k_j^*)}, \quad i,j = 1,2,3, \tag{27d}$$

where $\sigma_1=1$ and $\sigma_2=-1$. Here $\alpha_1^{(j)}$, $\alpha_2^{(j)}$, and $\alpha_3^{(j)}$, k_1, k_2 and k_3, $j=1,2,3$, are complex parameters.

The solution (26) also features singular and nonsingular behaviors, as in the case of one and two soliton solutions depending upon the values of the soliton parameters. Though the denominator D in the solution (26) is cumbersome, possible nonsingular conditions can be obtained with some effort. Equation (26b) can be rewritten as

$$\begin{aligned} D = {}& 2e^{\eta_{1R}+\eta_{2R}+\eta_{3R}}\{e^{(R_1+R_6)/2}\cosh[\eta_{1R} - \eta_{2R} - \eta_{3R} + (R_1 - R_6)/2] \\ &+ e^{(R_2+R_5)/2}\cosh[\eta_{2R} - \eta_{1R} - \eta_{3R} + (R_2 - R_5)/2] + e^{(R_3+R_4)/2}\cosh[\eta_{3R} - \eta_{1R} - \eta_{2R} + (R_3 - R_4)/2] \\ &+ 2e^{(\delta_{10R}+\tau_{30R})/2}[\cosh(X_1)\cos(Y_1)\cos(Z_1) - \sinh(X_1)\sin(Y_1)\sin(Z_1)] + 2e^{(\delta_{20R}+\tau_{20R})/2}[\cosh(X_2)\cos(Y_2)\cos(Z_2) \\ &- \sinh(X_2)\sin(Y_2)\sin(Z_2)] + 2e^{(\delta_{30R}+\tau_{10R})/2}[\cosh(X_3)\cos(Y_3)\cos(Z_3) - \sinh(X_3)\sin(Y_3)\sin(Z_3)] \\ &+ e^{R_7/2}\cosh(\eta_{1R} + \eta_{2R} + \eta_{3R} + R_7/2)\}, \end{aligned} \tag{28a}$$

where

$$X_1 = -\eta_{3R} + \frac{(\delta_{10R} - \tau_{30R})}{2}, \quad X_2 = -\eta_{2R} + \frac{(\delta_{20R} - \tau_{20R})}{2},$$

$$X_3 = -\eta_{1R} + \frac{(\delta_{30R} - \tau_{10R})}{2}, \quad Y_1 = \eta_{1I} - \eta_{2I} + \frac{(\delta_{10I} + \tau_{30I})}{2},$$

$$Y_2 = \eta_{1I} - \eta_{3I} + \frac{(\delta_{20I} + \tau_{20I})}{2}, \quad Y_3 = \eta_{2I} - \eta_{3I} + \frac{(\delta_{30I} + \tau_{10I})}{2},$$

$$Z_1 = \frac{(\delta_{10I} - \tau_{30I})}{2}, \quad Z_2 = \frac{(\delta_{20I} - \tau_{20I})}{2}, \quad Z_3 = \frac{(\delta_{30I} - \tau_{10I})}{2}. \tag{28b}$$

Here the suffixes R and I denote the real and imaginary parts, respectively. As in the case of two soliton solution here also we find the following conditions need to be satisfied for the solution to be regular:

$$e^{R_i} > 0, \quad i = 1,2,\ldots,7, \tag{29a}$$

$$e^{(R_1+R_6)/2}, e^{(R_2+R_5)/2}, e^{(R_3+R_4)/2}, e^{R_7/2} > 4\max\{e^{\delta_{10R}+\tau_{30R}}, e^{\delta_{20R}+\tau_{20R}}, e^{\delta_{30R}+\tau_{10R}}\}. \tag{29b}$$

Note that the conditions given in Eq. (29a) are necessary as the falsity of any of them always results in a singular solution and the last condition (29b) is sufficient to ensure that the given solution is regular. In fact these conditions can also be expressed in terms of soliton parameters, but due to their cumbersome nature we do not present them here. The appropriate choice of parameters can be made by carefully looking at the explicit forms of e^{R_i}, $e^{\delta_{j0}}$, and $e^{\tau_{j0}}$, $i=1,\ldots,7$, and $j=1,2,3$.

Such a nonsingular solution representing the shape changing (intensity redistribution) collision of three solitons S_1, S_2, and S_3 in the two components q_1 and q_2 is shown in Fig. 6 for the parameter choice $k_1=1+i$, $k_2=1.2-0.5i$, $k_3=1-i$, $\mu=1$, $\alpha_1^{(1)}=\cosh(\theta_1)e^{i\phi_1}$, $\alpha_2^{(1)}=\cosh(\theta_2)e^{i\phi_1}$, $\alpha_3^{(1)}=\cosh(\theta_3)e^{i\phi_1}$, $\alpha_1^{(2)}=\sinh(\theta_1)e^{i\phi_2}$, $\alpha_2^{(2)}=\sinh(\theta_2)e^{i\phi_2}$, $\alpha_3^{(2)}=\sinh(\theta_3)e^{i\phi_2}$,

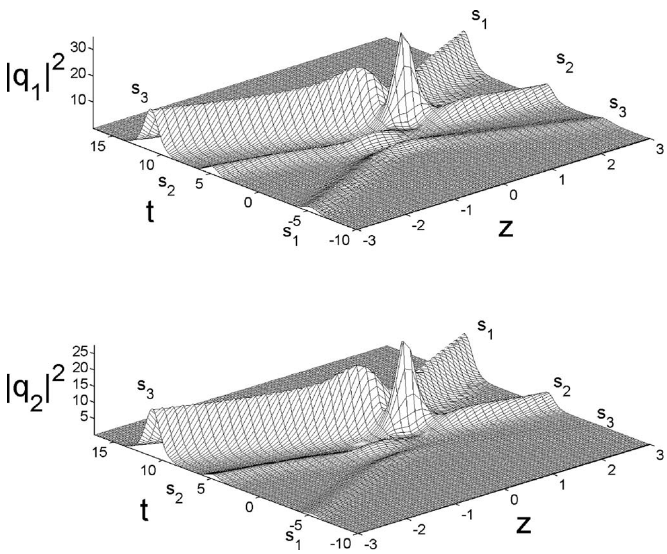

FIG. 6. Shape changing (intensity redistribution) collision of three solitons in the mixed CNLS system for the $N=2$ case.

where $\theta_1=0.8$, $\theta_2=0.4$, $\theta_3=0.2$, $\phi_1=0.5$, and $\phi_2=1.0$. From the figure we observe that after collision solitons S_1 and S_2 are enhanced in their intensities while there occurs suppression of intensity for soliton S_3 in both the components q_1 and q_2. It can be verified that before and after collision the conservation relation

$$|A_1^{j-}|^2-|A_2^{j-}|^2=|A_1^{j+}|^2-|A_2^{j+}|^2=\frac{1}{\mu}, \quad j=1,2,3 \qquad (30)$$

is satisfied, so that the difference of intensities of the solitons between the components q_1 and q_2 is preserved before and after the collision process. The standard elastic collision can be regained if $\alpha_1^{(1)}:\alpha_2^{(1)}:\alpha_3^{(1)}=\alpha_2^{(1)}:\alpha_2^{(2)}:\alpha_3^{(2)}$. Figure 7 illustrates such an elastic collision for the choice $\theta_1=\theta_2=\theta_3=0.4$, $\phi_1=\phi_2=0.5$, with same k_j's, $j=1,2,3$, as in Fig. 6.

In a similar manner the four soliton solution can be deduced from Eq. (A2) given in Ref. [24] by redefining κ_{ij} as in Eq. (27d) with i,j running from 1 to 4. We do not present the explicit form of it here because of its cumbersome nature.

B. Multicomponent case with $N>2$

The next step is to generalize the above results for the $N=2$ case to arbitrary N with $N>2$. To do this we follow the earlier work of two of the authors (T.K. and M.L.) [24] on the focusing type CNLS equations with all $\sigma_l=1$, $l=1,2,\ldots,N$. This study shows that the solutions of mixed CNLS equations with the $N=2$ case can be generalized to arbitrary N case just by allowing the number of components to run from 2 to N and redefining κ_{ij}'s suitably.

The procedure can be well understood by considering the example of writing down the soliton solutions of Eq. (1) for the case $N=3$.

1. One soliton solution

The one soliton solution of mixed 3-CNLS equations obtained by Hirota's method can be written as

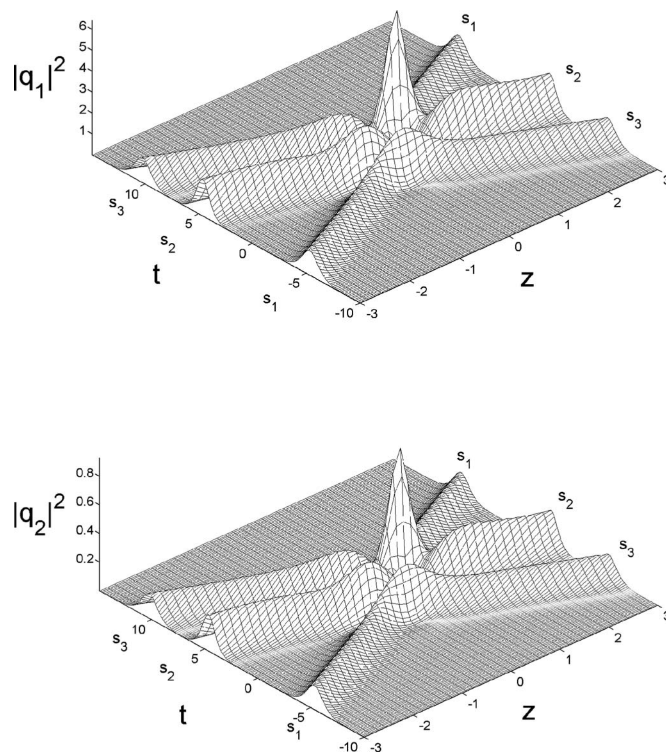

FIG. 7. Elastic collision of three solitons in the mixed CNLS system for the $N=2$ case.

$$\begin{pmatrix} q_1 \\ q_2 \\ q_3 \end{pmatrix}=\begin{pmatrix} \alpha_1^{(1)} \\ \alpha_1^{(2)} \\ \alpha_1^{(3)} \end{pmatrix}\frac{e^{\eta_1}}{1+e^{\eta_1+\eta_1^*+R}}, \qquad (31a)$$

where

$$\eta_1=k_1(t+ik_1z), \quad e^R=\frac{\kappa_{11}}{(k_1+k_1^*)}, \qquad (31b)$$

in which $\kappa_{11}=\mu(\sigma_1|\alpha_1^{(1)}|^2+\sigma_2|\alpha_1^{(2)}|^2+\sigma_3|\alpha_1^{(3)}|^2)/(k_1+k_1^*)$ and without loss of generality we assume either $\sigma_1=1$, $\sigma_2=\sigma_3=-1$ or $\sigma_1=\sigma_2=1$, $\sigma_3=-1$. As in the case of $N=2$, Sec. III A, the solution is singular if $\sigma_1|\alpha_1^{(1)}|^2+\sigma_2|\alpha_1^{(2)}|^2+\sigma_3|\alpha_1^{(3)}|^2\leqslant 0$. Otherwise the solution is regular. It can be noticed that for any other combination of σ_l's also the above solution satisfies Eq. (1), for $N=3$.

2. Two soliton solution

The two soliton solution for the $N=3$ case is found to possess the same form of Eq. (10), with $j=1,2,3$, and κ_{ij} is given by

$$\kappa_{ij}=\frac{\mu(\sigma_1\alpha_i^{(1)}\alpha_j^{(1)*}+\sigma_2\alpha_i^{(2)}\alpha_j^{(2)*}+\sigma_3\alpha_i^{(3)}\alpha_j^{(3)*})}{(k_i+k_j^*)}, \quad i,j=1,2, \qquad (32)$$

where σ_l's, $l=1,2,3$, can take the value either $+1$ or -1. Here also the nonsingular solution exists for the conditions (14b), (14c), and (14e) with the redefinition of κ_{ij}'s as in Eq. (32).

3. Three and multisoliton solutions

A similar analysis can be done for the multisoliton solutions of the multicomponent case with arbitrary N. Particularly the three soliton solution of the mixed 3-CNLS equations, Eq. (1) with $N=3$, can be identified to have the form of three soliton solution for the $N=2$ case with j running from 1 to 3 (that is, now we have three components q_1, q_2, and q_3) and here κ_{ij} is redefined as

$$\kappa_{ij}=\frac{\mu(\sigma_1\alpha_i^{(1)}\alpha_j^{(1)*}+\sigma_2\alpha_i^{(2)}+\sigma_3\alpha_i^{(3)}\alpha_j^{(3)*})}{(k_i+k_j^*)}, \quad i,j=1,2,3, \tag{33}$$

where σ_l's, $l=1,2,3$, can take the value either +1 or −1 [see also Eq. (10) of Ref. [24]].

It can also be noticed that the stationary singular solution for the $N=3$ case given in Ref. [38] results from the above mentioned three soliton solution for the choice

$$\alpha_1^{(1)}=-e^{\eta_{10}}, \quad \alpha_2^{(2)}=e^{\eta_{20}}, \quad \alpha_3^{(3)}=-e^{\eta_{30}}, \quad \alpha_i^{(j)}=0, \tag{34}$$

$$k_{jI}=0, \quad \mu=1, \quad i\neq j, \quad i,j=1,2,3,$$

where η_{j0}'s, $j=1,2,3$, are real parameters. The resulting limiting form reads in terms of hyperbolic functions as given in Appendix A. This singular solution at $z=0$ is shown in Fig. 8. The parameters are chosen as $\alpha_1^{(1)}=-1$, $\alpha_2^{(2)}=1$, $\alpha_3^{(3)}=-1$, $\alpha_i^{(j)}=0$, $i\neq j$, $i,j=1,2,3$, $k_{1R}=0.8$, $k_{2R}=0.5$, $k_{3R}=0.4$, and $\mu=1$.

This procedure can be generalized further to obtain multisoliton solutions of the multicomponent case with arbitrary N. For completeness we present the determinant form of the N-soliton solution of the N-component case in Appendix B, following the lines of Ref. [46] for the Manakov case.

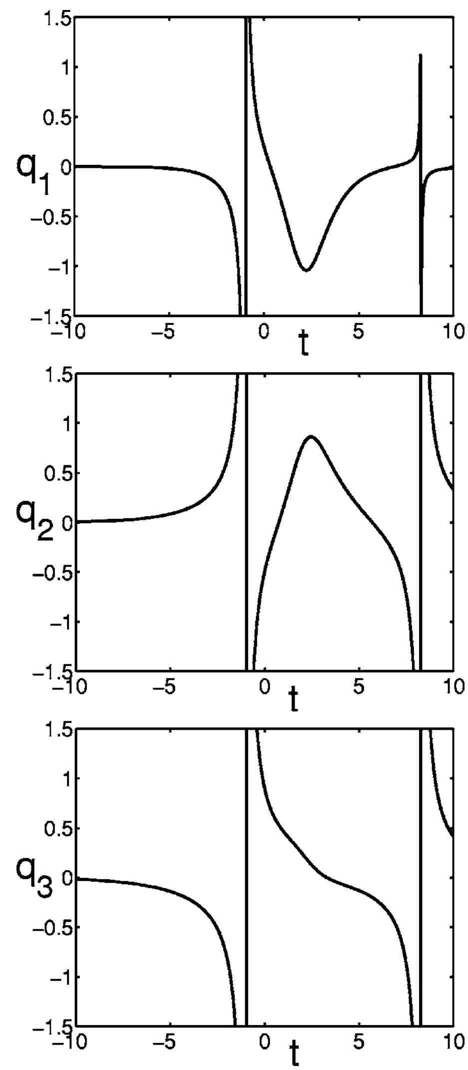

FIG. 8. Stationary singular three soliton solution for the $N=3$ case.

C. Collision scenario in multicomponent cases

As we increase the number of components the collision behavior becomes more interesting. For example, we consider the collision of two solitons in the three component ($N=3$) mixed CNLS system. We study the collision dynamics for the following two possible combinations of σ's. For illustration, we present two nontrivial scenarios with two different choices of σ_i's.

Case (i): $\sigma_1=1$, $\sigma_2=\sigma_3=-1$. For this case, one possible parametric choice for nonsingular solution is given by $k_1=1.0+i$, $k_2=0.9-i$, $\alpha_1^{(1)}=\alpha_2^{(1)}=1+i$, $\alpha_1^{(2)}=0.2+0.4i$, $\alpha_2^{(2)}=0.7+0.2i$, $\alpha_1^{(3)}=0.1+0.3i$, $\alpha_2^{(3)}=0.4+0.1i$, and $\mu=1$. We plot the two soliton solution corresponding to this parameter choice in Fig. 9. The figure shows that after collision there is an enhancement (suppression) of intensities (amplitudes) for a given soliton [say soliton S_1 (S_2)] in all the three components. Here also one can verify that the difference of intensities is conserved according to the conservation law

$$|A_1^{l\mp}|^2-|A_2^{l\mp}|^2-|A_3^{l\mp}|^2=\frac{1}{\mu}, \quad l=1,2. \tag{35}$$

Case (ii): $\sigma_1=\sigma_2=1,\sigma_3=-1$. Next we consider the above possible choice for σ's. The nonsingular intensity plots of solitons S_1 and S_2 are shown in Fig. 10. The parameters are chosen as $k_1=1.0+i$, $k_2=0.9-i$, $\alpha_1^{(1)}=1+i$, $\alpha_2^{(1)}=(39-80i)/89$, $\alpha_1^{(2)}=0.2+0.4i$, $\alpha_2^{(2)}=1$, $\alpha_1^{(3)}=(39+80i)/89$, $\alpha_2^{(3)}=0.3+0.2i$, and $\mu=1$. This figure shows that after collision the intensity of soliton S_1 (S_2) in the first and third components gets enhanced (suppressed) while in the second component S_1 (S_2) is suppressed (enhanced) in its intensity. This is a consequence of the conservation given by the relation

$$|A_1^{l-}|^2+|A_2^{l-}|^2-|A_3^{l-}|^2=|A_1^{l+}|^2+|A_2^{l+}|^2-|A_3^{l+}|^2=\frac{1}{\mu}, \quad l=1,2. \tag{36}$$

Thus for the two soliton solution of the N-component case the shape changing (intensity redistribution) collision occurs according to the relation

$$\sum_{l=1}^{N}\sigma_l|A_l^{j-}|^2=\sum_{l=1}^{N}\sigma_l|A_l^{j+}|^2=\frac{1}{\mu}, \quad j=1,2. \tag{37}$$

However, the elastic collision occurs for the choice

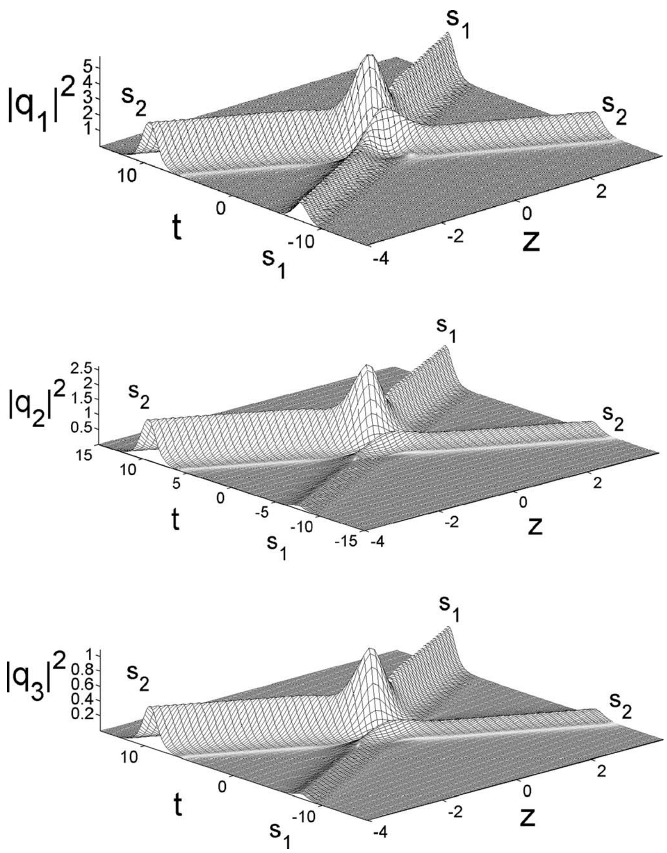

FIG. 9. Shape changing (intensity redistribution) collision of two solitons in the mixed CNLS system, for the $N=3$ case, exhibiting same kind of shape changes for a given soliton in all the three components.

$$\frac{\alpha_1^{(1)}}{\alpha_2^{(1)}} = \frac{\alpha_1^{(2)}}{\alpha_2^{(2)}} = \cdots = \frac{\alpha_1^{(N)}}{\alpha_2^{(N)}}. \qquad (38)$$

One can also observe that multisoliton solutions for the case $N>2$ also undergo the above kind of shape changing (intensity redistribution) collisions but with more possible ways of energy exchange.

VI. CONCLUSION

In this paper we have obtained the bright soliton type solutions of mixed CNLS, Eq. (1), by applying Hirota's bilinear method. These solutions admit both singular and nonsingular behaviors depending upon the choice of the soliton parameters. The condition for the existence of nonsingular one and two soliton solutions for the $N=2$ case are identified first. Analysing the corresponding collision behavior reveals the fact that the solitons undergo fascinating shape changing (intensity redistribution) collisions with similar changes occurring in both components, which is not possible in the well known Manakov system. This shape changing (intensity redistribution) collision occurs with a redistribution of intensities among the solitons, spread up in two components, in a particular fashion, where the intensity difference of the solitons between the two components is preserved after collision, and amplitude dependent phase shifts as well as change in relative separation distances also occur. We have extended this study to obtain multicomponent multisoliton solutions. Numerical plottings of the solutions show that similar shape changing (intensity redistribution) collision behavior is also observed for the multicomponent case with $N>2$ as in the case of $N=2$ but with many possible ways of shape variation. Still it is an open question to identify the regions in which system (1) admits bright-dark, dark-bright, dark-dark soliton solutions. Our study gives an adequate understanding of collision of bright-bright solitons arising in system (1) for mixed signs of nonlinearities. We believe that this kind of study will be of interest in the description of magnetic excitations over an antiferromagnetic vacuum, electromagnetic pulse propagation in left handed materials and so on. In particular one of the most interesting properties of the bright solitons that we have identified in the present work is that the two components of a soliton can be simultaneously amplified during a collision process. Using this property, in principle it

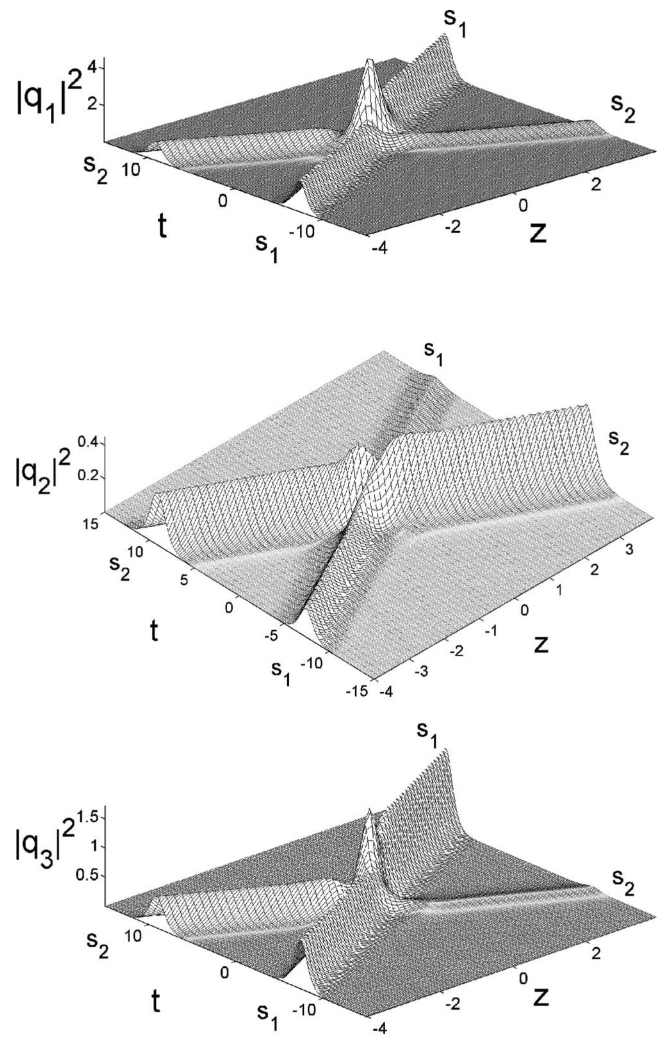

FIG. 10. Shape changing (intensity redistribution) collision of two solitons in the mixed CNLS system, for the $N=3$ case, exhibiting same kind of shape changes for a given soliton in the q_1 and q_3 components and an exactly opposite collision scenario in the q_2 component.

becomes possible to promote the collision process to the rank of a highly efficient amplification process without noise generation, in which the gain can be tuned over a relatively large range through a careful choice of precollision parameters. However, there still remains a lot of work to be done to make the fascinating concept of amplifiers with zero noise figure as practical device for optical communication systems. For example, an important and challenging issue will be to determine whether such amplification process can survive in the presence of strong perturbations or in the presence of propagation instabilities.

ACKNOWLEDGMENTS

T.K. acknowledges the Ministrie de l' Education Nationale, de la Recherche et de la Technology for financial support. The work of M.L. is supported by the Department of Science and Technology, Government of India, research project. N.A. acknowledges support from the Australian Research Council.

APPENDIX A: SINGULAR STATIONARY THREE SOLITON SOLUTION FOR $N=3$ CASE

In this appendix we present the singular stationary three soliton solutions of mixed 3-CNLS equations. Considering the three soliton solution given by Eq. (26) but now the κ_{ij}'s are defined as in Eq. (33), the limiting form for the specific choice of parameters given by Eq. (34) can be deduced as

$$q_1 = \frac{-2k_{1R}\sqrt{\left(\frac{(k_{1R}+k_{2R})(k_{1R}+k_{3R})}{(k_{2R}-k_{1R})(k_{3R}-k_{1R})}\right)}\left[\cosh(A_1) + \left|\frac{(k_{2R}+k_{3R})}{(k_{2R}-k_{3R})}\right|\cosh(B_1)\right]e^{ik_{1R}^2 z}}{D}, \tag{A1a}$$

$$q_2 = \frac{2k_{2R}\sqrt{\left(\frac{(k_{1R}+k_{2R})(k_{2R}+k_{3R})}{(k_{2R}-k_{1R})(k_{3R}-k_{2R})}\right)}\left[\cosh(A_2) - \left|\frac{(k_{1R}+k_{3R})}{(k_{1R}-k_{3R})}\right|\cosh(B_2)\right]e^{ik_{2R}^2 z}}{D}, \tag{A1b}$$

$$q_3 = \frac{2k_{3R}\sqrt{\left(\frac{(k_{1R}+k_{3R})(k_{2R}+k_{3R})}{(k_{3R}-k_{1R})(k_{3R}-k_{2R})}\right)}\left[\sinh(A_3) + \left|\frac{(k_{1R}+k_{2R})}{(k_{2R}-k_{1R})}\right|\sinh(B_3)\right]e^{ik_{3R}^2 z}}{D}, \tag{A1c}$$

where

$$D = \cosh(D_1) + \left|\frac{(k_{1R}+k_{2R})(k_{1R}+k_{3R})}{(k_{2R}-k_{1R})(k_{3R}-k_{1R})}\right|\cosh(D_2) - \left|\frac{(k_{1R}+k_{2R})(k_{2R}+k_{3R})}{(k_{2R}-k_{1R})(k_{2R}-k_{3R})}\right|\cosh(D_3) - \left|\frac{(k_{2R}+k_{3R})(k_{1R}+k_{3R})}{(k_{2R}-k_{3R})(k_{3R}-k_{1R})}\right|\cosh(D_4), \tag{A1d}$$

$$A_1 = (k_{2R}+k_{3R})t + \eta_{20} + \eta_{30} + \frac{1}{2}\ln\left[\frac{(k_{2R}-k_{1R})(k_{3R}-k_{1R})(k_{3R}-k_{2R})^2}{16k_{2R}^2k_{3R}^2(k_{1R}+k_{2R})(k_{1R}+k_{3R})(k_{2R}+k_{3R})^2}\right],$$

$$B_1 = (k_{2R}-k_{3R})t + \eta_{20} - \eta_{30} + \frac{1}{2}\ln\left[\frac{(k_{1R}-k_{2R})(k_{1R}+k_{3R})k_{3R}^2}{k_{2R}^2(k_{1R}+k_{2R})(k_{1R}-k_{3R})}\right],$$

$$A_2 = (k_{1R}+k_{3R})t + \eta_{10} + \eta_{30} + \frac{1}{2}\ln\left[\frac{(k_{2R}-k_{1R})(k_{3R}-k_{1R})^2(k_{3R}-k_{2R})}{16k_{1R}^2k_{3R}^2(k_{1R}+k_{2R})(k_{1R}+k_{3R})^2(k_{2R}+k_{3R})}\right],$$

$$B_2 = (k_{1R}-k_{3R})t + \eta_{10} - \eta_{30} + \frac{1}{2}\ln\left[\frac{(k_{1R}-k_{2R})(k_{2R}+k_{3R})k_{3R}^2}{k_{1R}^2(k_{1R}+k_{2R})(k_{2R}-k_{3R})}\right],$$

$$A_3 = (k_{1R}+k_{2R})t + \eta_{10} + \eta_{20} + \frac{1}{2}\ln\left[\frac{(k_{3R}-k_{1R})(k_{2R}-k_{1R})^2(k_{3R}-k_{2R})}{16k_{1R}^2k_{2R}^2(k_{1R}+k_{2R})^2(k_{1R}+k_{3R})(k_{2R}+k_{3R})}\right],$$

$$B_3 = (k_{1R}-k_{2R})t + \eta_{10} - \eta_{20} + \frac{1}{2}\ln\left[\frac{(k_{3R}-k_{1R})(k_{2R}+k_{3R})k_{2R}^2}{k_{1R}^2(k_{1R}+k_{3R})(k_{3R}-k_{2R})}\right],$$

$$D_1 = (k_{1R} + k_{2R} + k_{3R})t + \eta_{10} + \eta_{20} + \eta_{30} + \ln\left[\frac{(k_{1R} - k_{2R})(k_{1R} - k_{3R})(k_{2R} - k_{3R})}{8k_{1R}k_{2R}k_{3R}(k_{1R} + k_{2R})(k_{1R} + k_{3R})(k_{2R} + k_{3R})}\right],$$

$$D_2 = (k_{1R} - k_{2R} - k_{3R})t + \eta_{10} - \eta_{20} - \eta_{30} + \ln\left[\frac{2(k_{2R} + k_{3R})k_{2R}k_{3R}}{k_{1R}(k_{2R} - k_{3R})}\right],$$

$$D_3 = (k_{1R} - k_{2R} + k_{3R})t + \eta_{10} - \eta_{20} + \eta_{30} + \ln\left[\frac{(k_{3R} - k_{1R})k_{2R}}{2k_{1R}k_{3R}(k_{1R} + k_{3R})}\right],$$

$$D_4 = (k_{1R} + k_{2R} - k_{3R})t + \eta_{10} + \eta_{20} - \eta_{30} + \ln\left[\frac{(k_{2R} - k_{1R})k_{3R}}{2k_{1R}k_{2R}(k_{1R} + k_{2R})}\right]. \tag{A1e}$$

Particularly, the stationary solution corresponding to the choice given in Eq. (34) can be easily checked to be the same as the previously reported form given by Eq. (19) in Ref. [38]. This clearly shows that the more general soliton solutions presented in this paper admit singular solutions as special cases which behave as regular and bounded solutions in specific regions.

APPENDIX B: MULTICOMPONENT MULTISOLITON SOLUTIONS

To write down the multicomponent multisoliton solutions in a formal way we define the following $(1 \times N)$ row matrix C_s, $(N \times 1)$ column matrices ψ_j, ϕ, $j,s = 1,2,\ldots,N$, and the $(N \times N)$ matrix σ:

$$C_s = -(\alpha_1^{(s)}, \alpha_2^{(s)}, \ldots, \alpha_N^{(s)}), \quad \psi_j = \begin{pmatrix} \alpha_j^{(1)} \\ \alpha_j^{(2)} \\ \vdots \\ \alpha_j^{(N)} \end{pmatrix},$$

$$\phi = \begin{pmatrix} e^{\eta_1} \\ e^{\eta_2} \\ \vdots \\ e^{\eta_N} \end{pmatrix}, \quad j,s = 1,2,\ldots,N,$$

$$\sigma = \begin{pmatrix} \sigma_1 & 0 & \cdots & 0 \\ 0 & \sigma_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \sigma_N \end{pmatrix}, \tag{B1a}$$

where σ_j, $j = 1,2,\ldots,N$, can take value either +1 or −1. Then the N-soliton solution of N-CNLS system (1) with mixed signs of nonlinearities can be written as

$$q_s = \frac{g^{(s)}}{D}, \quad s = 1,2,3,\ldots,N, \tag{B1b}$$

where

$$g^{(s)} = \begin{vmatrix} A & I & \phi \\ -I & B & 0 \\ 0 & C_s & 0 \end{vmatrix}, \quad D = \begin{vmatrix} A & I \\ -I & B \end{vmatrix}, \tag{B1c}$$

in which s denotes the component. Here I is $(N \times N)$ unit matrix and the $(N \times N)$ matrices A and B are defined as

$$A_{i,j} = \frac{e^{\eta_i + \eta_j^*}}{k_i + k_j^*}, \quad B_{i,j} = \kappa_{ji} = \frac{\mu(\psi_i \dagger \sigma \psi_j)}{k_i^* + k_j}, \quad i,j = 1,2,\ldots,N, \tag{B1d}$$

where $\eta_i = k_i(t + ik_i z)$, k_i is complex, † represents the transpose conjugate. Here we remark that though presenting the solutions in determinant form seems to be compact, one has to explicitly write down the solutions as we have presented in Secs. II–V, for a complete characterization and analysis of the solution. This way of expressing the solutions explicitly is also useful to identify the particular parameter choice for which the singular stationary N-soliton solution of the N-component case results from the general solutions. In particular, by generalizing the Eqs. (11) and (34) one can identify that the singular stationary N-soliton solution of the N-component case results from the above solution (B1) for the choice $\alpha_i^{(i)} = (-1)^i e^{\eta_{i0}}$, $i = 1,2,\ldots,N$, and $\alpha_i^{(j)} = 0$, $k_{jI} = 0$, $\mu = 1$, where $i \neq j$, $i,j = 1,2,3,\ldots,N$, and $e^{\eta_{i0}}$'s are arbitrary real parameters.

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