

Classical Limit of Time-Dependent Quantum Field Theory-a Schwinger-Dyson Approach

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Abstract

We rewrite the Martin-Siggia-Rose (MSR) formalism for the statistical dynamics of classical fields in a covariant second order form appropriate for the statistical dynamics of relativistic field theory. This second order formalism is related to a rotation of Schwinger's closed time path (CTP) formalism for quantum dynamics, with the main difference being that certain vertices are absent in the classical theory. These vertices are higher order in an \hbar expansion. The structure of the second order formulation of the Schwinger Dyson (S-D) equations is identical to that of the rotated CTP formalism apart from initial conditions on the Green's functions and the absence of these vertices. We then discuss self-consistent truncation schemes based on keeping certain graphs in the two-particle irreducible effective action made up of bare vertices and exact Green's functions.

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I. INTRODUCTION

Recently there has been a lot of interest in connecting hot relativistic quantum field theory which is important for cosmology and heavy ion physics with classical field theory. Although there have been several insightful studies in perturbation theory [1], a systematic analysis has not been given for the Schwinger Dyson equations. One exception is the work of Wetterich [6], but that work only concerns itself with equal time correlation functions. Since formalisms already exist for obtaining Schwinger Dyson (S-D) equations for both the quantum [2] and classical problems [3], we thought it timely to review the MSR formalism, and cast it in a form so that it could be directly compared with the now more familiar CTP formalism [4]. We show here that the second order formulation of the S-D Equations in the MSR formalism is identical to that of the quantum theory (CTP formalism) apart from initial conditions on the Green's functions and the absence of those vertices which are higher order in \hbar .

II. REVIEW OF FIRST ORDER IN TIME MSR FORMALISM

In the paper of Martin-Siggia-Rose (MSR) [3], an operator formalism was developed which allows one to find the generating functional for both the correlation and response functions for first order classical field equations of the type:

$$\dot{x}(r, t) = A[x(r, t)] , \quad (1)$$

where $A[x(r, t)]$ is a local polynomial in the classical field $x(r, t)$. In the work of MSR, $A[x(r, t)]$ could contain dissipative terms as well as prescribed noise terms. The formalism presented in MSR is first order in time derivatives and not apparently covariant. What we will show is that there is a covariant subset of the MSR equations in terms of which all the MSR Green's functions can be recovered. This subset can be derived from a second order Lagrangian formulation which can be related to the $\hbar \rightarrow 0$ limit of the CTP formalism of Schwinger and Keldysh.

For the statistical classical field evolutions of N interacting classical fields ϕ_a , $a = 1, 2 \dots N$ then $x(r, t)$ is the $2N$ component field consisting of ϕ_a and the canonical momentum $\pi_a = \dot{\phi}_a$.

$$x = \begin{pmatrix} \phi_a \\ \pi_a \end{pmatrix}. \quad (2)$$

If for example we restrict ourselves to cubic interactions, then the vector A

is of the form

$$A_i = c_i(r, t) + d_{ij}x_j(r, t) + \frac{1}{2}e_{ijk}x_j(r, t)x_k(r, t) , \quad (3)$$

where $i = 1 \dots 2N$. In the MSR formalism one introduces the operator

$$\hat{x}(r, t) \equiv -\frac{\delta}{\delta x(r, t)} ,$$

such that the commutation rule

$$[x(r, t), \hat{x}(r', t)] = \delta(r - r') , \quad (4)$$

is true. Defining an operator Hamiltonian H

$$H(t) = \int dr' \hat{x}_i(r', t)A_i(r', t) , \quad (5)$$

the equations of motion can be written in the compact form

$$\dot{x}(r, t) = [x(r, t), H(t)] . \quad (6)$$

For Eq. (4) to be true at all times one needs that \hat{x} satisfies

$$\frac{d\hat{x}(r, t)}{dt} = [\hat{x}(r, t), H(t)] , \quad (7)$$

for consistency. Therefore $\hat{x}(r, t)$ is a functional of $x(r, 0)$ and $\hat{x}(r, 0)$. The formal solution to these equations is given by (in what follows we suppress the spatial coordinate r)

$$\begin{aligned} x(t) &= U^{-1}(t, 0)x(0)U(t, 0) \\ \hat{x}(t) &= U^{-1}(t, 0)\hat{x}(0)U(t, 0) , \end{aligned} \quad (8)$$

where

$$U^{-1}(t, t_0) = T \exp[-\int_{t_0}^t H(t')dt'] ; \quad U(t, t_0) = T^* \exp[\int_{t_0}^t H(t')dt'] .$$

The meaning of the expectation value $\langle x(t)\hat{x}(t') \rangle$ is as follows. Given an initial probability function $P[x(0)]$ then

$$\langle x(t)\hat{x}(t') \rangle = \int dx(0) x[t, x(0)] \hat{x}[t', x(0), \hat{x}(0)] P[x(0)] . \quad (9)$$

Wherever $\hat{x}(0)$ appears, it is replaced by

$$\hat{x}(0) \rightarrow -\frac{\delta}{\delta x(0)} ,$$

and it acts on everything to the right of itself. This definition of the extended averaging procedure has three important properties. (1) The average of a product of x 's agrees with the conventional definition. (2) The time dependence of $\langle x(t)\hat{x}(t') \rangle$ is consistent with the above equations of motion. (3) The expectation value of a product of x and \hat{x} which has an \hat{x} to the left vanishes if $P[x(0)]$ goes to zero fast enough at large $|x|$.

The last property is crucial for the tridiagonal form of the Green's functions and follows from the fact that

$$\int_{-\infty}^{\infty} \frac{d}{dx} (x^n P(x)) dx = 0 \quad \text{if} \quad \lim_{|x| \rightarrow \infty} x^n P[x] = 0 .$$

Thus in particular in the absence of external sources $\langle \hat{x}(t')x(t) \rangle = \langle \hat{x}(t')\hat{x}(t') \rangle = 0$. The meaning of the *hatted* operators is understood in terms of the response of the system to an external source. If one changes the Hamiltonian by

$$\delta H = \int dr' \hat{x}(r't) \delta f(r't) , \quad (10)$$

one then finds [3] that the response of any observable A is given by:

$$\begin{aligned} \frac{\delta \langle A(t) \rangle}{\delta f(t')} \Big|_{\delta f=0} &= \langle T (A(t)\hat{x}(t')) \rangle \\ \frac{\delta \langle A(t) \rangle}{\delta f(t')\delta f(t'')} \Big|_{\delta f=0} &= \langle T (A(t)\hat{x}(t')\hat{x}(t'')) \rangle , \end{aligned} \quad (11)$$

where T corresponds to the usual time ordered product operation. Since $x(t)$ and $x(t')$ always commute, it is clear that the generating functional for both the fluctuation and response functions is given by

$$\begin{aligned} W[\eta, \hat{\eta}] &= \ln Z[\eta, \hat{\eta}] = \ln \langle T(S[\eta, \hat{\eta}]) \rangle \\ S[\eta, \hat{\eta}] &= \exp \int dt dr [x(r, t)\eta(r, t) + \hat{x}(r, t)\hat{\eta}(r, t)] . \end{aligned} \quad (12)$$

In particular the one particle functions are

$$\frac{\delta W}{\delta \eta(t)} \Big|_{\eta=\hat{\eta}=0} = \langle x(t) \rangle , \quad \frac{\delta W}{\delta \hat{\eta}(t)} \Big|_{\eta=\hat{\eta}=0} = \langle \hat{x}(t) \rangle = 0 . \quad (13)$$

The two particle Green's function matrix is tridiagonal

$$G_2(t) = \begin{pmatrix} U(t) & D_R(t) \\ D_R(-t) & 0 \end{pmatrix} \quad (14)$$

where

$$G_{xx}(12) = U(12) = \langle x(1)x(2) \rangle - \langle x(1) \rangle \langle x(2) \rangle ,$$

and

$$G_{x\hat{x}}(12) = D_R(12) = \langle T(x(1)\hat{x}(2)) \rangle = \Theta(t_1 - t_2) \langle x(1)\hat{x}(2) \rangle .$$

One next doubles the space as in the CTP formalism by introducing the field $\Phi(r, t)$

$$\Phi(r, t) = \begin{pmatrix} x(r, t) \\ \hat{x}(r, t) \end{pmatrix} \quad (15)$$

Then the commutators of Φ satisfy in the (2×2) space

$$[\Phi_\alpha(r, t), \Phi_\beta(r', t)] = i(\sigma_2)_{\alpha\beta} \delta(r - r') . \quad (16)$$

In this larger space the equations of motion are

$$\dot{\Phi} = [\Phi, H] , \quad (17)$$

where the “non-Hermitian” operator H has the form

$$H = \int \gamma_a^1 \Phi_a(1) d1 + \frac{1}{2} \gamma_{ab}^2 \int d1 \Phi_a(1) \Phi_b(1) + \frac{1}{3!} \gamma_{abc}^3 \int d1 \Phi_a \Phi_b \Phi_c , \quad (18)$$

and the equations of motion in the presence of an external source J is

$$-i\sigma_2 \dot{\Phi}_a = J_a(1) + \gamma_a^1 + \gamma_{ab}^2 \Phi_b(1) + \frac{1}{2} \gamma_{abc}^3 \Phi_b \Phi_c , \quad (19)$$

where

$$J(r, t) = \begin{pmatrix} \eta(r, t) \\ \hat{\eta}(r, t) \end{pmatrix} . \quad (20)$$

The generating functional $W[\eta, \hat{\eta}]$ of Eq. 12 is now written in compact form by writing

$$S = \exp \left[\int dr dt \Phi(r, t) J(r, t) \right] . \quad (21)$$

The expectation value of the equation of motion in the presence of an external source leads to the equation

$$\begin{aligned}
-i\sigma_2\dot{G}_1(1) &= J(1) + \gamma^1(1) + \gamma^2 G_1(1) + \frac{1}{2}\gamma^3[G_2(11) + G_1(1)G_1(1)] , \\
G_m^J &= \frac{\delta^m W}{\delta J(1)\delta J(2)\cdots\delta J(m)} .
\end{aligned} \tag{22}$$

All the higher n-point functions are obtained by functional differentiation. The complete S-D equations are derived in the original MSR paper [3], and a path integral representation was derived by Jouvét and Phythian [5]. These coupled Green's functions are in a $2N \times 2N$ matrix space and obey first order in time equations rather than in the usual $N \times N$ space obeying second order equations. To see how to obtain the second order formalism we will simplify the discussion by considering a cubic plus quartic anharmonic oscillator.

III. SECOND ORDER MSR FORMALISM

For the anharmonic oscillator with equation of motion

$$\ddot{x} + m^2 x + gx^2 + \lambda x^3 = 0 ,$$

the above discussion leads to

$$H = \hat{x}p - \hat{p}(m^2 x + gx^2 + \lambda x^3) . \tag{23}$$

This yields the first order equations

$$\begin{aligned}
\frac{dx}{dt} &= [x, H] = p , & \frac{dp}{dt} &= [p, H] = -(m^2 x + gx^2 + \lambda x^3) , \\
\frac{d\hat{x}}{dt} &= [\hat{x}, H] = \hat{p}(m^2 + 2gx + 3\lambda x^2) , & \frac{d\hat{p}}{dt} &= [\hat{p}, H] = -\hat{x} .
\end{aligned} \tag{24}$$

The first order equations for the Green's functions are obtained by taking the expectation value of these equations with sources added and then using functional differentiation. The expectation value is over an initial distribution function $\rho[x_0, p_0]$. A simple example is to choose our ensemble of initial conditions at $t = 0$ from a thermal distribution of the free Hamiltonian, i.e.

$$\rho = N e^{-\beta H_0} , \quad H_0 = \frac{1}{2}p^2 + \frac{1}{2}m^2 x^2 , \quad \int \rho dx dp = 1 . \tag{25}$$

One can show that out of the twelve nonzero Green's Functions in the first order formalism only two are independent. These can be chosen as G_{xx} , and $G_{x\hat{p}}$.

The harmonic oscillator values of these Green's functions are easily determined from the operator solutions

$$x(t) = x_0 \cos mt + \frac{p_0}{m} \sin mt, \quad \hat{p}(t) = \hat{p}_0 \cos mt - \frac{\hat{x}}{m} \sin mt, \quad (26)$$

and the initial density matrix. For the thermal initial conditions described above we obtain

$$\langle x(t)x(t') \rangle = \frac{1}{\beta m^2} \cos m(t-t'), \quad \langle x(t)\hat{p}(t') \rangle = \frac{1}{m} \sin m(t-t'). \quad (27)$$

The response function is

$$\langle T(x(t)\hat{p}(t')) \rangle = D_{ret}^0(t-t') = \theta(t-t') \frac{1}{m} \sin m(t-t'). \quad (28)$$

In general, in the absence of sources

$$G_{\hat{p}\hat{p}}(t-t') = 0, \quad G_{\hat{p}x}(t-t') = G_{x\hat{p}}(t'-t) = D_{adv}^0(t-t').$$

The independent second order equations (adding sources) are

$$\begin{aligned} \left[\frac{d^2}{dt^2} + m^2 \right] x + gx^2 + \lambda x^3 &= j_x, \\ \left[\frac{d^2}{dt^2} + m^2 \right] \hat{p} + 2gx\hat{p} + 3\lambda x^2\hat{p} &= j_{\hat{p}}. \end{aligned} \quad (29)$$

Here $\hat{p}(0)$ is treated as an operator $-\frac{\delta}{\delta p}$ when one averages over the initial probability function in phase space.

These equations are derivable from the Lagrangian

$$L_{MSR} = \frac{1}{2} \left(\hat{p} \left[\frac{d^2}{dt^2} + m^2 \right] x + x \left[\frac{d^2}{dt^2} + m^2 \right] \hat{p} \right) + gx^2\hat{p} + \lambda x^3\hat{p} - j_{\hat{p}}x - j_x\hat{p}. \quad (30)$$

The vertices of the classical theory are

$$\gamma_{\hat{p}xx} = \gamma_{x\hat{p}x} = \gamma_{xx\hat{p}} = 2g, \quad \gamma_{\hat{p}xxx} = \gamma_{x\hat{p}xx} = \gamma_{xx\hat{p}x} = \gamma_{xxx\hat{p}} = 6\lambda.$$

A formal path integral formalism can be generated for the generating functional

$$Z[j_{\hat{p}}, j_x] = \left\langle \int dx d\hat{p} e^{-\int L(x, \hat{p}) dt} \right\rangle, \quad (31)$$

where the expectation value is over the initial density matrix.

We want to compare L_{MSR} with the Lagrangian for the CTP formalism. The result of the CTP formalism [2] is that the action is the difference of two terms, one for each branch of the closed time path contour. Explicitly for the anharmonic oscillator

$$L_{CTP} = \frac{1}{2} \left(x_+ \left[\frac{d^2}{dt^2} + m^2 \right] x_+ - x_- \left[\frac{d^2}{dt^2} + m^2 \right] x_- \right) + \frac{1}{3} g [x_+^3 - x_-^3] + \frac{1}{4} \lambda [x_+^4 - x_-^4] - j_+ x_+ + j_- x_- . \quad (32)$$

Introducing the change of variables

$$x_+ = x + \frac{\hbar \hat{p}}{2} , \quad x_- = x - \frac{\hbar \hat{p}}{2} , \quad (33)$$

we obtain in this new basis

$$\frac{L_{CTP}^{(2)}}{\hbar} = \frac{1}{2} \left(\hat{p} \left[\frac{d^2}{dt^2} + m^2 \right] x + x \left[\frac{d^2}{dt^2} + m^2 \right] \hat{p} \right) + g x^2 \hat{p} + \lambda x^3 \hat{p} + \frac{g \hbar^2}{12} \hat{p}^3 + \frac{\lambda \hbar^2}{4} x \hat{p}^3 - j_{\hat{p}} x - j_x \hat{p} , \quad (34)$$

where

$$j_{\hat{p}} = j_+ - j_- , \quad j_x = \frac{j_+ + j_-}{2} .$$

This formal manipulation can be justified by first obtaining the SD equations in the $(+, -)$ basis and making a rotation by $\pi/4$ as discussed in Ref. [1].

We notice that there are five extra vertices in the quantum case

$$\gamma_{\hat{p}\hat{p}\hat{p}} = \frac{g \hbar^2}{2} , \quad \gamma_{x\hat{p}\hat{p}\hat{p}} = \gamma_{\hat{p}x\hat{p}\hat{p}} = \gamma_{\hat{p}\hat{p}x\hat{p}} = \gamma_{\hat{p}\hat{p}\hat{p}x} = \frac{3\lambda \hbar^2}{2} .$$

not present in the MSR Lagrangian. Note that \hbar arises from the fact that the classical \hat{p} is the derivative operator $-d/dp$ which for quantum mechanics becomes $\frac{\hbar}{i} d/dp$. It may be noted that apart from these extra vertices in the quantum case, \hbar dependence also enters in the initial conditions on the Green's functions. This dependence is explicit in the commutator contributions and also occurs in the \hbar dependence on the initial distribution which is now a Bose-Einstein distribution rather than a Maxwell-Boltzmann one. As

an illustration consider the quantum version of Eq. (26) for the Harmonic oscillator with an initial Bose distribution. Then we obtain

$$x^Q(t) = x_0^Q \cos(mt) + \frac{p_0^Q}{m} \sin(mt). \quad (35)$$

From this we find

$$\begin{aligned} \langle x(t)x(t') \rangle &= \langle x_0^2 \rangle \cos(mt) \cos(mt') + \langle \frac{p_0^2}{m^2} \rangle \sin(mt) \sin(mt') \\ &\quad - i \frac{\hbar}{2} \sin[m(t-t')], \end{aligned} \quad (36)$$

where

$$\langle x_0^2 \rangle = \frac{\hbar}{2m} + \frac{\hbar}{m} [\exp(\hbar\beta m) - 1]^{-1}. \quad (37)$$

and

$$\langle p_0^2 \rangle = m^2 \langle x_0^2 \rangle.$$

So we see that \hbar enters in many ways in the initial conditions for the Green's functions, and also in the structure of the S-D equations where certain vertices are proportional to \hbar^2 . The way one derives the S-D equations from the action is identical for both classical and quantum mechanics. Thus we obtain the same structure classically, but there are fewer vertex functions.

We now derive the S-D equations for a generic cubic self-interacting field theory whether classical or quantum. For N interacting scalar fields, we introduce the column vector Φ_α composed of $\Phi_i^1 = \phi_i$ and $\Phi_i^2 = \hat{\pi}_i$ where $i = 1, 2, \dots, N$. We also need the metric $g_{\alpha\beta}$ which is just $\sigma_{\alpha\beta}^1$ as far as connecting the Φ^1 and Φ^2 sectors. Then we can write generically for cubic interactions:

$$\mathcal{L} = \frac{1}{2} \phi^\alpha (D_0^{-1})_{\alpha\beta} \Phi^\beta + \frac{1}{3!} \gamma_{\alpha\beta\rho} \Phi^\alpha \Phi^\beta \Phi^\rho - J_\alpha \Phi^\alpha, \quad (38)$$

where

$$D_0^{-1}{}_{\alpha\beta}(x-y) = g_{\alpha\beta} [\square + m^2] \delta(x-y).$$

The generating functional is formally

$$Z = \langle T(\exp[J_\alpha \Phi^\alpha]) \rangle = e^{W[J]}. \quad (39)$$

Defining the ‘‘classical’’ field Φ^α and the connected 2 point function G via

$$\Phi^\alpha = \frac{\delta W}{\delta J_\alpha} , \quad G(1,2)_{\alpha\beta} = \frac{\delta\Phi_\alpha(1)}{\delta J_\beta(2)} ,$$

one has that in the presence of sources

$$\begin{aligned} D_0^{-1}{}_{\alpha\beta}\Phi^\beta(1) + \frac{1}{2}\gamma_{\alpha\beta\rho}\left[\frac{\delta\Phi^\beta(1)}{\delta J_\rho(1)} + \Phi^\beta(1)\Phi^\rho(1)\right] &= J_\alpha(1) , \\ D_0^{-1}{}_{\alpha\beta}G^{\beta\rho}(12) + \frac{1}{2}\gamma_{\alpha\beta\sigma}\left[\frac{\delta G^{\beta\sigma}(11)}{\delta J_\rho(2)} + \Phi^\beta(1)G^{\sigma\rho}(12) + \Phi^\sigma(1)G^{\beta\rho}(12)\right] \\ &= \delta_\alpha^\rho\delta(1-2) . \end{aligned} \quad (40)$$

To obtain the S-D equation for the inverse two point function we first use the connection between the connected 3-point function and the 1-PI vertex function [8] obtained by first Legendre transforming from J to Φ and using the chain rule

$$\frac{\delta}{\delta J_\alpha(1)} = \int d2 \, G(12)^{\alpha\beta} \frac{\delta}{\delta\Phi(2)^\beta} , \quad (41)$$

$$\begin{aligned} \langle T\left(\Phi_\alpha(1)\Phi_\beta(2)\Phi_\rho(3)\right)\rangle_c &= \frac{\delta G_{\alpha\beta}(12)}{\delta J^\rho(3)} \\ &= \int d4 \, d5 \, d6 \, G_{\alpha\alpha'}(14)G_{\beta\beta'}(25)G_{\rho\rho'}(3,6)\Gamma^{\alpha'\beta'\rho'}(456) . \end{aligned} \quad (42)$$

where the 1-PI three point function is defined by

$$\Gamma_{\alpha\beta\rho}(123) = \frac{\delta G_{\alpha\beta}^{-1}(1,2)}{\delta\Phi^\rho(3)} . \quad (43)$$

The usual S-D is obtained by multiplying Eq. 40 on the right by G^{-1} to obtain

$$G_{\alpha\beta}^{-1}(1,2) = D_{0\alpha\beta}^{-1}(1,2) + \Sigma_{\alpha\beta}(1,2) + \gamma_{\alpha\beta\rho}\Phi^\rho(1)\delta(1-2) , \quad (44)$$

and

$$\Sigma_{\alpha\beta}(1,2) = \frac{1}{2} \int d3d4 \, \gamma_{\alpha\rho\sigma}G^{\rho\lambda}(13)G^{\sigma\nu}(14)\Gamma(342)_{\lambda\nu\beta} . \quad (45)$$

We also have

$$\Gamma_{\alpha\sigma\rho}(1,2,3) = \gamma_{\alpha\sigma\rho}\delta(1-3)\delta(1-2) + \frac{\delta\Sigma_{\alpha\sigma}(1,2)}{\delta\Phi^\rho(3)} . \quad (46)$$

However Σ is a proper self energy and can be related [7] to the effective action $\Gamma_2[G]$ for the 2-PI irreducible graphs of the theory via

$$\Sigma(12)_{\alpha\beta} = 2 \frac{\delta\Gamma_2[G]}{\delta G^{\alpha\beta}(1,2)} , \quad (47)$$

so that it is just a function of the bare vertices and the full Green's functions. Using

$$\frac{\delta\Sigma(12)_{\alpha\beta}}{\delta\Phi_\rho(3)} = \int d5d6 \frac{\delta\Sigma(12)_{\alpha\beta}}{\delta G^{\sigma\lambda}(5,6)} \frac{\delta G^{\sigma\lambda}(5,6)}{\delta\Phi_\rho(3)} , \quad (48)$$

we obtain that

$$\begin{aligned} \Gamma_{\alpha\beta\nu}(1,2,3) &= \gamma_{\alpha\beta\nu} \delta(1-2)\delta(1-3) \\ &- \int d4 d5 d6 d7 \Gamma_{\alpha\rho\sigma}(145) G^{\rho\eta}(46) G^{\sigma\lambda}(57) H(67;23)_{\eta\lambda;\beta\nu} . \end{aligned} \quad (49)$$

The scattering Kernel H is defined by

$$\frac{\delta\Sigma_{\alpha\beta}(12)}{\delta G^{\sigma\lambda}(5,6)} = H(1256)_{\alpha\beta\sigma\lambda} .$$

Self consistent approximations [7, 8] are determined by keeping a certain class of graphs in $\Gamma_2[G]$, the sum of all 2-PI graphs made from bare vertices and full propagators. For cubic interactions, the Bare Vertex Approximation (BVA) is obtained by keeping the graph

$$\int d1d2 \gamma_{ijk} G^{il}(12) G^{jm}(12) G^{kn}(12) \gamma_{lmn} ,$$

which then leads to the self energy being the one loop diagram, and the scattering Kernel being single particle exchange. By using the variational definitions of Σ and H one is guaranteed an internally consistent approximation. As an example of the difference between the quantum and classical S-D equations, let us restrict ourselves to a cubic anharmonic oscillator and make the bare vertex approximation. The S-D equation for the correlation function becomes

$$G_{xx}(12) = G_{xx}^0(12) + \int d3d4 G_{xi}^{(0)}(13) \gamma_{ijk} G^{jl}(34) G^{km}(34) \gamma_{lmn} G_{nx}(42) , \quad (50)$$

which can be identified with Kraichnan's Direct Interaction Approximation [9]. Expanding we can write this in terms of classical and quantum contributions. We have in symbolic form

$$G_{xx} = G_{xx}^0 + G_{xx}^0 \Sigma_A^{cl} G_{\hat{p}x} + G_{x\hat{p}}^0 \Sigma_R^{cl} G_{xx} + G_{x\hat{p}}^0 [\Sigma_F^{cl} + \Sigma_F^Q] G_{\hat{p}x} . \quad (51)$$

The classical contributions to the self-energy in the BVA are

$$\begin{aligned} \Sigma_F^{cl}(12) &= \frac{1}{2} \gamma_{\hat{p}xx}^2 G_{xx}^2(12) , & \Sigma_R^{cl}(12) &= \gamma_{\hat{p}xx}^2 G_{x\hat{p}}(12) G_{xx}(12) , \\ \Sigma_A^{cl}(12) &= \gamma_{\hat{p}xx}^2 G_{\hat{p}x}(12) G_{xx}(12) , \end{aligned} \quad (52)$$

and the quantum contribution is

$$\Sigma_F^Q(12) = \frac{1}{2} \gamma_{\hat{p}xx} \gamma_{\hat{p}\hat{p}\hat{p}} [G_{x\hat{p}}^2(12) + G_{\hat{p}x}^2(12)] , \quad (53)$$

which has formal order of \hbar^2 since $\gamma_{\hat{p}\hat{p}\hat{p}}$ is of order \hbar^2 .

We have recently shown that the *classical* BVA gives excellent agreement with Monte Carlo simulations in 1 + 1 dimensional ϕ^4 field theory [10].

IV. CONCLUSIONS

We have shown how the classical S-D equations can be derived from the quantum S-D equations by comparing the CTP formalism with the MSR formalism. Our hope is that this result will allow researchers to make clearer the connection between quantum field theory and classical field theory at high temperatures. We are in the process of comparing the BVA for the quantum and classical cases at high and low temperature to understand for what range of temperatures the classical approximation is valid.

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