ACCELERATOR DRIVEN SUB CRITICAL SYSTEMS WITH ENHANCED NEUTRON MULTIPLICATION

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Abstract

Utilisation of Thorium, by way of the ^{233}U-Th cycle, is of particular interest to the Indian Nuclear Power Programme because of large Thorium deposits and limited Uranium reserves. Several schemes, such as fast and advanced heavy water reactors, leading to Thorium utilisation, are under study at this centre.

The present paper discusses a scheme for evolving a practical accelerator driven sub critical ^{233}U-Th system with increased neutron multiplication and a consequent reduced requirement of the accelerator current. It is shown that the requirement of the accelerator current is considerably reduced if a sub critical assembly with a given K_{eff} is composed of two partially coupled regions.

1.0 INTRODUCTION

The accelerator driven sub critical system for power production has evoked considerable interest in recent years (Rubia et al, 1995). The scheme promises to burn Pu and other actinides and long lived fission products, and produce power with a high degree of inherent safety and at a reasonable cost. The principal difficulty continues to be the requirement of accelerator currents one order of magnitude over the best achieved so far. For this reason Daniel and Petrov (1996) have proposed a scheme for enhancing the multiplication of the system by having a central booster region having a $K_\infty \sim 1.2$ and an outer region having a $K_\infty \sim 0.975$. The idea is to introduce source neutrons in a region of high neutron importance and thereby get enhanced multiplication.

In the present paper we show that it is possible to achieve higher neutron multiplication if there is a one way coupling between the two regions. Neutrons from the inner region may leak out into the outer region. However the probability for outer region neutrons to re-enter the inner region is to be made as low as possible. For the purpose of the K eigenvalue, such a system is effectively a decoupled system and its K is determined by the larger of the two K. Two practical ways of achieving such a coupling are introduction of a sufficiently large gap between the two systems and having an inner fast system driving an outer thermal system with a thermal absorber in between them.

Our calculations indicate that such a scheme gives a neutron multiplication about four to five times larger than in the Energy Amplifier scheme described by Rubia et al (1995). This means that it should be possible to get the desired power level of 1.5 GW (t) with a 1 Gev proton beam current of as low as 2 mA.

2.0 SYSTEM DESCRIPTION AND COMPUTATIONAL MODEL

For simplicity of analysis we consider a system having spherical symmetry. There is an inner system having a $K_\infty > 1$

up to a radius R_1 followed by a gap up to radius R_2 and finally an outer region having $K_\infty < 1$ in the region between R_2 to R_3. Two cases are considered. One in which both regions are fast systems and the other in which the inner region is a fast reactor, while the outer region is thermal. In the latter case the inner booster is lined with a thermal neutron absorber such as Cd which prevents thermal neutrons from the outer reactor to enter the booster while allowing fast neutrons from the booster to enter the outer reactor.

We present the following discussion by defining three K_{eff}s: one each for the isolated inner and outer systems and the third for the combined system. In order to maintain adequate sub criticality margins such a single combined K_{eff} is found to be appropriate.

The system, described above, could be treated by a modification of the one or two group theory of a reflected reactor. However, the presence of the gap and the absorber lining, complicates matters. It is nevertheless possible to use simple diffusion theory if suitable interface conditions can be derived. It is then possible to write down analytical expressions for the flux and fundamental mode distributions. A simple numerical search yields the K_{eff} of the two individual systems as well as the combined K_{eff} (or one of the radii for a given K_{eff}). The diffusion theory approach is particularly useful for a quick search for a system with the desired K_{eff}. The details of this approach are discussed in the Appendices I and II. The results of the diffusion theory calculations were also checked using an integral transport theory program based on the collision probability method. Good agreement between the two is found. The program was also used for computing the flux distribution, and amplification properties of the selected cases.

3.0 RESULTS

Let us consider first the scheme described by Daniel and Petrov (1996) with an inner system with a $K_\infty = 1.2$ and an outer system having $K_\infty = 0.975$. For the mixture of Pb,

^{233}U and ^{232}Th described by Rubia et al (1995), the migration length M would be approximately 10 cm and 11 cm respectively for the inner and the outer regions. With the outer system having $K_{eff} = K_\infty/(1+M^2B^2) = 0.97$, it is clear that the buckling parameter $B = 6.52\times10^{-3}$ cm^{-1} which gives $r_2-r_1 = 288$ cm leading to $r_2 = 320$ cm and $r_1 = 32$ cm. The results for this case are summarized in Tables 1 and 2. Table 1 shows the K_{eff} for various inner and outer region sizes while Table 2 shows the energy amplification for a system with the above dimensions. Table 1 shows that there exists an inverse relation between K_1 and K_2 for maintaining the same value of K_{12}. Table 2 shows the difference between a distributed source and a point source.

We next consider the two sets of systems falling in our scheme. The first consists of two fast regions separated by a gap. The material properties are the same as in the previous scheme. The sizes are adjusted to give overall sub criticality of about 20 mK. Table 3 presents the K_{eff} for systems of various sizes. It clearly shows the effect of the gap in maintaining a constant $K_{eff} = 0.953$ of the inner region for a given $K_{eff} = 0.980$ of the combined system even when the K_{eff} of the outer region increases. This is a signature of the oneway coupling between the two systems. Note that this is in contrast with Table 1 where constancy of the overall K_{eff} can be maintained only if K_1 decreases while K_2 increases or vice versa. Table 4 shows the results obtained for neutron multiplication, source importance S_o, the overall energy gain and the required accelerator current for the various cases described in Table 3. This table clearly demonstrates the efficacy of the booster and the decoupling concept for giving enhanced multiplication.

Table 5 shows the effectiveness of the gap and the absorber lining in increasing the degree of decoupling for the fast-thermal systems. When the gap and the thermal absorbers are used the decoupling effect is stronger than that for the fast system. For the subcritical system (case 3 in table 1) the parameters used are R_1 = 60 cm, R_2 = 210 cm and R_3 = 310 cm. The neutron multiplication obtained in this case is 310 corresponding to an energetic gain of 682 and an accelerator current of about 1.1 mA for a 750 MW(th) reactor.

Finally Table 6 shows a comparison of the diffusion and transport theory values obtained for K_{eff}. It is seen that diffusion theory with appropriate internal boundary conditions is reasonably good in predicting the K_{eff}. Though we have used the transport theory for computing the energetic gain and multiplication, we could have used the diffusion theory as well.

Figs.1-3 show respectively the volumetric power density, the power density per unit radial interval and the flux distribution for a typical fast-fast system. Likewise Figs. 4-5 show the corresponding power distributions for a typical fast-thermal system. The power is normalised to a total power of 750 MW (th). Finally, Figs. 6-7 show the fast and thermal flux distribution respectively.

The power distribution in the case of the fast-fast system is strongly peaked in the inner region and the power density is about an order of magnitude lower in the outer region compared to that in the inner region and also falls off towards the periphery. While the power density is typical of fast reactors in the inner region, it is too low in the outer regions. If the outer region is to have a power density high enough to be economical, it is clear that a special fuel or cooling would have to be used in the inner region for withstanding the resultant high power density. Moreover since a large fraction of the power (~ 35%) is produced in the booster region, it will be necessary to have a smaller inner region (possibly with a higher enrichment) so as to reduce the fraction of power produced in it and the consequent burnup of fissile fuel. Finally it would also be necessary to flatten the power in the outer region by adding a reflector or by some other means.

For the fast-thermal system the situation is much better. The outer region shows power densities typical of thermal reactors while the ones in the inner region correspond to those in fast reactors. The fraction of the total power in the inner region is also smaller (15%) though ideally we would want it to be a few percent in order to minimise the burnup of fissile fuel. Power flattening in the outer region would also be desirable.

4.0 CONCLUSIONS

We have presented calculations which indicate that a booster region can be placed at the centre of an accelerator driven reactor provided the main reactor is coupled only one way with the booster so that the overall K_{eff} does not exceed unity. This could be achieved either by keeping a sufficiently large gap between the two or by having a thermal reactor coupled one way to a fast booster with a gap and a thermal absorber such as Cd in between the two. Neutron importance factors of about 5 to 10 can be achieved in this way. Our estimates show that an energy amplification can be obtained which is about two times that reported by Daniel and Petrov (1996) for the same K_{eff}. With such a system it should be possible to work with accelerator currents as low as 2 mA for a 1500 MW (th) reactor. However, this issue requires a detailed design of the two subsystems constituting an overall subcritical reactor.

Another problem associated with such a system is the strong power peaking in the booster region and adequate cooling would have to be provided for this purpose. However, this issue is similar to the problem of cooling a target region producing a spallation source of the same strength, since, in the latter, the energy per neutron is 30 to 40 Mev while for fission neutrons it is 80 Mev which is about twice as large.

REFERENCES

Daniel H. and Petrov Yu. V. (1996) Nucl. Instr. Meth. Phys. Res. A 373, 131

Davison (1957) Neutron Transport Theory, Oxford University Press, Oxford
Glasstone S. and Edlund (1952) The Elements of Nuclear Reactor Theory, D. Van Nostrand, NJ
Newmarch D.A.(1955) J. Nucl. Energy 2, 52
Rubia C., Rubio J.A., Buono S. Carmine F., Fieter N., Galvez J., Geles C., Kadi Y., Klapisch R., Mandrillon P. Revol J.P., and Roche C. (1995) Report CERN/AT/95-44 (ET)

Appendix I

A 1.1 Diffusion Theory: Boundary Conditions at the Gap

For spherically symmetrical systems, Davison (1957) has given the following physically obvious boundary condition at the two faces of an annular gap:

$$R_i^2 J_i = R_o^2 J_o \tag{A.1}$$

where the subscripts i and o stand for the inner and outer. To solve the diffusion equation one more interface condition is required. The flux is not expected to be equal at the inner and outer faces. Newmarch (1955) has given the correct boundary conditions for a cylindrical system in the context of computing the thermal utilisation factor when there is a gap between the fuel and the moderator.

By using a method similar to that described in elementary text books on reactor Physics (Glasstone and Edlund, 1952) we can derive the following equations for J^+ and J^- at the two faces for the spherically symmetrical reactor under consideration

$$J_i^+ = \frac{1}{4}\phi_i - \frac{1}{2}D_i\frac{\partial\phi_i}{\partial r} \tag{A.2a}$$

$$J_i^- = \frac{1}{4}\phi_o - \frac{1}{2}D_o\left(\frac{R_o}{R_i}\right)^2\left[1 - \left\{1 - \left(\frac{R_i}{R_o}\right)^2\right\}^{3/2}\right]\frac{\partial\phi_o}{\partial r} \tag{A.2b}$$

$$J_o^+ = \left(\frac{R_i}{R_o}\right)^2\left[\frac{1}{4}\phi_i - \frac{1}{2}D_i\frac{\partial\phi_i}{\partial r}\right] + \frac{1}{4}\left\{1-\left(\frac{R_i}{R_o}\right)^2\right\}\phi_o + \frac{1}{2}D_o\left\{1-\left(\frac{R_i}{R_o}\right)^2\right\}^{3/2}\frac{\partial\phi_o}{\partial r} \tag{A.2c}$$

$$J_o^- = \frac{1}{4}\phi_o + \frac{1}{2}D_o\frac{\partial\phi_o}{\partial r} \tag{A.2d}$$

The subscripts o and i refer to the inner and outer faces of the gap. Equation (1) can be easily seen to follow from (2) by subtracting (2a) from (2b) and (2c) from (2d). The first interface condition is obtained by replacing the currents by their diffusion theoretic expressions viz:

$$R_i^{\,2} D_i \frac{\partial \phi_i}{\partial r} = R_o^{\,2} D_o \frac{\partial \phi_o}{\partial r} \qquad \text{(A.3)}$$

The second interface condition follows on subtracting (2c) from (2d) and replacing the net J_o by its diffusion theoretic value. We thus get

$$\left(\frac{R_i}{R_o}\right)^2 \left[\frac{1}{4}\phi_i - \frac{1}{2} D_i \frac{\partial \phi_i}{\partial r}\right] = \frac{1}{4}\left(\frac{R_i}{R_o}\right)^2 \phi_o + \frac{1}{2} D_o \left[1 + \left\{1 - \left(\frac{R_i}{R_o}\right)^2\right\}^{3/2}\right] \frac{\partial \phi_o}{\partial r}$$

(A.4)

For the fast-thermal system under consideration, we describe it in terms of the fast inner flux ϕ_{i1} and the fast and thermal outer fluxes ϕ_{o1} and ϕ_{o2}. For the fast flux we use the above boundary conditions. For the thermal flux the boundary condition cannot be used since it is not permissible to replace J_i by its diffusion theoretic value. However the second one can be used after setting ϕ_i and its derivative to zero. In other words we have,

$$\frac{1}{4}\left(\frac{R_i}{R_o}\right)^2 \phi_o + \frac{1}{2} D_o \left[1 + \left\{1 - \left(\frac{R_i}{R_o}\right)^2\right\}^{3/2}\right] \frac{\partial \phi_o}{\partial r} = 0 \qquad \text{(A.5)}$$

for the thermal flux.

A 1.2 Solution of the Two Group Equations

The inner system has $K_\infty > 1$ and is essentially fast. Hence its solution is given by

$$\phi_i^{(f)} = AZ(r) = A\frac{\sin(\mu r)}{r} \qquad \text{(A.6)}$$

where,

$$\mu = \frac{(K_{\infty i}/\ K_{12} - 1)}{L_i^{(f)2}} \qquad \text{(A.7)}$$

The superscripts f and t refer to fast and thermal quantities while the subscripts (i) and (o) stand for the inner and the outer region quantities respectively. The outer system has a $K_{\infty o} < 1$ and if we assume further that $K_{12} > K_{\infty o}$, the flux would be a decaying function. With the boundary condition that the fluxes go to zero at R_3 the outer most boundary of the system, we can write the two group fluxes as follows:

$$\phi_o^{(f)} = CX(r) + EY(r) = C\frac{\sinh\nu_1(R_3-r)}{r} + E\frac{\sinh\nu_2(R_3-r)}{r} \quad (A.8)$$

$$\phi_o^{(t)} = CS_1X(r) + ES_2Y(r) \quad (A.9)$$

where $-\nu_1^2$ and $-\nu_2^2$ are the roots (both negative in this case) of the critical equation

$$K_{12} = \frac{K_{\infty o}}{(1+L_o^{(f)2}B^2)(1+L_o^{(t)2}B^2)} \quad (A.10)$$

given by

$$\nu_{1,2}^2 = \frac{-(L_o^{(f)2}+L_o^{(t)2}) \pm\sqrt{(L_o^{(f)2}+L_o^{(t)2})^2-4L_o^{(f)2}L_o^{(t)2}(1-K_{\infty o}/K_{12})}}{2L_o^{(f)2}\;L_o^{(t)2}} \quad (A.11)$$

We note here that $L_o^{(t)}$ for the thermal system is essentially the slowing down length. S_1 and S_2 are obtained by substituting these solutions in the two group equations and are given by

$$S_1 = \frac{\Sigma_{ro}}{\Sigma_{ao}^{(t)} - D_o^{(t)}\nu_1^2} \quad (A.12a)$$

$$S_2 = \frac{\Sigma_{ro}}{\Sigma_{ao}^{(t)} - D_o^{(t)}\nu_2^2} \quad (A.12b)$$

Imposing the three interface conditions, we get the following homogeneous system of equations for A, C and E:

$$C_{11}A + C_{12}C + C_{13}E = 0 \qquad \text{(A.13a)}$$
$$C_{21}A + C_{22}C + C_{23}E = 0 \qquad \text{(A.13b)}$$
$$C_{32}C + C_{33}E = 0 \qquad \text{(A.13c)}$$

This system has a non trivial solution only if

$$\begin{vmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ 0 & C_{32} & C_{33} \end{vmatrix} = 0 \qquad \text{(A.14)}$$

The coefficients are given by:

$$C_{11} = R_1{}^2 D_i^{(f)} Z'(R_1) \qquad \text{(A.15a)}$$

$$C_{12} = - R_2{}^2 D_o^{(f)} X'(R_2) \qquad \text{(A.15b)}$$

$$C_{13} = - R_2{}^2 D_o^{(f)} Y'(R_2) \qquad \text{(A.15c)}$$

$$C_{21} = \left(\frac{R_i}{R_o}\right)^2 \left(0.25\ Z(R_1) -).50\ D_o^{(f)} Z'(R_1)\right) \qquad \text{(A.15d)}$$

$$C_{22} = 0.25\left(\frac{R_i}{R_o}\right)^2 X(R_2) - 0.50\ D_o^{(f)}\left[1 + \left\{1 - \left(\frac{R_i}{R_o}\right)^2\right\}^{3/2}\right] X'(R_2) \qquad \text{(A.15e)}$$

$$C_{23} = 0.25\left(\frac{R_i}{R_o}\right)^2 Y(R_2) - 0.50\ D_o^{(f)}\left[1 + \left\{1 - \left(\frac{R_i}{R_o}\right)^2\right\}^{3/2}\right] Y'(R_2) \qquad \text{(A.15f)}$$

$$C_{32} = 0.25\left(\frac{R_i}{R_o}\right)^2 S_1 X(R_2) - 0.50\ D_o^{(f)}\left[1+\left\{1-\left(\frac{R_i}{R_o}\right)^2\right\}^{3/2}\right] S_1 X'(R_2) \qquad \text{(A.15g)}$$

$$C_{33} = 0.25\left(\frac{R_i}{R_o}\right)^2 S_2Y(R_2) - 0.50\ D_o^{(f)}\left[1+\left\{1-\left(\frac{R_i}{R_o}\right)^2\right\}^{3/2}\right]S_2Y'(R_2)$$

(A.15h)

Any desired value of K_{12} can be given as input and a search made for the outer radius R_3 once, R_1, R_2 $K_{\infty i}$, $K_{\infty o}$ are given. K_1 and K_2 individually are also easily determined. The case of two fast systems appears as special case.

Appendix II

A 2.1 Derivation of the Partial Currents

To derive Eqs(2) we use a modified version of the technique described by Glasstone and Edlund (1957). Consider three concentric spherical regions having radii R_1, R_2 and R_3 and having material 1 in the innermost sphere, vacuum in the region between R_1 and R_2 and material 2 in the region between R_2 and R_3 and vacuum outside R_3. We also assume that the radii R_1 and R_3-R_1 are sufficiently large compared to a mean free path and that the absorption is sufficiently weak so that diffusion theory is valid.

We see immediately that the first and last of the equations for the partial currents are the usual equations for these currents and follow from the assumptions made. So we only need to derive Eqs. (2b,2c). For the first of these for J_i^- we refer to Fig. 1a. By solving the triangles OPQ and OPR we can write down the following relation between x and δR correct to the lowest order in δR:

$$x = R_i \delta R \left(R_o^2 - R_i^2 \sin^2\vartheta \right)^{-1/2} \qquad \text{(A.16)}$$

The expression for J_i^- can be now written down as follows:

$$J_i^- = \frac{\Sigma_o}{2} \int_0^{\pi/2} d\vartheta \int_y^{\infty} dr \, \exp(-\Sigma_o x) \, \sin\vartheta \cos\vartheta \, \phi(R_o+\delta R) \qquad \text{(A.17)}$$

Expanding ϕ around R_o to the lowest order in δR and using Eq. (A1) the integrals are easily evaluated to give us the desired result.

To get an expression for J_o^+ we note (Figs. 1b and 1c) that this current has two components coming from the inner and the outer regions. Thus we have

$$J_o^+ = \frac{\Sigma_o}{2}\int_0^{\sin^{-1}(R_i/R_o)} d\vartheta \int_{y_2}^{\infty} dr\ \exp(-\Sigma_o x_1)\ \sin\vartheta\cos\vartheta\ \phi(R_i-\delta R_2)$$

$$+ \frac{\Sigma_o}{2}\int_{\sin^{-1}(R_i/R_o)}^{\pi/2} d\vartheta \int_{y_2}^{\infty} dr\ \exp(-\Sigma_o x_2)\ \sin\vartheta\cos\vartheta\ \phi(R_o+\delta R_2) \qquad (A.18)$$

The relations between x and δR to the lowest order in δR can be written down by solving the triangles OPQ and OPR in the two figures. We get

$$x_1 = R_i\delta R_1\left(R_i^2-R_o^2\sin^2\vartheta\right)^{-1/2} \qquad (A.19)$$

$$x_2 = \delta R_2/\cos\vartheta \qquad (A.20)$$

Employing these relations and expanding ϕ to the lowest order in δR_1 and δR_2 we get can carry out the integrations to yield the desired result.

Table 1

Two fast concentric spherical regions in close contact: K_{eff}

R_1	R_2	R_3	K_1	K_2	K_3
48	48	74	0.858	0.598	0.980
40	40	110	0.768	0.866	0.980
36.3	36.3	347	0.716	0.965	0.980

Table 2

Amplification results for a booster-reactor combination in close contact with one another (last case of Table 1)

Source description	Multiplication factor	Importance factor	Energetic gain
Uniform source in a 25 cm radius	180	3.6	396
Point Source at the centre	244	4.9	537

Table 3

Effect of gap on the K_{eff} of fast concentric spheres

S.No.	R_1	R_2	R_3	K_1	K_2	K_3
1a	60	60	67	0.953	0.125	0.980
1b	60	160	200	0.953	0.794	0.980
1c	60	210	284	0.953	0.910	0.980
1d	60	250	370	0.953	0.947	0.980
2a	52.5	52.5	69.5	0.900	0.421	0.980
2b	52.5	75.0	112	0.900	0.748	0.980
2c	52.5	100.0	175.0	0.900	0.900	0.980
2d	52.5	125.0	296.0	0.900	0.956	0.980

Table 4

Energy amplification and accelerator current requirements for a 750 MW (t) reactor driven by a 1Gev proton beam (Uniform source in the inner region up to a radius of 25 cm)

S.No.	R_1 (cm)	R_2 (cm)	R_3 (cm)	Multpl. factor	Import. factor	Energy gain	Accl. current (mA)
1a.	60	60	67	93	1.9	205	3.7
1b	60	16	200	152	3.0	334	2.2
1c.	60	210	284	216	4.3	475	1.6
1d.	60	250	370	318	6.3	699	1.1
2a.	53	53	70	89	1.8	196	3.8
2b.	53	75	112	119	2.4	262	2.9
2c.	53	100	175	178	3.6	396	1.9
2d.	53	125	296	230	4.6	512	1.5

Table 5

Effect of Gap and Thermal Absorber Lining on the Degree of Coupling between Inner (Fast) and Outer (Thermal) Regions

	Case 1	Case 2	Case 3
Gap(cm)	0	0	150
Absorber	Absent	Present	Present
K_1	0.953	0.953	0.953
K_2	0.962	0.962	0.963
K_3	1.052	1.010	0.976

Table 6

Comparison of Diffusion and Transport Theory Results

System	R_1	R_2	R_3	K(Diff)	K(Trans)
Fast (1Grp)	60	160	200	0.9800	0.9792
Fast (1Grp)	36	36	350	0.9792	0.9776
Fast (1Grp)	52.5	125	296	0.9800	0.9796

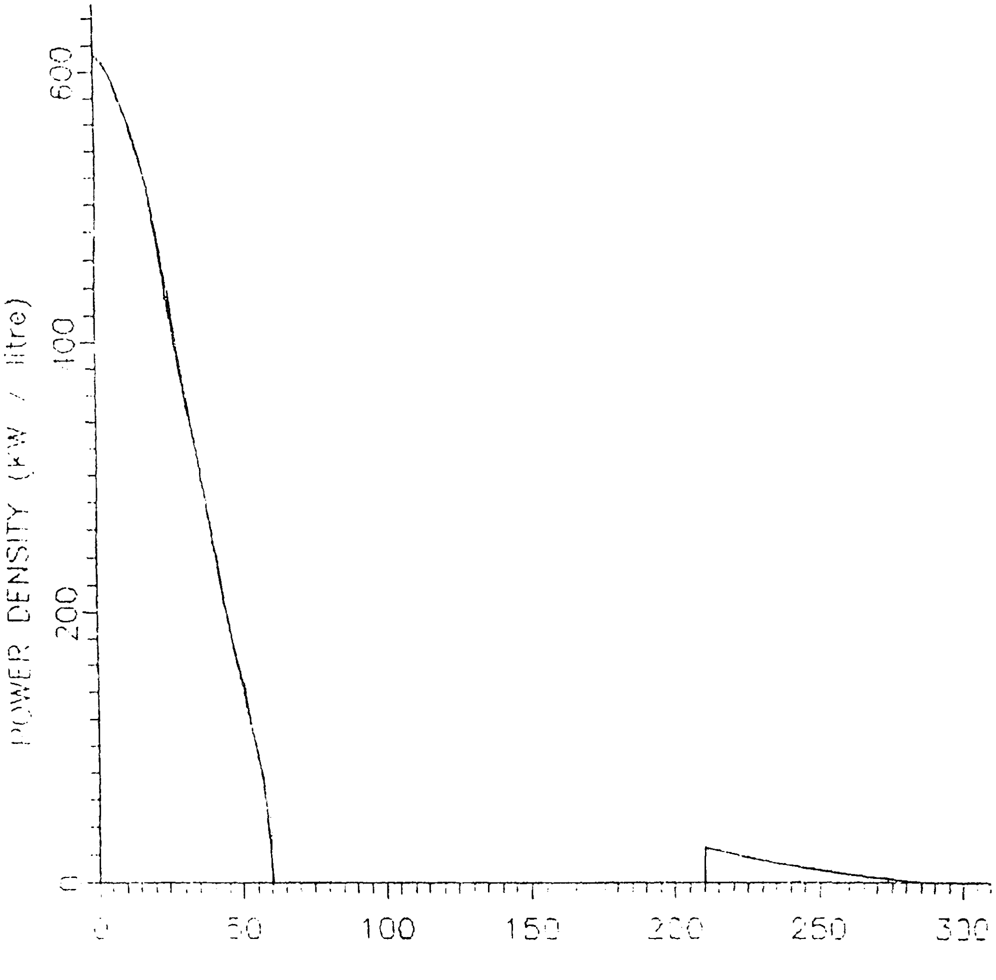

Fig. 1. Volumetric power density for a typical fast-fast system.

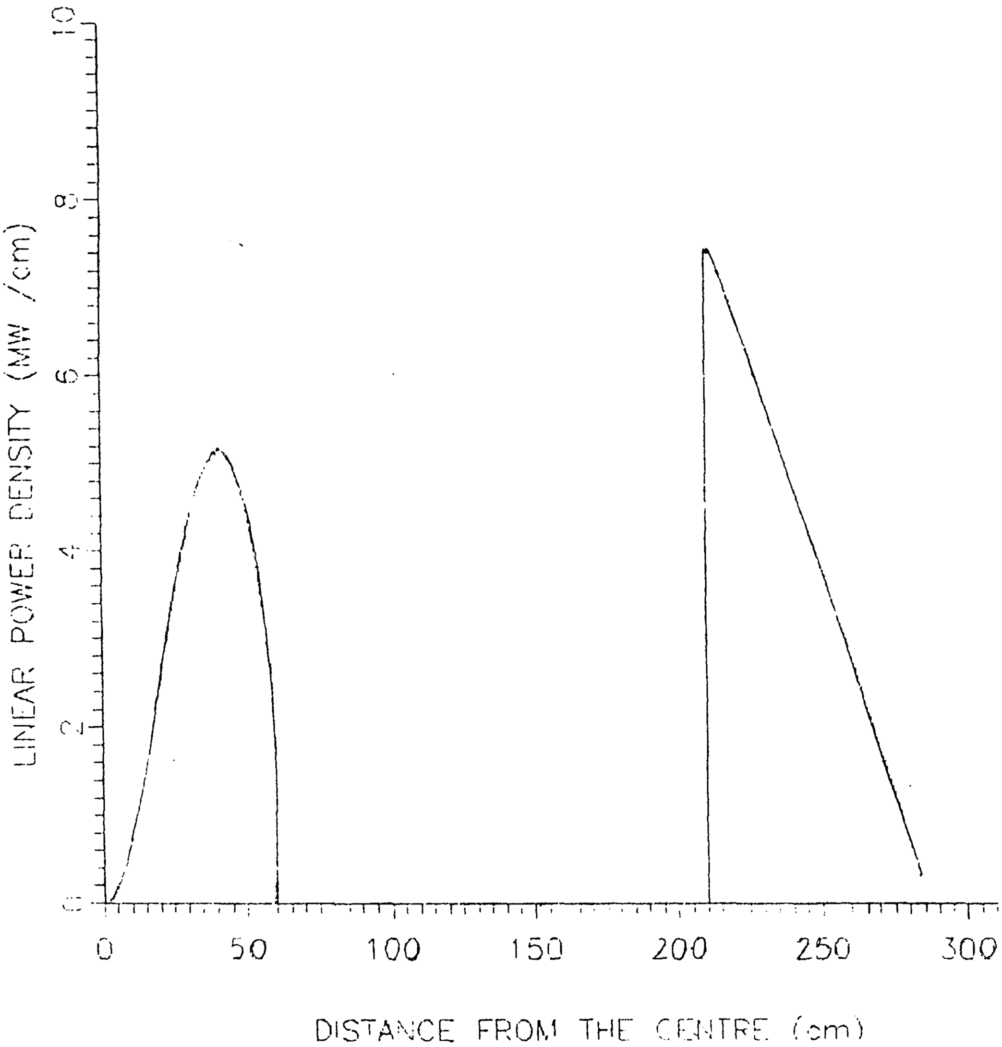

Fig. 2. Power density per unit radial interval for the fast-fast system.

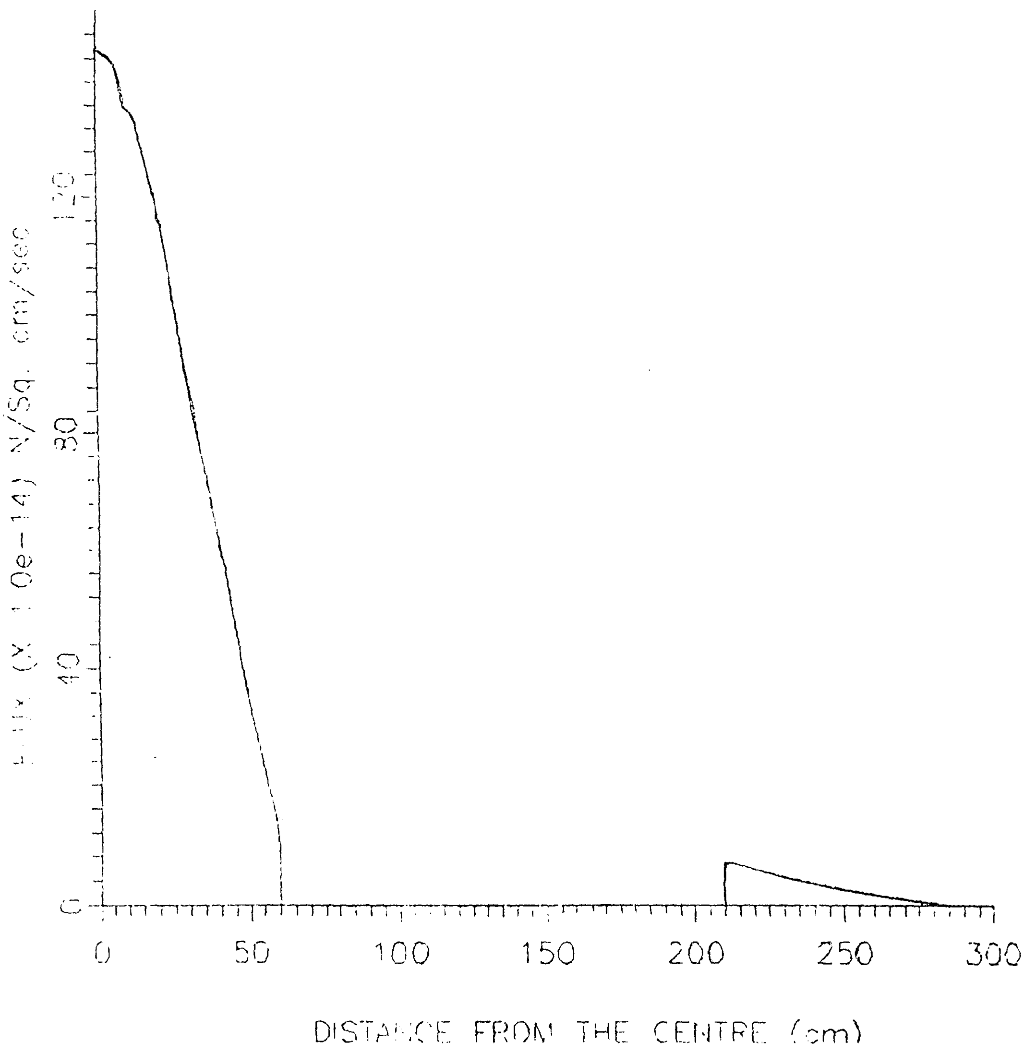

Fig. 3. Flux distribution for the fast-fast system.

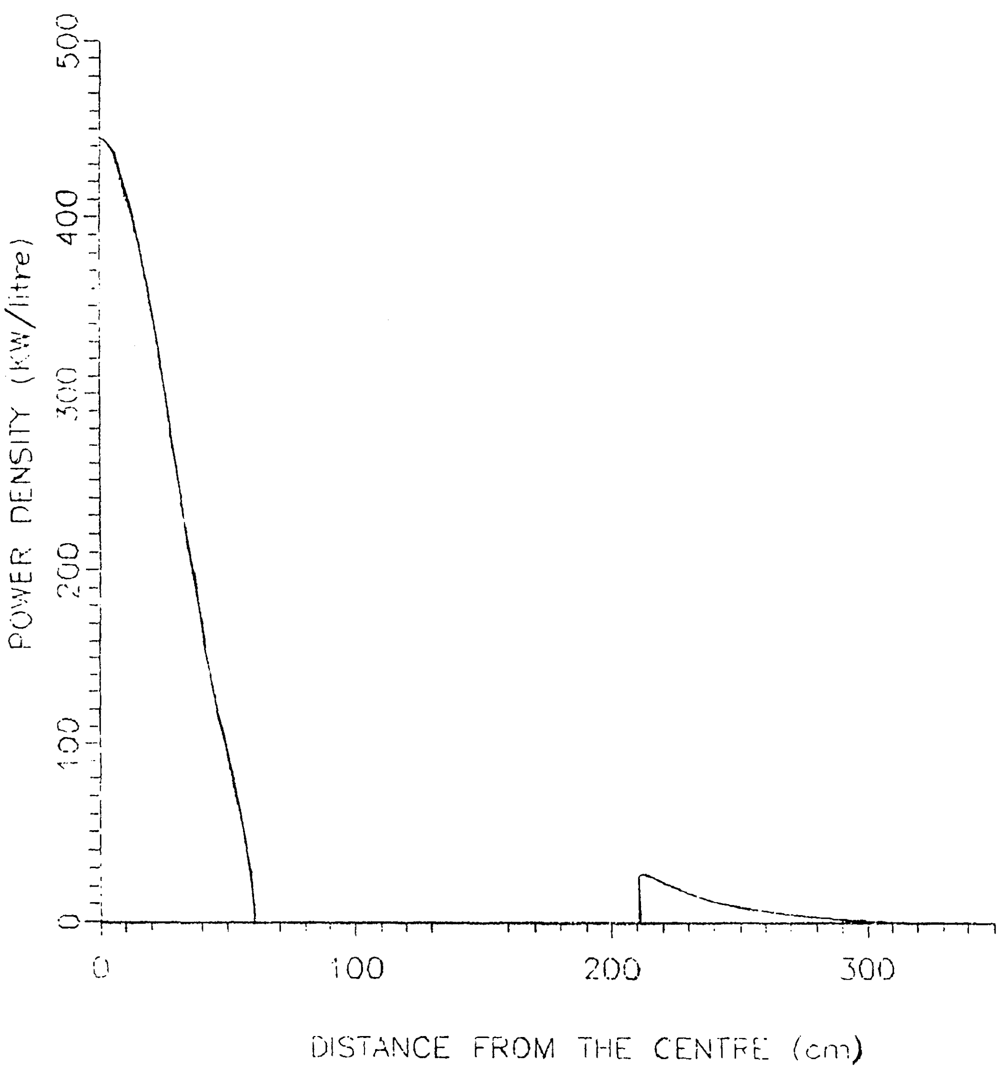

Fig. 4. Volumetric power density for the fast-thermal system.

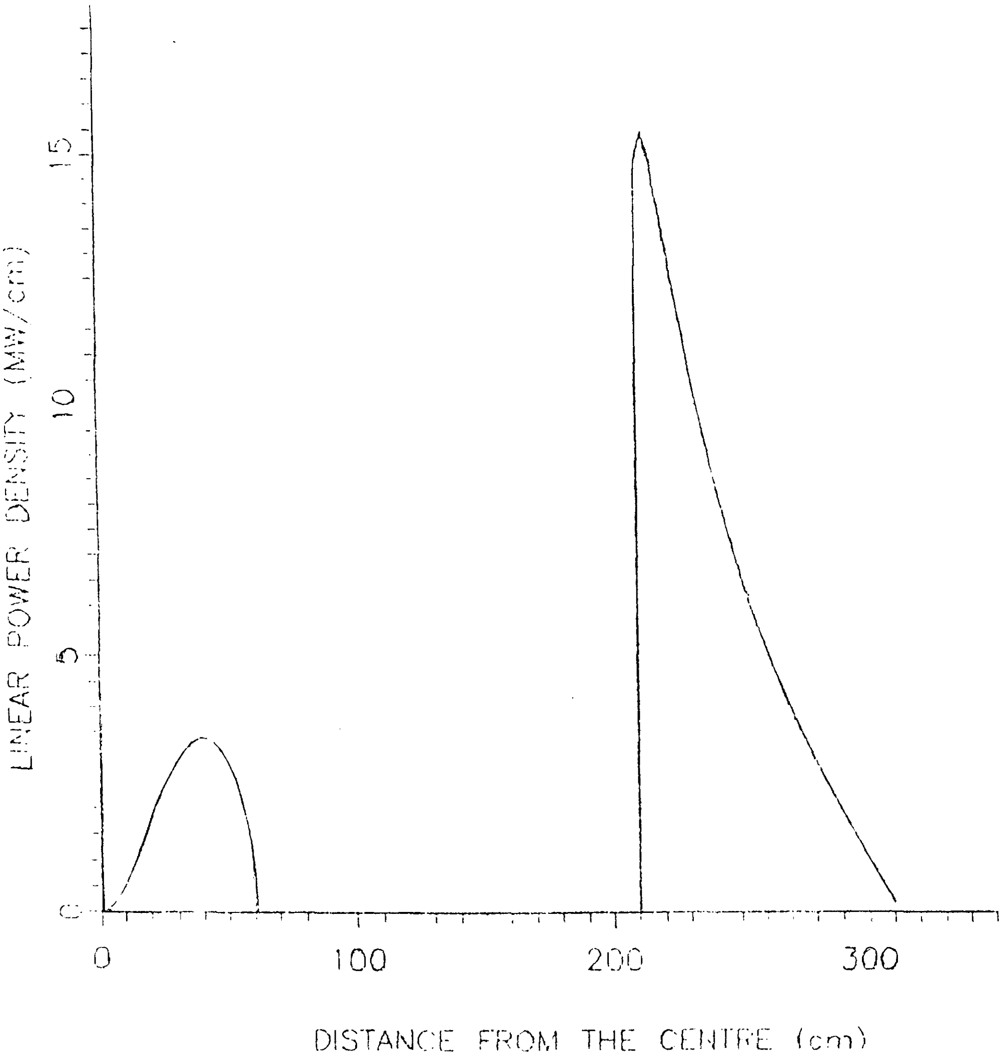

Fig. 5. Power density per unit radial interval for the fast-thermal system.

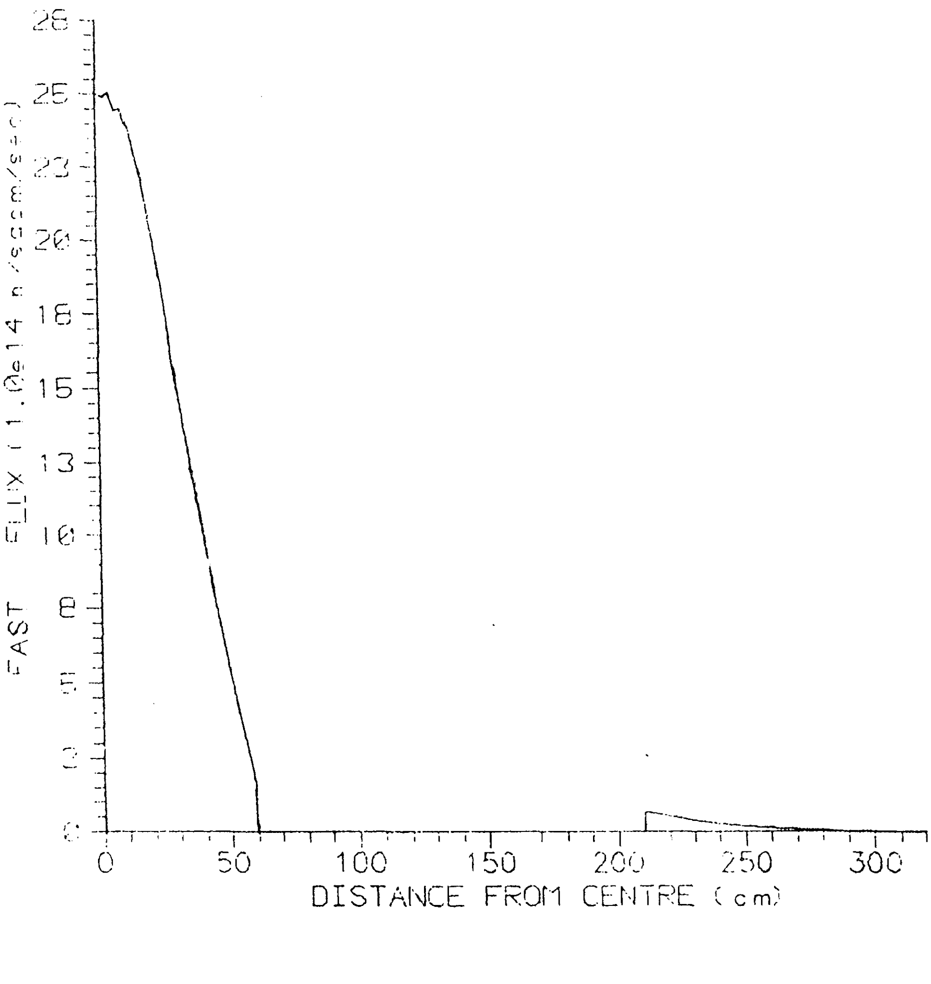

Fig. 6. Fast flux distribution for the fast-thermal system.

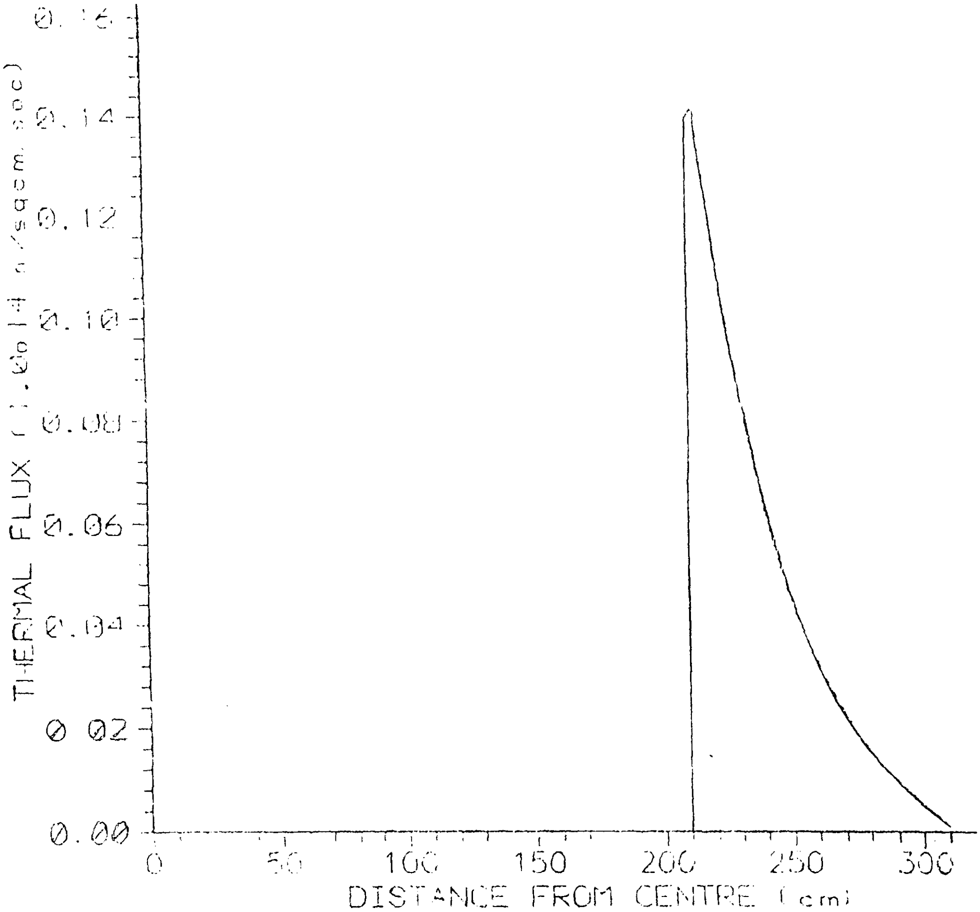

Fig. 7. Thermal flux distribution for the fast-thermal system.