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Dynamics of Shock Probes in Driven Diffusive Systems

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We study the dynamics of shock-tracking probe particles in driven diffusive systems and also in equilibrium systems. In a driven system, they induce a diverging timescale that marks the crossover between a passive scalar regime at early times and a diffusive regime at late times; a scaling form characterises this crossover. Introduction of probes into an equilibrium system gives rise to a system-wide density gradient, and the presence of even a single probe can be felt across the entire system.

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A commonly used method to investigate the properties of a system is to inject probe particles and monitor their behavior after they have equilibrated with the system under study. The dynamics of probe particles then yields important information about the static and dynamic properties of the medium and has been used to study diverse phenomena, ranging from the sol-gel transition in a polymer solution [1] to the growth mechanism of surfactant assemblies [2] and correlations present in bacterial motion [3]. The steady state involves the coupled probe-medium system, but if the concentration of probe particles is very low, it is generally assumed that the properties of the original system are not affected greatly [1, 2, 3]. In this paper, we discuss the validity of this assumption by studying simple lattice gas models, and find that in certain circumstances it can break down in interesting ways. We characterize the types of behaviour that occur by studying the displacement of a probe particle as a function of time, the correlations between different probes particles, and their effect on the medium. Depending on the system under study, different behaviours are found in the low-concentration limit, ranging from diverging correlation lengths and power law decays, to effects which are felt over macroscopic distances throughout the system. We relate these differences to an interesting interplay between the equilibrium and nonequilibrium characteristics of the medium and the probe particles.

We focus on the effects of probe particles on one-dimensional driven diffusive systems, and their equilibrium counterparts. We mainly consider probes which exchange with the particles and holes of the medium with equal rate but in opposite directions. Such probes tend to settle in regions of strong density variations (shocks) and thus serve to track the locations and movements of shocks. We consider primarily, but not exclusively, a medium in which the shock-tracking probes (STPs) reduce to second class particles [4, 5], *i.e.* they behave as holes for the particles and as particles for the holes. STPs are intrinsically nonequilibrium probes, as they evolve through moves which do not respect detailed balance. We will see below that they have very strong effects on a medium that is originally in equilibrium: even a single probe generates a density perturbation which extends over macroscopic distances. However, when immersed in a medium that is in a current-carrying nonequilibrium state, the perturbation produced by a single probe particle decays to zero, but as a slow power law; a finite density of probes brings in a correlation length which diverges strongly in the low concentration limit. We find that these broad conclusions remain valid for a class of models with nearest neighbor interaction and also with an extended range of particle hopping.

Recently, Levine *et al.* [6, 7] have studied the behavior of STPs in a driven diffusive system whose steady state has nearest neighbor Ising measure [8]. When only a finite number of probes are present, the medium induces a long-ranged attraction between pairs of successive STPs, as shown earlier by Derrida *et al.* [5]. The distribution of the separation between a pair follows a power law with a power which depends continuously on the strength of the Ising interaction [6]. Consequences for the phase diagram with a macroscopic number of STPs were explored in [7] . In this paper, we are interested in the *dynamical* properties of STPs primarily for the case of uncorrelated particle occupancies. We find that these properties are governed by a time-scale τ which diverges as the STP concentration approaches zero. This time-scale is related to a diverging correlation length which characterises the density profile around a probe particle [5]. τ enters into a scaling description of the crossover between a 'passive scalar advection' regime and a diffusive regime which sets in at longer times. Similar behaviour is found, though with some interesting variations, also in members of the class of k-hop models, which are driven diffusive systems with longer ranged hopping of particles [9].

Even more striking is the effect of STPs on a system which is initially in equilibrium. With the introduction of a single probe, the medium develops a density gradient across macroscopic distances and a concomitant shock around the probe. Interestingly, the resulting state shows a signature of an inhomogeneous product measure. The dynamical properties of the medium remain characteristic of local equilibrium, despite the fact that there is a small current in the system.

The system is defined on a 1-d lattice with periodic boundary conditions. We denote particles and holes in the original system by '+' and '−', respectively, and the probe particles by '0'. The dynamical moves involve the

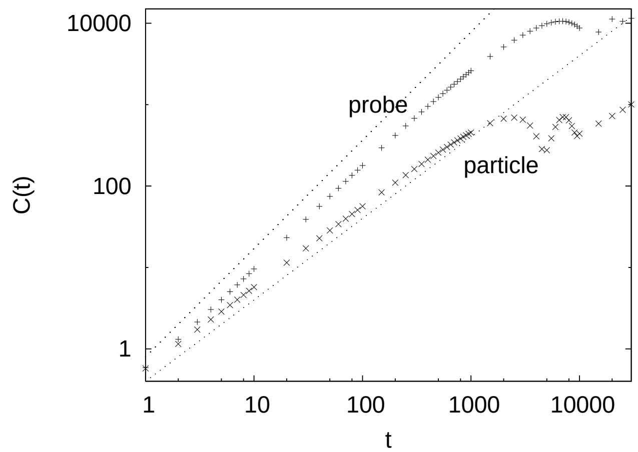

FIG. 1: *Mean squared displacement of particles and probes for $L = 2048$ and $\rho_0 = 0.15$, showing the initial linear growth of $C_+(t)$ and the nonlinear growth of $C_0(t)$. The dashed lines depict power laws with exponent $4/3$ and 1. The late-time oscillations are due to the existence of kinematic waves in a finite periodic system.*

exchanges [4, 5]:

$$\begin{array}{lrcl} (a) & +- & \rightarrow & -+ \\ (b) & +0 & \rightarrow & 0+ \\ (c) & 0- & \rightarrow & -0 \end{array} \tag{1}$$

All moves take place with equal rate. Since a probe exchanges with a particle and a hole in opposite directions, it tends to migrate to places where there is an excess of holes to its left and particles to its right, *i.e.* a shock.

In the absence of STPs the system reduces to the asymmetric simple exclusion process (ASEP), which is a paradigm of driven diffusive systems [10]. With probes present, we see that a particle (hole) exchanges with a hole (particle) and a probe in exactly the same way. The STPs in this case reduce to second class particles [4, 5]. If ρ and ρ_0 are the densities of particles and probes respectively, then a particle behaves as if in an ASEP with an effective hole density $(1-\rho)$, while a hole finds itself in an ASEP with an effective particle density $(\rho+\rho_0)$. STPs are correlated but the exact steady state measure for the system can be found using the matrix method [5]. Throughout this paper, we will consider equal densities of particles and holes in the medium, which implies $\rho_0 = 1-2\rho$.

To monitor the dynamics, we measured the mean squared displacement of tagged particles in the medium, and tagged STPs, using Monte Carlo simulation after the system has reached steady state. Let $Y_k(t)$ be the position of the kth particle at time t. We monitor the mean squared displacement defined as

$$C(t) = \langle (Y_k(t) - Y_k(0) - \langle Y_k(t) - Y_k(0)\rangle)^2 \rangle \tag{2}$$

where the average is over the steady state ensemble of initial configurations. We denote by $C_0(t)$ and $C_+(t)$ the mean squared displacements of probes and particles, respectively. Monte Carlo simulation results show interesting differences between $C_+(t)$ and $C_0(t)$ with a low but finite density of probes [Fig.(1)]. In contrast to the linear increase of $C_+(t)$, $C_0(t)$ increases non-linearly in the range of time considered. The behaviors of both C_+ and C_0, including long-time crossovers, are discussed in detail below.

Tagged particles in the medium behave as if in a regular ASEP, for which $C_+(t)$ is known to grow linearly in time for an infinite system [11]. In a finite system of size L, $C_+(t)$ is nonmonotonic due to the existence of a kinematic wave which carries density fluctuations through the system with speed $\frac{dJ}{d\rho} = (1-2\rho)$ [12, 13], where J is the current, given by $\rho(1-\rho)$. Since the average speed of the tagged particle is $J/\rho = (1-\rho)$, it moves from one density patch to the other with relative speed ρ; the mean squared displacement increases linearly, since each patch contributes a random excess to the relative velocity of the tagged particle. The subsequent dips in the mean squared displacement, as seen in Fig.(1), correspond to the return of a tagged particle to its initial environment in time L/ρ; the lower envelope of the curve in Fig.(1) grows as $t^{2/3}$ as long as $t \ll L^z$ [14, 15].

For tagged probe particles, $C_0(t)$ shows a crossover from an initial passive scalar advection regime to a long-time diffusive regime. The associated crossover occurs on a timescale τ that diverges strongly as the probe density approaches zero. Ferrari and Fontes have calculated the asymptotic ($t \to \infty$) behavior of $C_0(t)$ for STPs [16] and

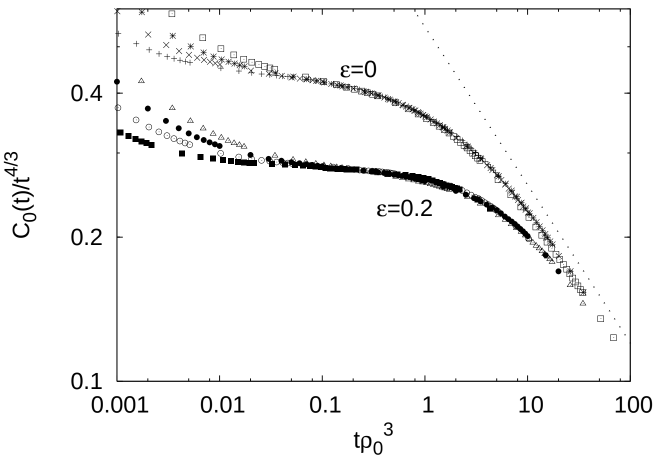

FIG. 2: *Scaling collapse for mean squared displacement of tagged probe particles with densities* $0.08, 0.1, 0.12, 0.15$ (*moving upwards*). *The upper curve shows* $C_0(t)$ *for* $\epsilon = 0$ ($L = 131072$, *averaged over* 100 *histories*) *and the lower curve shows* $C_0(t)$ *for* $\epsilon = 0.2$ ($L = 16384$, *averaged over* 1000 *histories*). *The dashed line shows a power law decay with exponent* $-1/3$.

shown that $C_0(t) \approx Dt$ with diffusion constant $D = [\rho(1-\rho) + (\rho+\rho_0)(1-\rho-\rho_0)]/\rho_0$. For small times, in the limit of low concentration of the probe particles, one would expect that each STP would behave as an individual non-interacting particle, subject only to the fluctuations of the medium. The mean squared displacement of a single probe has been shown analytically to grow as $t^{4/3}$ using the matrix product method [17]. This behavior agrees with the results for particles sliding down on surface evolving through Kardar-Parisi-Zhang dynamics, where $C_0(t)$ grows as $t^{2/z}$, with $z = 3/2$ [18]. We propose the following scaling form in the limit of large t and τ

$$C_0(t) \sim t^{4/3} F\left(\frac{t}{\tau}\right) \tag{3}$$

in terms of the crossover time τ whose dependence on ρ_0 is given below. Here $F(y)$ is a scaling function which approaches a constant as $y \to 0$. For $y \gg 1$, we must have $F(y) \sim y^{-1/3}$, in order to reproduce $C(t) \approx Dt$. The timescale τ is related to the correlation length: at sufficiently large distance r from an STP, the shock profile decays as $r^{-1/2}\exp(-r/\xi)$ where ξ has been calculated by Derrida *et al.* [5]. In the limit of small ρ_0, it reduces to $\xi \approx 4\rho(1-\rho)/\rho_0^2$ which diverges as $\rho_0 \to 0$. Relating τ to ξ through $\tau \sim \xi^z$ with $z = 3/2$ then implies $\tau \sim \rho_0^{-3}$. We verified the scaling form by Monte Carlo simulation. We were able to circumvent equilibrating by directly generating steady state initial configurations, following the prescription in [19]. In Fig.(2), we plot $C(t)/t^{4/3}$ vs t/τ for various values of ρ_0 and obtain a good scaling collapse, except for very small values of t which fall outside the scaling regime.

A nontrivial check of the scaling form comes from examining the dependence of τ on ρ_0. Matching the early and late time form for $C_0(t)$ at $\tau \sim \rho_0^{-3}$ then yields $D \sim \rho_0^{-1}$, in agreement with [16]. Yet another check comes from considering the implication for a system with a finite size L. Finite size scaling would suggest that once L is smaller than ρ_0^{-2}, the behavior $D \sim \xi^{1/2}$ found above should give way to $D \sim L^{1/2}$. This is in conformity with the calculation of [20].

To track dissipation in a single component system, one can perform a Galilean shift to keep up with the kinematic wave [14]. This is not possible in our system since there are two kinematic waves with different velocities $\pm(1-2\rho)$ corresponding to the two ASEPs with densities ρ and $\rho+\rho_0$. We thus use the method of van Beijeren [21], wherein we monitor the quantity

$$B(t) = \overline{\left(Y_k(t) - Y_k(0) - \overline{(Y_k(t) - Y_k(0))}\right)^2} \tag{4}$$

where the overhead bar denotes averaging over different evolution histories, starting from a *fixed* initial configuration drawn from the steady state ensemble. Note that in this averaging process, the initial pattern of density fluctuations around a particular tagged probe is identical for all evolution histories. The mean $\overline{(Y_k(t) - Y_k(0))}$ shows fluctuations superposed on a linear growth law. These fluctuations are determined by the density pattern in the initial configuration [22]. $B(t)$ therefore gives the spread of this pattern with time. We find that $B(t)$ increases as $t^{2/3}$ for large t, and would expect a scaling function to connect this regime to the small time regime $B(t) \sim t^{4/3}$, characteristic of

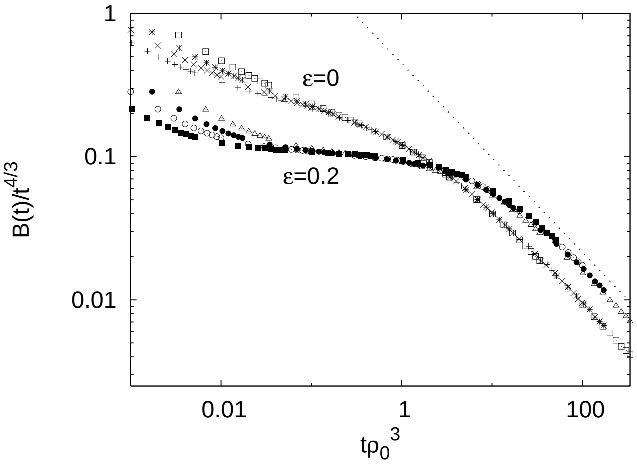

FIG. 3: *Scaling collapse for $B(t)$ with probe densities* $0.08, 0.1, 0.12, 0.15$ *(moving upawrds). The two curves correspond to* $\epsilon = 0$ *and* $\epsilon = 0.2$ *(*$L = 16384$*, averaged over* 25 *initial configurations and* 40 *evolution histories for each). The dashed line shows a power law decay with exponent* $-2/3$.

single particle behavior. In the limit of large t and small ρ_0, we expect

$$B(t) \sim t^{4/3} G\left(\frac{t}{\tau}\right). \tag{5}$$

The scaling function $G(y)$ should approach a constant as $y \to 0$ while for $y \gg 1$, one expects $G(y) \sim y^{-2/3}$. Our simulation results [Fig.(3)] are consistent with this scaling form.

Similar results are found for shock trackers in a driven diffusive system in which there is an Ising interaction $V = -\epsilon[(n(i) - \frac{1}{2})(n(j) - \frac{1}{2})]$ between the neighboring particles $\langle ij \rangle$ in the medium [8]. The dynamical moves are as specified in Eq.(1) but the rates are different: while the moves (b) and (c) occur with rate 1, move (a) takes place with a rate $(1 - \Delta V)$ where ΔV is the change in Ising energy [8]. Note that in this case, STPs do not behave as second class particles. In [6], it has been shown that with a finite number of probes, the medium induces an attraction and the separation s between two consecutive probes follows the distribution $P(s) \sim s^{-b}$ for large s. The coefficient b varies continuously with ϵ; in the non-interacting case, $b = 3/2$. If s_i is the separation between the i-th and $(i+1)$-th probe and R_m is the distance between the first and the $(m+1)$-th probe, then $R_m = \sum_{i=1}^{m} s_i$. The quantity R_m then follows a Lévy distribution with a norming constant $\sim m^{1/(b-1)}$, so long as $R_m < \xi$. This in turn implies that the correlation length $\xi \sim \rho_0^{-1/(2-b)}$ and hence $\tau \sim \xi^{z_0} \sim \rho_0^{-z_0/(2-b)}$, where z_0 is the dynamical critical exponent of the system. Our numerical simulations indicate that for the ranges of ϵ, ρ_0 and t studied (Fig.(2)and (3)), $C_0(t)$ and $B(t)$ continue to show a crossover at a timescale $\tau \sim \rho_0^{-3}$, surprisingly similar to the non-interacting case [23].

To investigate the effect of probes in an equilibrium system, we studied STPs in a symmetric exclusion process (SEP). The dynamical rules remain the same as in Eq.(1), except that (a) changes to $+- \rightleftharpoons -+$. All moves take place with equal rate. This model is a special case of the model proposed by Arndt *et al.* with the asymmetry parameter set equal to its critical value $q = 1$ [24].

In the absence of probes the system obeys detailed balance and the state is described by a uniform product measure. With the introduction of a single probe, the condition of detailed balance is violated and there is a small ($\sim 1/L$) current in the system. In this nonequilibrium steady state, there is a system-wide density gradient around the probe.

Figure (4) shows the density $\rho(r) \equiv \langle n(r) \rangle$, at a distance r away from the probe, where $n(r)$ is the occupancy at r. We find that $\rho(r) = A(1 - r/L)$ with $A \simeq 1$. The inset in Fig.(4) shows the spatial two point correlation function defined as $g(r, \Delta r) = \langle n(r)n(r + \Delta r) \rangle - \rho(r)\rho(r + \Delta r)$, for a fixed value of r. The fact that $g(r, \Delta r)$ is close to zero points to the existence of an inhomogeneous product measure state.

The mean squared displacement of the probe grows diffusively with a diffusion constant $D \sim 1/L$ (Fig.(5)). This follows from the fact that for equal densities of particles and holes in the medium, the probe has an equal probability to move to the left or to the right and from the form of $\rho(r)$ given above, it follows that this probability $\sim 1/L$ in each Monte Carlo step. Far from the probe, the local properties of the medium still resemble those of the SEP. We demonstrate this by measuring the mean squared displacement of a tagged particle $C_+(r, t)$ and comparing with the SEP result $C_+(t) \approx \sqrt{\left(\frac{2}{\pi}\right)}(1 - \rho)/\rho \; t^{1/2}$ [10]. In Fig.(5), we present data for $C_+(r, t)$ for different values of local densities.

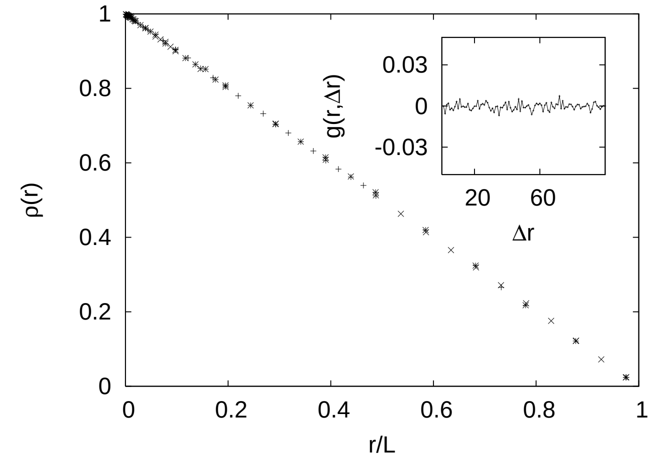

FIG. 4: *Density profile as a function of the scaled distance away from the probe in the SEP for* $L = 513, 1025, 2049$. *The data is averaged over* 50000 *histories. The inset shows* $g(r, \Delta r)$ *for* $r = 1024$ ($L = 2049$, *averaged over* 10^4 *histories) and illustrates that the pair correlation is close to zero.*

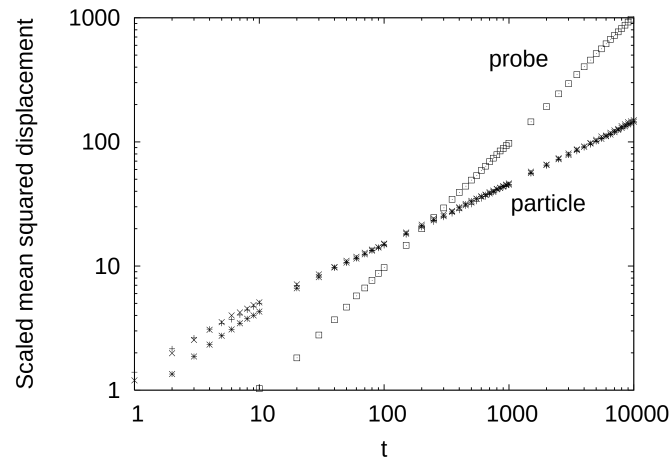

FIG. 5: *Scaled mean squared displacement of tagged particles at distances* $r = 580, 985, 1390$, *away from the probe, corresponding to* $\rho(r) = 0.72, 0.52, 0.32$. ($L = 2049$, *averaged over* 10^4 *histories). The curves are seen to merge when the coefficient* $\left(\frac{2}{\pi}\right)^{1/2} \frac{1-\rho(r)}{\rho(r)}$ *is divided out. Also shown is the mean squared displacement of a single probe scaled up by a factor of 100.*

This agreement should hold as long as the tagged particle remains in the region where the local density is $\rho(r)$. Since the density changes over a length scale $\sim L$ and the average velocity of the tagged particle is $\sim 1/L$, the region of validity extends upto a time scale $\sim L^2$.

When a macroscopic number of STPs is introduced, they are phase separated and form essentially a single cluster. The other phase is comprised of the medium, which continues to remain in local equilibrium with a similar density gradient as in the single probe case. The probe cluster is found to diffuse slowly through the medium with a diffusion constant $D \sim 1/L$.

So far we have considered the effect of shock-tracking probes on systems described by the simple exclusion models and the model with Ising interactions. Our broad conclusions from these studies remain valid even for systems with an extended range for particle hops. We have studied the behavior of STPs in a driven diffusive system described by the k-hop model, in which the particle hopping range is k [9]. Our simulation shows that in this case also, the effect produced by the probe is long-ranged—there is a diverging correlation length in the low concentration limit of STPs [23]. In the symmetric version of the k-hop model, a single STP gives rise to a macroscopic density gradient, as before.

In this paper, we have discussed the effect of shock-tracking probes on one dimensional equilibrium and nonequilibrium systems. An equilibrium system is found to be affected very strongly by the presence of even a single probe—a density gradient is generated that extends over the entire system. For probes in nonequilibrium systems, the effect is less drastic: the shock around a single probe decays as a slow power law. With a macroscopic number of probes, this decay extends upto the correlation length, which in turn diverges in the low probe-density limit. The question arises: How general are these conclusions—do they remain valid even with other nonequilibrium probes in one-dimensional systems? We have studied another example which falls into this general framework, namely directed probes driven by an external field. In an ASEP, such a probe behaves like a tagged particle, but when immersed in a symmetric exclusion process, it is again found to give rise to a macroscopic density gradient [23]. It would be interesting to have a general criterion which sets conditions for the occurrence of such phenomena.

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