Minijets, soft gluon resummation and photon cross-sections

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Abstract

We compare the high energy behaviour of hadronic photon-photon cross-sections in different models. We find that the photon-photon cross-section appears to rise faster than the purely hadronic ones (pp and $p\bar{p}$).

1 Introduction

Experimentally, all total cross-sections rise asymptotically with energy, but it is not clear whether the rate of increase is the same for different processes. To appreciate it at a glance, we show in Fig. 1 a compilation of available data on pp, $p\bar{p}$, γp and $\gamma\gamma$. The data span an energy range of four orders of magnitude. To plot them all on the same scale, we have multiplied the relevant cross-section by a constant factor $1/R_\gamma$ for each incident photon. In this figure, we have taken R_γ to be $1/330$, purely on the grounds to have all the cross-sections in the low energy region to be close in value to each other. One simple way to estimate this value consists in counting the fermion lines in the proton and the photon and the probability of basic quark-antiquark scattering. Through this the factor is found to be

$$R_\gamma \approx \alpha_{QED} \left(\frac{N^{photon}_{fermion\ lines}}{N^{hadron}_{fermion\ lines}} \right)^2 \approx \frac{1}{300} \quad (1)$$

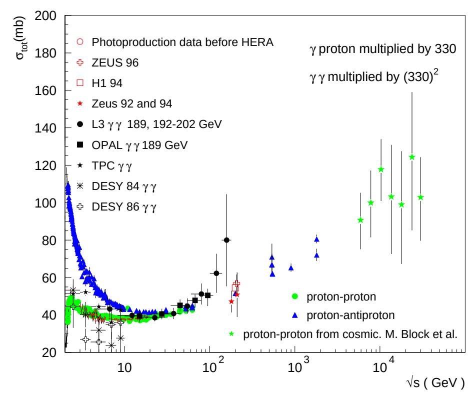

Figure 1: Proton [2] and photon [3, 4] normalized total cross-sections

The same result is obtained using Vector Meson Dominance (VMD) with pure $\gamma - \rho$ coupling [5]. This scaling is very approximate. Further there is no reason to expect the scaling factor to be energy independent. This can be easily understood by noting that at low energy the photon behaves like a vector meson in its interactions with matter, while at high energies QCD phenomena which are energy dependent will appear. Thus while at low energies the factor R_γ can be evaluated through VMD considerations [6, 7] which may include other Vector Mesons beyond the ρ, at high energy it is likely to be different [8] due to the difference in the quark and gluon content of photons [9] and that of the hadrons.

To understand the role played by the quark-parton structure of the photon, one can use a QCD model such as the one developed in [10, 11, 12, 13, 14], and apply it to evaluate cross-sections for processes involving photons. In the following, we shall describe the model for protons and then apply it to photons.

2 The Bloch-Nordsieck model for proton processes

This model is based on the following :

1. QCD mini-jets to drive the rise of the total cross-section in the QCD asymptotic freedom regime;

2. resummation of soft gluon emission down to zero momentum to soften the rise due to the increasing number of gluon-gluon collisions between hard perturbative, but low-x, gluons;

3. eikonal representation for the total cross-section (with $\Re e\chi \approx 0$) to incorporate the mini-jet cross-section, using an impact parameter distribution obtained as the Fourier transform of resummed soft gluon transverse momentum distribution.

Each of these components will be discussed in detail, before applying it to obtain the total cross-section.

2.1 The mini-jet cross-section

The mini-jet cross-section is obtained by integrating the standard QCD inclusive jet cross-section, using a lower cutoff $p_{tmin} \approx 1\ GeV$, and is given by:

$$\sigma_{hard} \equiv \sigma^{AB}_{\rm jet}(s, p_{tmin}) = \int_{p_{tmin}}^{\sqrt{s}/2} dp_t \int_{4p_t^2/s}^{1} dx_1 \int_{4p_t^2/(x_1 s)}^{1} dx_2 \times \sum_{i,j,k,l} f_{i|A}(x_1, p_t^2) f_{j|B}(x_2, p_t^2) \frac{d\hat{\sigma}^{kl}_{ij}(\hat{s})}{dp_t} \quad (2)$$

where A and B are the colliding hadrons. This cross-section strongly depends on the value chosen for such p_{tmin} and -for a fixed cut off- it increases with energy, reflecting the sharp increase in the number of low-x gluons with increasing energy. This increase is very rapid and, if left unchecked, the mini-jet cross-section would surpass the observed total cross-section. The saturation mechanism which restores the Froissart bound comes from another QCD mechanism, soft gluon emission, which will be described in the next subsection.

The mini-jet cross-sections are calculated using realistic parton densities (PDFs). The most common ones for the proton are GRV [15], MRST [16], CTEQ [17], whereas those for the photon are GRV[18], GRS [19], CJKL [20]. These densities are available both at leading order (LO) or higher, but in our model we use only the LO given the fact that part of the NLO effects are described by the soft gluon resummation discussed in the next section. We show in Figure 2 the energy dependence of the mini-jet cross-sections for $\gamma\gamma$ collisions, for three different sets of parton densities, GRV, GRS and CJKL, for a typical value of the cut-off, $p_{tmin} = 1.3\ GeV$.

2.2 Soft Gluons and the infrared limit

We stress the distinction between low-x gluons which participate in hard parton-parton scattering described by the mini-jet cross-section discussed in the previous section, and the soft gluons emitted in any given parton-parton process. Soft gluons by definition need to be resummed, and hence

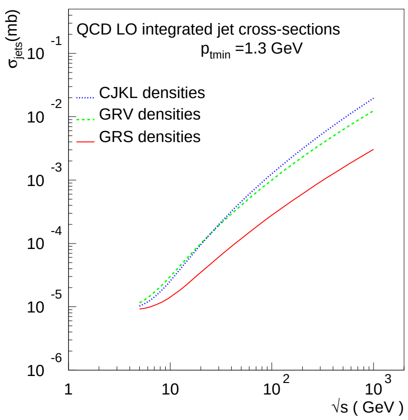

Figure 2: Photon-photon jet cross-sections for different densities and a typical p_{tmin} value.

their momentum integrated up to a maximum value. The maximum value should typically be $10 \div 20\%$ of the emitting parton energy, the lower value is usually taken to correspond to the intrinsic transverse momentum scale of the scattering hadron. Instead we extend the integration down to the zero momentum modes. To do so, we need therefore to make an ansätz as to the behaviour of the strong coupling constant in the infrared region, where the usual asymptotic freedom expression for $\alpha_s(Q^2)$ cannot be used. Our proposal is that in the infrared limit, one can phenomenologically use the expression

$$\alpha_s(k_t) = constant \times \left(\frac{\Lambda}{k_t}\right)^{2p} \qquad k_t \to 0 \quad (3)$$

where Λ is the QCD constant in the scheme chosen for the hard scattering calculation, and p is a parameter which embeds the infrared behavior with $p < 1$ so that the soft gluon integrals converge. The constant in front of Eq. 3 should be chosen to provide a smooth extrapolation to the perturbative expression for α_s. Our choice for the interpolating function is

$$\alpha_s = \frac{12\pi}{33 - 2N_f} \frac{p}{\ln[1 + p(\frac{k_t}{\Lambda})^{2p}]} \qquad (4)$$

This expression is used in the soft gluon resummation formula in the transverse momentum variable which reads:

$$d^2P(\mathbf{K}_\perp) = d^2\mathbf{K}_\perp \int \frac{d^2\mathbf{b}}{(2\pi)^2} e^{i\mathbf{K}_\perp \cdot \mathbf{b} - h(b, q_{max})} \qquad (5)$$

with

$$h(b, q_{max}) = \int_0^{q_{max}} d^3\bar{n}(k)[1 - e^{-\mathbf{k_t}\cdot\mathbf{b}}] \quad (6)$$

In QED $d^3\bar{n}(k) \propto \alpha \log(\frac{2q_{max}}{m_{electron}})$ and resummation in transverse momentum variable is well approximated by first order expansion in α. In QCD, the situation is completely different because α_s is (i) not a constant and (ii) can become very large as the gluon transverse momentum goes to zero. In phenomenological applications, resummation is typically exploited by splitting the integral between the very soft and the non-infrared region so that

$$\begin{aligned} h(b, q_{max}) &= c_0 b^2 + \int_\mu^{q_{max}} d^3\bar{n}(k)[1 - e^{-i\mathbf{k_t}\cdot\mathbf{b}}] \\ &\approx c_0 b^2 + c_1 \int_\mu^{q_{max}} \frac{dk_t^2}{k_t^2} \alpha_s(k_t^2) \log(\frac{2q_{max}}{k_t}) \quad (7) \end{aligned}$$

As mentioned before, our approach is different. We use Eq. 4 and

$$\begin{aligned} h(b, q_{max}) &= \frac{16}{3} \int_0^{q_{max}} \frac{dk_t}{k_t} \frac{\alpha_s(k_t^2)}{\pi} \\ &\left(\log \frac{2q_{\max}}{k_t}\right) [1 - J_0(k_t b)]\,. \qquad (8) \end{aligned}$$

The energy dependent quantity $q_{\max}$ represents the maximum energy allowed to a soft gluon in a given parton-parton interaction, and it depends upon the kinematics, i.e. the parton energy fractions x_1 and x_2, and from the emitted parton momentum p_t. (This is described explicitly in [11]).

2.3 Embedding QCD in the eikonal

A convenient formalism to calculate total cross-sections is provided by the eikonal integral for the elastic amplitude, valid for small angle scattering. Using the optical theorem, the total cross-section can be written as

$$\sigma_{tot} = 2\int d^2\mathbf{b}[1 - e^{-\Im m\chi(b,s)}\cos\Re e\chi(b,s)] \tag{9}$$

A simple model for calculating the eikonal function $\chi(b,s)$ consists in evaluating the probability of inelastic processes, obtained by summing over all possible Poisson distributed collisions. One then obtains

$$\sigma_{inel} = \int d^2\mathbf{b}[1 - e^{-n(b,s)}] \tag{10}$$

where $n(b,s)$ is the average number of inelastic collisions. Neglecting the real part of the eikonal, one finds

$$\sigma_{tot} = 2\int d^2\mathbf{b}[1 - e^{-n(b,s)/2}] \tag{11}$$

This expression has the advantage of satisfying unitarity, but it requires knowledge of the impact parameter distribution of the scattering particles. We propose this distribution to be given by the Fourier transform of the soft gluon distribution discussed in the previous section, namely

$$A_{BN}(b,s) = N\int d^2\mathbf{K}_\perp \frac{d^2P(\mathbf{K}_\perp)}{d^2\mathbf{K}_\perp} e^{-i\mathbf{K}_\perp\cdot\mathbf{b}} = \frac{e^{-h(b,q_{max})}}{\int d^2\mathbf{b} e^{-h(b,q_{max})}} \tag{12}$$

and approximate $n(b,s)$ as

$$n(b,s) = n_{soft}(b,s) + n_{hard}(b,s) = n_{soft}(b,s) + A_{BN}(b,s)\sigma_{jet}(s,p_{tmin}) \tag{13}$$

where $n_{hard}(b,s)$ represents the average number of collisions with outgoing partons with $p_t > p_{tmin}$, with all other collisions of non perturbative description included in $n_{soft}(b,s)$. This quantity is often parametrized by factorizing completely the $b-$ and $s-$ dependence, and using the Fourier transform of the relevant hadron form factor to describe the impact parameter distribution. Instead, in our model, we do not use a factorised form in b and s, and write

$$n_{soft}(b,s) = A_{BN}^{soft}(b,s)\sigma_0[1 + \epsilon\frac{2}{\sqrt{s}}] \tag{14}$$

with σ_0 a constant to fix the overall normalization, and $\epsilon = 0, 1$ depending upon the process being proton-proton or proton-antiproton respectively. Here, $A_{BN}^{soft}(b,s)$ is obtained from the Bloch-Nordsieck (BN) model with q_{max}^{soft} parametrized so as to always remain $< 10 \div 20\%$ of the value of p_{tmin}. For proton-proton and proton-antiproton scattering we show the results of our model in Figure 3, where we also show comparison with other models [21].

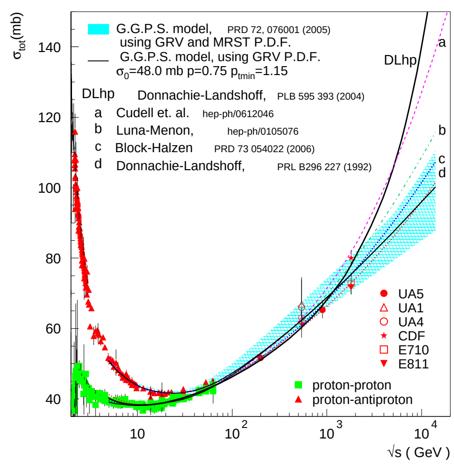

Figure 3: Proton-proton and proton-antiproton total cross-sections from the Bloch-Nordsieck model described in the text. A discussion of comparison with other models and explanation of symbols can be found in [14].

3 Photon processes

The model described previously can now be applied to the photon processes. An extremely simple exercise is to use factorization as

$$\sigma_{\gamma\gamma} = \frac{(\sigma_{\gamma p})^2}{\sigma_{pp}}. \quad (15)$$

One can then either parametrize $\sigma_{\gamma p}$ or choose an appropriate constant as we did in Fig. 1. The result is shown in Figs. 4 and 5 where the green band corresponds to the band for proton-proton scattering of Fig. 3 multiplied by $1/330$ and $(1/330)^2$ respectively.

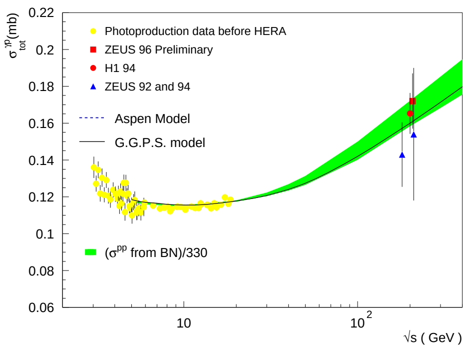

Figure 4: Total photon-proton cross-sections with the full line corresponding to GGPS model full line of Fig. 3 and the dotted line to [22].

A more involved exercise is to follow the same eikonalization procedure as outlined for the proton cross-sections, using the experimentally determined photonic parton densities, and then compare it with data, as well as other model predictions. In such case, one needs to know how to apply to the photon a typically hadronic description like the eikonal representation. Quite a while ago [6], the following expression was proposed for photon processes:

$$\sigma_{tot} = 2[P_{had}]^l \int d^2\mathbf{b}[1 - e^{-n^\gamma(b,s)/2}] \quad (16)$$

with

$$n^\gamma(b,s) = [\frac{2}{3}]^l n_{soft}(b,s) + n^\gamma_{hard} \quad (17)$$

and with

$$n^\gamma_{hard} = A^\gamma_{BN}(b,s)\frac{\sigma_{jet}(s,p_{tmin})}{[P_{had}]^l} \quad (18)$$

with $l = 1, 2$ for γp and $\gamma\gamma$ respectively, $\sigma_{jet}(s, p_{tmin})$ to be calculated using current photon densities, and $A^\gamma_{BN}(b,s)$ given by Eq.12 with the appropriate q_{max} for the photon processes.

In the above described expressions, there appears the quantity P_{had}, which represents the probability that the photon behaves like a hadron in the eikonal formulation. A possibility is to use VMD models, namely

$$P_{had} = P_{VMD} = \sum_{V=\rho,\omega,\phi} \frac{4\pi\alpha}{f_V^2} = \frac{1}{250} \quad (19)$$

where the sum extends to all vector mesons, not just the ρ. As the photon energy increases, the contribution from the other, resolved, components also increases and one can expect P_{had} to differ from the VMD proposal. Using a phenomenologically fixed value $P_{had} = 1/240$ we obtain the results shown in Fig. 5 for the $\gamma\gamma$ case, using GRS densities.

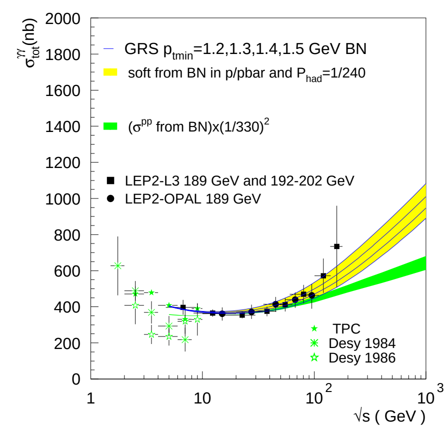

Figure 5: $\gamma\gamma$ total cross-sections from factorization (green band) or using the Eikonal with GRS densities and soft gluon resummation.

Please notice that the parameter R_γ appearing elsewhere in this note is a purely phenomenological multiplicative factor between the total photon and proton cross-sections, and it is different from P_{had}, which is defined through the eikonal and *not* through the cross-sections. It is however to be expected that both parameters be of the same order of magnitude, namely of $O(\alpha)$.

4 Conclusions

We notice that while factorization a' la Gribov could still hold for γp, the same cannot be said for $\gamma\gamma$ where present data do appear to be higher than the curve obtained by the simple factorization hypothesis. On the other hand a more detailed model, like the Mini-jet cum soft gluon resummation appears better suited to describe the present photon-photon data.

5 Acknowledgments

G. P. thanks the organizers of Photon2007 for the hospitality and the very beautiful venue of the Conference. This work has been partially supported by MEC (FPA2006-05294) and by Junta de Andalucía (FQM 101 and FQM 437).

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