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ORIGINAL PAPER

Modeling of Gel Layer-Controlled Fruit Juice Microfiltration in a Radial Cross Flow Cell

Sourav Mondal · Alfredo Cassano · Sirshendu De

Received: 21 August 2012 / Accepted: 19 February 2013 / Published online: 8 March 2013 © Springer Science+Business Media New York 2013

Abstract An analysis of gel layer-controlled microfiltration in a radial cross flow cell is presented in this study. Clarification of a real fruit juice, i.e., cactus pear juice is considered. An expression of Sherwood number is derived using an integral method under the framework of boundary layer theory. The effects of viscosity and temperature are incorporated in the Sherwood number relationship through the Sieder-Tate type correction factor and Stokes-Einstein equation, respectively. The transient flux behavior is modeled successfully both for the total recycle mode and batch concentration mode of operation. The model parameters are evaluated from the total recycle mode and are used in the predictive calculation of the batch concentration mode. In batch concentration mode of filtration, the model predicted results match excellently with the experimental data.

Keywords Membranes · Cactus pear juice · Mathematical modeling · Transport processes · Radial cross flow cell · Gel controlling filtration

Nomenclature

Α	Constant in Eq. 8 (=3Re Sc h/R)
a_1, a_2, a_3	Constants in Eq. 9
A_m	Effective membrane area (square meter)
С	Bulk concentration in batch recycle mode
	(kilogram per cubic meter)

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C^*	Non-dimensional concentration			
C_0	Initial bulk concentration (kilogram per cubic			
	meter)			
C_b	Bulk concentration in batch mode (kilogram per			
	cubic meter)			
C_g	Gel concentration (kilogram per cubic meter)			
C_g^*	Non-dimensional gel layer concentration			
C_m	Mean concentration in the boundary layer			
	(kilogram per cubic meter)			
C_m^*	Non-dimensional mean concentration			
C_p	Permeate concentration (kilogram per cubic			
	meter)			
D	Diffusivity of solute (square meter per second)			
D_0	Diffusivity at the reference temperature T_0			
	(square meter per second)			
h	Half height of channel (meter)			
h_0	Initial channel thickness (meter)			
H_b	Gel layer thickness in batch mode (meter)			
H_t	Thickness of gel layer in total recycle mode			
	(meter)			
k	Mass transfer coefficient (meter per second)			
т	Geometric factor in Eq. 42			
n	Exponent in viscosity–temperature relation $\mu \alpha \frac{1}{T^n}$			
P_{ew}	Non-dimensional flux			
Q	Volumetric flow rate (cubic meter per second)			
r	Radial coordinate (meter)			
r	Non-dimensional radial coordinate $(=r/R)$			
Re	Reynolds number $\left(\frac{\rho uh}{\mu}\right)$			
R_{gb}	Gel layer resistance defined by Eq. 36 (per meter)			
R_{gt}	Gel layer resistance in total recycle mode			
0	(per meter)			
R_m	Membrane hydraulic resistance (per meter)			
Sc	Schmidt number $\left(\frac{\mu}{\rho D}\right)$			
Sh	Sherwood number (kr/D_0)			
Sh	Length averaged Sherwood number (Eq. 15)			

- TTemperature in absolute scale (kelvin)tTime of operation (second) T_0 Reference temperature (=298 K)uEffective transverse velocity (meter per second)
defined in Eq. 2 u_0 Initial linear velocity (meter per second)VRetentate volume (cubic meter)
- V_0 Feed volume in batch mode (cubic meter)
- *v v*-component velocity (meter per second)
- v_w Permeate flux (cubic meter per square meter per second)
- *y y*-coordinate direction (meter)
- y^* Non-dimensional *y*-coordinate (=*y/h*)

Greek Symbols

- α Parameter in viscosity–temperature relation $\mu = \mu_0 e^{\alpha C}$
- β Specific gel resistance in Eq. 26, meter per kg
- δ Concentration boundary layer thickness (meter)
- $\boldsymbol{\delta^*} \quad \text{Non-dimensional concentration boundary layer} \\ \text{thickness}$
- ΔP Transmembrane pressure drop (pascal)
- ε_g Gel layer porosity
- ξ Parameter in Eq. 27 (per square meter)
- μ Viscosity (pascal-second)
- μ_0 Reference bulk viscosity when concentration of suspended solids is zero (pascal-second)
- μ_m Viscosity corresponding to mean concentration C_m in the boundary layer (pascal-second)
- μ_w Viscosity at the wall (pascal-second)
- ρ_f Density of feed (kilogram per cubic meter)
- ρ_g Gel layer density (kilogram per cubic meter)
- ρ_p Density of permeate (kilogram per cubic meter)
- τ Non-dimensional time $\left(=\frac{tD}{h^2}\right)$

Introduction

Membrane filtration processes are becoming popular these days, due to several advantages over the conventional separation processes, such as absence of thermal degradation, no phase change, no addition of chemicals, physical separation, etc. Cross flow ultrafiltration is widely used in food processing industries (Girard and Fukumoto 2000), biotechnology (Cheryan 1998), the pharmaceutical sector (Wang and Chung 2006), clarification and concentration of fruit juice (Rai et al. 2010; Mondal et al. 2011b; Thomas et al. 1987; Mohammad et al. 2012). However, efficient design of such systems for large scale is based on the prediction and detailed understanding of the mass transfer phenomena with coupled fluid flow. The relevant flow configuration and flow regimes are extremely important in modeling the process performance. The mass transfer coefficient is generally calculated from different Sherwood number correlations,

obtained from heat and mass transfer analogies. However, these correlations fail to take into account the property variation due to solute interactions, changes due to developing mass transfer boundary layer, effects of transmembrane pressure drop on the mass transfer coefficient and local geometric effects on the hydrodynamics of the flow regime. The available mass transfer correlations for membrane separation processes along with their shortcomings have been already reviewed in detail (van Den Berg et al. 1989; Gekas and Hallstrom 1987).

The significant phenomena leading to the decline in flux is concentration polarization (Porter 2005; Sablani et al. 2001). A simple description of concentration polarization is obtained from a stagnant film model, used by Sherwood et al. (1965) to analyze reverse osmosis. Many researchers (Opong and Zydney 1991; Zydney 1997; Johnston and Deen 1999) have used stagnant film model that considers a thin layer of solute adhered to the membrane surface, leading to one-dimensional problem in which the solute concentration depends only on distance from the membrane surface. To overcome this problem, a detailed numerical solution of the governing momentum and solute mass balance equation with pertinent boundary conditions may be used (Kleinstreuer and Paller 1983; Bouchard et al. 1994; De and Bhattacharya 1997a). But this method seems unattractive for design purposes owing to its inherent complexities and rigorous computational requirements.

A number of authors have computed two-dimensional concentration fields for laminar cross flow ultrafiltration in tubes or parallel-plate channels (Shen and Probstein 1977; Gill et al. 1988; Denisov 1999; Bhattacharjee et al. 1999; Madireddi et al. 1999), spiral-wound membrane modules (Kozinski and Lightfoot 1971). Field and Aimar have modified Leveque's correlation for laminar flow in rectangular channel by using a viscosity correction factor (Field and Aimar 1993). However, the effects of suction were not considered in their study which has been incorporated later by De and Bhattacharya (1997a). Sherwood number relationship incorporating the effects of suction (in presence of a membrane) for laminar flow in rectangular, radial, and tubular geometries, have been formulated starting from first principles (De and Bhattacharya 1997a). However, that study includes the osmotic pressure controlled filtration only and the effect of developing mass transfer boundary layer. Variation of physical properties at high polarized condition (gel layer controlling case) is not attempted. It has been shown that due to concentration polarization, the variation of the physical properties with concentration is significant in the performance of the ultrafiltration and subsequent development of the boundary layer (Gill et al. 1988; De and Bhattacharya 1999; Bowen and Williams 2001). In case of osmotic pressure controlled filtration, effects of property variation are considered in a simplistic way by De and

Bhattacharya (1996) and in detailed way by Bhattacharya et al. (2001).

Apart from osmotic pressure dominating, gel layer controlling filtration is another prevalent mechanism of ultrafiltration. This occurs in case of filtration of high molecular weight proteins, polymer, paint, clay, etc. It is assumed that a highly viscous solid-like layer is formed over the membrane surface with uniform solute concentration (commonly known as gel concentration) and it obeys the classical gel filtration theory. The primitive gel layer model is derived from conventional film theory (Blatt et al. 1970). However, this model suffers from three limitations. First, a uniform thickness of mass transfer boundary layer was considered instead of developing boundary layer which is more fundamentally correct. Secondly, solution viscosity is a strong function of solute concentration and it varies significantly within the mass transfer boundary layer, as the solute concentration increases from bulk to gel layer concentration. Gel layer concentration is quite often three to seven times of bulk concentration. This variation of viscosity as a function of concentration was not included in the film model. Thirdly, variation of feed temperature affects solution viscosity and solute diffusivity significantly and these effects were not considered in the film theory. The first limitation was circumvented by Probstein et al. (1978) by considering a two-dimensional, developing mass transfer boundary layer under laminar flow condition in a rectangular channel. Clarification of fruit juice by ultrafiltration has been found to be gel controlling in many occasions due to presence of protein, cell debris, cellulose, etc. (Rai et al. 2010; Sarkar et al. 2008; Mondal et al. 2012a). Second and third drawbacks of film theory have been addressed by Mondal et al. (2011a) in a tubular module, Mondal et al. (2012a) in stirred cell and Mondal et al. (2012b) in a rectangular cell.

One of the popular geometry in juice processing industry is the radial cross flow cell. Ganguly and Bhattacharya (1994) have solved the convective momentum equations numerically to obtain the ultrafiltration flux. De and Bhattacharya (1997a) and Minikanti et al. (1999) have shown the convective-diffusive equation in radial cross flow cell under laminar and turbulent flow regimes, respectively, for osmotic pressure controlling only and not for gel controlling filtration. Moreover, their analysis does not take into account the effect of concentration on the property variation. Solution of convective-diffusive equation in case of a radial cross flow cell for gel polarized filtration is not available. The present model addresses this case under the framework of boundary layer analysis. Similar model for tubular cross flow system is available (Mondal et al. 2011a) and that model can easily be extended to rectangular geometry. For radial cross flow system, the velocity field is entirely different, leading to a new expression of Sherwood number compared to tubular and rectangular geometry and that is attempted in this work. The model includes the developing mass transfer boundary layer over the gel layer, effects of concentration dependence on viscosity and effects of feed temperature on solution viscosity and solute diffusivity. The present model is further extended to quantify the flux decline as well as the volume reduction factor (VRF) during batch mode of operation from the first principles by solving the overall material balance, overall solute balance and solute balance within the mass transfer boundary layer. A numerical solution of these balance equations leads to the flux decline and VRF profile. Therefore, the present model is a comprehensive one including various fundamental transport aspects of transport phenomena in a radial geometry. The extensive analytical treatment makes the model easy to estimate the steady-state flux values in either of the operation modes by simple computational techniques.

The model is successfully applied for ultrafiltration of cactus pear juice (Cassano et al. 2007). Since fruit juice is a complex mixture, some of physical properties are unknown and have never been reported prior to this study. These property values are estimated by minimizing the sum of square errors of the experimental and predicted values. However, the model analysis is applied completely in predictive mode for the batch concentration using the determined physical constants.

Theoretical Development

Cross flow experiments were carried out in radial cross flow cell in both total recycle and batch concentration mode. In the total recycle mode of operation, both the permeate and retentate streams are recycled back to the feed tank so that a steady state is attained with fixed concentration of the feed. This is a popular mode of operation in any membrane-based process in order to evaluate the effects of operating parameters (feed concentration, cross flow velocity, transmembrane pressure drop) on steady-state permeate flux and permeate quality (Rai et al. 2006). In the batch concentration mode, retentate stream is recycled back to the feed tank but the permeate is continuously taken out. This results in an increase in feed concentration accompanied by a reduction in feed volume with time of operation.

Total Recycle Mode

In order to quantify the permeate flux, mass transfer coefficient has to be estimated. In this regard, an analysis in a radial cell is presented in this section.

During microfiltration of juice, high molecular weight solutes are transported towards the membrane wall, forming a gel layer over the membrane surface. The gel concentration is assumed to be constant within this layer. Therefore, there exists a concentration boundary layer next to the gel layer with a variation in concentration from bulk to gel concentration. For convenience, we assume a twodimensional Cartesian-coordinate system, where the radial direction, r, is not a radial coordinate but a Cartesian axis. This simplification makes the model equations simpler and do not alter the results to any significant extent (Ganguly and Bhattacharya 1994). Following the convention as given in Fig. 1, the convective-diffusive flux equation for gel-forming solute within concentration boundary layer is given for a radial cell as (Mondal et al. 2011a),

$$\frac{\partial(Cu)}{\partial r} + \frac{\partial(Cv)}{\partial y} = \frac{\partial}{\partial y} \left(D \frac{\partial C}{\partial y} \right) \tag{1}$$

The radial-velocity profile in the flow channel, within the boundary layer $(y \ll h)$ can be expressed as (Ganguly and Bhattacharya 1994),

$$u(r,y) = \frac{3Qy}{4\pi rh^2}.$$
(2)

It must be emphasized here, that the rheology of fruit juice typically exhibits power law behavior, being pseudoplastic in nature. The effect of rheology, in terms of function of the power law index k and exponent n $\left[\tau = k\left(\frac{du}{dy}\right)^{n-1}\left(-\frac{du}{dy}\right)\right]$, has to be incorporated for evaluation of the velocity field for this particular geometry. It should also be noted here that the rheological constants k and n would also be dependent on the total solid concentration of the juice. This makes the analysis complex and for the sake of simplicity, Newtonian rheology is considered. To formulate the above equation (Eq. 2), it is also assumed that the permeate flux is negligible as compared to feed velocity (u) so that the profile of u is not distorted by the permeation at the



Fig. 1 Flow configuration of the radial cross flow cell

wall. The average volumetric flow rate (Q) is given by,

$$Q = 4\pi Rhu_0 \tag{3}$$

where, u_0 is the average velocity in the conduit and h is the channel half height.

Since, the concentration boundary layer thickness is extremely small, it can be assumed that the *y*-component velocity is equal to the permeation velocity at the wall (De et al. 1997) and within small concentration boundary layer, vis not a function of *y*. Therefore, the *y*-component velocity becomes,

$$v = -v_w \tag{4}$$

Inserting the velocity profiles into the Eq. 1, the following equation is obtained.

$$\frac{3u_0 Ry}{hr} \frac{\partial C}{\partial r} - v_w \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2}$$
(5)

In the present analysis, variation of diffusion coefficient (D) with concentration is ignored as an assumption. Defining non-dimensional parameters as $y^* = y/h$; $r^* = r/R$ and $C^* = C/C_0$, the above equation is made dimensionless as,

$$\frac{3u_0h^2}{DR}\frac{y^*}{r^*}\frac{\partial C^*}{\partial r^*} - \frac{v_wh}{D}\frac{\partial C^*}{\partial y^*} = \frac{\partial^2 C^*}{\partial {y^*}^2} \tag{6}$$

It is to be noted here that $\frac{u_0h^2}{DR}$ is $Re \cdot Sc \cdot \frac{h}{R}$ and $\frac{v_wh}{D}$ is the non-dimensional flux (denoted as P_{ew}). Thus, Eq. 6 becomes,

$$3 \operatorname{Re} \cdot \operatorname{Sc} \frac{\mathrm{h}}{\mathrm{R}} \frac{\mathrm{y}^*}{\mathrm{r}^*} \frac{\partial \mathrm{C}^*}{\partial \mathrm{r}^*} - \operatorname{Pe}_{\mathrm{w}} \frac{\partial \mathrm{C}^*}{\partial \mathrm{y}^*} = \frac{\partial^2 \mathrm{C}^*}{\partial \mathrm{y}^{*2}}$$
(7)

The coefficient $3 \operatorname{Re} \cdot \operatorname{Sc} \cdot \frac{h}{R}$ is represented as *A*. Thus, Eq. 7 can be rearranged as,

$$A\frac{y^*}{r^*}\frac{\partial C^*}{\partial r^*} - Pe_w\frac{\partial C^*}{\partial y^*} = \frac{\partial^2 C^*}{\partial y^{*2}}$$
(8)

The relevant boundary conditions of above equation are:

at
$$r^* = 0, \quad C^* = 1$$
 (8a)

at
$$y^* = \delta^*$$
, $C^* = 1$ (8b)

where, δ^* is non-dimensional thickness of concentration boundary layer. At $y^*=0$, solute flux towards membrane surface is zero. This leads to, at y=0, $v_w C_g + D \frac{\partial C}{\partial y} = 0$. The non-dimensional of this boundary condition becomes

at
$$y^* = 0$$
, $P_{ew}C_g^* + \frac{\partial C^*}{\partial y^*} = 0$ (8c)

Equation 8 along with its boundary conditions is solved using an integral method, assuming a concentration profile within mass transfer boundary layer.

A quadratic concentration profile within boundary layer is considered,

$$C^* = \frac{C}{C_0} = a_1 + a_2 y^* + a_3 y^{*2}$$
(9)

where a_1 , a_2 and a_3 are the constant coefficients. Using the following boundary conditions, the constants of the above equation are determined,

at
$$y^* = 0$$
, $C^* = C_g^* = C_g/C_0$ (9a)

at
$$y^* = \delta^*$$
, $C = C_0$; $C^* = 1$ (9b)

at
$$y^* = \delta^*$$
, $\frac{\partial C^*}{\partial y^*} = 0$ (9c)

Evaluating the constants a_1 to a_3 , Eq. 9 is rewritten as

$$C^* = C_g^* - 2\left(C_g^* - 1\right)\left(\frac{y^*}{\delta^*}\right) + \left(C_g^* - 1\right)\left(\frac{y^*}{\delta^*}\right)^2 \tag{10}$$

A similar analysis is provided for the tubular geometry in Mondal et al. (2011a). The detailed solution steps are presented in Appendix A. This analysis provides a closed form expression of the mass transfer boundary layer profile as a function of radial coordinate and other physical and flow parameters, as presented below,

$$\delta^* = \left(\frac{6}{Re \cdot \mathrm{Sc} \cdot \frac{\mathrm{h}}{\mathrm{r}} \mathrm{C}_{\mathrm{g}}^*}\right)^{1/3} r^{*2/3} \tag{11}$$

Estimation of the Mass Transfer Coefficient

The definition of mass transfer coefficient can be written as (Incropera and Dewitt 1996),

$$k(C_g - C_0) = -D \frac{\partial C}{\partial y}\Big|_{y=0}$$
(12)

Non – dimensionalizing the above equation and substituting $\frac{\partial C^*}{\partial y^*}\Big|_{y^*=0}$ (refer to Eq. 9) leads to the following expression of Sherwood number,

$$\mathrm{Sh} = \frac{2}{\delta^*} \tag{13}$$

Substituting the profile of δ^* from Eq. 11, the expression of Sherwood number becomes,

$$\operatorname{Sh}(\mathbf{r}^{*}) = 2 \left(\frac{\operatorname{Re} \cdot \operatorname{Sc} \cdot \mathbf{h} \cdot \operatorname{C}_{g}^{*}}{18 \operatorname{R}} \right)^{1/3} \mathbf{r}^{*-2/3}$$
(14)

The length averaged Sherwood number thus, becomes

$$\overline{Sh} = \int_0^1 Sh(\mathbf{r}^*) d\mathbf{r}^* = 1.65 \left(Re \cdot Sc \cdot \frac{h}{R} \right)^{1/3} C_g^{*1/3}$$
(15)

It is to be noted that the leading coefficient in Sherwood number relation using classical Leveque solution is 1.86 for rectangular, 1.62 for tubular geometry (van Den Berg et al. 1989) and 1.47 for radial cell geometry (De and Bhattacharya 1997a). It may also be noted that these coefficients remain unaltered in case of incorporation of Sieder–Tate correction factor (Incropera and DeWitt 1996). However, the coefficient increases if one includes the developing mass transfer boundary layer. In case of rectangular geometry, the coefficient increases from 1.85 to 2.10 (Probstein et al. 1978). In case of tubular geometry, the Sherwood number coefficient increases from 1.62 to 1.816 (Mondal et al. 2011a). In the present study, for radial cross flow cell, the coefficient increases from 1.47 to 1.65.

Since, the gel layer concentration is several order of magnitude higher than the bulk concentration it is obvious that the viscosity variation within concentration boundary layer is significant. The viscosity and temperature effects are included in Sherwood number expression following the derivation as elaborated in a recent study (Mondal et al. 2011a), which was developed for a tubular geometry. The final expression of Sherwood number is, therefore, in this case, is presented below,

$$\overline{Sh} = 1.65 \left(Re \cdot Sc \frac{h}{R} \right)^{\frac{1}{3}} \left(e^{-\frac{2}{3}\alpha C_0 \left(C_g^* - 1 \right)} \right)^{0.14} C_g^{*1/3}$$
(16)

Estimation of Gel Layer Thickness and Permeate Flux

Performing a material balance of the gel-forming component in the concentration boundary layer results in the following equation (Mondal et al. 2012b; De and Bhattacharya 1997b):

for
$$0 < y < \delta$$
, $j_1 = \text{mass flux} = \rho_g \frac{dH_t}{dt} = v_w C_1 - D \frac{dC_1}{dy}$
The particular boundary conditions are (17)

The pertinent boundary conditions are,

$$C_1 = C_0(t) \quad \text{at} \quad \mathbf{y} = 0 \tag{17a}$$

$$C_1 = C_g \quad \text{at} \quad \mathbf{y} = \boldsymbol{\delta} \tag{17b}$$

The solution of the Eq. 17 within the boundary layer (0 to δ), using the above stated boundary conditions, represents the variation of the gel layer thickness (*H*) with time as,

$$\rho_g \frac{dH_t}{dt} = v_w \frac{C_g - C_0 \exp\left(\frac{v_w}{k}\right)}{1 - \exp\left(\frac{v_w}{k}\right)} \tag{18}$$

where, k is the mass transfer coefficient defined as D/δ . It must be noted here, that the Eq. 18 transforms into the form at the steady state,

$$v_w = k \ln C_o^* \tag{19}$$

$$v_w C_g = k \left(C_g - C_0 \right) \tag{20}$$

Non-dimensional form of the above equation is

$$P_{ew} = \operatorname{Sh}\left(\frac{\operatorname{C}_{g}^{*}-1}{\operatorname{C}_{g}^{*}}\right)$$
(21)

Combining Eqs. 16 and 21, the length averaged permeate flux becomes,

$$\overline{P_{ew}} = 1.65 \left(Re \cdot \text{Sc} \,\frac{\text{h}}{\text{R}} \right)^{\frac{1}{3}} \left(e^{-\frac{2}{3}\alpha C_0 \left(C_g^* - 1 \right)} \right)^{0.14} \left(\frac{T_0}{T} \right)^{\frac{\mu+1}{3}} \left(C_g^{*1/3} - C_g^{*-2/3} \right)$$
(22)

For $C_g^* \ll 20$, $\left(C_g^{*1/3} - C_g^{*-2/3}\right)$, is reduced to $\ln C_g^*$, thus

$$\overline{P_{ew}} = 1.65 \left(Re \cdot \text{Sc} \frac{\text{h}}{\text{R}} \right)^{\frac{1}{3}} \left(e^{-\frac{2}{3}\alpha C_0(C_g^* - 1)} \right)^{0.14} \left(\frac{T_0}{T} \right)^{\frac{n+1}{3}} \ln C_g^*$$
(23)

Therefore, Sherwood number can be expressed as (using Eqs. 19 and 23),

$$Sh = 1.65 \left(Re \cdot Sc \frac{h}{R} \right)^{\frac{1}{3}} \left(e^{-\frac{2}{3}\alpha C_0 \left(C_g^* - 1 \right)} \right)^{0.14} \left(\frac{T_0}{T} \right)^{\frac{n+1}{3}}$$
(24)

The flux v_w can be expressed using the phenomenological equation,

$$v_w = \frac{\Delta P}{\mu \left(R_m + R_{gt} \right)} \tag{25}$$

where, R_m is the hydraulic membrane resistance determined experimentally and R_{gt} is the gel layer resistance. The gel layer is assumed to be a deposit of porous gel, using the filtration concept. Hence, the gel layer resistance and its characteristics are described with the platform of traditional gel filtration theory (Bhattacharjee et al. 1996). So, R_{gt} is expressed as,

$$R_{gt} = \beta (1 - \varepsilon_g) \rho_g H_t \tag{26}$$

where, β is the specific gel resistance, ε_g is the porosity of the gel, ρ_g is the density of the gel layer. Since ε_g , ρ_g are all constants during the experiment, the product $\beta(1-\varepsilon_g)\rho_g$ is clubbed together into a single parameter and is treated as another constant (ξ) during the course of the simulation.

Thus, $R_{gt} = \xi H_t$ (27)

Combining Eqs. 18 and 25, the governing equation of gel layer thickness becomes,

$$\rho_g \frac{dH_t}{dt} = v_w \frac{C_g - C_0 \exp\left(\frac{\Delta P}{k\mu(R_m + R_{g_t})}\right)}{1 - \exp\left(\frac{\Delta P}{k\mu(R_m + R_{g_t})}\right)}$$
(28)

The initial condition of the above equation is $H_t=0$ at t=0

Batch Concentration Mode

Considering an overall material balance, the following equation is obtained,

$$\frac{d}{dt}\left(\rho_{f}V\right) = -\nu_{w}A_{m}\rho_{p} \tag{29}$$

where, ρ_f and ρ_p are densities in feed and permeate streams; V is the feed volume and A_m is the effective membrane area. Assuming $\rho_f \simeq \rho_p$ (density is a weak function of concentration and both feed and permeate are diluted solutions) the above equation is modified as,

$$\frac{dV}{dt} = -v_w A_m \tag{30}$$

Using overall species balance of gel-forming component, the following equation is obtained,

$$\frac{d}{dt}(C_b V) = -v_w A_m C_p \tag{31}$$

Since, concentration of the gel-forming material in the permeate is zero ($C_p=0$) (Cheryan 1998), the above equation reduces to a simple algebraic equation

$$C_b V = C_0 V_0 \tag{32}$$

With initial boundary condition as $C=C_0$ and $V=V_0$ at t=0

Now, following the material balance for the gel-forming component in the concentration boundary layer results in the following equation (Mondal et al. 2012b):

for
$$0 < y < \delta$$
, $j_1 = mass flux = \rho_g \frac{dH_b}{dt} = v_w C_1 - D \frac{dC_1}{dy}$

(33)

The pertinent boundary conditions are,

$$C_1 = C_b(t) \quad \text{at} \quad \mathbf{y} = 0 \tag{33a}$$

$$C_1 = C_g \quad \text{at} \quad \mathbf{y} = \boldsymbol{\delta}$$
 (33b)

The solution of the Eq. 33 using the above stated boundary conditions represents the variation of the gel layer thickness (H) with time (Mondal et al. 2011a),

$$\rho_g \frac{dH_b}{dt} = v_w \frac{C_g - C_b \exp\left(\frac{v_w}{k}\right)}{1 - \exp\left(\frac{v_w}{k}\right)} \tag{34}$$

where, k is the mass transfer coefficient defined as D/δ . In this case, the expression of mass transfer coefficient is different from Eq. 24. This is because the boundary condition of the concentration profile within concentration boundary layer at the edge is no longer initial feed concentration (C_0). It becomes bulk concentration that is a function of time, $C_b(t)$. The mass transfer analysis and expression of corresponding average permeate flux in this case are derived in Appendix B. The expression of length averaged permeate flux, is presented as,

$$\overline{P_{ew}} = 1.65 \left(Re \cdot \operatorname{Sc} \frac{\mathrm{h}}{\mathrm{R}} \right)^{\frac{1}{3}} \left(e^{-\frac{2}{3}\sigma C_0 \left(C_g^* - C_b^* \right)} \right)^{0.14} \left(\frac{T_0}{T} \right)^{\frac{n+1}{3}} \ln \left(\frac{C_g^*}{C_b^*} \right)$$
(35)

The flux v_w can be expressed using the phenomenological equation,

$$v_w = \frac{\Delta P}{\mu \left(R_m + R_{gb} \right)} \tag{36}$$

where, R_m is the hydraulic membrane resistance determined experimentally and R_{gb} is the gel layer resistance in batch concentration mode. The gel layer resistance and its characteristics are described with the platform of traditional gel 361

filtration theory (Bhattacharjee et al. 1996). So, R_{gb} is expressed as,

$$R_{gb} = \beta (1 - \varepsilon_g) \rho_g H_b \tag{37}$$

where, β is the specific gel resistance, ε_g is the porosity of the gel, ρ_g is the density of the gel layer. Since ε_g , ρ_g are all constants during the experiment, the product $\beta(1-\varepsilon_g)\rho_g$ is clubbed together into a single parameter and is treated as another constant (ξ) during the course of the simulation.

Thus,
$$R_{gb} = \xi H_b$$
 (38)

Combining Eqs. 34 and 36, the governing equation of gel layer thickness becomes (Mondal et al. 2011a),

$$\rho_g \frac{dH_b}{dt} = v_w \frac{C_g - C_b \exp\left(\frac{\Delta P}{k\mu (R_m + R_{gb})}\right)}{1 - \exp\left(\frac{\Delta P}{k\mu (R_m + R_{gb})}\right)}$$
(39)

The initial condition of the above equation is $H_b=0$ at t=0

It may be noted that as time of operation proceeds, the channel height, h(t), of radial cell diameter decreases by deposition of gel layer and it is quantified as,

$$h(t) = h_0 - \frac{1}{2}H_b(t) \tag{40}$$

Consequently, the cross flow velocity $u_0(t)$ inside the radial cell changes as,

$$u_0(t) = \frac{Q}{4\pi Rh(t)} \tag{41}$$

The above expressions of channel height and cross flow velocity within the radial cell have been utilized to evaluate the non-dimensional permeate flux in Eq. 35. The governing equation of volume and bulk concentration at any time point is given by Eqs. 30 and 32, respectively.

Equations 30, 32, 36, 38, and 39 present a system of differential-algebraic equations. These are solved numerically using fourth order Runge–Kuta method. Thus, the time profiles of VRF, retentate concentration, gel layer thickness, permeate flux, etc., are obtained.

Introduction of the Local Geometric Effect for Mass Transfer Enhancement

The experimental setup comprised grooved circular channels at the base of the membrane support, which are responsible for increase in mass transfer and correspondingly permeate flux. This geometric configuration is supposed to alter the flow hydrodynamics that can be accounted by variation in Reynolds number (Prabhavathy and De 2011;





Das and De 2009). Considering the present context, the average Sherwood number is corrected as,

$$\overline{\mathrm{Sh}_{\mathrm{ac}}} = \overline{\mathrm{Sh}}(\mathrm{Re})^m \tag{42}$$

The functionality of Sh with Re can be analyzed from the slope of the log–log plot of flux with flow rate. In case of negligible geometric effect, the value of m is 0 and Eq. 42 reverts back to Eq. 16.

Experimental Details

Cactus pear juice was clarified by using a MF-UF laboratory plant unit (Permalab) supplied by Permeare S.r.l. (Milan, Italy). The unit is mainly composed of a

10-l stainless steel feed tank, a feed pressure pump, two pressure cells for flat sheet membranes, piping and instrumentation for feed, concentrate and permeate streams and an electric board. A cooling device, placed after the feed tank, is used to maintain the temperature of the feed juice constant. A schematic diagram of the experimental setup is depicted in Fig. 2a.

Each cell is composed of two stainless steel parts to be piled with 42-mm-diameter membrane disk (effective membrane area 13.85 cm²) and o-rings in between; a bottom part with two ports for pre-piped feed inlet and concentrate outlet, and one threaded 1/4" port for feed pressure gauge; an upper part with one threaded 1/8" port and stainless steel adapter for flexible permeate outlet hose, 4 mm inside diameter, that can be directed either to the process tank, to a sample collector or to drain. Membrane is placed on the top part. The feed flows into the filtration cell through a 5mm-diameter tube and then in radial direction in the cell. The bottom part of the cell has five concentric grooves each having a height of 1 mm. These grooves provide localized turbulence to induce higher mass transfer. The total height of the channel is 2 mm. Details of the radial cell are illustrated in Fig. 2b.

Pressure cells were equipped with polyvinylidenfluoride flat sheet membranes (Nadir MV020) having a nominal pore size of 0.20 μ m and supplied by Microdyn-Nadir (Wiesbaden, Germany).

MF experiments were performed according two types of operating mode: the total recycle mode and the batch concentration mode. In the former, the permeate was continuously recycled to the feed tank to ensure a steady state in the volume and composition of the feed. In the batch concentration procedure, the permeate was collected separately and the retentate was recycled to the feed tank. The permeate flux was determined by measuring the volume of permeate collected in a certain time through the membrane surface area.

Duration of the cross flow experiments was 120 min for total recycle mode. The effect of transmembrane pressure (TMP) on the permeate flux was investigated in the range 140–230 kPa at 460 L/h and 20 °C. The effect of temperature was investigated in the range 20– 35 °C at 500 L/h and 200 kPa. The feed flow rate was investigated in the range 400–500 L/h at 26 °C and 200 kPa.

In the batch concentration mode, the MF system was operated at a TMP of 220 kPa, an axial feed flow rate of 500 L/h and a temperature of 25 °C to clarify the juice up to a volume reduction factor of 1.4. Duration of experiments was 480 min.

The initial hydraulic permeability of the MF membranes was 705.0 L/m^2 hbar at 25 °C.

At the end of each run, cells were dismantled and the gel formed on the membrane surface was removed by rinsing with water (20 min). After that a cleaning process was used with an alkaline solution (Ultraclean WA, 1 %, 40 °C, 60 min). After the above-mentioned procedure the hydraulic permeability was totally restored.

 Table 1 Properties of the cactus-pear juice by microfiltration

Parameter	Feed	Permeate	Retentate
Total soluble solids (°Brix)	9.5±0.8	9.5±0.8	10.0±0.8
pH	$5.71 {\pm} 0.2$	$5.58 {\pm} 0.2$	$5.54 {\pm} 0.2$
Suspended solids (%w/w)	6.55±0.5	0	4.79 ± 0.5
Total acidity (% citric acid)	0.037±0.005	0.024±0.005	0.029±0.005



Fig. 3 Variation of steady-state flux with TMP

Results and Discussion

Physicochemical Characterization of Various Streams During Microfiltration

Various properties of the feed, permeate, and retentate are presented in Table 1. Since the suspended solids are completely rejected by the membrane and also the concentration of it in retentate stream is less compared to feed, this establishes that a significant proportion of suspended solids form a gel layer over the membrane surface.

Identification of Modeling Domain

Variation of steady-state permeate flux with transmembrane pressure drop is shown in Fig. 3. It is observed from this



Fig. 4 Variation of the steady-state permeate flux with flow rate (ΔP =200 kPa, T=26 °C).

figure that permeate flux increases with pressure drop and beyond 200 kPa, the flux is invariant of pressure drop. The pressure-independent permeate flux is a strong indicator that the filtration domain is entirely gel layer controlling (Trettin and Doshi 1981; Porter 2005). The analysis of the present work is in this domain of filtration.

Hydrodynamic Effect of Turbulence Enhancers on Mass Transfer Coefficient

It is observed from Eq. 19 that permeate flux is directly proportional to the mass transfer coefficient. Thus, steadystate permeate flux at different Reynolds number is plotted in log-log scale in Fig. 4. It is seen that the slope of the best fit straight line is 1.1. Thus, in Eq. 42, the exponent of Revnolds number is 1.1. It may be noted here that this exponent includes the geometric effect of the cell on the promotion of turbulence in the flow path.

Total Recycle Mode

Calculation procedure of total recycle mode and the corresponding flow chart is presented in Fig. 5. The optimized values of various parameters are: $D_0 = (2.4 \pm 0.05) \times 10^{-11} \text{ m}^2/\text{s};$ $C_{\sigma}=389\pm 6 \text{ kg/m}^3$; $n=4.3\pm 0.5$; $\alpha=0.016\pm 0.002 \text{ m}^3/\text{kg}$; $\rho_{\sigma}=$ $1680\pm10 \text{ kg/m}^3$ and $\xi = (5.1\pm0.8) \times 10^{15} \text{ m}^{-2}$. Out of six parameters, four parameters, namely solute diffusivity, gel concentration, gel layer density, and gel resistance constant (ξ) are physical properties of the solute and the gel layer. By examining the order of magnitude of these parameters and comparing



with other similar fruit juices (Mondal et al. 2011a; Sarkar et al. 2008; Mondal et al. 2012a), it is confirmed that these values are physically meaningful. The other two parameters n and α which account for viscosity–temperature and viscosity–concentration variation are the adjustable parameters. Thus, the first four parameters are those of the physical property values which should not be misinterpreted as system adjustable parameters.

With the optimized values of the parameters, mass transfer coefficient is computed for different Reynolds number and feed temperature. The results are presented in Fig. 6. It is observed from this figure that the mass transfer coefficient increases with Reynolds number. Reynolds number is increased with cross flow rate, imparting more forced convection effects, leading to curbing of growth of mass transfer boundary layer. Thus, mass transfer coefficient increases with Reynolds number. On the other hand, the solute diffusivity is directly proportional to temperature and solution viscosity is inversely proportional to temperature. Therefore, backward diffusion of solutes from the gel layer to the bulk increases and the reduction in viscosity increases more forced convection (as the Reynolds number increases). These two effects work in tandem and as a result, mass transfer coefficient increases with temperature. These effects of Reynolds number and temperature on mass transfer coefficient are shown in this figure.

Using the optimized parameter values, the permeate flux profiles for various cross flow rates are shown in Fig. 7a. It is observed in this figure that a steady state is attained within 20 min of operation. The permeate flux increases with cross flow rate. As cross flow rate increases, mass transfer coefficient increases and therefore, growth of the gel layer is arrested due to more forced convection. Permeate flux at steady-state increases from about 50 to 63 L/m^2h (26 % increase) as the cross flow rate increases from



Fig. 6 Variation of the mass transfer coefficient with Reynolds number (ΔP =200 kPa)



Fig. 7 a Effect of flow rate on the flux profile (temperature=26 °C; ΔP =200 kPa). b Effect of temperature on the flux profile (flow rate= 500 L/h; ΔP =200 kPa)

400 to 500 L/h. It is noted that above calculation is done using mean value of the measurement. The figure also shows the matching between the experimental and calculated values of permeate flux. Variation between these two is well within ± 10 %.

Figure 7b shows the effect of feed temperature on the permeate flux decline profiles. It is observed that the permeate flux increases with temperature. As discussed earlier in Fig. 6, mass transfer coefficient increases with temperature. With temperature, both forced convection effect of retentate flow rate and backward diffusion of solutes from the gel layer increase. As a result, the gel layer thickness decreases and the permeate flux increases. The steady-state permeate flux increases from 57 to 74 L/m^2h (about 30 % increase) as the feed temperature increases from 20 to 35 °C.

Comparison of calculated flux and experimental data is also presented in this figure. It is observed that the agreement between the two is excellent.

Since steady state is attained within 20 min, steady-state flux values are important in the long run. Thus, Figs. 8a and b present a comparison of experimental and calculated steady-state flux values for different values of temperature and cross flow rates. It can be seen that the agreement between the two is well within ± 10 %.

Development of gel layer thickness and gel layer resistance are shown in Fig. 9a and b, respectively. From Fig. 9a, it is seen that gel layer thickness decreases with flow rate



Fig. 8 a Comparison of the predicted and experimental steady-state permeate flux variation with temperature (flow rate 500 L/h; ΔP =200 kPa). b Comparison of the predicted and experimental steady-state permeate flux variation with flow rate (temperature=26 °C; ΔP =200 kPa)



Fig. 9 a Growth of gel layer with time during microfiltration under total recycle mode (ΔP =200 kPa). b Ratio of the gel and membrane hydraulic resistance with time of operation (ΔP =200 kPa)

(comparing curves 1 and 3). As discussed earlier, higher cross flow rate increases the mass transfer coefficient resulting to reduced gel layer thickness. For example, the steady-state gel layer thickness decreases from 300 to 220 μ m (27 % decrease) as the flow rate increases from 400 to 500 L/h. Also, it is discussed previously that feed temperature enhances the permeate flux by reducing viscosity and increasing the solute diffusivity. Thus, the gel layer thickness is reduced from 255 to 180 μ m (36 % decrease) as temperature increases from 20 to 35 °C. Corresponding variations are shown in Fig. 9b for gel layer resistance. Since the gel layer characteristics remain same, the gel layer resistance is proportional to gel layer thickness. Hence, the gel layer resistance varies with the operating conditions similar to gel layer thickness. Thus, gel layer resistance also decreases with temperature and cross

flow rate. It is observed from this figure that the steady-state value of gel layer resistance lies between 1.5 to about 3.0 times of membrane resistance in the range of operating conditions considered herein. At the same temperature (26 °C). non-dimensional gel layer resistance decreases from 2.8 to 2.2 when the cross flow rate increases from 400 to 500 L/h. For the same cross flow rate 500 L/h, the gel layer resistance decreases from 2.5 to 1.6 as temperature increases from 20 to 35 °C.

Batch Concentration Mode

а

Permeate flux (l/m²h)

b

In this mode of operation, all the parametric values evaluated in total recycle mode are used as they are. Eqs. 30, 32, 35 to 42 are solved simultaneously in entirely predictive mode and the profiles of permeate flux, volume reduction factor of the feed, gel layer thickness and gel layer resistance are evaluated. The profile of permeate flux is presented in Fig. 10a. In the batch mode, permeate is not recycled back to the feed tank and therefore, the feed concentration keeps on increasing, leading to gradual increase in concentration polarization in situ. This makes the gradual decrease in predicted permeate flux with time. VRF is defined as the ratio of initial feed volume to the feed volume at any point of time. Since, feed volume decreases continuously as permeate is extracted without recycle, VRF increases with time. It is observed that both permeate flux and VRF show close agreement between the predicted and experimental data in Fig. 10a and b.



Fig. 10 a Comparison of the predicted and experimental permeate flux profile during batch mode of operation (ΔP =220 kPa). **b** Comparison of the predicted and experimental volume reduction factor during batch mode of operation ($\Delta P=220$ kPa)

Fig. 11 a Effect of temperature and flow rate on the growth of gel layer profile during batch operation (ΔP =220 kPa). b Effect of temperature and flow rate on the growth of gel layer resistance during batch operation ($\Delta P=220$ kPa)

Profiles of gel laver thickness and gel laver resistance are shown in Figs. 11a and b, respectively, for batch operation. Gel layer thickness increases with time and exhibits expected variation with cross flow rate and feed temperature. However, since time of operation is quite long, after 6 h, gel layer thickness varies in between 380 to 620 µm for different operating conditions. In this mode operation, since permeate is withdrawn from the cell, the feed concentration keeps on increasing and hence leads to enhanced deposition of solute particles on the gel layer, thereby increasing its thickness. For example, gel layer thickness decreases from 620 to 500 µm as the cross flow rate increases from 400 to 500 L/h at the same temperature (25 °C). At the same cross flow rate (500 L/h), temperature increase from 25 to 40 °C, causes an enhancement in mass transfer coefficient as discussed in Fig. 6. This leads to enhanced convection and the gel layer growth is arrested. Thus, gel layer thickness decreases from 500 to 400 µm when temperature increases from 25 to 40 °C. Corresponding variation of gel layer resistance is presented in Fig. 11b. As discussed earlier, gel layer resistance is proportional to the gel layer thickness and hence, gel layer resistance decreases with feed temperature and cross flow rate. Gel layer resistance varies between 4 and 6.5 times of membrane resistance for various operating conditions after 6 h of filtration. It is noted from both these figures that the gel layer thickness and resistance increase with time monotonically. As the permeate is drawn continuously in this mode of operation, feed concentration increases as feed volume is reduced. This leads to enhanced concentration polarization and a steady state is never attained in batch concentration mode of filtration. Non-dimensional gel layer resistance decreases from about 6.4 to 5.2 when the cross flow rate increases from 400 to 500 L/h at the same feed temperature and it decreases from 5 to 4 when feed temperature increases from 25 to 45 °C.

Conclusion

A generalized Sherwood number is derived for a radial cross flow membrane filtration from the first principle for gel layer controlling case. The derived expression includes the effects of developing mass transfer boundary layer, effects of solute concentration and feed temperature. Separate models are formulated for total recycle and batch concentration mode of filtration. The model parameters are estimated in total recycle mode. These are: solute diffusivity, $D_0=(2.4\pm0.05)\times10^{-11}$ m²/s; gel concentration, $C_g=389\pm6$ kg/m³; exponent of temperature variation of viscosity, $n=4.3\pm0.5$; concentration variation factor of viscosity, $\alpha=0.016\pm0.002$ m³/kg; gel layer density, $\rho_g=1680\pm10$ kg/m³ and gel characteristic

 $\xi = (5.1 \pm 0.8) \times 10^{15} \text{ m}^{-2}$. The model for batch concentration mode is run entirely in predictive mode and excellent matching with experimental data is observed. The presented model can be utilized for efficient design of gel layer controlling membrane filtration in radial cross flow cell and subsequent scaling up. Fruit juice rheology is important in mass transfer analysis, and considering cactus pear juice as a non-Newtonian fluid would certainly improve the mathematical analysis and accuracy of the model. Nevertheless, it could be attempted as a future scope in further refinement of the present model.

Appendix A

The derivatives, $\frac{\partial C^*}{\partial r^*}$, $\frac{\partial C^*}{\partial y^*}$ and $\frac{\partial^2 C^*}{\partial y^{*2}}$ in Eq. 8 are evaluated using Eq. 10. These partial derivatives are inserted in Eq. 8 and after simplification the following equation is obtained,

$$\frac{A}{r^*} \left(\frac{y^{*2}}{\delta^{*2}} - \frac{y^{*3}}{\delta^{*3}} \right) \frac{d\delta^*}{dr^*} - Pe_w \left(\frac{y^*}{\delta^{*2}} - \frac{1}{\delta^*} \right) = \frac{1}{\delta^{*2}}$$
(A.1)

Taking the zeroth moment of above equation by multiplying both sides by dy^* and integrating across the boundary layer thickness from 0 to δ^* , the following equation is obtained.

$$\frac{A}{r^*} \left(\frac{d\delta^*}{dr^*}\right) \int_0^{\delta^*} \left(\frac{y^{*2}}{\delta^{*2}} - \frac{y^{*3}}{\delta^{*3}}\right) dy^* - P_{ew} \int_0^{\delta^*} \left(\frac{y^*}{\delta^{*2}} - \frac{1}{\delta^*}\right) dy^*$$
$$= \frac{1}{\delta^{*2}} \int_0^{\delta^*} dy^* \tag{A.2}$$

On solving the above integral the final equation is arrived

$$\frac{A}{12} \frac{\delta^{*2}}{r^*} \left(\frac{d\delta^*}{dr^*} \right) + \frac{P_{ew}\delta^*}{2} = 1$$
(A.3)

Substituting $\frac{\partial C^*}{\partial y^*}$ from Eq. 10 in Eq. 8c, the following expression is obtained.

$$P_{ew}\delta^* = 2\left(\frac{C_g^* - 1}{C_g^*}\right) \tag{A.4}$$

Replacing this value of $P_{\rm ew}\delta^*$ in Eq. A.3, results to the following governing equation of concentration boundary layer thickness,

$$\frac{A}{12}\frac{\delta^{*2}}{r^*}\left(\frac{d\delta^*}{dr^*}\right) = \frac{1}{C_g^*} \tag{A.5}$$

Integration of the above equation leads to the profile of concentration boundary layer thickness with r^* as,

$$\delta^* = \left(\frac{18}{AC_g^*}\right)^{1/3} r^{*2/3}$$
(A.6)

Appendix **B**

The non-dimensional solute balance equation within concentration boundary layer can be written as,

$$\frac{\partial C^*}{\partial \tau} + A \frac{y^*}{r^*} \frac{\partial C^*}{\partial r^*} - P_{ew} \frac{\partial C^*}{\partial y^*} = \frac{\partial^2 C^*}{\partial y^{*2}}$$
(B.1)

where, the non-dimensional time is defined as, $\tau = tD/h^2$. Next, an order of magnitude analysis of Eq. B.1 is carried out term wise. $O(x^*)$ is 1; order of *y* is same as that of thickness of concentration boundary layer, $\delta \approx \frac{D}{k} = \frac{10^{-11}}{10^{-6}} = 10^{-5}$. Thus, $O\left(\frac{y^*}{r^*}\right)$ is $\frac{10^{-5}/10^{-3}}{10^{-1}/10^{-1}} = 10^{-2}$. O(A) is $\frac{u_0h^2}{DR} = \frac{1 \times 10^{-6}}{10^{-11} \times 10^{-1}} = 10^6$. $O(Pe_w)$ is $\frac{v_wh}{R} = \frac{10^{-6} \times 10^{-3}}{10^{-11}} = 10^2$. Therefore, order of the terms, T_2 , T_3 , and T_4 is 10^4 . Thus, it may be noted that T_1 has significant magnitude compared to other three terms up to a time of operation of 100 s. Beyond 100 s, it is reduced in order of magnitude. Hence, comparing the full operation time in this experiment (360 min), T_1 is small enough to be ignored. Therefore, we can take recourse to a quasi-steady-state analysis for estimation of concentration boundary layer profile. The governing equation of solute mass balance is same as Eq. 8. The concentration profile can be approximated as described in Appendix A, with the following boundary conditions,

at
$$y^* = \delta^*$$
, $C = C_b$; $C^* = C_b^*$ (B.2)

The concentration profile within the boundary layer now becomes,

$$C^{*} = C_{g}^{*} - 2\left(C_{g}^{*} - C_{b}^{*}\right)\left(\frac{y}{\delta^{*}}\right) + \left(C_{g}^{*} - C_{b}^{*}\right)\left(\frac{y}{\delta^{*}}\right)^{2}$$
(B.3)

The mean concentration within boundary layer is (Mondal et al. 2011a),

$$C_m^* = \frac{1}{3} \left(C_g^* + 2C_b^* \right)$$
(B.4)

and,

$$\frac{\mu_m}{\mu_w} = e^{-\frac{2}{3}\alpha C_0 \left(C_g^* - C_b^*\right)} \tag{B.5}$$

The Sherwood number relation is modified as,

$$\overline{\mathrm{Sh}} = 1.65 \left(Re \cdot \mathrm{Sc} \,\frac{\mathrm{h}}{\mathrm{R}} \right)^{\frac{1}{3}} \left(e^{-\frac{2}{3}\alpha C_0 \left(C_g^* - C_b^* \right)} \right)^{0.14} \left(\frac{C_g^*}{C_b^*} \right)^{\frac{1}{3}} \quad (\mathrm{B.6})$$

Including temperature correction, the average Sherwood number is,

$$\overline{\mathrm{Sh}} = 1.65 \left(Re \cdot \mathrm{Sc} \frac{\mathrm{h}}{\mathrm{R}} \right)^{\frac{1}{3}} \left(e^{-\frac{2}{3}\alpha C_0 \left(C_g^* - C_b^* \right)} \right)^{0.14} \left(\frac{T_0}{T} \right)^{\frac{n+1}{3}} \left(\frac{C_g^*}{C_b^*} \right)^{\frac{1}{3}}$$
(B.7)

Average dimensionless permeate flux becomes,

$$\overline{P_{ew}} = 1.65 \left(Re \cdot \text{Sc} \frac{\text{h}}{\text{R}} \right)^{\frac{1}{3}} \left(e^{-\frac{2}{3}\alpha C_0 \left(C_g^* - C_b^* \right)} \right)^{0.14} \left(\frac{T_0}{T} \right)^{\frac{n+1}{3}} \left[\left(\frac{C_g^*}{C_b^*} \right)^{\frac{1}{3}} - \left(\frac{C_g^*}{C_b^*} \right)^{-\frac{2}{3}} \right]$$
(B.8)

For $\frac{C_g^*}{C_b^*} << 20$, $\left(\frac{C_g^*}{C_b^*}\right)^{\frac{1}{3}} - \left(\frac{C_g^*}{C_b^*}\right)^{-2/3}$, is reduced to $\ln\left(\frac{C_g^*}{C_b^*}\right)$. Under this condition, the final expression of length averaged permeate flux is presented in Eq. 35.

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