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A New Class of Uniform Continuous Higher Order Sliding Mode Controllers

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This paper proposes a new class of uniform continuous higher order sliding mode controllers (UCHOSMA) for the arbitrary relative degree systems. The proposed methodology is a combination of two controllers where one of the components is an uniform super twisting control which acts as the disturbance compensator and the second part gives the uniform finite time convergence for the disturbance free system. This algorithm provides uniform finite time convergence of the output and its higher derivatives using an absolutely continuous control signal and thus alleviating the chattering phenomenon. The attractive feature of the proposed controller is that irrespective of the different initial conditions, the control is able to bring the states of the system to the equilibrium point uniformly in finite time. The effectiveness of the proposed controller has been demonstrated with both simulation as well as experimental results.

1 Introduction

A significant amount of research effort has been focused in the area of robust control due to both their practical potential chattering in various applications and theoretical challenges. Sliding mode control, which is one of the most actively studied topics within the realm of robust control techniques, generally aims to achieve finite time or asymptotic convergence of state variables in the presence of matched disturbances/uncertainties. Depending on the finite time or asymptotic stability of state variables, a variety of sliding mode algorithms have been reported in literature.

Despite of the various intriguing aspects of sliding mode control such as finite time convergence, compensation of matched disturbances, reduced order design etc., practical realization of sliding mode control still requires some more classes of novel algorithms. This is due to the discontinuous nature of the control action or less flexibility and restrictions of the existing sliding mode algorithms that could generate an absolutely continuous control signal.

The main disadvantage of sliding mode control is the chattering effect [1]. It is an undesirable phenomenon generated due to high frequency oscillations of control signal when the system trajectories slide along the sliding manifold [1]. To avoid this effect, super-twisting algorithm (STA) [2] has been proposed for the sliding manifold having relative degree one with respect to control.

Super-twisting control [2], [3] is a continuous controller, ensuring all the main properties of first order sliding mode control for the system with Lipschitz (in time) matched bounded uncertainties/disturbances. The superior property of this algorithm has been exploited for the development of continuous integral sliding mode controller for various control applications [4], [5], [6].

Most of the practical systems are represented as a sec-

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ond (for example: Mechanical systems and Electrical systems) and higher order systems. The main control objective for these plants are, to provide high performance in a specified time in spite of uncertainties/perturbations and also avoid chattering. Existing second order sliding mode controllers (for example: twisting, terminal sliding mode, controller with prescribed convergence, sub-optimal etc.) are directly applicable to second order uncertain plants [1]. But the main drawback of these controllers is that they provide discontinuous control signal, which generates unnecessary chattering.

To ensure finite time convergence using continuous control, Levant [2] recommended use of a third order sliding mode controller. But such controller not only uses the output and its derivative, but also the second derivative of the output. To realize this control, the exact information about the uncertainties are required. In this case, it is possible to compensate the uncertainties even without the use of a sliding mode controller [7].

In the past years, several uniform algorithms have been proposed and used in different applications like multi-agent systems [25–27], neural networks [28,29] and stochastic synchronization of complex networks [30]. These existing controllers based on fixed time stability are discontinuous in nature and are homogeneous. Some controllers are continuous but fail to reject the disturbances/uncertainties.

It is clear from the above discussion that the finite time control under the absolutely continuous control signal without explicit knowledge of the disturbance is still unexplored. Similar kind of situation is also true for the system with a higher relative degree. To avoid the above mentioned drawbacks and to generate an absolutely continuous control signal for the arbitrary relative degree system with respect to the output, some of the generalized family has been recently studied [9]- [11]. However, the convergence time of the above mentioned algorithms are not uniform with respect to the initial conditions.

For the higher order systems, it is possible to use the STA to obtain a continuous control and therefore compensate the chattering problem [12]. However, this has the disadvantage that it requires the design of a first order sliding surface resulting in asymptotic convergence instead of a finite time convergence of the states.

Motivation

The main motivation of designing uniform continuous higher order sliding mode algorithm (UCHOSMA) is that the controllers are continuous, nonhomogenous and uniform with respect to the initial conditions and are capable of eliminating Lipschitz disturbances/uncertainties.

Main Contributions

The goal of this paper is to suggest a generalized order UCHOSMA which has the following properties:

Absolutely continuous control signal for the arbitrary relative degree which is more desirable from the actuator point of view for chattering minimization.

Uniform finite time convergence for the set $\widetilde{\sigma},\dot{\widetilde{\sigma}},...,\widetilde{\sigma}^{(r)}$ where $\widetilde{\sigma}$ represents the output and r is the relative degree of the system with respect to output (i.e., irrespective of the initial conditions of the states of the systems, the trajectories will converge to origin at the same upper bounded time. Such phenomenon is called as fixed time stability [20–24]). Existing controllers based on fixed time stability are discontinuous in nature. But our proposed controllers are absolutely continuous which are independent of the initial conditions.

Extra information does not require other than $\widetilde{\sigma},\dot{\widetilde{\sigma}},...,\widetilde{\sigma}^{(r-1)}$, to generate the absolutely continuous control signal for the arbitrary relative degree.

Compensates those uncertainties/perturbations which are Lipschitz in time and output.

This new class of continuous controllers are nonhomogeneous or homogeneous in the bi-limit.

The organization of the paper is as follows. In Section 2, a brief background of the globally uniform super twisting algorithm as a disturbance observer is given. Section 3 discusses the main results. Section 4 discusses a detailed illustrating example and Section 5 presents the concluding remarks.

2 Globally uniform super twisting algorithm as a disturbance observer

The super twisting algorithm is considered as the most prominent type of second order sliding mode algorithm for achieving robustness against matched disturbances/uncertainties along with chattering free control by generating absolutely continuous control signal. Consider the following first order system

$$\dot{z}_1 = u + d \tag{1}$$

where $z_1 \in \mathbb{R}$ is the state variable, $d \in \mathbb{R}$ is the matched non vanishing disturbance and u is defined as,

$$u = -k_1\phi_1(z_1) + v, \;\; \dot{v} = -k_2\phi_2(z_1) \tag{2}$$

where k_1 and k_2 are positive gains to be designed and

$$\begin{aligned}\phi_1(z_1) &= \mu_1|z_1|^{\frac{1}{2}}sign(z_1) + \mu_2|z_1|^{\frac{3}{2}}sign(z_1)\\ \phi_2(z_1) &= \frac{1}{2}\mu_1{}^2sign(z_1) + 2\mu_1\mu_2|z_1| + \frac{3}{2}\mu_2^2|z_1|^2sign(z_1)\end{aligned} \tag{3}$$

with $\mu_1,\mu_2 \geq 0$. After substituting control input (2) in (1), we get

$$\dot{z}_1 = -k_1\phi_1(z_1) + v + d, \;\; \dot{v} = -k_2\phi_2(z_1) \tag{4}$$

Now, let us define $z_2 := v + d$ then, $\dot{z}_2 = \dot{v} + \dot{d}$,then the above system can be written as

$$\dot{z}_1 = -k_1\phi_1(z_1) + z_2, \;\; \dot{z}_2 = -k_2\phi_2(z_1) + \dot{d} \tag{5}$$

where $|\dot{d}| \leq \Delta_1$. The above algorithm is known as the uniform super twisting algorithm. The following definition and lemma states the finite time convergence properties of this algorithm.

Definition 1. *[14] The origin $z_1 = z_2 = 0$ for a system (5) is globally uniformly stable if all trajectories starting in $\mathbb{R}^2$ converge to a neighborhood of $z = 0$ in finite time and the convergence time is uniformly bounded with respect to the initial conditions.*

The convergence of the system (5) will be uniform within a finite time, which means that all the trajectories converge to zero at a time smaller than a constant irrespective of the initial conditions.

Lemma 1. *[14] If the gains $k_1 > 2\sqrt{\Delta_1}$, $k_2 > 2\Delta_1$ and $\mu_1, \mu_2 > 0$, then the trajectories of the system (5) starting at $z_0 \in \mathbb{R}^2$ converge to the origin in finite time and they reach that point at most after a time*

$$T(z_0) = \frac{6}{k_2}\left(\frac{1}{\mu^{\frac{1}{6}}} - \frac{1}{W^{\frac{1}{6}}(z_0)}\right) + \frac{2}{k_1}\mu^{\frac{1}{2}} \tag{6}$$

where μ is any value satisfying $0 < \mu < W(z_0)$, k_1, k_2 are constants and $W(z) = V_Q(z) + V_N(z)$. Here $V_Q(z) = \zeta^T P\zeta$, where $\zeta^T = \phi^T(z) = [\phi_1(z_1),\ z_2]$, and $V_N(z) = \alpha|\phi_1(z_1)|^2 - \beta|\phi_1(z_1)|^{\frac{2}{3}} sign(z_1)|z_2|^{\frac{1}{3}} sign(z_2) + \delta z_2{}^2$, where $\alpha = k_2\delta$, $\beta = 1$, $\delta > 0$ is a global strong Lyapunov function for system (5) for sufficiently large δ and positive symmetric matrix P. Moreover, the convergence time is uniformly bounded by $T_{max} = \frac{6}{k_2}\left(\frac{k_2}{k_1}\right)^4 + \frac{2}{k_1}\left(\frac{k_1}{k_2}\right)^{3/4}$, which implies any trajectory converges to zero in a time smaller than T_{max}.

Once the states reach the sliding surface, then from (5), $z_2 = v + d = 0$,

$$d = -v = \int_0^t k_2\phi_2(z_1)d\tau \tag{7}$$

The above property leads to use the super twisting algorithm as a controller as well as a disturbance observer. In the next section we are going to propose a new class of uniform continuous finite time controller for the uncertain chain of integrators. The proposed methodology is based on the combination of two controllers where the first controller is able to stabilize the system uniformly in the absence of disturbance and the second part of the controller is inspired from the above disturbance observation property of the uniform super twisting algorithm. The main idea behind the above proposal is that, the effect of disturbance is taken care by the uniform super twisting control and the closed loop system response is always governed by the proposed uniform controller. Hence this control strategy can be considered as an absolutely continuous control law for compensating Lipschitz perturbations exactly and ensuring finite time convergence.

In this manuscript, the r^{th} order sliding mode control with respect to output is equivalent to the finite time stabilization of output and its $(r-1)^{th}$ derivatives to origin of the uncertain chain of integrators using the non-smooth controller where the solution is interpreted in the sense of Filippov [8].

3 Main Results

Consider the following uncertain chain of integrators

$$\begin{aligned}\dot{x}_1 &= x_2\\ \dot{x}_2 &= x_3\\ &\vdots\\ \dot{x}_n &= f(x) + g(x)(u+d)\end{aligned} \tag{8}$$

where $x \in \mathbb{R}^n$, output $y = x_1 = \widetilde{\sigma}$, $f(x) \in \mathbb{R}$, $g(x) \in \mathbb{R}$, $d \in \mathbb{R}$ and $u \in \mathbb{R}$. Before proceeding to the controller design, it is important to mention that the present manuscript is based on the following assumptions:

Assumption 1. *Since this paper seeks a non-smooth controller for solving the problem, solutions of the systems are defined in the Filippov sense [8], i.e., letting x denote the state of the entire system $\dot{x}(t) = f(x(t), d(t))$, solutions are defined with the differential inclusion*

$$\dot{x} \in \bigcap_{\delta>0}\bigcap_{\mu N=0} \mathrm{cl}(\mathrm{co}(f(B_\delta(x)\setminus N)))$$

where cl *and* co *denote the closure and the convex hull respectively, $B_\delta(x)$ is the unit ball and the sets N are all sets of zero Lebesgue measure.*

Assumption 2. *It is assumed that $f(x)$ and d contain all kinds of uncertainties/disturbances, whose derivative satisfy $|\dot{f}(x) + \dot{\overbrace{g(x)d}}| < \Delta$, although it might not be necessary that $f(x) + g(x)d$ is bounded and also $(g^{-1}(x)) \neq 0$.*

The main aim of the work is to propose a uniform continuous finite time stabilizing control for an uncertain chain of integrators so that irrespective of the initial conditions of the states of the systems the trajectories will converge to origin at the same upper bounded time. For fulfilling the above mentioned goal the feedback control u is proposed as $u = g^{-1}(x)(u_0 + u_D)$, where

$$\begin{aligned}u_0 = -k_1|s_1|^{\alpha_1}\mathrm{sign}(s_1) - k_2|s_2|^{\alpha_2}\mathrm{sign}(s_2) - \cdots\\ - k_n|s_n|^{\alpha_n}\mathrm{sign}(s_n)\end{aligned} \tag{9}$$

with $s_i = x_i + \eta_i|x_i|^{\frac{1}{\alpha_i}}\mathrm{sign}(x_i)$, $\eta_i = \chi_i/k_i$ and

$$u_D = -\hat{k}_1\phi_1(\sigma) + \nu,\ \ \dot{\nu} = -\hat{k}_2\phi_2(\sigma) \tag{10}$$

where sliding or coupling variable is defined as

$$\sigma = x_n - \int_0^t u_0 d\tau, \tag{11}$$

and

$$\begin{aligned} \phi_1(\sigma) &= \mu_1|\sigma|^{\frac{1}{2}} sign(\sigma) + \mu_2|\sigma|^{\frac{3}{2}} sign(\sigma); \quad \mu_1,\mu_2 > 0 \\ \phi_2(\sigma) &= \frac{1}{2}\mu_1{}^2 sign(\sigma) + 2\mu_1\mu_2|\sigma| + \frac{3}{2}\mu_2^2|\sigma|^2 sign(\sigma) \end{aligned} \tag{12}$$

Now, the next aim is to design appropriate $k_i, \chi_i, \hat{k}_1, \hat{k}_2$ and α_i for $i = 1,2,\cdots,n$ such that origin of the closed loop system after applying the proposed controller u is globally uniformly finite time stable. Following theorem gives the convergence conditions for the proposed controller.

Theorem 1. *For the uncertain system* (8)*, under the feedback control* $u = g^{-1}(x)(u_0 + u_D)$*, there exists a nonempty set of gains* $k_i, \chi_i, \hat{k}_1, \hat{k}_2$ *and parameter* α_i *for* $i = 1,2,\cdot,n$

$$\begin{cases} \hat{k}_1 > 2\sqrt{\Delta}, \hat{k}_2 > 2\Delta \\ \alpha_{i-1} = \frac{\alpha_i\alpha_{i+1}}{2\alpha_{i+1}-\alpha_i}, \quad i = 2,3,\ldots,n. \\ \qquad with\ \alpha_{n+1} = 1 \quad and \quad \alpha_n = \alpha \\ k_i > \chi_i\Delta_0^{(1-\alpha)}, \chi_i > 0,\ and\ 1 \geq \Delta_0 > 0 \end{cases} \tag{13}$$

such that (13) *is satisfied. Furthermore, if* (13) *is satisfied and* k_i *and* $\chi_i > 0$ *will be selected such that the polynomial* $s^n + a_n s^{n-1} + \cdots + a_2 s + a_1$ *is Hurwitz with coefficient,* $a_i = k_i\Delta_0^{(\alpha_i-1)} + \chi_i$*,* $\Delta_0 > 0$ *and there exists* $\varepsilon \in (0,1)$ *such that for every* $\alpha \in (1-\varepsilon,1)$*, the origin is globally uniformly finite time stable.*

Proof. Taking the time derivative of sliding variable (11) and including it into an uncertain system (8), one can write

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_3 \\ &\vdots \\ \dot{\sigma} &= \dot{x}_n - u_0 = u_D + f(x) + g(x)d \end{aligned} \tag{14}$$

Substituting u_D from (10) into (14)

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_3 \\ &\vdots \\ \dot{\sigma} &= -\hat{k}_1\phi_1(\sigma) + \nu + f(x) + g(x)d \\ \dot{\nu} &= -\hat{k}_2\phi_2(\sigma). \end{aligned} \tag{15}$$

Let us define $\Xi := \nu + f(x) + g(x)d$, time derivative of Ξ is given as, $\dot{\Xi} = -\hat{k}_2\phi_2(\sigma) + \overbrace{f(x)+g(x)d}^{\cdot}$.

One can further write (15) using Ξ and $\dot{\Xi}$ as

$$\dot{x}_1 = x_2 \tag{16a}$$
$$\dot{x}_2 = x_3 \tag{16b}$$
$$\vdots \tag{16c}$$
$$\dot{\sigma} = -\hat{k}_1\phi_1(\sigma) + \Xi \tag{16d}$$
$$\dot{\Xi} = -\hat{k}_2\phi_2(\sigma) + \overbrace{f(x)+g(x)d}^{\cdot} \tag{16e}$$

First, we analyze (16d) and (16e) together. Then we investigate (16a)-(16c) which depends on σ generated by (16d)-(16e). Let us define $\hat{\sigma} := [\sigma, \Xi]^\top \in \mathbb{R}^2$. Equation (16e) has a discontinuous right hand side. Depending on $\overbrace{f(x)+g(x)d}^{\cdot}$, $\hat{\sigma} = 0$ is not an equilibrium point of (16d) and (16e). For proving the stability of (16d)-(16e), the same candidate Lyapunov function is used as proposed in [14]

$$\begin{aligned} W(\hat{\sigma}) &= [\phi_1(\sigma), \Xi]^\top P [\phi_1(\sigma), \Xi] + \alpha|\phi_1(\sigma)|^2 \\ &\quad - \beta|\phi_1(\sigma)|^{\frac{2}{3}} sign(\sigma)|\Xi|^{\frac{1}{3}} sign(\Xi) + \delta\Xi^2 \end{aligned} \tag{17}$$

where P is positive symmetric definite matrix and $\alpha = k_2\delta$, $\beta = 1$, $\delta > 0$, one can easily prove that all the trajectories of the subsystem (16d)-(16e) starting at $\hat{\sigma}_0 \in \mathbb{R}^2$ converge to the origin in finite time and reach that point at most after a time,

$$T(\hat{\sigma}_0) = \frac{6}{k_2}\left(\frac{1}{\mu^{\frac{1}{6}}} - \frac{1}{W^{\frac{1}{6}}(\hat{\sigma}_0)}\right) + \frac{2}{k_1}\mu^{\frac{1}{2}} \tag{18}$$

where μ is any value satisfying $0 < \mu < W(\sigma_0)$, $k_1 > 2\sqrt{\Delta}$, $k_2 > 2\Delta$.

Since $\sigma = \Xi = 0$ after at most time $T(\hat{\sigma}_0)$ and maintained $\hat{\sigma} = 0$ irrespective of the disturbance. So one can write

$$\dot{\sigma} = -\hat{k}_1\phi_1(\sigma) + \Xi = 0 \tag{19}$$

which further implies,

$$\dot{\sigma} = 0 \Rightarrow \dot{x}_n = u_0 \tag{20}$$

It means that after fixed time $T(\hat{\sigma})$ the closed loop system (16a)-(16e) becomes free from the disturbances and the closed-loop system trajectories are only governed by the control u_0. In the light of the above discussion, one can express the closed-loop system (16a)-(16e) as

$$\begin{aligned} \dot{x}_1 &= x_2 \\ &\vdots \\ \dot{x}_n &= u_0 \end{aligned} \tag{21}$$

Now our aim is to prove the claim that the proposed controller u_0 for the system (21), forces the trajectories to converge globally uniformly to the origin in finite time. After substituting u_0 from (9) into (21), one can write

$$x_1^{(n)} = -\sum_{i=1}^{n} k_i |s_i|^{\alpha_i} \operatorname{sign}(s_i)$$
$$= -\sum_{i=1}^{n} k_i |(x_i + \eta_i |x_i|^{\frac{1}{\alpha_i}} \operatorname{sign}(x_i))|^{\alpha_i} \operatorname{sign}(x_i + \eta_i |x_i|^{\frac{1}{\alpha_i}} \operatorname{sign}(x_i)) \quad (22)$$

where (n) represents the n^{th} derivative of x_1. Substituting $\eta_i = 0$ for $i = 1, 2, \cdots, n$, the proposed controller is similar to the work of Bhat and Bernstein [13], which is stated as:

Let $k_1, \ldots, k_n > 0$ be such that the polynomials $s^n + k_n s^{n-1} + \cdots + k_2 s + k_1$ is Hurwitz, and there exists $\varepsilon \in (0,1)$ such that, for every $\alpha \in (1-\varepsilon, 1)$, the origin is a globally finite time stable equilibrium for the system $x_1^{(n)} = \hat{u}$ under the feedback control

$$\hat{u} = -k_1 |x_1|^{\alpha_1} \operatorname{sign}(x_1) - \cdots - k_n |x_n|^{\alpha_n} \operatorname{sign}(x_n). \quad (23)$$

Actually the proposed controller satisfies the property of homogeneity in the bi-limit, as defined by Vincent Andrieu et.al, [15] and all solutions of the system (22) converge in finite time to the origin, uniformly with respect to initial condition because the degree of homogeneity in the 0-limit is negative (inside the sphere $|x_i| < \Delta_0 \leq 1$) and in the ∞-limit is positive (outside the sphere $|x_i| \geq 1 > \Delta_0$) [15], where $\Delta_0 > 0$ is some positive constant.

Due to the property of homogeneity in the bi-limit, the convergence proof of closed loop system (22) is analyzed in two separate parts; first when trajectories lie inside the sphere $|x_i| < \Delta_0 \leq 1$ and the second when they lie outside the sphere $|x_i| \geq 1 > \Delta_0$.

The main idea behind introducing the extra term $\eta_i |x_i|^{\frac{1}{\alpha_i}} \operatorname{sign}(\alpha_i))$ in the control is that for all initial conditions of states $|x_i| \geq 1 > \Delta_0$,

$$u_0 \approx -\sum_{i=1}^{n} k_i \eta_i x_i \quad (24)$$

because $|(x_i + \eta_i |x_i|^{\frac{1}{\alpha_i}} \operatorname{sign}(x_i))|^{\alpha_i} \leq |x_i|^{\alpha_i} + \eta_i |x_i|$ and also $\operatorname{sign}(x_i + \eta_i |x_i|^{\frac{1}{\alpha_i}} \operatorname{sign}(x_i)) = \operatorname{sign}(x_i)$. Since we are interested in $|x_i| \geq 1 > \Delta_0$, $\eta_i |x_i|$ is dominant over term $|x_i|^{\alpha_i}$ and $|x_i|^{\alpha_i} + \eta_i |x_i| \approx \eta_i |x_i|$. Due to this approximation, outside the sphere $|x_i| \geq 1 > \Delta_0$, the closed loop system is approximately governed by (24) which yields a faster asymptotic convergence of the states variables towards the homogeneous sphere $|x_i| < \Delta_0 \leq 1$, rather than the control $\hat{u}$ as proposed by Bhat and Bernstein [13]. (One can also select $s_i = x_i + \eta_i |x_i|^{\frac{3}{\alpha}} \operatorname{sign}(x_i)$ in place $s_i = x_i + \eta_i |x_i|^{\frac{1}{\alpha}} \operatorname{sign}(x_i)$ for more faster convergence. Then also same analysis is valid.) However, inside the homogeneous sphere $|x_i| < \Delta_0 \leq 1$, $u_0 \approx -\sum_{i=1}^{n} k_i |x_i|^{\alpha_i} \operatorname{sign}(x_i)$ which is same as Bhat and Bernstein [13], it yields the finite time convergence of trajectories to the origin.

Now our next aim is to analyze the choice of gain such that finite time convergence is not violated. For this purpose, suppose in the first case that trajectories are confined in the homogeneous sphere of radius Δ_0 i.e., $|x_i| \leq \Delta_0 \leq 1$. Then we have to show that $-k_i |x_i|^{\alpha} \operatorname{sign}(x_i)$ term is dominant over the linear term $k_i \eta_i x_i$. One can easily show that the above mentioned condition is always satisfied, when trajectories will stay inside the $|x_i| \leq \Delta_0 \leq 1$. For showing this, one can write

$$k_i |(x_i + \eta_i |x_i|^{\frac{1}{\alpha_i}} \operatorname{sign}(x_i))|^{\alpha_i} \leq k_i |x_i|^{\alpha_i} + \chi_i |x_i|$$
$$= (k_i |x_i|^{\alpha_i - 1} + \chi_i)|x_i| \quad (25)$$

which further implies $u_o \approx -(k_i |x_i|^{\alpha_i - 1} + \chi) x_i$. Therefore, by selecting

$$k_i \geq \chi_i \Delta_0^{(1-\alpha_i)} \quad (26)$$

the nonlinear term $-k_i |x_i|^{\alpha_i} \operatorname{sign}(x_i)$ will be dominant over linear term. This is always possible because Δ_0 is a very small quantity. Similarly for the case when $|x_i| \geq \Delta_0$, the linearized system about the point $x_i \approx \Delta_0$ is stable provided the characteristic polynomial $s^n + a_n s^{n-1} + \cdots + a_1$ is Hurwitz. Stability of system when the system trajectories are outside the homogeneous sphere $|x_i| \geq \Delta_0$ can also be easily proved using the quadratic Lyapunov $V = x^T P x$, where P is positive symmetric matrix. Using Lyapunov methodology also, the gain condition remains the same as Hurwitzness of polynomial $s^n + a_n s^{n-1} + \cdots + a_1$, because outside the unit sphere linear term is dominant over the nonlinear term and behavior of closed loop system is governed by the linear control $u_0 \approx -\chi_i x_i$. This completes the proof of the theorem.

Remark 1. *It might be possible that one can think of a proof based on construction of strict continuously differential Lyapunov function, but it is not an easy task. Therefore in the present paper, a logical proof is adopted which is based on disturbance observation property and homogeneity in bi-limit.*

4 Illustrative examples

For validating the capabilities of the proposed control algorithm, we have taken three examples of second, third and fourth-order complex systems. Further, a comparative study has been done with some existing algorithms.

4.1 Second-order system

In the first example consider a robotic manipulator as below [16].

$$\dot{x}_1(t) = x_2(t), \;\; \dot{x}_2(t) = \frac{1}{ml^2 + I}\left(u - gl\cos(x_1(t)) + d(t)\right)$$

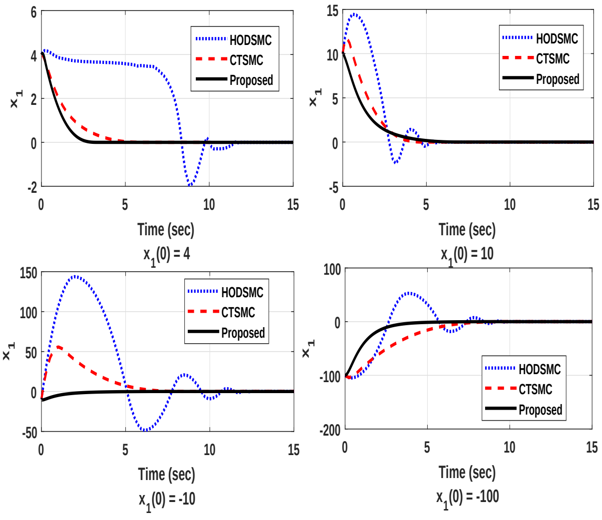

Fig. 1. Evolution of state 1 for different controllers

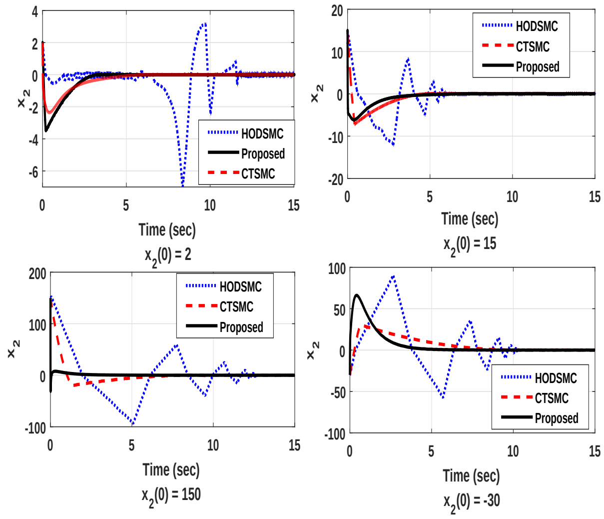

Fig. 2. Evolution of state 2 for different controllers

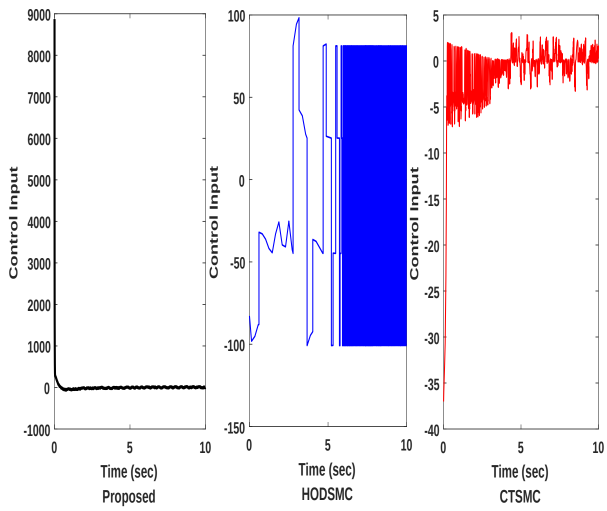

Fig. 3. Control input for different controllers

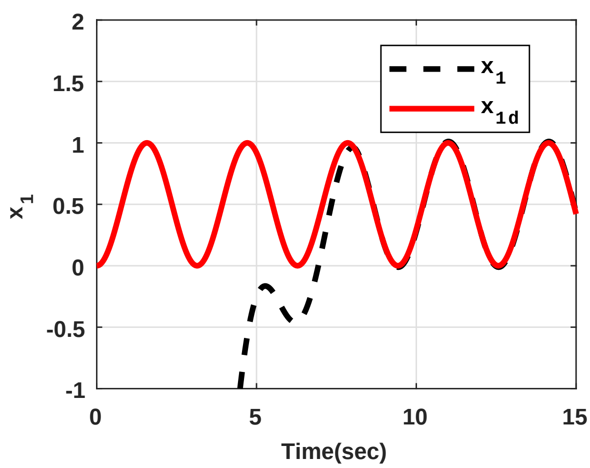

Fig. 4. Tracking response of state x_1 for $x_1(0) = -100$

and $y(t) = x_1(t)$, where, $x_1(t)$ is the joint angle, $x_2(t)$ is the angular velocity, $m = 3kg$ is the mass, $l = 1m$ is the length, $I = 0.5kg.m^2$ is the moment of inertia, u is the joint input and $d(t) = 2+4\sin(t/2)+0.6\sin(10t)$ is a disturbance. The output of the system is $x_1(t)$ and the desired output trajectory is $\sin^2(t)$. Substituting the parameter values in the above equation,

$$\begin{aligned} \dot{x}_1(t) &= x_2(t) \\ \dot{x}_2(t) &= -2.8\cos(x_1(t)) + 0.2857u_1(t) + d(t). \end{aligned} \tag{27}$$

Now control $u_1(t)$ is taken as, $u_1(t) = \frac{1}{0.2857}(2.8\cos(x_1(t)) + u(t))$. Then the above system (27) will become

$$\dot{x}_1(t) = x_2(t), \ \ \dot{x}_2(t) = u(t) + d(t) \tag{28}$$

where $x = [x_1 \ \ x_2]^T$ is the state vector, $u = u_0 + u_D$ is the control input and $d(t)$ is the matched disturbance. Consider two arbitrary variables $s_1(x_1) = x_1 + \eta_1 |x_1|^{\frac{1}{\alpha_1}} \text{sign}(x_1)$ and $s_2(x_2) = x_2 + \eta_2 |x_2|^{\frac{1}{\alpha_2}} \text{sign}(x_2)$ where $\alpha_1 = \frac{1}{3}$, $\alpha_2 = \frac{1}{2}$ and $\eta_1 = \eta_2 = 1$. Then the control u_0 is expressed as $u_0 = -k_1|s_1(x_1)|^{\frac{1}{3}}\text{sign}(s_1(x_1)) - k_2|s_2(x_2)|^{\frac{1}{2}}\text{sign}(s_2(x_2))$ where $k_1 = 5$ and $k_2 = 6$.

Now constructing an arbitrary sliding surface $\sigma \in \mathbb{R}$ as $\sigma = x_2 - \int_0^t u_0 d\tau$. Then, $\dot{\sigma} = u + d - u_0 = 0 \Rightarrow u_0 + u_D + d - u_0 = 0 \Rightarrow u_D + d = 0$. Therefore, when the system is on the sliding surface, the disturbance has to be canceled out by the control u_D, which is defined as below $u_D = -\lambda_1\phi_1(\sigma) + v$, $\dot{v} = -\lambda_2\phi_2(\sigma)$ where $\lambda_1 = 4.4$, $\lambda_2 = 3$, $\phi_1(\sigma) = \mu_1|\sigma|^{\frac{1}{2}}\text{sign}(\sigma) + \mu_2|\sigma|^{\frac{3}{2}}\text{sign}(\sigma)$, $\phi_2(\sigma) = \frac{1}{2}\mu_1{}^2\text{sign}(\sigma) + 2\mu_1\mu_2|\sigma| + \frac{3}{2}\mu_2^2|\sigma|^2\text{sign}(\sigma)$, $\mu_1 = 2$ and $\mu_2 = 4$.

The simulation is carried out for different initial conditions of the states of the system. The proposed controller is compared with the existing controllers. For this, Higher Order Discontinuous Sliding Mode Controller (HODSMC) [2]

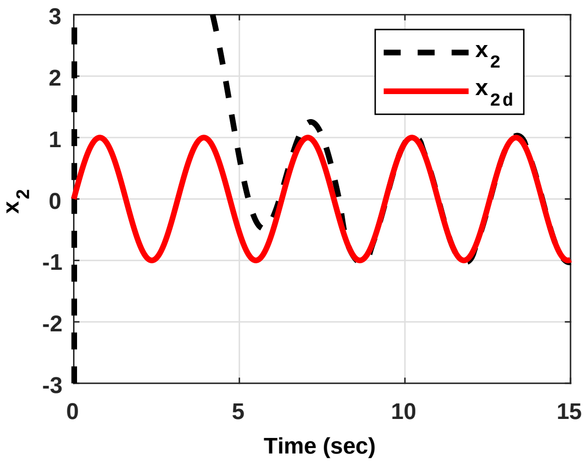

Fig. 5. Tracking response of state x_2 for $x_2(0) = -30$

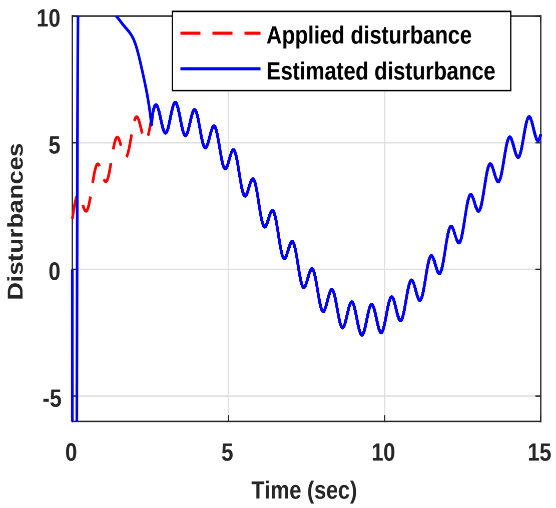

Fig. 6. Evolution of estimated and applied disturbance

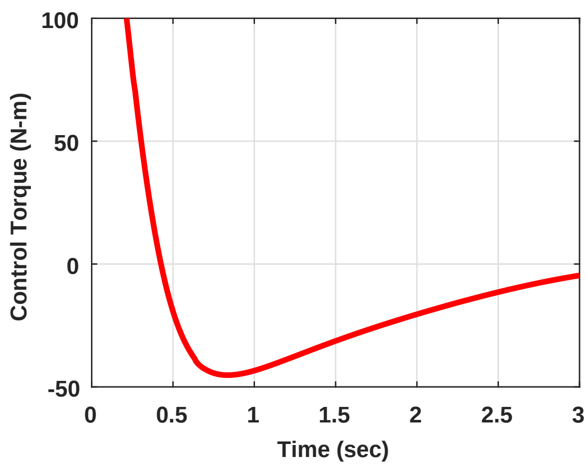

Fig. 7. Evolution of control input

i.e., $u_{HODSMC} = -K_1\text{sign} - K_2\text{sign}$ with different controller gains, i.e., $K_1 = 10$ and $K_2 = 5$ and Continuous Terminal Sliding Mode Controller (CTSMC) proposed in [5] with controller gains, $k_1 = 10, k_2 = 5, L = 5, \alpha = 1$ have been designed. The obtained results are shown in the figures Fig.1-2. It can be seen that HODSMC is dependent on the initial conditions and the nature of control signal is discontinuous which causes chattering effect. In case of CTSMC, the control input is continuous but states take a longer time to converge to zero as compared to the proposed controller. In the proposed one, states are converging to zero uniformly irrespective of different initial conditions. Also, from Fig. 3, it is observed that HODSMC is discontinuous in nature while the CTSMC and proposed are continuous. A tradeoff is well seen from the same figure that there is a huge initial magnitude but it is uniform and acts independently to the different initial conditions. For the same values of tuned parameters of the controller, the system states successfully track a set of desired trajectories as shown in Fig. 4-5. The disturbance observation capability of the proposed controller can be inferred from Fig.6. The Fig.7 shows the control torque for the tracking problem with initial conditions of $x_1(0) = -30$ and $x_2(0) = -100$ which shows that control is continuous and hence chattering is alleviated.

4.2 Third-order system

For further verification we consider a second example of the kinematic model of a car (as given in [1]), $\dot{x} = \upsilon\cos\psi$, $\dot{y} = \upsilon\sin\psi$, $\dot{\psi} = \frac{\upsilon}{l}\tan\theta$ $\dot{\theta} = u$; where x and y are the Cartesian coordinates of the midpoint of the rear axle, ψ is the orientation angle, υ is the longitudinal velocity, l is the length between the two axles and θ is the steering angle which is the control input. The control aim is to steer the car from a given initial position to the desired trajectory $y = g(x)$, where $g(x)$ and y are assumed to be available in real time.

Now define $x_1 = y - g(x)$, $\upsilon = 10m/s$, $l = 5m$, $x = y = \upsilon = \theta = 0$ at $t = 0$, $g(x) = 10\sin(0.05x) + 5$. From the state equations of the system it is obvious that the control appears for the first time explicitly in the third-order derivative of x_1. Hence the relative degree of the system is 3 with respect to the control.

Now redefining the state equation by differentiating $x_1 = y - g(x)$, which is given as $\dot{x}_1 = x_2$, $\dot{x}_2 = x_3$, and $\dot{x}_3 = f(x) + g_1(x)u_1$. Now control $u_1(t)$ is taken such that, $u_1(t) = \frac{1}{g_1(x)}(-f(x) + u(t))$. Then the above system will become $\dot{x}_1 = x_2$, $\dot{x}_2 = x_3$, and $\dot{x}_3 = u(t) + d$, where $x = [x_1 \quad x_2 \quad x_3]^T$ is the state vector, $u = u_0 + u_D$ is the control input and $d(t)$ is the matched disturbance.

Consider the arbitrary variables $s_1(x_1) = x_1 + \eta_1|x_1|^{\frac{1}{\alpha_1}}\text{sign}(x_1)$, $s_2(x_2) = x_2 + \eta_2|x_2|^{\frac{1}{\alpha_2}}\text{sign}(x_2)$ and $s_3(x_3) = x_3 + \eta_3|x_3|^{\frac{1}{\alpha_3}}\text{sign}(x_3)$, where $\eta_1 = \eta_2 = \eta_3 = 1$, $\alpha_1 = \frac{1}{2}$, $\alpha_2 = \frac{3}{5}$ and $\alpha_3 = \frac{3}{4}$. Then the control u_0 is expressed as $u_0 = -k_1|s_1(x_1)|^{\frac{1}{2}}\text{sign}(s_1(x_1)) - k_2|s_2(x_2)|^{\frac{3}{5}}\text{sign}(s_2(x_2)) - k_3|s_3(x_3)|^{\frac{3}{4}}\text{sign}(s_3(x_3))$, where $k_1 = 1$, $k_2 = 1.5$ and $k_3 = 1.5$ are positive constants.

Now constructing an arbitrary sliding surface $\sigma \in \mathbb{R}$

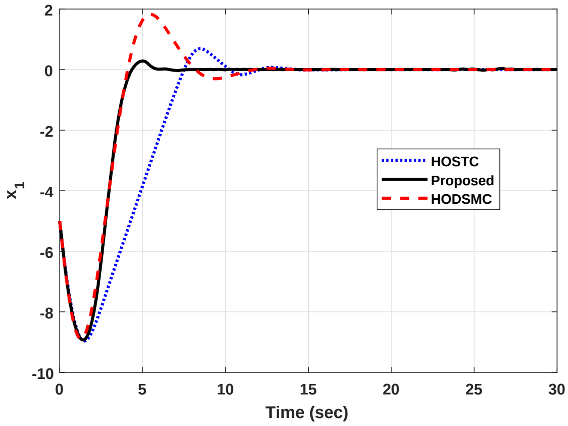

Fig. 8. Tracking deviations

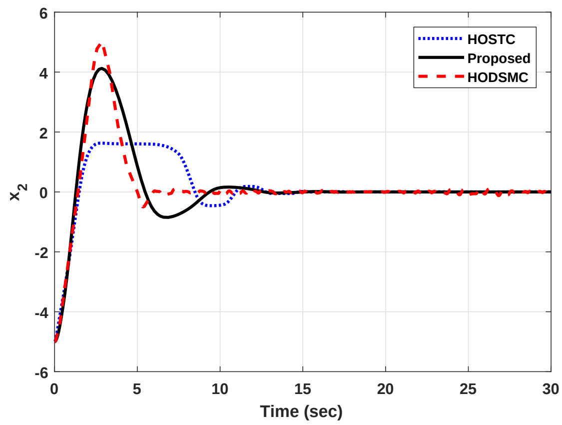

Fig. 9. Tracking deviations

as $\sigma = x_3 - \int_0^t u_0 d\tau$ and defining the second part of the control law as, $u_{\mathrm{D}} = -\lambda_1\phi_1(\sigma) + v, \quad \dot{v} = -\lambda_2\phi_2(\sigma)$ where $\lambda_1 = 4.4$, $\lambda_2 = 3$, $\phi_1(\sigma) = \mu_1|\sigma|^{\frac{1}{2}}\mathrm{sign}(\sigma) + \mu_2|\sigma|^{\frac{3}{2}}\mathrm{sign}(\sigma)$, $\phi_2(\sigma) = \frac{1}{2}{\mu_1}^2\mathrm{sign}(\sigma) + 2\mu_1\mu_2|\sigma| + \frac{3}{2}\mu_2^2|\sigma|^2\mathrm{sign}(\sigma)$, $\mu_1 = 1$ and $\mu_2 = 2$. The simulation is carried out for different initial conditions of the states with an additional disturbance of $2 + 0.2\sin(t)$ and it can be observed that tracking deviations or redefined states $x_i, i = 1, \cdots, 3$ are converging to zero irrespective of the disturbances/uncertainties as shown in Fig. 8-10. A comparative study between the Higher Order Super-Twisting Controller (HOSTC) [9], HODSMC [2] and the proposed controller shows least settling time for the proposed one. The car trajectory tracking response and steering angle derivative (control) are shown in Fig.11 and Fig.12 respectively. One can clearly observe that the control is continuous in Fig.12, hence it is more desirable for the mechanical actuator. Fig. 13 shows the discontinuous nature of the control input for HODSMC leading to a chattering effect.

Furthermore, for the demonstration of the efficacy of the proposed controllers, a two-degree of freedom (2-DOF) he-

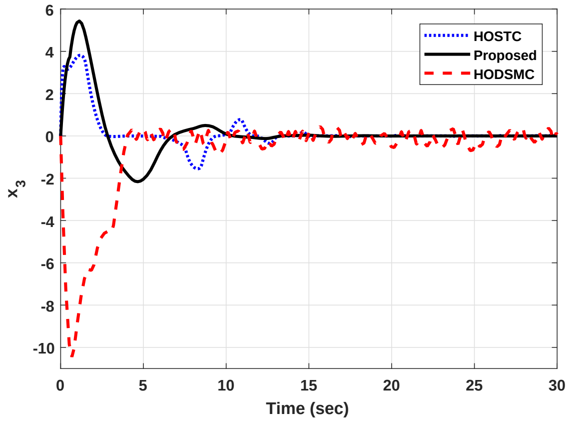

Fig. 10. Tracking deviations

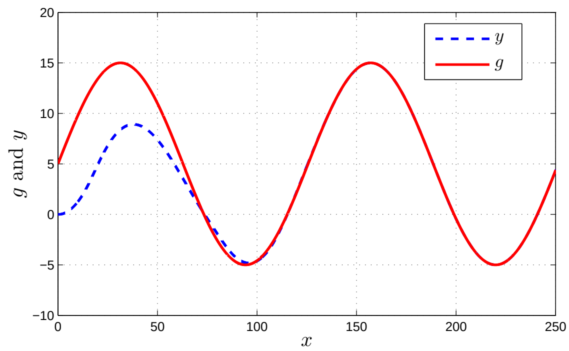

Fig. 11. Car trajectory tracking

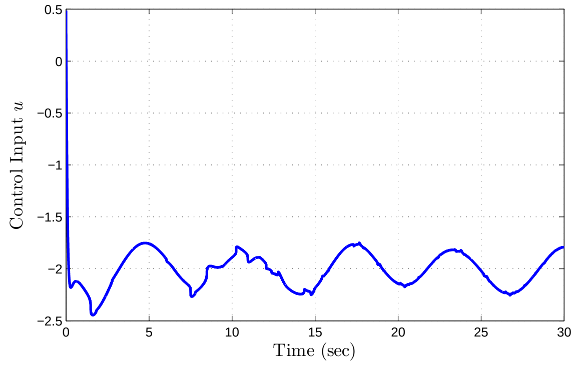

Fig. 12. Steering angle derivative (control) Proposed

licopter model which is an example of fourth-order system is considered and implementation of the proposed controllers on the same model in real time is shown in the next subsection.

4.3 Fourth-order System

4.3.1 Dynamic model of a 2-DOF Helicopter

The 2-DOF helicopter is a non-linear and multivariable unstable system with cross couplings and unmodeled dynam-

$$
\begin{aligned}
\dot{x_1} =& x_2, \\
\dot{x_2} =& \frac{-Bpx_2 + (M_y g d_2 - M_p g d_1)\cos x_1 - (M_y d_2^2 + M_p d_1^2) x_4^2 \cos x_1 \sin x_1}{M_p d_1^2 + M_y d_2^2 + I_T} + \frac{\tau_1}{M_p d_1^2 + M_y d_2^2 + I_T}, \\
\dot{x_3} =& x_4, \\
\dot{x_4} =& \frac{-B_y x_4 + 2(M_p d_1^2 + M_y d_2^2) x_2 x_4 \cos x_1 \sin x_1}{M_p d_1^2 + M_y d_2^2 + I_T} + \frac{\tau_2}{M_p d_1^2 + M_y d_2^2 + I_T},
\end{aligned} \tag{29}
$$

Table 1. System Specifications

S.No.	Symbol	Description	Value	Unit
1.	K_{pp}	Thrust torque constant for pitch motor assembly	0.204	N-m/V
2.	K_{yy}	Thrust torque constant for yaw motor assembly	0.072	N-m/V
3.	K_{py}	Cross-torque constant, acting along pitch axis from yaw motor	0.0068	N-m/V
4.	K_{yp}	Cross-torque constant, acting along yaw axis from pitch motor	0.0219	N-m/V
5.	B_p	Equivalent damping about pitch axis	0.800	N/V
6.	B_y	Equivalent damping about yaw axis	0.318	N/V
8.	M_h	Mass of helicopter	1.3872	kg
9.	d_1	Distance of pitch motor from hinge along the helicopter body	0.186	m
10.	d_2	Distance of yaw motor from hinge along the helicopter body	0.186	m
11.	J_p	Moment of inertia about pitch axis	0.0384	kg-m^2
12.	J_y	Moment of inertia about yaw axis	0.0432	kg-m^2
13.	M_p	Total moving mass about pitch axis	0.633	kg
14.	M_y	Total moving mass about yaw axis	0.667	kg
15.	g	Acceleration due to gravity	9.8	m/sec^2

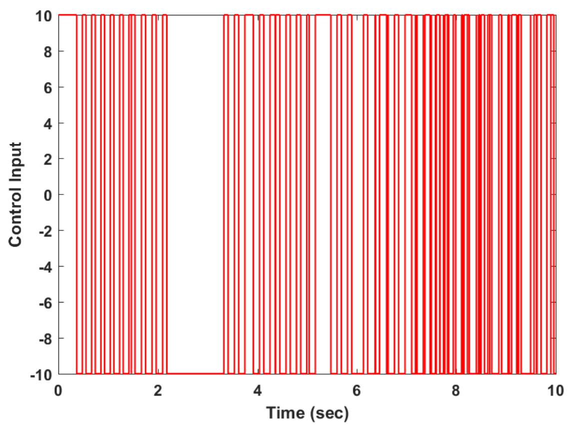

Fig. 13. Steering angle derivative (control) HODSMC

ics widely used in mechatronics' and aerodynamic applications [18, 19]. A free body diagram of 2-DOF helicopter model is shown in Fig. 14. It is mounted on a fixed base with two propellers that are driven by DC motors. The front propeller controls the elevation of the nose over the pitch axis while the tail propeller guides the rotation motion around the yaw axis. The motors correspond to each one of the actuators of the propellers. It is assumed that the pitch and yaw thrust forces i.e. F_p and F_y always remain positive when pitch angle and yaw angle increase ($\theta_1 > 0$, $\theta_2 > 0$). F_g represents the thrust force due to gravity.

The nonlinear dynamic model [17] of the system in state-space form is given by (29). Here, x_1, x_2, x_3 and x_4 are the states of the system representing pitch angle, pitch velocity, yaw angle and yaw velocity, respectively. I_T represents the total moment of inertia of the system. $\tau_1 = K_{pp}V_p + K_{py}V_y$ is the total input torque along the pitch axis and is the sum of thrust torque from pitch motor and cross torque acting along the pitch axis from yaw motor. Similarly, $\tau_2 = K_{yp}V_p + K_{yy}V_y$ is the total input torque along the yaw axis and is the sum of thrust torque from yaw motor and cross torque acting along the yaw axis from pitch motor V_p and V_y are the input voltages to pitch and yaw motors.

Assume, $x_1 = \theta_1$, $x_2 = \dot{\theta}_1$, $x_3 = \theta_2$, $x_4 = \dot{\theta}_2$. Thus, by carrying out simple manipulations, one can obtain the nonlinear dynamical model in state space form represented by (29). For simplicity, (29) can be rewritten in terms of two separate dynamics i.e., pitch (θ_1) and yaw (θ_2), treating two

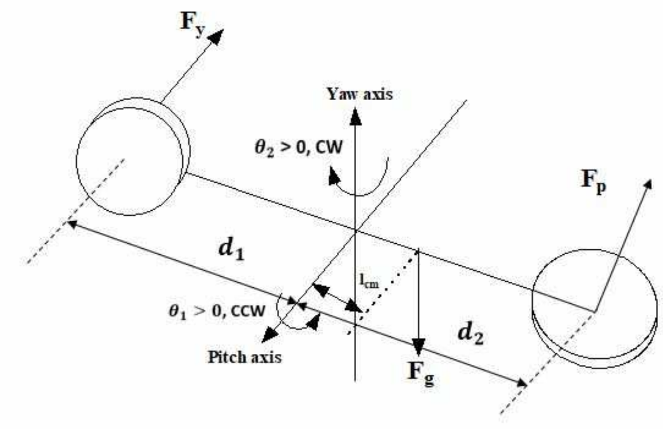

Fig. 14. Free body diagram of a generalised twin rotor MIMO system

Fig. 15. Experimental Setup

Single Input Single Output (SISO) systems that can be easily controlled.
For pitch,

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= u_p + \zeta_1(t, x_1, x_4, V_y, g) \end{aligned} \tag{30}$$

For Yaw,

$$\begin{aligned} \dot{x}_3 &= x_4 \\ \dot{x}_4 &= u_y + \zeta_2(t, x_1, x_2, x_4, V_p) \end{aligned} \tag{31}$$

where $\zeta_1 = \frac{(M_y g d_2 - M_p g d_1)\cos x_1 - (M_y d_2^2 + M_p d_1^2) x_4^2 \cos x_1 \sin x_1 + K_{py} V_y}{J_p}$ and $\zeta_2 = \frac{2(M_p d_1^2 + M_y d_2^2) x_2 x_4 \cos x_1 \sin x_1 + K_{yp} V_p}{J_y}$ are taken as an uncertainties/disturbances acting on the system. Substituting, $V_p = (\frac{K_{pp}}{J_p})^{-1}(u_p + \frac{B_p}{J_p} x_2)$ and $V_y = (\frac{K_{yy}}{J_y})^{-1}(u_y + \frac{B_y}{J_y} x_4)$ in (29), we obtain two second-order pitch and yaw dynamics separately which are in the chain of integrators form (30) and (31). u_p and u_y are the control inputs which are to be designed. Our objective is to design a robust control law for the given model that can stabilize the system and achieve the desired pitch and yaw angle. The numerical values of the design parameters for the exemplary Quanser 2-DOF helicopter [17] is tabulated in Table 1.

In the next section, both the simulation and experimental test have been carried out for the 2-DOF helicopter model as discussed in earlier section and the obtained results are shown.

4.4 Experimental Setup

The experimental setup of 2-DOF helicopter consists of two brushless DC motors with $\pm 24V$ and $\pm 15V$, one for pitch and another for yaw followed by two optical encoders for measuring the pitch and yaw angular positions with a resolution of 4096 and 8192 counts/revolution respectively, as shown in the Fig. 15. The yaw and pitch encoders are connected directly to the two channel data-acquisition board (DAQ board), which has 8 digital inputs and 8 pulse width modulated digital outputs, and it is capable of reaching 4 kHz sampling rate which is further wired with two channel two Volt-PAQ power amplifiers which provide a regulated $\pm 30V$ at 3 A, amplify the output voltages and thus drive the pitch and yaw motors.

4.4.1 Controllers Design

Consider the pitch and yaw dynamics given by (30) and (31). Then, a proposed controller for the pitch and yaw control are designed separately considering an arbitrary variable as $s_1(x_1) = x_1 + \eta_1 |x_1|^{\frac{1}{\alpha_1}} \operatorname{sign}(x_1)$, and $s_2(x_2) = x_2 + \eta_2 |x_2|^{\frac{1}{\alpha_2}} \operatorname{sign}(x_2)$ for pitch. Similarly, $s_3(x_3) = x_3 + \eta_3 |x_3|^{\frac{1}{\alpha_3}} \operatorname{sign}(x_3)$ and $s_4(x_4) = x_4 + \eta_4 |x_4|^{\frac{1}{\alpha_4}} \operatorname{sign}(x_4)$ for yaw where η_1, η_2, η_3, η_4, α_1, α_2, α_3 and α_4 are design parameters which are to be chosen. Then, the pitch controller is designed as $u_p = u_{p0} + u_{Dp}$ where control u_{p0} is expressed as $u_{\mathrm{p0}} = -k_1 |s_1(x_1)|^{\frac{1}{3}} \operatorname{sign}(s_1(x_1)) - k_2 |s_2(x_2)|^{\frac{1}{2}} \operatorname{sign}(s_2(x_2))$, where k_1 and k_2 are controller gains. Now constructing an arbitrary sliding surface $\sigma \in \mathbb{R}$ as $\sigma = x_2 - \int_0^t u_{p0} d\tau$, $\dot{\sigma} = u_p + \zeta_1 - u_{\mathrm{p0}} = 0 \Rightarrow u_{\mathrm{p0}} + u_{\mathrm{Dp}} + \zeta_1 - u_{\mathrm{p0}} = 0 \Rightarrow u_{\mathrm{Dp}} + \zeta_1 = 0$. Therefore, when the system is on the sliding surface the disturbance has to be cancelled out by the control u_{Dp}, which is defined below $u_{\mathrm{Dp}} = -\lambda_1 \phi_1(\sigma) + v$, $\dot{v} = -\lambda_2 \phi_2(\sigma)$ $\phi_1(\sigma) = \mu_1 |\sigma|^{\frac{1}{2}} \operatorname{sign}(\sigma) + \mu_2 |\sigma|^{\frac{3}{2}} \operatorname{sign}(\sigma)$, $\phi_2(\sigma) = \frac{1}{2}\mu_1{}^2 \operatorname{sign}(\sigma) + 2\mu_1\mu_2|\sigma| + \frac{3}{2}\mu_2^2 |\sigma|^2 \operatorname{sign}(\sigma)$ where λ_1, λ_2, μ_1 and μ_2 are tunable gains.

Similarly the yaw controller is designed as $u_y = u_y 0 + u_D y$ where control u_{p0} is expressed as $u_{\mathrm{y0}} = -k_3 |s_3(x_3)|^{\frac{1}{3}} \operatorname{sign}(s_3(x_3)) - k_4 |s_4(x_4)|^{\frac{1}{2}} \operatorname{sign}(s_4(x_4))$ where k_3 and k_4 are controller gains. Now constructing an arbitrary sliding surface $\sigma \in \mathbb{R}$ as $\sigma = x_4 - \int_0^t u_{y0} d\tau$, $\dot{\sigma} = u_y + \zeta_2 - u_{\mathrm{y0}} = 0 \Rightarrow u_{\mathrm{y0}} + u_{\mathrm{Dy}} + \zeta_2 - u_{\mathrm{y0}} = 0 \Rightarrow u_{\mathrm{Dy}} + \zeta_2 = 0$. Therefore, when the system is on the sliding surface the disturbance has to be cancelled out by the control u_{Dy}, which is defined below $u_{\mathrm{Dy}} = -\lambda_3 \phi_3(\sigma) + v$, $\dot{v} = -\lambda_4 \phi_4(\sigma)$ $\phi_3(\sigma) = \mu_3 |\sigma|^{\frac{1}{2}} \operatorname{sign}(\sigma) + \mu_4 |\sigma|^{\frac{3}{2}} \operatorname{sign}(\sigma)$, $\phi_4(\sigma) = \frac{1}{2}\mu_3{}^2 \operatorname{sign}(\sigma) + 2\mu_3\mu_4|\sigma| + \frac{3}{2}\mu_4^2 |\sigma|^2 \operatorname{sign}(\sigma)$ where λ_3, λ_4, μ_3 and μ_4 are tunable gains.

4.4.2 Simulation Results

In simulation, we consider the problem of stabilization at the origin and set-point tracking of the pitch and yaw angles, simultaneously and perform the simulation test for different initial conditions in the presence of matched uncertainties present in the model (ζ_1, ζ_2). The control design parameters are chosen as $k_1 = k_3 = 10, k_2 = k_4 = 5$, $\lambda_1 = \lambda_3 = 2$, $\lambda_2 = \lambda_4 = 4$, $\eta_1 = \eta_3 = 1$, $\eta_2 = \eta_4 = 2$, $\mu_1 = \mu_3 = 1$ and $\mu_2 = \mu_4 = 1$. The obtained results are shown in Figures 16, 17, 18 and 19.

It can be observed that states are converging to zero uniformly irrespective of different initial conditions as shown in Fig. 16. For the same values of tuned parameters of the controller, the system states successfully track a desired set-point trajectory shown in Fig. 18 and Fig. 19. From Fig. 17, it can be seen that the control input is continuous in nature and free from chattering.

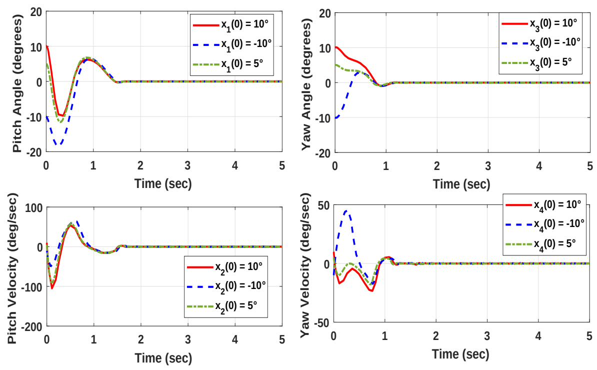

Fig. 16. Time evolution of states for different initial conditions

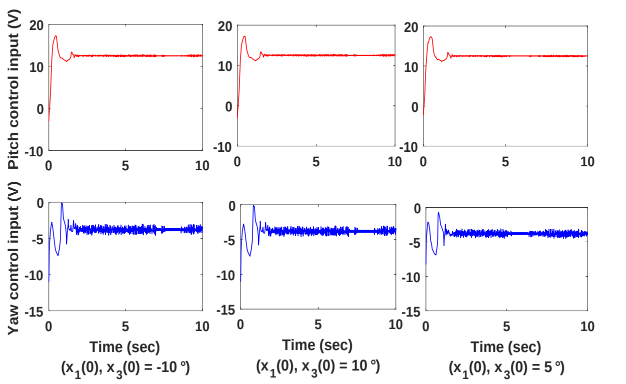

Fig. 17. Time evolution of control input for different initial conditions

4.4.3 Experimental Results

In experimental case, we consider the problem of set-point tracking of the desired pitch and yaw angles of the 2-DOF helicopter model. Selecting the control design parameters as $k_1 = k_3 = 0.2, k_2 = k_4 = 0.15$, $\lambda_1 = \lambda_3 = 9$,

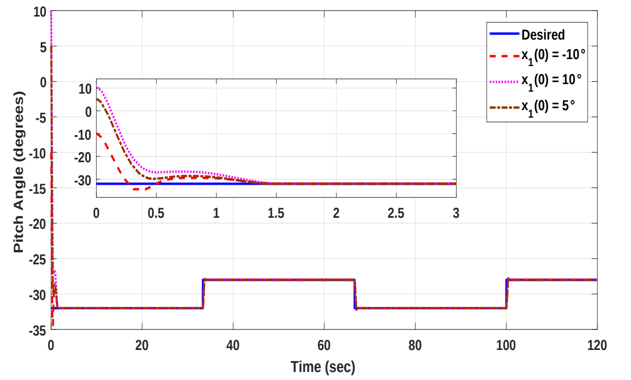

Fig. 18. Set-point tracking of pitch angle for different initial conditions

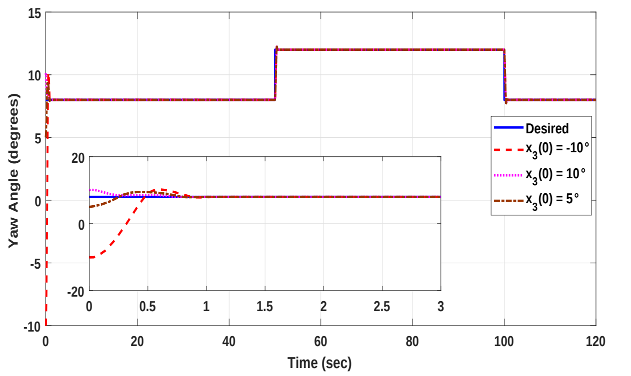

Fig. 19. Set-point tracking of yaw angle for different initial conditions

$\lambda_2 = \lambda_4 = 6$, $\eta_1 = \eta_3 = 2$, $\eta_2 = \eta_4 = 4$, $\mu_1 = \mu_3 = 1$ and $\mu_2 = \mu_4 = 1$, the pitch controller (u_p) and yaw controller (u_y) are designed following the procedure given in above controller design section. Then, the designed controller is applied on the 2-DOF helicopter model in real time. The experimental test is carried out for different initial conditions to show the effectiveness of the proposed controllers. The obtained results are shown in the Figures 20, 21 and 22.

Fig. 20 shows the set-point tracking response of the pitch angle. Similarly, the set-point tracking response of the yaw angle is shown in the Fig. 21. It can be observed that the desired control objectives, i.e., tracking of the pitch and yaw angle are achieved successfully using the proposed controllers. Additionally, another property, i.e., the controller acting independently to the different initial conditions is also verified through the results obtained in the experiment. Fig. 22 shows the continuous nature of the controllers.

5 Conclusions

This paper proposes a new class of uniform finite time higher order sliding mode control. The proposed control is a combination of a modified uniform finite time continuous controller and the fixed time super twisting algorithm. Hence, due to the combination of two continuous controllers, the overall controller is continuous in nature eliminating the chattering effect completely. The proposed control is suc-

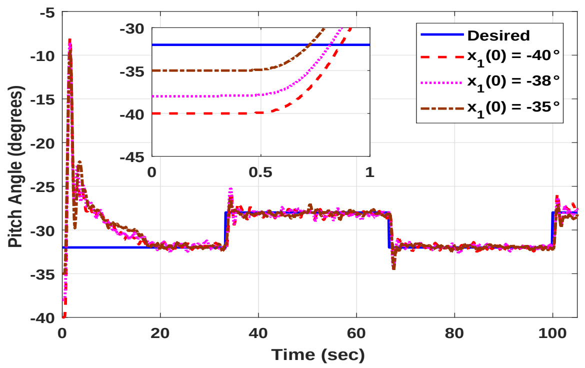

Fig. 20. Set-point tracking of pitch angle for different initial conditions

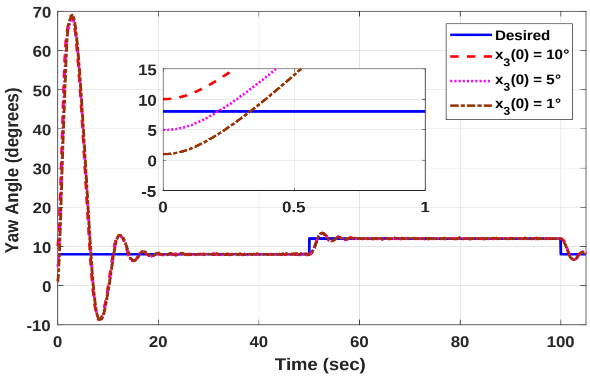

Fig. 21. Set-point tracking of yaw angle for different initial conditions

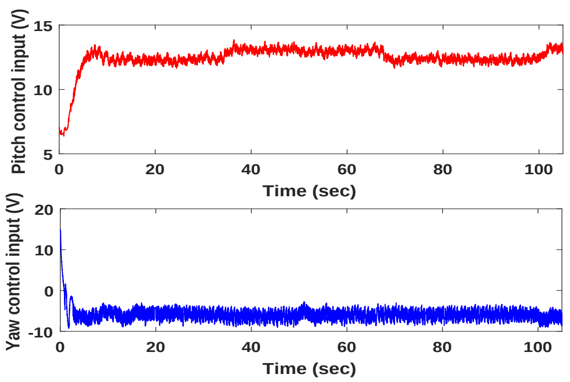

Fig. 22. Time evolution of control input

cessful in driving the states to the equilibrium point in finite time independent of the initial conditions. This robust control technique can also be used as a disturbance observer. The superior properties of the proposed controller have been demonstrated with simulations as well as experimental results.

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