A general geomorphological recession flow model for river basins

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[1] Recession flows in a basin are controlled by the temporal evolution of its active drainage network (ADN). The geomorphological recession flow model (GRFM) assumes that both the rate of flow generation per unit ADN length (q) and the speed at which ADN heads move downstream (c) remain constant during a recession event. Thereby, it connects the power law exponent of $-dQ/dt$ versus Q (discharge at the outlet at time t) curve, α, with the structure of the drainage network, a fixed entity. In this study, we first reformulate the GRFM for Horton-Strahler networks and show that the geomorphic α (α_g) is equal to $\mathcal{D}/(\mathcal{D}-1)$, where D is the fractal dimension of the drainage network. We then propose a more general recession flow model by expressing both q and c as functions of Horton-Strahler stream order. We show that it is possible to have $\alpha = \alpha_g$ for a recession event even when q and c do not remain constant. The modified GRFM suggests that α is controlled by the spatial distribution of subsurface storage within the basin. By analyzing streamflow data from 39 U.S. Geological Survey basins, we show that α is having a power law relationship with recession curve peak, which indicates that the spatial distribution of subsurface storage varies across recession events.

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1. Introduction

[2] Recession flows are characterized by continuously decreasing streamflow over time occurring during dry or no-rain periods. Drainage basins, irrespective of their location and size, follow an interesting recession flow pattern that $-dQ/dt$ and Q (Q being discharge at the outlet at time t) exhibit a power law relationship [*Brutsaert and Nieber*, 1977]:

$$-\frac{dQ}{dt} = kQ^{\alpha} \tag{1}$$

[3] *Biswal and Marani* [2010] suggested that recession flow in a basin is controlled by the dynamics of its active drainage network or ADN (the part of the drainage network that is actively draining at time t). In particular, they hypothesized that the gradual shrinking of the ADN [see, e.g., *Gregory and Walling*, 1968; *Blyth and Rodda*, 1973; *Day*, 1983] originates the power law relationship between $-dQ/dt$ and Q (equation (1)). In order to prove their hypothesis, they proposed a conceptual model, the geomorphological recession flow model (GRFM), by making two simple assumptions that during a recession event: (i) the rate of flow generation per unit ADN length (q) remains constant in both space and time and (ii) the speed at which the heads of the ADN configuration move downstream (c) also remains constant in both space and time. With these two assumptions, one can find that the power law exponent α in equation (1) is equal to the power law exponent of the $N(t)$ versus $G(t)$ curve [*Biswal and Marani*, 2010], where $N(t)$ and $G(t)$ are, respectively, the number of heads and the total length of the ADN configuration at time t. Note that as c is constant, $t = l/c$, l being the distance of any ADN head from its farthest source or channel head at time t. Because of the linear relationship between l and t, both the $N(t)$ versus $G(t)$ curve and the $N(l)$ versus $G(l)$ curve give the same power law exponent, the geomorphic α (α_g). Thus, α_g depends only on the structure of the channel network, an entity that remains unchanged over time.

[4] *Biswal and Marani* [2010] observed that $-dQ/dt$ versus Q curves from a basin can be different for different recession events, which implies that α must be computed individually for the available recession curves. They then considered the median of the distribution of the α values as the representative α (α_r) of the basin, as the distribution is often skewed. Considering streamflow data from 67 basins situated across the United States, they found that α_r is nearly equal to 2, particularly when the basins are steep and free from significant human interventions (see also *Shaw and Riha* [2012]). Then they used digital elevation models (DEMs) for the same set of basins and found that their α_g values are nearly equal to 2.1. The observation α_r being nearly equal to α_g, or the error ϵ ($\alpha_r - \alpha_g$) being nearly equal to zero, implies that the assumptions made by the GRFM (both q and c remain constant during a recession

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event) are adequate for the majority of recession events in a basin. However, the GRFM does not explain why some recession events do not give α equal to α_g. Also, it was found that some natural basins display large ϵ values (see the inset of Figure 4, *Biswal and Marani* [2010]). We propose that, when observational errors are negligible, α deviates from α_g because either q or c, or both vary during the recession event.

[5] In this study, we propose a broader theoretical framework to explain the deviation of observed α from α_g by allowing both q and c to vary along stream channels. First, we reformulate the GRFM in the context of Horton-Strahler tree networks. We then generalize the model by expressing both q and c as functions of Horton-Strahler stream order. We show that the constant q and constant c assumptions, as adopted by the GRFM, are not the necessary conditions for having $\alpha = \alpha_g$. We then analyzed observed recession curves from 39 U.S. Geological Survey (USGS) basins and found that there exists a power law relationship between α and the recession flow curve peak. Our analysis attempts to explain how the variation in subsurface storage distribution across streams of different orders controls the value of α.

2. Horton-Strahler Tree and the GRFM

[6] River networks are classic examples of a binary tree. They observe some deep structural regularities that can be expressed quantitatively by means of the well-known Horton-Strahler ordering scheme [*Horton*, 1945; *Strahler*, 1952] (see Figures 1a and 1b). In the Horton-Strahler ordering scheme, a stream that does not receive flow from any other stream is called a first-order stream. Two streams of order ω join to form a stream of order $\omega+1$. If two streams of different orders join, then the resulting stream will have the order that of the higher order stream. *Horton* [1945] found that in a typical river network, the ratio of average length of streams of order $\omega+1$ ($\overline{L}_{\omega+1}$) to that of streams of order ω ($\overline{L}_{\omega}$), R_L, and the ratio of number of streams of order ω (N_ω) to that of order $\omega+1$ ($N_{\omega+1}$), R_B, are approximately constant for any chosen ω. That means, $\overline{L}_\omega$ and N_ω for a river network with its highest order stream having order Ω can be expressed as:

$$\overline{L}_\omega = L_\Omega R_L^{-(\Omega-\omega)} \tag{2}$$

$$N_\omega = R_B^{\Omega-\omega} \tag{3}$$

where L_Ω is the length of the Ω order stream or the main stream of the channel network. Note that the number of streams of order Ω is always equal to 1 for a real river network.

[7] In a channel network, a channel reach can only drain into its downstream channel reach; thus during a recession event, the channel reach will dry or stop flowing before its downstream channel reach. It also means that the drying pattern of the channel reach will be determined by its upstream channel reach and not by its downstream channel reach. This property of recession dynamics suggests that the channel reach can be relocated to any other place in the channel network as long as the drying pattern of its upstream channel reach remains the same. That means it is possible to restructure a channel network such that its drying patterns remain the same as those of the original network. The method of restructuring a drainage network according to a specific use is also known as a "dynamic tree" approach [*Zaliapin et al.*, 2010]. Our objective here is to restructure a drainage network such that it gives the same $N(t)$ versus $G(t)$ curve or $N(l)$ versus $G(l)$ curve as that of the original network. In the new structure, we allow a stream of order ω to drain only into a stream of order $\omega+1$ (see Figure 1b). We also assume that streams of a certain order have equal length; that means, $\overline{L}_\omega$ is equal to L_ω, length of any chosen stream of order ω. So if the constant c assumption is applied, all streams of a certain order will take the same amount of time to dry up. Considering that at time t, only the streams of order greater than or equal to ω are contributing (i.e., they are actively draining), application of the constant q assumption gives the expression for Q as:

$$Q = q \cdot L_\Omega \sum_{n=\Omega}^{\omega} \left(\frac{R_B}{R_L}\right)^{\Omega-n} = q \cdot G(\omega) \tag{4}$$

where $G(\omega)$ is the length of the ADN at time t or the total length of the streams with order greater than or equal to ω.

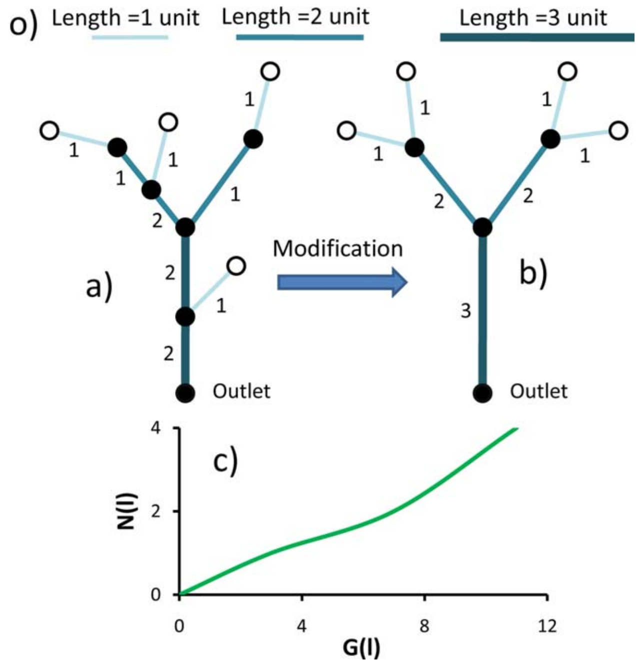

Figure 1. (a) A graphical illustration of a group of stream reaches with different lengths (o) joining one another to form a drainage network ($\Omega = 2$). Closed circles represent active ends (ends that are receiving flow) and open circles represent inactive ends (ends that are not receiving flow). (b) The network has been modified following the rule that a stream of order ω can only drain into a stream or order $\omega+1$ (Ω is 3 now). In either case, the application of the assumption that both q and c are constant gives the same $N(l)$ versus $G(l)$ curve, implying that the modification scheme preserves recession curve characteristics. (c) The $N(l)$ versus $G(l)$ curve for the two networks when both q and c are 1.

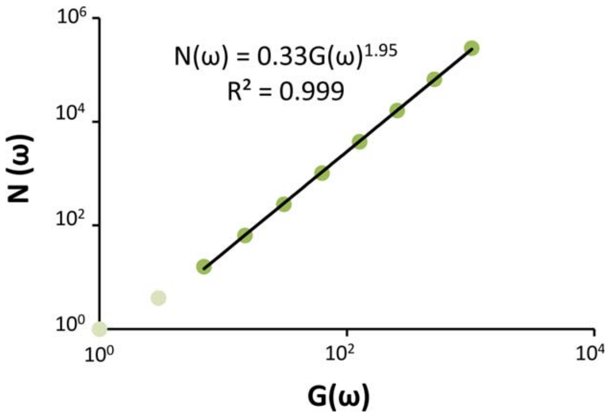

Figure 2. The $N(\omega)$ versus $G(\omega)$ curve for a modified Horton-Strahler network with $R_B = 4$ and $R_L = 2$ when L_Ω, q, and c are equal to 1. The slope of the curve in log-log plot, the geomorphic α (α_g), decreases as ω approaches Ω. Thus, α_g was computed by excluding the last two points (shown as lighter olive green dots). The value of α_g for the network is equal to 1.95, which is close to the value obtained by the analytical expression in this study, 2.

$-dQ/dt$ is then defined as:

$$-\frac{dQ}{dt} = q \cdot \frac{G(\omega) - G(\omega+1)}{\Delta t_\omega} \tag{5}$$

where $\Delta t_\omega = L_\omega / c$ is the time taken by a stream of order w to dry up, which is also equal to $L_\Omega R_L^{-(\Omega-\omega)}/c$ (equation (2)), owing to the constant c assumption. Thus, by combining equations (2)–(5), we obtain:

$$-\frac{dQ}{dt} = q \cdot c \cdot R_B{}^{\Omega-\omega} = q \cdot c \cdot N(\omega) \tag{6}$$

[8] where $N(\omega) = N_\omega$.

[9] If the constant c and the constant q assumptions remain valid, then the exponent of the $N(\omega)$ versus $G(\omega)$ curve should be the same as that of the $-dQ/dt$ versus Q curve as $Q \propto G(\omega)$ (equation (4)) and $-dQ/dt \propto N(\omega)$ (equation (6)). Figure 2 shows $N(\omega)$ versus $G(\omega)$ curve for a Horton-Strahler tree network with $R_B = 4$ and $R_L = 2$.

2.1. An Analytical Expression

[10] Because L_Ω is constant for a basin and q and c are assumed to be constant during a recession event, equation (4) gives the expression for Q as:

$$Q \propto G(\omega) \propto \frac{1 - (R_B/R_L)^{\Omega-\omega+1}}{1 - (R_B/R_L)} \tag{7}$$

[11] When $R_B/R_L \geq 1$ (which is true for real river basins) and $(\Omega - \omega) \to \infty$ [*Rodriguez-Iturbe and Rinaldo*, 1997]:

$$G(\omega) \propto \left(\frac{R_B}{R_L}\right)^{\Omega-\omega} \tag{8}$$

[12] Manipulation of equation (2) gives

$$-(\Omega - \omega) \propto \frac{\log L_\omega}{\log R_L} \tag{9}$$

as L_Ω is a constant. Now using the expression for $\Omega - \omega$ (equation (9)) in equation (8) and taking logarithms of both the sides:

$$\log G(\omega) \propto \log L_\omega \left(1 - \frac{\log R_B}{\log R_L}\right) \tag{10}$$

[13] The term $\log R_B / \log R_L$ is known as the fractal dimension of the drainage network, $\mathcal{D}$ [*La Barbera and Rosso*, 1989]. Exponentiating both sides of equation (10), it can be found that

$$G(\omega) \propto L_\omega^{1-\mathcal{D}} \tag{11}$$

[14] Noting that $L_{\omega+1} = L_\omega \cdot R_L$ and $\Delta t_\omega = L_\omega / c \propto L_\omega$, equations (5) and (11) give

$$-\frac{dQ}{dt} \propto N(\omega) \propto \left(1 - R_L^{1-\mathcal{D}}\right) L_\omega^{-\mathcal{D}} \tag{12}$$

[15] Then combining equations (11) and (12):

$$N(\omega) \propto G(\omega)^{\frac{\mathcal{D}}{\mathcal{D}-1}} \tag{13}$$

because R_L is a constant. Thus, the expression for the geomorphic α (α_g) is:

$$\alpha_g = \frac{\mathcal{D}}{\mathcal{D} - 1} \tag{14}$$

[16] Equation (14) suggests that the fractal geometry of the drainage network [e.g., *Rinaldo et al.*, 2006; *Mantilla et al.*, 2010] gives rise to the power law relationship between $-dQ/dt$ and Q. From Figure 2, α_g obtained for the drainage network having $R_B = 4$ and $R_L = 2$ is 1.95, which is close to the value predicted by equation (14), 2. The discrepancy, although small, is introduced during the transformation of equation (7) to equation (8).

[17] According to *de Vries et al.* [1994], $\beta = 1 - 1/\mathcal{D}$, where β is defined as $P[A \geq a] \propto a^{-\beta}$, with $P[A \geq a]$ being the probability of a randomly chosen stream pixel having contributing area A greater than or equal to a [*Rodriguez-Iturbe et al.*, 1992]. From *Rigon et al.* [1996], $\beta = 1 - h$, where h is the Hack's exponent [*Hack*, 1957]. So one can now find that α_g for the Horton-Strahler stream network is equal to $1/(1 - h)$, a relationship which was also obtained by *Biswal and Marani* [2010]. This proves that the modified Horton-Strahler network mathematically preserves recession flow characteristics of the original network. For real basins, $\mathcal{D} \approx 2$ [*Rodriguez-Iturbe and Rinaldo,* 1997], which gives $\alpha_g = 2$, a value consistent with the earlier observation [*Biswal and Marani*, 2010].

3. A General GRFM: *q* and *c* Following Horton's Laws

[18] Geomorphological and ecological properties vary gradually along a stream channel in a drainage basin, suggesting that, in a relatively homogeneous hydro-geomorphological

region, channel reaches located at equal distance from their respective channel heads are likely to have the same geometrical and physical properties [*Vannote et al.*, 1980]. Often, the variation of physical properties along stream channels is characterized by power law relationships, e.g., between drainage area, distance *l*, channel slope, and channel cross section [*Leopold and Miller*, 1956; *Hack*, 1957; *Montgomery and Foufoula-Georgiou*, 1993; *Rodriguez-Iturbe and Rinaldo*, 1997]. The existence of power law relationships suggest that river basins exhibit self-similar property manifested in terms of Horton's laws [*Peckham and Gupta*, 1999]. Horton's laws can be expressed in a general form:

$$\frac{\mathcal{X}(\omega)}{\mathcal{X}(\omega+1)} = const \tag{15}$$

valid for any ω, where $\mathcal{X}(\omega)$ is the average value of the random variable $\mathcal{X}$ for streams of order ω. Equation (15) can also be written as $\mathcal{X}(\omega) = \mathcal{X}(\Omega) \cdot const^{\Omega-\omega}$ or $\mathcal{X}(\omega) \propto const^{\Omega-\omega}$, because $\mathcal{X}(\Omega)$ is a constant for the basin. It is found that equation (15) is valid not only for stream length and stream number, as originally suggested by *Horton* [1945], but also for many other physical variables like drainage area, channel cross section, discharge, channel slope, and vegetation indices [*Schumm*, 1956; *Leopold et al.*, 1964; *Dunn et al.*, 2011]. In this context, we assume that $q(\omega)/q(\omega+1) = \Theta$ and $c(\omega)/c(\omega+1) = \Psi$, where Θ and Ψ are constants; that means, $q(\omega) \propto \Theta^{\Omega-\omega}$ and $c(\omega) \propto \Psi^{\Omega-\omega}$. From equation (4), the expression for Q is:

$$Q \propto L_\Omega \sum_{n=\Omega}^{\omega} \left(\frac{R_B}{R_L}\Theta\right)^{\Omega-n} \tag{16}$$

[19] The term $L_\Omega \cdot \sum_{n=\Omega}^{\omega} \left(\frac{R_B}{R_L}\Theta\right)^{\Omega-n}$ can be considered as the summation of lengths of the streams with order greater than or equal to ω in a Horton-Strahler network having either bifurcation ratio equal to $R_B \cdot \Theta$ or length ratio equal to $R_L \cdot 1/\Theta$, denoted here as $G^*(\omega)$. Similar to the expression for $G(\omega)$ (equation (7)), when $(\Omega-\omega) \to \infty$ and $R_B\Theta/R_L \geq 1$, $G^*(\omega)$ can be expressed as:

$$G^*(\omega) \propto \left(\frac{R_B}{R_L}\Theta\right)^{\Omega-\omega} \tag{17}$$

[20] Combining equations (9) and (17), we find:

$$G^*(\omega) \propto L_\omega{}^{1-\mathcal{D}-\mathcal{E}} \tag{18}$$

where $\mathcal{E} = \log\Theta/\log R_L$. The expression for $-dQ/dt$ similarly will be

$$-\frac{dQ}{dt} \propto \frac{G^*_\omega - G^*_{\omega+1}}{\Delta t^*_\omega} \propto N^*(\omega) \tag{19}$$

where Δt^*_ω is the time taken by a stream of order ω to dry up when $c(\omega) \propto \Psi^{\Omega-\omega}$ or $c(\omega) \propto L_\omega^{-\log\Psi/\log R_L}$ (obtained by using equation (2)). Thus, the expression for N^*_ω becomes (recalling that $L_{\omega+1} = L_\omega \cdot R_L$ and $\Delta t^*_\omega = L_\omega/c$):

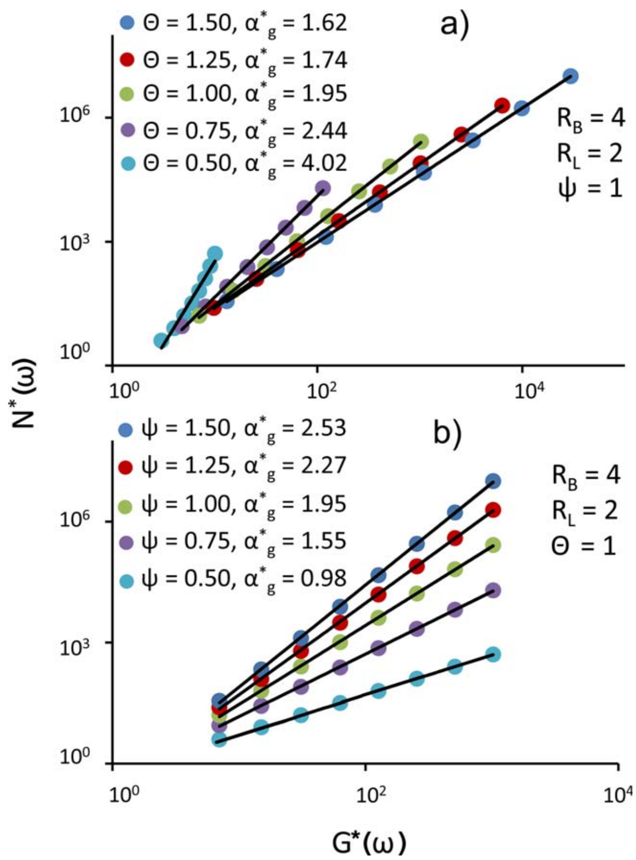

Figure 3. $N^*(\omega)$ versus $G^*(\omega)$ curves for different values of Θ and Ψ: when (a) $\Psi = 1$ and (b) $\Theta = 1$. In all cases, R_B and R_L were 4 and 2, respectively, and L_Ω, q, and c are equal to 1. It can be observed that the power law exponent α^*_g is increasing with Ψ but decreasing with Θ.

$$N^*(\omega) \propto \left(1 - R_L^{1-\mathcal{D}-\mathcal{E}}\right) L_\omega^{-(\mathcal{D}+\mathcal{E}+\mathcal{F})} \tag{20}$$

where $\mathcal{F} = \log\Psi/\log R_L$. As R_L is a constant, the combination of equations (17) and (20) gives the expression for the generalized GRFM as:

$$N^*(\omega) \propto G_\omega^{*\frac{\mathcal{D}+\mathcal{E}+\mathcal{F}}{\mathcal{D}+\mathcal{E}-1}} \tag{21}$$

which gives the expression for geomorphic α for the modified Horton-Strahler tree as:

$$\alpha^*_g = \frac{\mathcal{D}+\mathcal{E}+\mathcal{F}}{\mathcal{D}+\mathcal{E}-1} \tag{22}$$

[21] *D* will remain constant for a basin. Figure 3 shows $N^*(\omega)$ versus $G^*(\omega)$ curves for different combinations of Θ and Ψ. It can be noticed that α^*_g increases with decreasing $\mathcal{E}$ and/or increasing $\mathcal{F}$. Increasing $\mathcal{E}$ and increasing $\mathcal{F}$ imply increasing Θ and increasing Ψ, respectively, and vice versa.

3.1. The Condition of "Pseudoequilibrium"

[22] We define "equilibrium" as the state of a basin in which different subsurface storage systems interact with each other such that both q and c become constant during a recession event. In this case, the power law exponent of the $-dQ/dt$ versus Q curve is α_g, which is equal to $\mathcal{D}/(\mathcal{D}-1)$.

However, one can envision a scenario where both q and c vary during a recession event but still give $\alpha = \alpha_g$, i.e., a scenario in which $\alpha_g^* = \alpha_g$. Thus,

$$\frac{\mathcal{D}+\mathcal{E}+\mathcal{F}}{\mathcal{D}+\mathcal{E}-1} = \frac{\mathcal{D}}{\mathcal{D}-1} \tag{23}$$

[23] Simplifying equation (23), we obtain

$$\mathcal{D}-1 = \frac{\mathcal{E}}{\mathcal{F}} \tag{24}$$

[24] This means that the basin is in a "pseudoequilibrium" condition when $\mathcal{D}-1 = \mathcal{E}/\mathcal{F}$. Therefore, constant q and constant c, as assumed by *Biswal and Marani* [2010], are not the necessary conditions for having $\alpha = \alpha_g$ for a recession event. When $\mathcal{D} = 2$, $\mathcal{E}$ must be equal to $\mathcal{F}$, which also means that Θ must be equal to Ψ.

4. Analysis of Observed Recession Curves

[25] Flow of water beneath the earth surface is largely unknown due to technological limitations, e.g., it is not yet fully understood why "old water" dominates streamflows during flood events [e.g., *Botter et al*., 2010]. As recession flows occur during dry periods, they provide key information about the basin's subsurface storage systems [e.g., *Krakauer and Temimi*, 2011; *Biswal and Nagesh Kumar*, 2013]. The knowledge of the spatial distribution and movement of subsurface water is essential to efficiently manage water resources as well as to study the transport of solutes [*Cardenas*, 2007; *Welch and Allen*, 2012]. Groundwater may follow short (local) flow paths or long (regional) flow paths to reach stream channels [*Toth*, 1963]. Thus, the portion of water infiltrating into the subsurface of a hillslope adjacent to a lower order stream may follow longer flow paths and reach a higher order stream channel. As a consequence, base flow generation per unit channel length, which is also active subsurface storage per unit length (s) will increase in the downstream direction, a hypothesis supported by experimental evidence [*Ophori and Toth*, 1990]. For a channel reach of length dl situated at a distance l from its farthest source (Figure 4a), s is equal to the product of its q and the time period for which the channel reach drains, t: $s = q \cdot t$. The expression for the gradient of s at the channel reach, ds/dl, can then be found as:

$$\frac{ds}{dl} = q \cdot \frac{dt}{dl} = \frac{q}{c} \tag{25}$$

[26] Thus, for a channel reach of order ω, $ds/dl \propto (\Theta/\Psi)^{\Omega-\omega}$. When both Θ and Ψ are 1 (i.e., when both q and c are constant), ds/dl remains constant along a stream channel (see Figure 4b) or s increases linearly with l (as $t = c \cdot l$ if c remains constant). In this case, the power law exponent $\alpha = \alpha_g^* = \alpha_g$ according to equation (22). If either $\Theta < 1$ and/or $\Psi > 1$, ds/dl increases in the downstream direction. In this case, $\alpha = \alpha_g^* > \alpha_g$. Similarly when $\Theta > 1$ and/or $\Psi < 1$, ds/dl decreases in the downstream direction, which gives $\alpha = \alpha_g^* < \alpha_g$. The spatial distribution of subsurface storage in a basin depends on the topography and the distribution of hydraulic conductivity

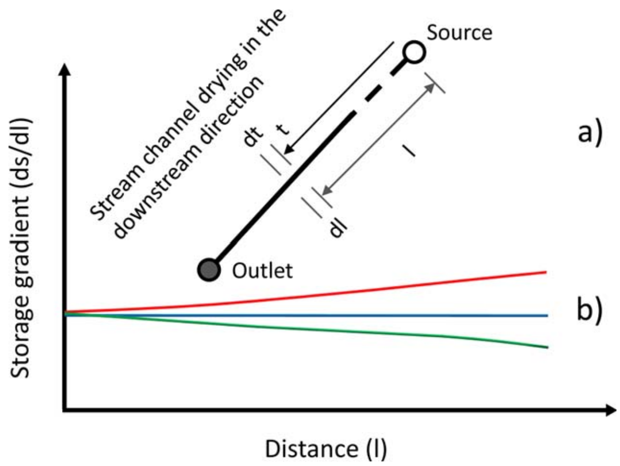

Figure 4. (a) A hypothetical single stream channel that connects a source (open circle) and the basin outlet (closed circle) undergoing desaturation (in the downstream direction) during a recession event. (b) Storage gradient (ds/dl) versus l curves when: both Θ and Ψ are 1 (ds/dl is constant along the stream channel, blue line), either $\Theta < 1$ or $\Psi > 1$ (ds/dl increases with l, red line), and either $\Theta > 1$ or $\Psi > 1$ (ds/dl decreases with l, green line).

in the subsurface zones [e.g., *Toth*, 1963; *Sophocleous*, 2002; *Haitjema and Mitchell-Bruker*, 2005; *Cardenas*, 2007]. Some subsurface storage zones may store more water during a rainfall event than others. Therefore, with increase in effective rainfall volume characterized by the peak discharge (Q_P), which is also the peak of the associated recession curve, ds/dl may either decrease or increase with l depending on the distribution of hydraulic conductivity.

[27] *Biswal and Nagesh Kumar* [2013] defined a recession curve as a continuously decreasing streamflow time series lasting for at least 5 days and found no appreciable correlation between α and Q_P. The reason might be that observational and other errors can significantly affect the value of α. Therefore, we followed a more stringent criterion in this study to select recession flow curves by defining a recession curve as a discharge time series lasting at least for 5 days during which both Q and $-dQ/dt$ decrease continuously [*Shaw and Riha*, 2012]. We then computed Q and $-dQ/dt$ following *Brutsaert and Nieber* [1977] as: $Q_{t+\Delta t/2} = (Q_t + Q_{t+\Delta t})/2$ and $-dQ/dt(t+\Delta t/2) = (Q_t - Q_{t+\Delta t})/\Delta t$, where Δt is the time step. We used daily discharge data for 39 relatively steep USGS basins (see Table 1 of the online supporting information; discharge data were obtained from http://waterwatch.usgs.gov/) and computed α for each recession event of a basin using the least square regression method. Basins with significant human interventions were avoided as activities like the presence of cities or dams can considerably alter recession curve properties [e.g., *Wang and Cai*, 2009; *Biswal and Marani*, 2010]. Although α_r obtained by considering the new definition is not very different from that by considering the earlier definition ($R^2 = 0.80$), by following the new definition, we found that α and Q_p exhibit a power law relationship: $\alpha \propto Q_p^{-\sigma}$ (see Figure 5). The R^2 correlation of α versus Q_p curve was found to be less than 0.1 for 21 study basins. The weak

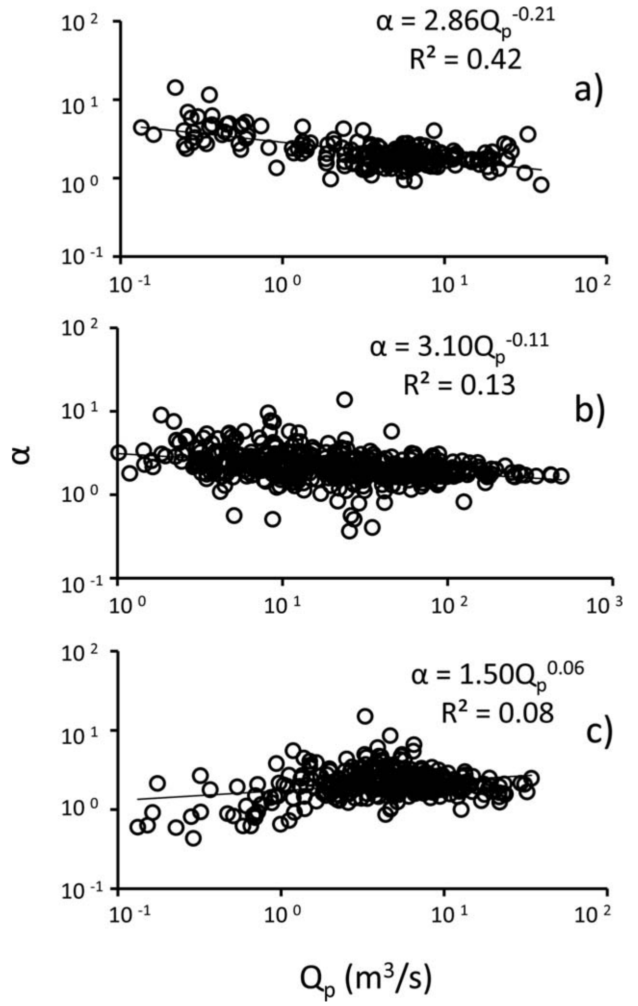

Figure 5. The power law exponent (α) of a recession curve versus its recession peak (Q_p) curve ($\alpha \propto Q_p^{-\sigma}$) for: (a) Bear (USGS id: 03076600; area = 126.65 sq km), (b) Tunkhannock (USGS id: 01534000; area = 991.75 sq km), and (c) Paluxy (USGS id: 08091500; area = 1061.90 sq km). Positive value of sigma indicates *ds / dl* increasing with *l*, and vice versa.

value of correlation may indicate that the distribution of subsurface storage along stream channels is unaffected by the amount of effective rainfall volume. Remarkably, the value of σ is positive for 31 basins, i.e., α decreases with increase in Q_p, which may imply that high-intensity rainfall events cause Θ to increase and/or Ψ to decrease. Similarly, negative values of σ for the remaining eight basins may imply that high-intensity rainfall events are causing Θ to decrease and/or Ψ to increase. The modified GRFM can thus be used to obtain information on subsurface storage distribution, potentially for many practical applications like stream restoration [e.g., *Bukaveckas*, 2007].

5. Summary

[28] The interplay between various subsurface storage units within a basin determines the drainage of water during no-rain or recession periods. Possibly, the channel network reflects the distribution of subsurface storage. The GRFM suggests that the exponent of the power law relationship between $-dQ/dt$ and Q, α, has links with the channel network structure. In particular, this model assumes that both q and c remain constant in a basin during individual recession events. For most steep and natural basins, the median of the observed α values or the representative α, α_r, is nearly equal to the power law exponent of the modeled recession curve ($N(l)$ versus $G(l)$), α_g, which indicates that the GRFM is generally able to capture the real recession flow characteristics. However, the GRFM cannot explain the discrepancy (if any), either between α, the power exponent of an individual recession event, and α_g or between α_r, the representative power law exponent, and α_g.

[29] In this study, we reformulated the GRFM for Horton-Strahler tree networks. Particularly, we restructured Horton-Strahler trees such that a stream of order ω can only drain into a stream of order $\omega + 1$. This scheme simplifies the computations while preserving the original recession flow characteristics. We found that the geomorphic α of a basin, α_g, is related to its fractal dimension D (which is equal to $\log R_B / \log R_L$) as: $\alpha_g = D/(D-1)$. We showed that this expression for α_g also leads to the relationship: $\alpha_g = 1/(1-h)$ (h being Hack's exponent), a relationship which was previously obtained and experimentally confirmed by *Biswal and Marani* [2010]. Hence, our network modification scheme preserves the recession characteristics of a drainage basin.

[30] We then proposed a broader conceptual framework to study the exponent $-dQ/dt$ versus Q curve by allowing both q and c to vary across streams of different Horton-Strahler stream order (ω). Particularly, we assumed that both q and c follow the generalized Horton's law as: $q(\omega)/q(\omega+1) = \Theta$ and $c(\omega)/c(\omega+1) = \Psi$, which also means $q(\omega) \propto \Theta^{\Omega-\omega}$ and $c(\omega) \propto \Psi^{\Omega-\omega}$, where Θ and Ψ are constants for the drainage network. The modified model gives geomorphic (modeled) α, α_g^*, equal to $(D + \mathcal{E} + \mathcal{F})/(D + \mathcal{E} - 1)$. We showed that it is not necessary that both q and c remain constant ("equilibrium" condition) during a recession event for α to be equal to α_g. There can be a "pseudoequilibrium" condition in which α_g^* is equal to α_g when $D - 1 = \mathcal{E}/F$.

[31] The modified GRFM suggests that the value α depends on the distribution of subsurface storage along the stream channels. If both Θ and Ψ are 1, i.e., when both q and c remain constant, the subsurface storage gradient (ds/dl) remains constant along a stream channel, giving $\alpha = \alpha_g$. If $\Theta < 1$ and $\Psi > 1$, ds/dl increases with l and $\alpha > \alpha_g$. Similarly, if $\Theta > 1$ and $\Psi < 1$, ds/dl decreases with l, which gives $\alpha < \alpha_g$. We found that α and recession curve peak (Q_p) exhibit a power law relationship: $\alpha \propto Q_p^{-\sigma}$, which possibly indicates that ds/dl is sensitive to effective rainfall intensity. Results obtained in this study are indicative of the possibility that information on the subsurface storage distribution of a basin can be obtained by analyzing its recession flow curves.

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