Diffraction tomography using power extinction measurements

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We propose a new method for determining structures of semitransparent media from measurements of the extinguished power in scattering experiments. The method circumvents the problem of measuring the phase of the scattered field. We illustrate how this technique may be used to reconstruct both deterministic and random scatterers. © 1999 Optical Society of America [S0740-3232(99)00211-2]
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1. INTRODUCTION

In the short-wavelength limit, tomographic reconstruction of two- and three-dimensional media has long been carried out from intensity measurements. More accurate methods of reconstruction that take into account diffraction[1] require knowledge of both the field amplitude and the phase of the scattered field. For rapidly varying fields such as optical fields the phase may be prohibitively difficult to measure and presents, at best, a technical challenge at lower frequencies. In this paper we propose a method to circumvent the phase problem. We will show that one can determine a function that is related to the scattering amplitude and makes it possible to reconstruct the scattering object for certain model media.

We begin by recalling a well-known result in scattering theory, the optical cross-section theorem.[2] It relates the total power extinguished from a plane wave on scattering to the scattering amplitude in the forward (incident) direction. More explicitly, let

$$\Psi^{(i)}(\mathbf{r}, t) = \psi^{(i)}(\mathbf{r})\exp(-i\omega t) \tag{1.1}$$

be a monochromatic field incident on the scatterer. We assume that it is a plane wave that propagates in the direction of a unit vector $\mathbf{s}_0$,

$$\psi^{(i)}(\mathbf{r}) = a \exp(ik\mathbf{r} \cdot \mathbf{s}_0), \tag{1.2}$$

with $k = \omega/c$, c being the speed of light in vacuum. Let

$$\Psi^{(s)}(\mathbf{r}, t) = \psi^{(s)}(\mathbf{r})\exp(-i\omega t) \tag{1.3}$$

represent the scattered wave. The total field [with time dependence $\exp(-i\omega t)$ being omitted from now on] is then given by the expression

$$\psi(\mathbf{r}) = \psi^{(i)}(\mathbf{r}) + \psi^{(s)}(\mathbf{r}). \tag{1.4}$$

In the far zone in a direction specified by the unit vector $\mathbf{s}$, the scattered field has the asymptotic form

$$\psi^{(s)}(r\mathbf{s}) \sim a \frac{\exp(ikr)}{r} f(\mathbf{s}, \mathbf{s}_0), \tag{1.5}$$

$f(\mathbf{s}, \mathbf{s}_0)$ being the so-called scattering amplitude.

The total power extinguished from the incident field as a result of scattering and absorption is given by the formula

$$P^{(e)} = |a|^2 \frac{4\pi}{k} \Im f(\mathbf{s}_0, \mathbf{s}_0), \tag{1.6}$$

where $\Im$ denotes the imaginary part. In general, one needs to know the scattering amplitude for all directions of incidence and scattering in order to reconstruct the low-pass-filtered version of the scattering object; however, equation (1.6) gives information only about the imaginary part of the scattering amplitude $f(\mathbf{s}, \mathbf{s}_0)$ in the forward direction $\mathbf{s} = \mathbf{s}_0$. Within the accuracy of the first-order Born approximation, $f(\mathbf{s}_1, \mathbf{s}_2)$ is related to the Fourier transform of the susceptibility $\eta(\mathbf{r})$ of the medium by the formula[1]

$$f(\mathbf{s}_1, \mathbf{s}_2) = k^2 \int \eta(\mathbf{r})\exp[-ik\mathbf{r} \cdot (\mathbf{s}_2 - \mathbf{s}_1)]\mathrm{d}^3 r, \tag{1.7}$$

and consequently Eq. (1.6) yields information only about the volume integral of the imaginary part of the susceptibility of the scattering object.

We will make use of a recent generalization of the optical cross-section theorem to introduce a method of determining a complex function that is related to the scattering amplitude of the object whose structure is to be determined. It is possible to determine this function experimentally from measurements of power alone. In many cases this function is simply related to the structure of the object.

2. THE DATA FUNCTION

Let us consider the power extinguished from a coherent beam consisting of two monochromatic plane waves,

$$\psi^{(i)}(\mathbf{r}) = a_1 \exp(ik\mathbf{r}\cdot\mathbf{s}_1) + a_2 \exp(ik\mathbf{r}\cdot\mathbf{s}_2), \quad (2.1)$$

propagating in the directions specified by the unit vectors $\mathbf{s}_1$ and $\mathbf{s}_2$. In this case the extinguished power, $P^{(e)}(a_1, a_2)$, is given by the expression[3]

$$P^{(e)}(a_1, a_2) = \frac{4\pi}{k}\Im[|a_1|^2 f(\mathbf{s}_1, \mathbf{s}_1) + a_1^* a_2 f(\mathbf{s}_1, \mathbf{s}_2) + a_2^* a_1 f(\mathbf{s}_2, \mathbf{s}_1) + |a_2|^2 f(\mathbf{s}_2, \mathbf{s}_2)]. \quad (2.2)$$

By making two measurements of the total extinguished power with different relative phases between the two incident waves, one can determine the cross terms by using the formula

$$P^{(e)}(a_1, a_2) - P^{(e)}(a_1, -a_2) = \frac{8\pi}{k}\Im[a_1^* a_2 f(\mathbf{s}_1, \mathbf{s}_2) + a_2^* a_1 f(\mathbf{s}_2, \mathbf{s}_1)]. \quad (2.3)$$

By making two additional measurements with plane waves of different relative phases, one can determine the quantity

$$D(\mathbf{s}_1, \mathbf{s}_2) = \frac{k}{8\pi a_1^* a_2}\{P^{(e)}(a_1, ia_2) - P^{(e)}(a_1, -ia_2) + i[P^{(e)}(a_1, a_2) - P^{(e)}(a_1, -a_2)]\}. \quad (2.4)$$

We will refer to $D(\mathbf{s}_1, \mathbf{s}_2)$ as the data function. It may be seen that the data function is related to the scattering amplitude by the expression

$$D(\mathbf{s}_1, \mathbf{s}_2) = f(\mathbf{s}_1, \mathbf{s}_2) - f^*(\mathbf{s}_2, \mathbf{s}_1). \quad (2.5)$$

3. DETERMINATION OF OBJECT STRUCTURE FROM THE DATA FUNCTION

We will now describe a method for determining the structure of the object from knowledge of the data function. While it is, in principle, possible to determine the exact structure of the object from complete knowledge of the scattering amplitude (or even from knowledge of the scattering amplitude over a continuous segment of the scattering angles) e.g., by use of the three-dimensional Marchenko method,[4] to do so presents a computational challenge in the volume of calculations required as well as in the regularization of the data and convergence problems. It seems unlikely that such a technique would be useful in practice. However, as we will see, the data function provides sufficient information to calculate the structure function, at least for some scattering media.

We assume that the data function has been determined by continuous sampling of all available real directions of propagation, i.e., that $D(\mathbf{s}_1, \mathbf{s}_2)$ is known for all values of the real unit vectors $\mathbf{s}_j$ ($s_{jx}^2 + s_{jy}^2 + s_{jz}^2 = 1$, $j = 1, 2$, the subscript labeling the Cartesian components).

A. Absorptive Part of the Susceptibility in the First-Order Born Approximation

We consider scattering on a medium with complex dielectric susceptibility $\eta(\mathbf{r})$. The total field (incident plus scattered) satisfies the equation

$$\nabla^2\psi(\mathbf{r}) + k^2\psi(\mathbf{r}) = -4\pi k^2\eta(\mathbf{r})\psi(\mathbf{r}). \quad (3.1)$$

Within the accuracy of the first-order Born approximation the scattering amplitude is given by the expression

$$f(\mathbf{s}_1, \mathbf{s}_2) = k^2\tilde{\eta}[k(\mathbf{s}_1 - \mathbf{s}_2)], \quad (3.2)$$

where

$$\tilde{\eta}(\mathbf{K}) = \int \eta(\mathbf{r})\exp(-i\mathbf{K}\cdot\mathbf{r})\mathrm{d}^3r \quad (3.3)$$

is the three-dimensional spatial Fourier transform of the dielectric susceptibility.

Let α be the imaginary part of the generally complex dielectric susceptibility,

$$\alpha(\mathbf{r}) \equiv \Im\eta(\mathbf{r}). \quad (3.4)$$

The absorbed power, often characterized by the absorption cross section, is proportional to the volume integral of $\alpha(\mathbf{r})$, and so we will refer to α as the absorptive part of the susceptibility (Ref. 5, p. 219). The data function is related to the Fourier transform of α by the simple formula

$$D(\mathbf{s}_1, \mathbf{s}_2) = 2ik^2\tilde{\alpha}[k(\mathbf{s}_1 - \mathbf{s}_2)], \quad (3.5)$$

valid for real unit vectors $\mathbf{s}_1$ and $\mathbf{s}_2$, $\tilde{\alpha}(\mathbf{K})$ denoting the three-dimensional spatial Fourier transform of $\alpha(\mathbf{r})$.

We may now reconstruct a low-pass-filtered version of $\alpha(\mathbf{r})$ from the data available within the sphere of radius $2k$ in the Fourier space, centered on the origin. Since the data function D is a function of only the difference of two unit vectors, we will formally integrate out an unnecessary variable. More precisely, we integrate D over the average vector variable $\mathbf{S} = (\mathbf{s}_1 + \mathbf{s}_2)/2$ of the two unit vectors $\mathbf{s}_1$ and $\mathbf{s}_2$, which is orthogonal to the difference vector $\mathbf{s} = \mathbf{s}_1 - \mathbf{s}_2$:

$$\mathcal{D}(\mathbf{s}) = \frac{1}{2\pi}\int_{(2\pi)} \mathrm{d}\phi D[\mathbf{S}(\phi) + \mathbf{s}/2, \mathbf{S}(\phi) - \mathbf{s}/2]. \quad (3.6)$$

Here $\mathbf{S}$ is a vector of length $\sqrt{1 - \mathbf{s}^2/4}$ lying in the plane perpendicular to $\mathbf{s}$ and making an angle ϕ with respect to some arbitrary reference line in the plane.[6] It follows from Eqs. (3.5) and (3.6) that the low-pass (subscript LP) reconstruction of $\alpha(\mathbf{r})$ is given by the formula

$$\alpha_{LP}(\mathbf{r}) = \frac{k}{i16\pi^3}\int_{|\mathbf{s}|\leq 2} \exp(ik\mathbf{r}\cdot\mathbf{s})\mathcal{D}(\mathbf{s})\mathrm{d}^3s. \quad (3.7)$$

To demonstrate the feasibility of this inverse technique, we first calculate the solution of the direct (forward) problem without any approximation (i.e., to all orders of perturbation) in the form of an infinite series. We then apply the inverse technique described above to that solution.

In Fig. 1, computer simulations of the low-pass reconstruction of the absorptive part of the susceptibility of a homogeneous sphere are shown. We first calculated the scattering amplitude for spheres of various radii and susceptibilities by the method of partial waves, with 35 terms in the series, thus including the effects of multiple scattering (see Appendix A). We then applied the algorithm described above to the calculated data in order to reconstruct the potential. It is seen that for media that are

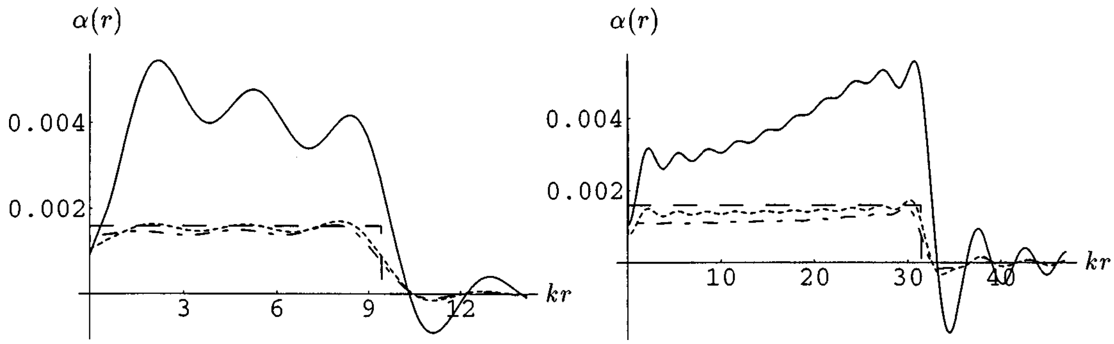

Fig. 1. Left, sphere with size parameter $ka = 3\pi$ (radius $a = 1.5\lambda$); right, sphere of radius $ka = 10\pi$ ($a = 5\lambda$). In all cases $\alpha = 0.01/2\pi$; the original profile is shown by long-dashed curves. Short–long-dashed curves, reconstruction of spheres with susceptibility $\eta = 0.01i/2\pi$; short-dashed curves, reconstruction of spheres with susceptibility $\eta = (0.01 + 0.01i)/2\pi$; solid curves, reconstruction of spheres with susceptibility $\eta = (0.0512 + 0.01i)/2\pi$.

purely absorptive (real part of $\eta = 0$) the reconstruction is quite accurate. The effects of multiple scattering begin to distort the reconstruction as the real part of the susceptibility (representing dispersion) increases. As can be seen by comparing the reconstructions, the size of the scatterer as well as the refractive index determine whether the single-scattering approximation (i.e., the first-order Born approximation) gives reasonably accurate results. This was perhaps to be expected.

B. Spatial Variations of a Dielectric Random Scatterer

Suppose that the scatterer is dielectric (i.e., is nonabsorbing) such that

$$\langle \Im \eta(\mathbf{r}) \rangle = 0, \tag{3.8}$$

the angle brackets denoting the ensemble average. We assume that the fluctuations of the medium are characterized by correlations that have a very short range in comparison with the scale of variations of the average susceptibility. We may then model the scatterer by the quasi-homogeneous approximation.[7–9] The correlation function of the dielectric susceptibility can then be taken to have the form

$$\langle \eta(\mathbf{r}_1)\eta(\mathbf{r}_2) \rangle \equiv C(\mathbf{r}_1, \mathbf{r}_2) \approx \Gamma\left(\frac{\mathbf{r}_1 + \mathbf{r}_2}{2}\right) g(\mathbf{r}_2 - \mathbf{r}_1), \tag{3.9}$$

where g is the degree of correlation normalized so that $g(0) = 1$. We will refer to Γ as the intensity (strength) of the susceptibility, and it may be seen that when $\mathbf{r}_1 = \mathbf{r}_2 = \mathbf{r}$, Γ is given exactly by the expression $\Gamma(\mathbf{r}) = \langle \eta(\mathbf{r})\eta(\mathbf{r}) \rangle$. Moreover, $\Gamma(\mathbf{R})$ is assumed to vary much more slowly with $\mathbf{R}$ than $g(\mathbf{r})$ varies with $\mathbf{r}$. In this case the scattering amplitude is given, to leading order in the susceptibility, by the formula[3]

$$f(\mathbf{s}_1, \mathbf{s}_2) = k^4 \tilde{\Gamma}[k(\mathbf{s}_1 - \mathbf{s}_2)] \tilde{\mathcal{G}}\left[-k\left(\frac{\mathbf{s}_1 + \mathbf{s}_2}{2}\right)\right], \tag{3.10}$$

where

$$\mathcal{G}(\mathbf{r}) \equiv g(\mathbf{r}) G(\mathbf{r}), \tag{3.11}$$

$\tilde{\mathcal{G}}(\mathbf{K})$ being the spatial Fourier transform of $\mathcal{G}(\mathbf{r})$, and $G(\mathbf{r})$ is the out-going free-space Green function, viz.,

$$G(\mathbf{r}) = \frac{\exp(ikr)}{r}. \tag{3.12}$$

In this case the data function is given by the expression

$$D(\mathbf{s}_1, \mathbf{s}_2) = 2ik^4 \tilde{\Gamma}[k(\mathbf{s}_1 - \mathbf{s}_2)] \Im \tilde{\mathcal{G}}\left(-k\frac{\mathbf{s}_1 + \mathbf{s}_2}{2}\right). \tag{3.13}$$

By making use of the Fourier representation of the Green function and the convolution theorem, it can be seen that

$$\tilde{\mathcal{G}}(\boldsymbol{\kappa}) = \frac{1}{2\pi^2} \int \frac{\tilde{g}(\boldsymbol{\kappa} - \boldsymbol{\kappa}')}{\kappa'^2 - (k + i\epsilon)^2} \mathrm{d}^3\kappa', \tag{3.14}$$

it being understood that $\epsilon \to 0^+$ on the right-hand side of this equation.

If $\tilde{g}(\boldsymbol{\kappa})$ is isotropic, i.e., if $\tilde{g}(\boldsymbol{\kappa}) \equiv \tilde{g}(|\boldsymbol{\kappa}|)$, one finds that the quantity $\Im \tilde{\mathcal{G}}[-k(\mathbf{s}_2 + \mathbf{s}_2)/2]$ is a simple multiplicative factor in the data function at a fixed frequency and depends only on k. Explicitly,

$$\tilde{\mathcal{G}}(-k\mathbf{s}) = \frac{1}{\pi} \int_0^1 \mathrm{d}x \int_{-\infty}^{\infty} \mathrm{d}\kappa' \kappa'^2 \times \frac{\tilde{g}[(k^2 + \kappa'^2 + 2xk\kappa')^{1/2}]}{\kappa'^2 - (k + i\epsilon)^2}. \tag{3.15}$$

If, in addition, $\tilde{g}[(k^2 + \kappa'^2 + 2xk\kappa')^{1/2}]$ is analytic in κ' in the upper half of the complex κ' plane[10] for all values of x in the range $0 \leqslant x \leqslant 1$, then

$$\tilde{\mathcal{G}}(-k\mathbf{s}) = i \int_{\sqrt{2}}^{2} \mathrm{d}y k y \tilde{g}(ky), \tag{3.16}$$

and, consequently,

$$D(\mathbf{s}_1, \mathbf{s}_2) = 2ik^5 \tilde{\Gamma}[k(\mathbf{s}_1 - \mathbf{s}_2)] \int_{\sqrt{2}}^{2} \tilde{g}(ky) y \mathrm{d}y. \tag{3.17}$$

Let us compare this situation with the case when the medium is delta correlated, i.e., when

$$g(\mathbf{r}) = \delta^{(3)}(\mathbf{r}). \tag{3.18a}$$

Then

$$\tilde{g}(\boldsymbol{\kappa}) = 1. \tag{3.18b}$$

In this case data function can again be shown to be related to a Fourier component of the object. Specifically,

$$D(\mathbf{s}_1, \mathbf{s}_2) = 2ik^5\tilde{\Gamma}[k(\mathbf{s}_1 - \mathbf{s}_2)], \tag{3.19}$$

for all real unit vectors $\mathbf{s}_1$ and $\mathbf{s}_2$. We see that as long as the magnitude of the degree of correlation, $|g(\mathbf{r})|$, is narrow enough for the quasi-homogeneous approximation to apply, the data function represents Fourier components of the intensity $\Gamma(\mathbf{r})$ of the susceptibility. The effect of a finite but small correlation length in the random medium is to multiply the Fourier components by some constant, as can be seen on comparing Eqs. (3.17) and (3.19). This effectively amplifies or deamplifies the intensity of the object on reconstruction but otherwise has no significant effect.

A low-pass-filtered version of the intensity (strength of the susceptibility) of the object can now be determined. It is given by the formula

$$\Gamma_{LP}(\mathbf{r}) = \frac{1}{i16\pi^3k^2}\int_{|\mathbf{s}|\leq 2} \exp(ik\mathbf{r}\cdot\mathbf{s})\mathcal{D}(\mathbf{s})\mathrm{d}^3s, \tag{3.20}$$

where $\mathcal{D}(\mathbf{s})$ is as defined in Eq. (3.6).

As an example, consider a model scatterer characterized by an ensemble of independent homogeneous spheres. The spheres are assumed to have the same radius a and the same susceptibility η_0 and are centered at a point specified by the position vector $\mathbf{r}_0$, distributed with a probability density $p(\mathbf{r}_0)$. In this case the correlation function of the susceptibility of the medium is given by the expression

$$C(\mathbf{r}_1, \mathbf{r}_2) = \eta_0^2\int B_a(\mathbf{r}_1 - \mathbf{r}_0)B_a(\mathbf{r}_2 - \mathbf{r}_0) \times p(\mathbf{r}_0)\mathrm{d}^3r_0, \tag{3.21}$$

where

$$B_a(\mathbf{r}) = \begin{cases} 1 & \text{if } |\mathbf{r}| \leq a \\ 0 & \text{otherwise} \end{cases}, \tag{3.22}$$

and the integration is taken over all space. Equation (3.21) can be expressed in terms of the average and difference coordinates, $\mathbf{R} = (\mathbf{r}_1 + \mathbf{r}_2)/2$ and $\mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2$, respectively, as

$$C(\mathbf{r}_1, \mathbf{r}_2) = \eta_0^2\int B_a(\mathbf{r}' + \mathbf{r}/2)B_a(\mathbf{r}' - \mathbf{r}/2) \times p(\mathbf{r}' + \mathbf{R})\mathrm{d}^3r'. \tag{3.23}$$

If $p(\mathbf{r})$ varies slowly with $\mathbf{r}$ on the scale of the radius a, one can replace $p(\mathbf{r}' + \mathbf{R})$ by the first terms of its Taylor expansion about $\mathbf{R}$, and one finds that

$$C(\mathbf{r}_1, \mathbf{r}_2) \approx \Gamma(\mathbf{R})g(\mathbf{r}) + \eta_0^2\int B_a(\mathbf{r}' + \mathbf{r}/2) \times B_a(\mathbf{r}' - \mathbf{r}/2)\mathbf{r}'\cdot\nabla p(\mathbf{R})\mathrm{d}^3r' + \ldots, \tag{3.24}$$

where

$$\Gamma(\mathbf{R}) = \frac{4\pi}{3}a^3\eta_0^2p(\mathbf{R}). \tag{3.25}$$

The degree of correlation, g, is related to the geometrical overlap of two spheres[11] and is given by the expression

$$g(\mathbf{r}) = \begin{cases} 1 - \dfrac{3r}{4a} + \dfrac{r^3}{16a^3} & \text{for } r \leq 2a \\ 0 & \text{for } r > 2a \end{cases}. \tag{3.26}$$

Retaining only the first term in Eq. (3.24), one obtains the quasi-homogeneous approximation for the correlation function of the random scatterer.

The scattering amplitude of a sphere of radius a centered on the point $\mathbf{r}_0$, $f(\mathbf{s}_1, \mathbf{s}_2; \mathbf{r}_0)$, is given in terms of the scattering amplitude $f_a(\mathbf{s}_1 - \mathbf{s}_2)$ of a sphere of radius a centered on the origin by the expression

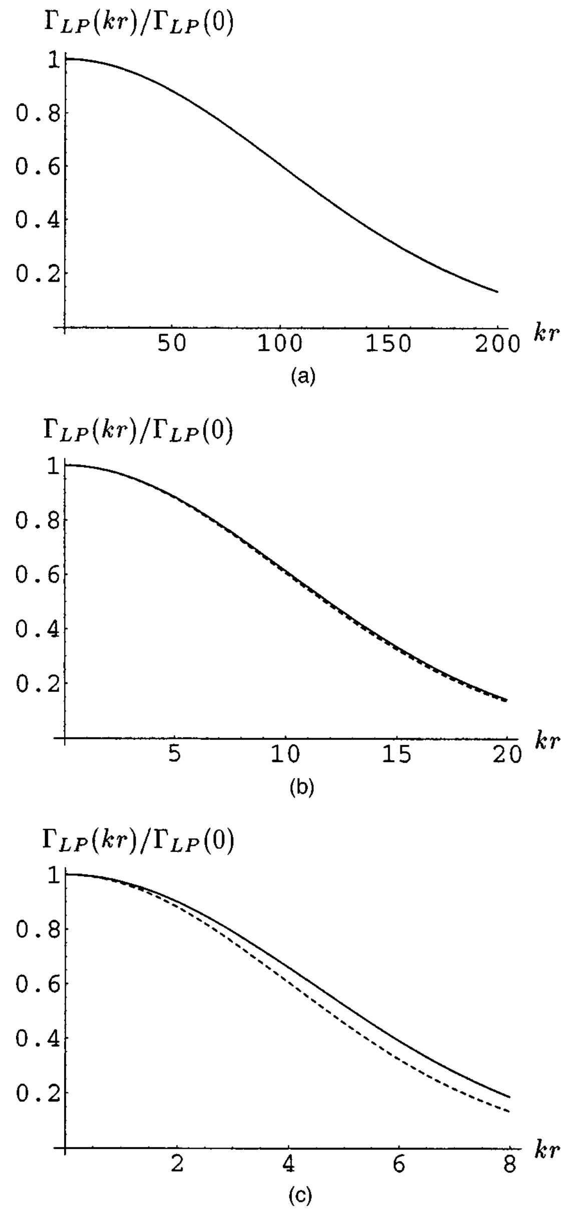

Fig. 2. Susceptibility intensity function Γ for a medium characterized by an ensemble of independent homogeneous spheres each with susceptibility $\eta = 0.21/4\pi$ and size parameter $ka = 4$. Dashed curves, original probability distribution, with (a) $k\sigma = 100$, (b) $k\sigma = 10$, and (c) $k\sigma = 4$. Solid curves, reconstructed fluctuation structure function. Both types of curves have been normalized so that the peak values are unity.

$$f(\mathbf{s}_1, \mathbf{s}_2; \mathbf{r}_0) = \exp[ik\mathbf{r}_0 \cdot (\mathbf{s}_2 - \mathbf{s}_1)] f_a(\mathbf{s}_1 - \mathbf{s}_2). \tag{3.27}$$

Thus the ensemble-average scattering amplitude for such a scatterer is given by the formula

$$\langle f(\mathbf{s}_1, \mathbf{s}_2; \mathbf{r}_0)\rangle = f_a(\mathbf{s}_1 - \mathbf{s}_2)\tilde{p}[k(\mathbf{s}_1 - \mathbf{s}_2)], \tag{3.28}$$

where the tilde indicates, as usual, the three-dimensional spatial Fourier transform. It follows directly from Eq. (3.28) that the data function is given by the expression

$$D(\mathbf{s}_1, \mathbf{s}_2) = [f_a(\mathbf{s}_1 - \mathbf{s}_2) - f_a^*(\mathbf{s}_2 - \mathbf{s}_1)] \times \tilde{p}[k(\mathbf{s}_1 - \mathbf{s}_2)]. \tag{3.29}$$

In this case the function $\mathcal{D}(\mathbf{s})$, defined by Eq. (3.6) has the value

$$\mathcal{D}(\mathbf{s}) = [f_a(\mathbf{s}) - f_a^*(-\mathbf{s})]\tilde{p}(k\mathbf{s}). \tag{3.30}$$

According to Eq. (3.20) the reconstructed intensity of the susceptibility is then given by the expression

$$\Gamma_{LP}(\mathbf{r}) = \frac{1}{k^3}\int \alpha_{LP}(\mathbf{r} - \mathbf{r}')p(\mathbf{r}')\mathrm{d}^3r', \tag{3.31}$$

where $\alpha_{LP}(\mathbf{r})$ is the reconstructed absorptive part of the susceptibility of the homogeneous sphere of radius a and susceptibility η_0 given in Eq. (A5) of Appendix A.

Figure 2 shows the numerically reconstructed intensity of the susceptibility Γ obtained by the use of Eq. (3.31) with a Gaussian probability distribution $p(\mathbf{r})$, viz.,

$$p(\mathbf{r}) = (\pi\sigma^2/2)^{-3/2}\exp(-r^2/2\sigma^2). \tag{3.32}$$

In Fig. 2(a), which corresponds to the case $k\sigma = 100$, the reconstruction is so accurate that the curves are indistinguishable. In Fig. 2(b) ($k\sigma = 10$), some deviation from the original profile is evident. In Fig. 2(c) ($k\sigma = 4$), the width of the probability distribution is equal to the radius of the spheres. One might expect that the quasi-homogeneous approximation will break down in this limit; indeed the figure shows that in this case the reconstruction differs significantly from the original function.

4. CONCLUSION

We have proposed a new technique for diffraction tomography that makes use of a generalization of the optical cross-section theorem and avoids the problems of measuring the phase of the scattered field as well as measuring the directional dependence of the scattered field. We have also shown that by using this method, one can determine the absorptive part of the susceptibility of a deterministic scatterer and the fluctuation strength function of a random scatterer. We have demonstrated the feasibility of the proposed method by numerical simulations.

APPENDIX A: THE HOMOGENEOUS SPHERE

In this appendix, formulas are given that relate the forward and inverse problems. We first determine the scattering amplitude associated with a homogeneous sphere of refractive index $N = (4\pi\eta + 1)^{1/2}$ and of radius a in the form of an infinite series. The formulas for the inverse problem in the single-scattering approximation are then expressed in terms of the (exact) series for the scattering amplitude. That is, the low-pass reconstructed object function α_{LP} and the averaged data function $\mathcal{D}$ are then expressed in terms of the series for the scattering amplitude.

The calculation of the scattering amplitude (usually referred to as the forward problem) can be performed by the method of partial waves (Ref. 12, p. 932). The scattering amplitude is then expressed in series form as

$$f(\mathbf{s}_1, \mathbf{s}_2) = \frac{1}{k}\sum_{m=0}^{\infty}(2m + 1)\frac{i\beta_m}{\beta_m - i\gamma_m}P_m(\mathbf{s}_1\cdot\mathbf{s}_2), \tag{A1}$$

where

$$\beta_m = kj_m(Nka)j'(ka) - nkj_m'(Nka)j_m(ka), \tag{A2}$$

$$\gamma_m = nkj_m'(Nka)n_m(ka) - kj_m(Nka)n_m'(ka). \tag{A3}$$

In these formulas n_m and j_m are the spherical Neumann functions and the spherical Bessel functions, respectively, of order m, and the prime indicates the derivative with respect to the argument. Further, P_m is the Legendre polynomial of order m. The series converges rapidly when the number of terms retained exceeds the size parameter ka. In this case the averaged data function can be expressed in the form

$$\mathcal{D}(\mathbf{s}) = \frac{2i}{k}\sum_{m=1}^{\infty}(2m + 1)\Re\left(\frac{\beta_m}{\beta_m - i\gamma_m}\right)P_m(1 - s^2/2). \tag{A4}$$

Using formula (3.7), one finds that

$$\alpha_{LP}(\mathbf{r}) = \sum_{m=0}^{\infty}\frac{(2m + 1)}{\pi^2 kr}\Re\left(\frac{\beta_m}{\beta_m - i\gamma_m}\right) \times \frac{\mathrm{d}}{\mathrm{d}(kr)}[krj_m(kr)n_m(kr)]. \tag{A5}$$

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