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Erratum

# Higher-order radiative corrections to low-angle Bhabha scattering: the YFS Monte Carlo approach [Phys. Lett. B 353 (1995) 362] ★

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We would like to report an error in the matrix element squared $D^{(r)}_{[1,0]}$ for the final state with one photon virtual and one photon real in Eq. (4) of paper [1]. The virtual correction in this function should read as follows:

$$v^{(2)}_{[1,0]} = (\gamma_p + \gamma_q)\ln\Delta + \tfrac{3}{2}\gamma - \frac{\alpha}{\pi} - \tfrac{3}{4}\gamma\ln(1-\tilde{\beta}_1) - \tfrac{1}{4}\gamma\ln(1-\tilde{\alpha}_1)$$
$$+ (\gamma_p - \gamma)\left(\tfrac{1}{4} - \ln(1-\tilde{\beta}_1)\right). \qquad (4)$$

Several comments and explanations have to be added immediately. First of all, although, as we shall see later, the above corrections do change the numerical results in the paper, the change is not big enough and has *no effect on any conclusion of the paper, in particular the estimate of the total theoretical error, 0.16%, remains valid.* Secondly, as soon as the error was found and corrected all users of the program BHLUMI were notified. The error was found at the beginning of the LEP2 workshop – results in the workshop and workshop proceedings [2] from the BHLUMI program correspond to version 4.03 (or 2.02a) in which the error is already corrected. It is worth stressing that the error was found not by chance, but during the routine round of tests on the matrix element in BHLUMI which lead to a precise estimate of the missing third-order leading-log contribution in BHLUMI [1].

Let us now comment on the corrections in $v^{(2)}_{[1,0]}$. The replacement of $-\frac{1}{2}\gamma\ln(1-\tilde{\alpha}_1-\tilde{\beta}_1+\ln(1-\tilde{\alpha}_1\tilde{\beta}_1)$ with $-\frac{3}{4}\gamma\ln(1-\tilde{\beta}_1)-\frac{1}{4}\gamma\ln(1-\tilde{\alpha}_1)$ corrects for a genuine second-order leading-log error. This error was not noticed earlier because for a non-calorimetric event selection like the "academic trigger" in [1] or the BARE1 of [2] its effect cancels exactly between initial and final state emission – *only for the calorimetric kind of event selection it does affect the total cross section.* The first round of comparisons of the Monte Carlo with semi-analytical calculations [4] was done for a non-calorimetric event selection, for which the error is numerically invisible. For instance, although the formula of Eq. (15) in [4] will change, the plot in Fig. 1 in [4] is unaffected. The error showed up for the first time in the second round of tests [3], in which the example of calorimetric event selection was used. Strictly speaking, the additional term $+(\gamma_p-\gamma)\left(\frac{1}{4}-\ln(1-\tilde{\beta}_1)\right)$ is not an error but an adjustment of the non-exponentiated matrix element such that the difference between the exponentiated and non-exponentiated cross sections does include *only* $\mathcal{O}(\alpha^3)$ contributions and none of

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[1] The numerical results of these estimates were shown in [2] and the calculations will be published elsewhere [3].


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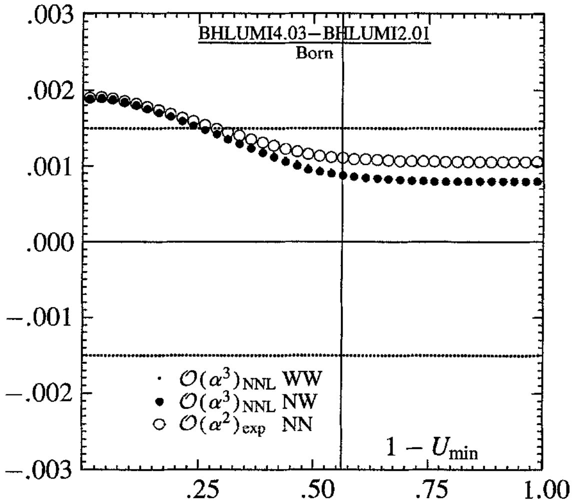


Fig. 3. For the ALEPH SICAL detector we plot $c_{4-2}$ = (BHLUMI.4 − BHLUMI.2)/Born as a function of the energy cut $1-U_{\min}$, where $U_{\min} = \min(E_1^{\rm cl}, E_2^{\rm cl})/E_{\rm beam}$. The standard value $(1-U_{\min})^{\rm CUT} = 0.561341$ is marked with the vertical line. Three curves plotted with small dots, open circles and big dots represent angular cuts WW, NN and NW, respectively, where W and N denote wide or narrow angular ranges on one side of the detector. The wide (W) angular range is $\theta_A + \Delta < \theta_1^{\rm cl} < \theta_B - \Delta$, and the narrow (N) angular range is $\theta_A + 2\Delta < \theta_2^{\rm cl} < \theta_B - 4\Delta$, where $\theta_A = 0.024$, $\theta_B = 0.058$ and $\Delta = (\theta_B - \theta_A)/16$. The other cuts are: (a) auxiliary energy cuts $Y_{\min} = 0.60315$, $Z_{\min} = 0$, (b) acoplanarity cut $\Delta\phi_{\max} = 0.52359$, see Fig. 2 for cut-off definitions.

$\mathcal{O}(\alpha^2)$. This improvement is necessary to make the discussion of the sub-leading $\mathcal{O}(\alpha^2)$ contributions easier and cleaner, see [2]. As a result of this improvement the curve in Fig. 5 of Ref. [1] will get slightly shifted upwards. The same kind of correction/adjustment was done in second-order virtual correction in Eq. (2) of Ref. [1], which now reads

$$v^{(2)} = 2\gamma\ln\Delta + \tfrac{1}{2}\gamma^2\ln^2\Delta + (1+2\gamma\ln\Delta)\left(\tfrac{3}{2}\gamma - \frac{\alpha}{\pi}\right) + \tfrac{9}{8}\gamma^2 - \frac{3}{2}\frac{\alpha}{\pi}\gamma. \tag{2}$$

Let us now present the corrected figures. Figs. 3, 4 and 5 of Ref. [1] should be replaced by the present Figs. 3, 4 and 5.

The formula of Eq. (15) of Ref. [1] should now read as follows:

$$\begin{aligned}
\rho^{(2)}_{\rm tot} &= \rho^{(r)}_{\bar\beta_0}(t,V) + 2\rho^{(2)}_{1U}(t,V) + 2\rho_{\bar\beta_{2UU}}(t,V) + \rho_{\bar\beta_{2UL}}(t,V)\\
&= b_0\, F(2\gamma)\, e^{2\Delta_{\rm YFS}(\gamma)}\, 2\gamma V^{2\gamma-1}\left\{1+\gamma+\tfrac{1}{2}\gamma^2\right\}\\
&+ b_0\, F(2\gamma)\, e^{2\Delta_{\rm YFS}(\gamma)}\, V^{2\gamma}\Big\{\gamma(-2+V) + \frac{\alpha}{\pi}\ln(1-V)(-4+4V-2V^{-1})\\
&+\gamma^2(-2) + \gamma^2\ln(1-V)(3-3V/2-2V^{-1})\\
&+\gamma^3(-9V/8) + \gamma^3\ln(1-V)[2+1/8V-2V^{-1}-(1/4)(2-V)^{-1}]\\
&+\gamma^3\ln(1-V)^2[-7/8+7V/16+(1/2)V^{-1}] + \gamma^3{\rm Li}_2(V)(2-V)\\
&+\gamma^3\ln(1-V)\ln(2-V)(-1/4+V/8) + \gamma^3{\rm Li}_2((1-V)/(2-V))(1/4-V/8)\\
&+\gamma^3{\rm Li}_2(1/(2-V))(-1/4+V/8)\\
&+\gamma\frac{\alpha}{\pi}[1/4+11V-(13/4)(2-V)^{-1}+(1/2)(2-V)^{-2}-6(2-V)^{-3}+2(1-V)^{1/2}]\\
&+\gamma\frac{\alpha}{\pi}\ln(1-V)[39/4-19V/4-2V^{-1}\\
&-2(2-V)^{-1}+(2-V)^{-2}-(1/2)(2-V)^{-3}-(3/2)(1-V)^{1/2}]\\
&+\gamma\frac{\alpha}{\pi}\ln(1-V/2)[-9/2+3V/4-4(2-V)^{-1}+2(2-V)^{-2}-4(2-V)^{-3}]
\end{aligned}$$

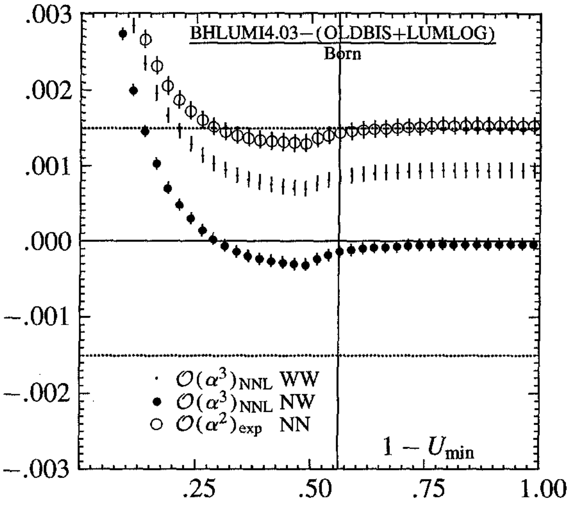


Fig. 4. The difference $d_3$ = (BHLUMI.4x − OLDBIS −LUMLOG)/Born for the ALEPH SICAL detector as a function of the energy cut $1-U_{\rm min}$. Cuts are the same as in Fig. 3.

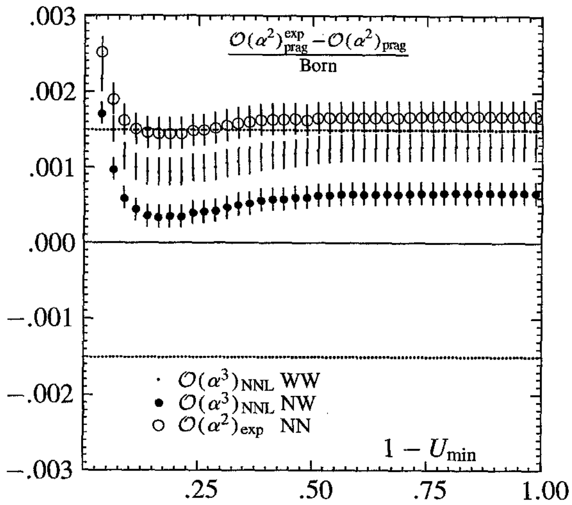


Fig. 5. Difference between the cross sections calculated with the $\mathcal{O}(\alpha^2)_{\rm prag}$ matrix element, with YFS exponentiation and without exponentiation, for the SICAL trigger. The difference divided by the Born cross section is plotted as a function of the energy cut $1-U_{\rm min}$. The other cuts are the same as in Fig. 3.

$$
\begin{aligned}
&+\gamma\frac{\alpha}{\pi}\ln(1-V)^2[27/8-49V/16]+\gamma\frac{\alpha}{\pi}\ln(1-V)\ln(2-V)(-1/2+V/4)\\
&+\gamma\frac{\alpha}{\pi}\ln(1-V)\ln(V)(12-10V)+\gamma\frac{\alpha}{\pi}\ln(1-V)\ln(V/2)(-6+5V)\\
&+\gamma\frac{\alpha}{\pi}\ln(1-V)\ln[1-(1-V)^{1/2}](-6+5V)\\
&+\gamma\frac{\alpha}{\pi}\ln(2-V)\ln(1-V/2)(3/2-11V/4)\\
&+\gamma\frac{\alpha}{\pi}\ln(1-V/2)^2(3/4-5V/8)+\gamma\frac{\alpha}{\pi}\mathrm{Li}_2(1/2)(-3/2+11V/4)\\
&+\gamma\frac{\alpha}{\pi}\mathrm{Li}_2[(1-V)/(2-V)](1/2-V/4)+\gamma\frac{\alpha}{\pi}\mathrm{Li}_2(1/(2-V))(1-5V/2)\\
&+\gamma\frac{\alpha}{\pi}\mathrm{Li}_2(-V/2/(1-V))(6-5V)+\gamma\frac{\alpha}{\pi}\mathrm{Li}_2[1-(1-V)^{-1/2}](6-5V)\\
&-\zeta\gamma\chi(V)/(1-V)\Big\}. \qquad (15)
\end{aligned}
$$

Note that terms of $\mathcal{O}(\gamma^2)$ did not change because, as we pointed out, the error is not visible for the non-calorimetric type of event selections. The terms of $\mathcal{O}(\gamma^3)$ and $\mathcal{O}(\gamma\frac{\alpha}{\pi})$ did change, but the numerical difference is about $10^{-4}$ so it does not affect Fig. 1 of Ref. [1].

Finally we would like to apologize for any inconvenience the above errors may have caused. We did what we could to minimize the impact of these errors by contacting all concerned users of our calculations as soon as it was possible. The forthcoming published version 4.04 and higher version of BHLUMI [5] will have all above corrections included (corrections are already included in versions 4.02a and 4.03 already available on WWW).

## References

[1] S. Jadach, E. Richter-Wąs, B.F.L. Ward, and Z. Wąs, Phys. Lett. B 353 (1995) 362, CERN preprint CERN-TH/95-38.
[2] S. Jadach and O. Nicrosini (convenors of Bhabha Working Group), in: Physics at LEP2, CERN Yellow Report 96-01, eds. G. Altarelli, T. Sjöstrand, and F. Zwirner (CERN, Geneva, 1996) Vol. 2, p. 229, hep-ph/9602393.
[3] S. Jadach and B.F.L. Ward, CERN preprint TH-96-156, June 1996, submitted to Phys. Lett. (unpublished).
[4] S. Jadach and B.F.L. Ward, Semi-Analytical Third Order Calculations of the Small Angle Bhabha Cross Sections, 1996, to be published in Acta Phys. Pol.
[5] S. Jadach et al., preprint CERN TH/96-158, June 1996, Upgrade of the Monte Carlo program BHLUMI for Bhabha scattering at low angles to version 4.04, to be submitted to Comput. Phys. Commun.; available from the WWW location http://hpjmiady.ifj.edu.pl (unpublished).