

# Chapter 9

## $S_3$ flavor symmetry at the LHC

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### Abstract

Discrete symmetries employed to explain neutrino mixing and mass hierarchies are often associated with an enlarged scalar sector which might lead to exotic Higgs decay modes. We explore such a possibility in a scenario with  $S_3$  flavor symmetry which requires three scalar  $SU(2)$  doublets. The spectrum is fixed by minimizing the scalar potential, and we observe that the symmetry of the model leads to tantalizing Higgs decay models potentially observable at the CERN Large Hadron Collider (LHC).

### 9.1. Introduction

The permutation group  $S_3$  is an attractive candidate to address the flavor puzzle. It was introduced in [15] and explored further in the article this work is based upon [24]. In this contribution we study the exciting prospect that such flavor models can predict *enlarged Higgs sectors with non-standard couplings to fermions and gauge bosons*. A full list of references can be found in the article this contribution is based upon [22].

The motivation for choosing  $S_3$  is that it is the smallest non-abelian discrete symmetry group that contains a 2-dimensional irreducible representation which can connect two maximally mixed generations. It has three irreducible representations:  $\mathbf{1}$ ,  $\mathbf{1}'$  and  $\mathbf{2}$ , with multiplication rules:  $\mathbf{2} \times \mathbf{2} = \mathbf{1} + \mathbf{1}' + \mathbf{2}$  and  $\mathbf{1}' \times \mathbf{1}' = \mathbf{1}$ . Besides facilitating maximal mixing through its doublet representation,  $S_3$  provides two inequivalent singlet representations which play a crucial role in reproducing fermion masses and mixing. To accomplish the latter, three scalar  $SU(2)$  doublets are introduced, which couple to the fermions as dictated by  $S_3$  symmetry. It so happens that large mixing among up- and down-type quarks cancel each other in the Cabibbo matrix. Neutrino masses are separately generated by a type-II see-saw mechanism using scalar  $SU(2)$  triplets, so that the mismatch between the large mixing of the charged leptons and the diagonal neutrino masses translates directly into the Pontecorvo-Maki-Nakagawa-Sakata matrix. In this paper we do not deal with those triplets, but explore the following avenues: (i) the mass spectrum and mixing of the scalars (ii) the gauge and Yukawa interactions of the neutral scalars, and (iii) different nonstandard production and decay modes of the neutral CP-even scalars leading to the possibility of their detection at the LHC.

For definiteness, we study the  $S_3$  model pursued in [24] to explain the leptonic flavor structure. We con-

centrate on the complementary aspects by exploring the scalar sector. The assignments of the fermion and scalar fields are as follows:

$$\begin{aligned}
(L_\mu, L_\tau) &\in \mathbf{2} & L_e, e^c, \mu^c &\in \mathbf{1} & \tau^c &\in \mathbf{1}', \\
(Q_2, Q_3) &\in \mathbf{2} & Q_1, u^c, c^c, d^c, s^c &\in \mathbf{1} & b^c, t^c &\in \mathbf{1}', \\
(\phi_1, \phi_2) &\in \mathbf{2} & \phi_3 &\in \mathbf{1}, & &
\end{aligned} \tag{9.1}$$

where the notations are standard and self-explanatory. The vacuum expectation values (VEVs) of the three scalar doublets  $\phi_{1,2,3}$  induce spontaneous electroweak symmetry breaking (SSB).

## 9.2. Scalar potential and spectrum

We use the most general  $S_3$  invariant scalar potential involving three scalar doublet fields given by [24]. It depends on two mass-like parameters  $m, m_3$  and eight couplings  $\lambda_{\{1,\dots,8\}}$ . After SSB, nine degrees of freedom are left: three neutral scalars, two neutral pseudoscalars and two charged scalars with two degrees of freedom each. We denote the VEVs of  $\phi_i$  by  $v_i$  and assume the  $\lambda_i$ 's to be real. For the purpose of generating maximal mixing in the lepton sector, we choose the vacuum alignment  $v_1 = v_2 = v$ . To ensure that the chosen vacuum alignment actually corresponds to a minimum of the potential, we adjust parameters to make sure that the determinant of the Hessian matrix is positive. In this case, the function is the scalar potential and the Hessian is just the mass matrix of the scalars. The positivity of the eigenvalues guarantees that the potential is minimized.

We now set out to find the spectrum of the three CP-even neutral scalars. We insert the expansion  $\phi_i^0 = v_i + h_i$  into the scalar potential to obtain the mass matrix. After its diagonalization the weak basis scalars  $h_{1,2,3}$  are expressed in terms of the physical scalars  $h_{a,b,c}$  as

$$\begin{aligned}
h_1 &= U_{1b} h_b + U_{1c} h_c - \frac{1}{\sqrt{2}} h_a, \\
h_2 &= U_{2b} h_b + U_{2c} h_c + \frac{1}{\sqrt{2}} h_a, \\
h_3 &= U_{3b} h_b + U_{3c} h_c,
\end{aligned} \tag{9.2}$$

where  $U_{ib}$  and  $U_{ic}$  are analytically tractable but complicated functions of  $\lambda_i$ 's,  $v$  and  $v_3$ , which we do not display. The condition  $v_1 = v_2$  immediately leads to  $U_{1b} = U_{2b}$  and  $U_{1c} = U_{2c}$ . The masses  $m_{a,b,c}$  of the three CP-even neutral scalars are a result of the diagonalization of the mass matrix.

A few things are worth noting at this stage: (i) Since  $\phi_{1,2,3}$  are all weak  $SU(2)$  doublets, their VEVs are related as:  $2v^2 + v_3^2 = v_{\text{SM}}^2$ , where  $v_{\text{SM}} \approx 246$  GeV.

(ii) One of the physical scalars is given by  $h_a = (h_2 - h_1)/\sqrt{2}$ , i.e. there is no dependence on  $\lambda_{\{1,\dots,8\}}$  or on the VEVs.

(iii) We randomly vary the *magnitude* of the  $\lambda_i$ 's in the range  $[0, 1]$ , although slightly larger (but  $< 4\pi$ ) values of  $|\lambda_i|$  would have still kept the couplings perturbative. We *accept* a given set of  $\{\lambda_1, \dots, \lambda_8, v\}$  only if it satisfies the minimization conditions.

(iv) It has been suggested in [24] that with *order one* Yukawa couplings, the ratio  $v_3/v \sim 0.1$  reproduces the correct Cabibbo angle in the quark sector. We require  $v_3/v \geq 0.6$  to ensure that  $m_b^2$  stays above the accepted limit. Since  $h_b$  and  $h_c$  have similar gauge and Yukawa properties, quite different from those of  $h_a$  (see discussions later), we show the mass splitting  $(m_c - m_b)$  against  $m_b$  in Fig. 9.1(a), and the relation between  $m_b$  and  $m_a$  in Fig. 9.1(b).

(v) The two CP-odd neutral scalars  $\chi_a$  and  $\chi_b$  can be light, with  $\chi_a$  being lighter than 100 GeV and  $\chi_b$  being lighter than 250 GeV. The charged scalars  $h_a^+$  and  $h_b^+$  are less restricted: They can be lighter than 250 GeV and 300 GeV respectively.

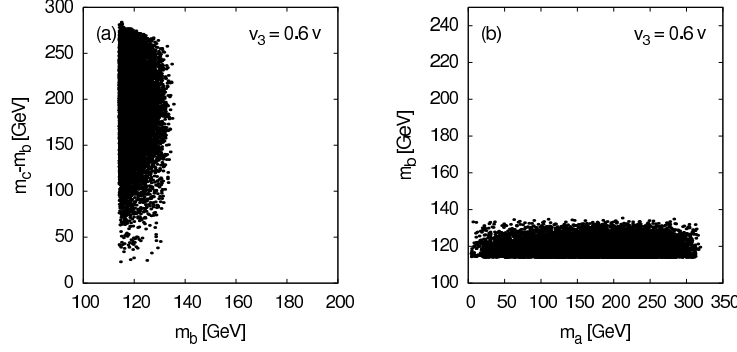


Figure 9.1. Results of a random search for allowed scalar masses for a fixed  $v_3/v = 0.6$ . In the left panel (a), we exhibit the splitting  $(m_c - m_b)$  for different choices of  $m_b$ . In the right panel (b), we show the allowed range of  $m_a$ .

### 9.3. Scalar couplings to gauge and matter fields

The couplings of the symmetry basis  $h_i$  to  $W^\pm$  and  $Z$  are modified by a factor of  $v_i/v_{\text{SM}} < 1$  compared to their SM expressions. In terms of the mass basis scalars, we observe the following: (i) The coupling of  $h_b$  to  $W^+W^-$  (or,  $ZZ$ ) is the corresponding SM coupling multiplied by  $(2vU_{1b} + v_3U_{3b})/v_{\text{SM}}$  and the corresponding factor for  $h_c$  is  $(2vU_{1c} + v_3U_{3c})/v_{\text{SM}}$ . (ii) The scalar  $h_a$  does not have  $h_aZZ$  or  $h_aWW$  couplings, unlike the other two scalars. This can be understood as follows. The gauge couplings of  $h_i$  arise from the linear expansion  $\phi_i^0 = v_i + h_i$  in the kinetic term. Since  $v_1 = v_2 = v$ , the combination  $(h_1 + h_2)$  will couple to gauge bosons as proportional to  $v$ . The orthogonal combination  $(h_2 - h_1)$  that represents the physical scalar  $h_a$  – see Eq. (9.2) and point *ii* following it – will not have the usual scalar-gauge-gauge vertex. The four-point  $h_a^2ZZ$  and  $h_a^2WW$  couplings will, however, exist.

The couplings of  $h_{b,c}$  to the quarks and leptons depend on the parameters  $v, v_3, \lambda_i$  and  $f_i$  (or,  $g_i^{u/d}$ ), while the couplings of  $h_a$  to fermions depend only on  $f_i$  (or,  $g_i^{u/d}$ ). The *physical* scalar couplings to the *mass basis* fermions are given by the following Yukawa matrices, displayed for the charged leptons as an example (the structures for the quark sector are similar *modulo* Cabibbo mixing):

$$Y_{h_a} = \begin{pmatrix} 0 & 0 & Y_{eL\tau R}^a \\ 0 & 0 & Y_{\mu L\tau R}^a \\ Y_{\tau L e R}^a & Y_{\tau L \mu R}^a & 0 \end{pmatrix}, \quad Y_{h_{b,c}} = \begin{pmatrix} Y_{eL e R}^{b,c} & Y_{eL \mu R}^{b,c} & 0 \\ Y_{\mu L e R}^{b,c} & Y_{\mu L \mu R}^{b,c} & 0 \\ 0 & 0 & Y_{\tau L \tau R}^{b,c} \end{pmatrix}. \quad (9.3)$$

The position of the zeros in the matrices deserves some attention. It turns out that  $h_{a,b,c}$  have off-diagonal fermion couplings at tree level due to the absence of any natural flavor conservation.  $h_a$  couples *only* off-diagonally and one of the two fermions has to be from the third generation.  $h_{b,c}$  couple diagonally as in the SM, *but also* possess small, numerically insignificant, off-diagonal couplings involving the first two generations.

In the present case,  $S_3$  symmetry, under which both scalars and fermions transform nontrivially, is instrumental in suppressing the off-diagonal couplings. To provide intuitive understanding, we take, as an example, only the two-flavor  $\mu\text{-}\tau$  sector together with two neutral scalars  $h_1$  and  $h_2$ . It is not difficult to see that the combination  $(h_2 - h_1)$ , which corresponds to  $h_a$ , couples only off-diagonally, as mentioned earlier. But the other combination  $(h_2 + h_1)$ , which corresponds to  $h_{b,c}$  following Eq. (9.2), couples only diagonally to physical

$\mu$  or  $\tau$ . When we consider the quark sector,  $\mu$  and  $\tau$  would be replaced by second and third generation quarks which will have CKM mixing. This will yield off-diagonal entries for  $h_{b,c}$  couplings to quarks suppressed by the off-diagonal CKM elements. The same happens for off-diagonal couplings involving the first two generations as well. In some setups where the fermion transformations under  $S_3$  are not appropriately adjusted, the off-diagonal Yukawa couplings may become order one which induce sizable neutral scalar mediated rare processes, like  $K_L \rightarrow \mu e$  or  $K_L \rightarrow 2\pi$ , at tree level. This requires those neutral scalars to lie beyond several TeV. But in our case, once we adjust the couplings to reproduce the fermion masses and mixing, the off-diagonal Yukawa couplings are determined too. The largest of them corresponds to  $\bar{c}_L t_R h_a$ , which is about 0.8. The second largest off-diagonal coupling is that for  $\bar{s}_L b_R h_a$ , and is about 0.02. The next in line is  $\bar{\mu}_L \tau_R h_a$ , whose coefficient is about 0.008. The others are orders of magnitude smaller, and are of no numerical significance. Although FCNC processes like  $B_d - \bar{B}_d$  and  $B_s - \bar{B}_s$  mixings proceed at tree level, the contributions are adequately suppressed even for light scalar mediators.

#### 9.4. Collider signatures

The perturbativity condition  $|\lambda_i| \leq 1$  and the requirement  $m_{b/c} \geq 114 \text{ GeV}$  (for which we set  $v_3/v \simeq 0.6$ ) yields  $m_b$  in the neighbourhood of 120 GeV and  $m_c$  within 400 GeV – see the scatter plots in Fig. 9.1. Both  $h_b$  and  $h_c$  would decay into the *usual*  $ZZ$ ,  $WW$ ,  $b\bar{b}$ ,  $\gamma\gamma$ ,  $\dots$  modes, but the dominant decay mode of  $h_b$  (or  $h_c$ ) for the case of  $m_a < m_b/2$  (or  $m_a < m_c/2$ ) would be into  $h_a h_a$ . Recall that the existing limits on the Higgs mass depend crucially on the gauge coupling of the Higgs. Since  $h_a ZZ$  or  $h_a WW$  couplings are nonexistent, the mass of  $h_a$  is unconstrained, i.e.  $m_a$  can be lower than 114 GeV or larger than 200 GeV. We numerically calculate the strength of the  $h_b h_a h_a$  coupling from the set of *acceptable* parameters characterizing the potential, and introduce a parameter  $k$  which is the ratio of the  $h_b h_a h_a$  coupling and the  $h_b WW$  coupling. The magnitude of  $k$  depends on the choice of  $\lambda_i$  and  $v_3$ . Assuming  $m_a = 50 \text{ GeV}$ , we obtain  $k$  in the range of (5 – 30). Just to compare with a 2HDM for illustration, the corresponding  $k$  value, when the heavier Higgs weighing around 400 GeV decays into two lighter Higgs weighing 114 GeV each, is about 10.

In Fig. 9.2(a) we have plotted the branching ratio of  $h_b \rightarrow h_a h_a$  as a function of  $m_b$  for two representative values  $m_a = 50, 75 \text{ GeV}$ , and for  $k \sim 5$  and 30, which correspond to the smallest and largest  $k$  obtained from the set of *accepted* scalar parameters. We observe that till the  $WW$  or  $ZZ$  decay modes open up, the branching ratio  $h_b \rightarrow h_a h_a$  is almost 100%.

As Fig. 9.2(b) suggests, as long as  $m_a < m_t$ ,  $h_a$  will dominantly decay into jets, and one of them can be identified as the  $b$ -jet. The branching ratio of  $h_a \rightarrow \mu\bar{\tau}$  is, nevertheless, not negligible (about 0.1). As shown in Fig. 9.2(c), for  $m_a \ll m_t$ , the branching ratio of  $t \rightarrow h_a c$  is quite sizable, which decreases with increasing  $m_a$ . It may be possible to reconstruct  $h_a$  from  $h_a \rightarrow \mu\bar{\tau}$ . In fact, a light  $h_a$  would be copiously produced from the top decay at the LHC. On the other hand, if  $m_a > m_t$ , as can be seen again from Fig. 9.2(b),  $h_a$  decays to  $t\bar{c}$  with an almost 100% branching ratio.

If  $k$  is large, then there is an interesting twist to the failed Higgs search at LEP-2. In this case,  $h_b \rightarrow h_a h_a$  would overwhelm  $h_b \rightarrow b\bar{b}$ , and hence the conventional search for the SM-like scalar ( $h_b$ , as the lighter between  $h_b$  and  $h_c$ ) would fail. This is similar to what happens in the next-to-minimal supersymmetric models, when the lightest scalar would dominantly decay into two pseudoscalars, and each pseudoscalar would then decay into  $2b$  or  $2\tau$  final states. In view of these possible  $4b$  or  $4\tau$  Higgs signals, LEP data have been reanalyzed putting constraints on the Higgs production cross section times the decay branching ratios. The possibility of the Higgs cascade decays into  $4j$  ( $j = \text{quark/gluon}$ ),  $2j + 2$  photons and 4 photons has been studied too. From a study of  $4b$  final states, a limit  $m_h > 110 \text{ GeV}$  (for a SM-like Higgs) has been obtained (references can be found in [22]). From all other cascade decays the limit on  $m_h$  will be considerably weaker. Our  $h_a$  has the special feature that it has only off-diagonal Yukawa couplings involving one third-family fermion. If  $h_b$  is lighter than the top quark, it would decay as  $h_b \rightarrow h_a h_a \rightarrow 2b + 2j$ , and into  $b + 1j + \mu + \tau$ , the latter constituting a spectacular signal with two different lepton flavors  $\mu$  and  $\tau$ . The standard  $2b$  and cascade  $4b$

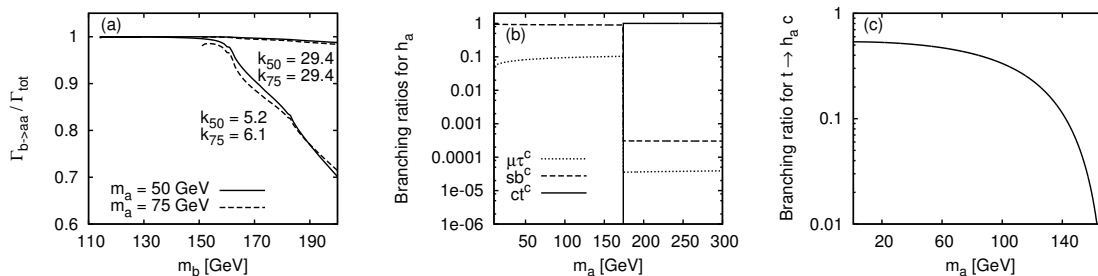


Figure 9.2. (a) Branching ratio of  $h_b \rightarrow h_a h_a$  for two representative values of  $m_a$ , and in each case for smallest / largest values of  $k$  in the set of accepted scalar parameters which compares the strength of the  $h_b h_a h_a$  coupling to the strength of  $h_b WW$  coupling; (b) branching ratios for the decay of  $h_a$ , and (c) branching ratio of the top quark decay into  $h_a$  and charm quark.

decay searches are not sensitive to our final states, and so a value of  $m_b$  much lighter than 110 GeV is not ruled out.

## 9.5. Conclusions

The discrete flavor symmetry  $S_3$ , besides successfully reproducing fermion masses and mixing, provides an extended Higgs sector having unconventional decay properties. We assume all the couplings to be real, and do not deal with the possibility of CP violation in this paper. The potential has been minimized requiring maximal mixing for the atmospheric neutrinos. In our setup, there are two scalars which are SM Higgs like, *except* that each of them can have a dominant decay into the third ( $h_{b,c} \rightarrow h_a h_a$ ). The latter, i.e.  $h_a$ , has no  $h_a VV$ -type gauge interactions, and has *only* flavor off-diagonal Yukawa couplings with one fermion from the third generation. It is not unlikely that by evading the conventional search strategies, both  $h_b$  and  $h_a$  are already buried in the existing LEP and Tevatron data.

## REFERENCES

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