Matching characteristics of the physically short linear impedance transformer

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Abstract: In contrast to the conventional quarter-wavelength transformer for matching two transmission lines of different characteristic impedance, it has been found that the length can be significantly shortened by using two stratified line sections in cascade. The frequency response of this short-length transformer (SLT) is investigated and it is shown that asymmetry and a reduction in matching bandwidth is the price to be paid for a shorter length. A comparison has also been made with the short-step transformer of Matthaei, which also provides a reduction in length.

1 Introduction

Conventionally, to match a transmission line of characteristic impedance Z_A to one of characteristic impedance *Z,* at a wavelength &, another transmission line of characteristic impedance $(Z, Z_B)^{1}$'2 and length I = Ad4 between the two is used. A perfect match is obtained at the frequency at which the length is exactly AD4, while mismatch occurs at both lower and higher frequencies. The mismatch characteristics, best displayed by a plot of lrI^2 or 1 - $lr1^2$, r being the reflection coefficient, against electrical length *8 = 2NIIA,* is symmetrical about *8* = *d2* (corresponding to A = &, at which T = 0).

McGinn and Moran [l] proposed an alternative matching section, shown in Fig. 1, which uses two equal-length sections of the same characteristic impedances as those of the lines to be matched, but in an altemating manner. They demonstrated, by analysis, that the maximum electrical length of the matching section required is *d3,* which is two thirds of the quarter-wave transformer (QWT). They did not, however, investigate the frequency response of this short-length transformer (SLT). The present investigation was undertaken to find out what price one has to pay for achieving the advantage of a shorter length. The results show that asymmetry of the matching characteristics and a reduction in matching bandwidth are the main disadvantages of the SLT as compared with the QWT.

In 1966, Matthaei [2] investigated what he called the short-step Chebyshev impedance transformer, consisting of an even number of transmission line sections, each of length W 1 6 . The two-section case has an electrical length of *d4* and is therefore a competitor to the SLT. This configuration is referred to as the SST, and its performance is compared with that of the SLT.

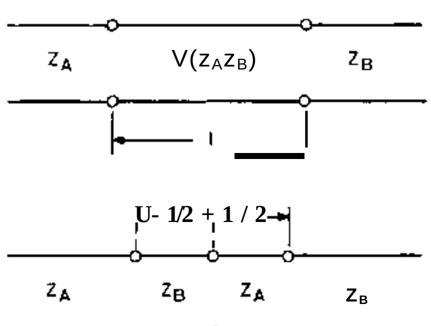

Fig. 1 *Short-length trmformer (SLT)*

2 Frequency response of SLT

By routine analysis of Fig. 1 we get the following expression for Irj^2:

$$iri^2_{1^{\bullet} 2} = \frac{-a-2}{a+2} \frac{[I-(a+1)\tan^2(O/2)I^2}{[I-(a-1)\tan^2(O/2)]^2+4\tan^2(O/2)} \quad (1)$$

where *a* = *r* + *lir* and *Y*= *ZdZ,.* Note that the minimum value of *a* is 2, which corresponds to *r* = 1 i.e. *2,* = *Z,* for which no matching section is needed. Observe that Irl' = 0, i.e. perfect matching occurs when *8* = 8, where

$$\tan(\sim_o/2) = *1/J;\overline{y+I} \quad (2)$$

The positive sign in eqn. 2 corresponds to

$$0_0 = 2\tan^{-'}(1/J\overline{X+}\ ) \quad (3)$$

Since the minimum value of *a* is 2, the maximum value of *8,* is 2 $\tan^{-'}$ (lid3) = *D3,* as observed in [l]. The negative sign in eqn. 2 corresponds to a value of *8,* equal to *2n* minus that given by eqn. 3, which falls outside the range 0 to *a,* and hence is of no interest. The values of JIT^2 at *8* = 0 and *8*= *n* are, respectively,

$$|\Gamma_0|^2 = \frac{\alpha-2}{\alpha+2} \quad (4)$$

and

$$|\Gamma_\pi|^2 = \frac{\alpha-2}{\alpha+2}\left(\frac{\alpha+1}{\alpha-1}\right)^2 > |\Gamma_0|^2 \quad (5)$$

By differentiating eqn. 1 with respect to *8,* it can be shown

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that Irl' does not have any maximum or minimum in the range $0 < 8 < n$ excepting the minimum at 8, given by eqn. 3 where $1JJ^2$ reaches the value zero. Hence lFI^2 decreases monotonically from $|1`_0|^2$ to zero at *0,* and then monotonically increases to the value *Ir,J'.* Fig. 2 shows a typical characteristic where the attenuation is plotted in decibels, defined by

$$i_A = -ioio_{g},(i- pi^2) \qquad (6)$$

against *Bl N* for $r = 8$.

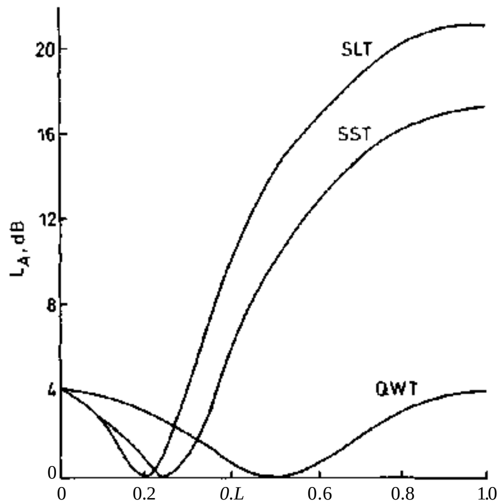

Fig. 2 *Matching characteristicx of QW, SLT and SSTfor r = 8*

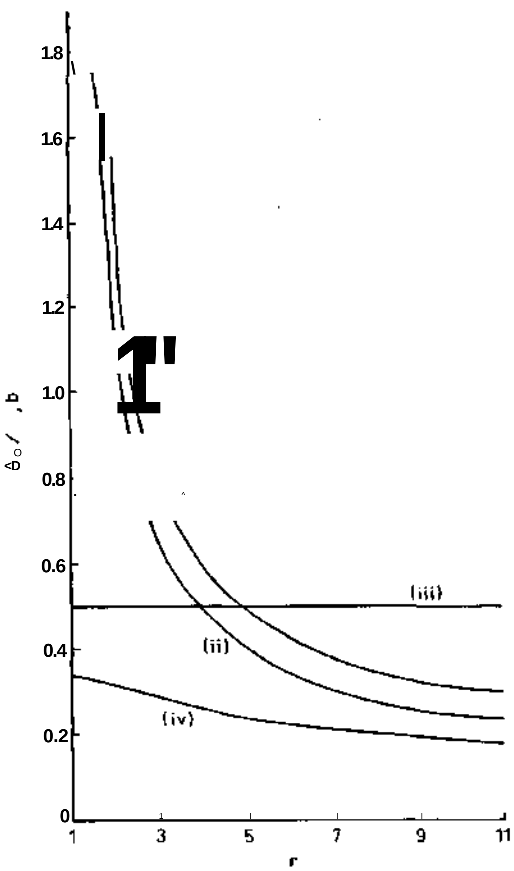

Fig. 3 *Plots of B&andb against rjOr QWTandSLT*
(i) *b* (QWT); (ii) b (SLT); (iii) @y'ir(QWr); (iv) Adn(SLT)

The matching bandwidth *B* can be defined for a chosen passband tolerance or return loss. In this paper l0dB retum loss is chosen for this definition, which corresponds to $1rI^2 = 0.1$ and a passband tolerance of 0.4676dB. Also, instead of B, the fractional bandwidth $h = Ble_0$ is used as the index of performance. Fig. 3 shows the variation of *h* and B_o with *r* for the SLT.

3 Comparison with the QWT

For the conventional QWT, standard analysis [3] shows that the fractional bandwidth for lOdB retum loss is given by

$$6 = 2[1 -(2/7r)tan^{-1}(\overline{\&^{G}T5/2})] \qquad (7)$$

A plot of *h* against *r* is included in Fig. 3, which clearly shows that the SLT achieves a reduced length at the cost of a reduction in bandwidth. However , this reduction is never more than 22"h in the range $2 < r < 11$. Note that for $1 < r$ s 1.925, *h* has a value of 2 because $/r,I^2 = lr,I^2$ s 0.1. Fig. 2 shows the attenuation characteristics of the QWT for $r = 8$.

4 Comparison with SST

As mentioned in Section I, the two-section SST of Matthaei [2] is a competitor to the SLT. For comparing the performance of these two impedance transformers analyse Fig. 1 again but with the characteristic impedances $Z_A z_I$ and *Z_ArlZ,,* instead of *Z,* and *Z,,* respectively, in the matching part, and obtain the following expression for lr1':

$$\frac{[illegible] + (z_1^4 - r)\tan^2(\theta/2)]^2}{[illegible]/2)}$$

Since $(TI^2$ is required to be zero when *8* assumes the value *d* 4, then

$$z;(i - r.) + (zt - T) \tan^2(7r/8) = o \qquad (9)$$

The positive solution of this quadratic equation gives the required value of zI^2. For $r = 8$ for example, *z1* is computed as 6.402. Using this value of z_1 in eqn. 8, the attenuation has been computed fix various values of *8* and the results are shown plotted in .Fig. 2.

Note that in the tables given in [2], z_1 is listed as a function of both r and the fractional bandwidth. The reason for this is that the centre frequency of the passband is not taken as d4, but is 'defined, arbitrarily, as the arithmetic mean of the bandedges, and the fractional bandwidth has been computed on tlhis basis. While this facilitates treatment of the general case with four or more sections, no such arbitrary definition is necessary for the two-section SST. For comparison, therefore, the values of the fractional bandwidth for the SLT are recalculated on the same basis. It is a matter of odd coincidence that the results so obtained are almost jdentical to those of the SST for *r* in the range 1.5 to 11. Also note that the electrical length of both the SLT and thce SST are the same i.e. n14 when $r = 4.6$. Computing z_1 from eqn. 9 for this value of r gives, precisely, $z_1 = 4.6$, i.e. the SLT and the SST become identical when $r = 4.6$.

In view of the preceding discussion it is concluded that for the same fractional bandwidth, the SST will require a shorter length than the SLT when $r < 4.6$, while for $r > 4.6$, the reverse will be the case. While the length of the SST is constant at *~14,* the SLT continues to be shorter and shorter as *r* increases, tending to zero as *r* - W. On the other hand, the maximum reduction in length provided by the SST for *r* in the range 1 to 4.6 is only nil2 i.e. 25%.

5 Concluding remarks

From the results presented it is clear that the SLT suffers from two main disadvantages compared to the QWT *viz.* that the matching characteristics are asymmetrical and that there is a reduction in the matching bandwidth. However, the advantages of a smaller total length, and the fact that no third line with a different characteristic impedance is needed for matching, may outweigh the disadvantages, at least in some situations.

With regard to the SST, if length is the only consideration, clearly it is to be preferred over the SLT for $r < 4.6$. However, besides length, one should also consider the fact that in the SST, two line sections of nonstandard characteristic impedances for matchmg standard lines would usually be required. In the SLT, on the other hand, line sections which have the same characteristic impedances as the lines to be matched are used. Thus no non-standard lines have to be fabricated, and the technique can be applied to coaxial cables, twin lines, microstrips and fin lines with much less difficulty than the QWT or the SST. Another advantage of the SLT over the SST is the simplicity of its design as compared with the table-based design of the SST.

6 Acknowledgment

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7 References

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