# Electron delocalization in disordered films induced by magnetic field and film thickness

R. K. Brojen Singh and Deepak Kumar
*School of Physical Sciences, Jawaharlal Nehru University, New Delhi 110067, India*
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We have studied the delocalization transition of noninteracting electrons in disordered thin films induced by magnetic field and film thickness. We also report results for two-dimensional systems. We have used for this purpose (i) a numerical technique based on transfer-matrix method for quasi-one-dimensional systems; (ii) self-consistent theory of localization for weak fields generalized to situations lacking time-reversal invariance. Numerical results provide strong evidence for a zero-temperature insulator-to-metal transition (MIT) with both field and film thickness. In self-consistent theory we adopt two procedures which give different results on MIT induced by field, temperature, and thickness. The variance between numerical and analytical results is analyzed.

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## I. INTRODUCTION

In this paper we examine the localization and transport behavior of noninteracting electrons in weakly disordered two-dimensional layers and thin films in the presence of a perpendicular magnetic field. The two-dimensional electron systems (2DES) have continued to be a source of very rich and fascinating physics for the past twenty-five years. The most outstanding example of this is the celebrated quantum Hall effect (QHE).[1] A more recent one is the vanishing of longitudinal resistance of 2DES by application of microwaves in resonance with the cyclotron frequency.[2] These and other phenomena in 2DES involve an interplay of one-electron Landau levels, disorder-induced localization, and electron-electron interactions. The present work is a contribution to elucidate one aspect of this complex physics, namely, the question of insulator-to-metal transition (MIT) in films induced by magnetic field and by increasing film thickness.

To place our work in perspective to the vast amount of work done in this area, we review the main results of relevance. First, we recall the remarkable prediction of the scaling theory by Abrahams *et al.*,[3,4] according to which all single-particle states in a two-dimensional disordered layer of electrons are localized, no matter how weak is the disorder. Physically this result is understood to arise from the quantum interference of amplitudes of time-reversed paths, which leads to an enhanced probability for the particle to return to its starting point.[5–8] The two important physical consequences of this result that have been verified are logarithmic temperature dependence of resistance[9] and negative magnetoresistance.[4,8,10,11] The latter effect supports the interference picture in a rather transparent way. The field introduces a phase difference in the time-reversed paths, thereby weakening the interference effect and the localization tendency.

The effect of field on the localization characteristics of the 2D states is a complex one due to formation of Landau bands. The simple understanding of QHE is based on the existence of extended states in the mid portion of each Landau band flanked by localized states and gaps (depending on the relative magnitude of disorder and field) on each side. From the intensive numerical studies[12–16] and theoretical arguments[17–20] it is now believed that at high fields, there is just one extended state in the middle of each Landau band. This raises the much debated question, as to what happens to the extended states as the field is reduced toward zero. Khmelnitskii[21] and Laughlin[22] argued that with the decreasing field the extended states float up in energy going beyond the Fermi energy. In this picture the system becomes insulating when the extended state at the middle of the lowest Landau level floats up to Fermi energy, which is expected to happen when the magnetic length $l_B^2=\hbar c/eB$ is of order $l$, the elastic mean-free path. The idea of floating up of extended states has recently been nicely confirmed numerically by Yang and Bhatt[16] in a tight-binding model, where the extended states get annihilated at the edges and get concentrated at the band center as the field is reduced.

Here our focus is on delocalization of states at weak disorders by a magnetic field, a topic which we feel has not been fully investigated. In particular, the question whether the field can induce an insulator-to-metal transition in two dimensions has not been unequivocally answered. For example, one might ask how well is the metal-insulator phase boundary given by Khmelnitskii's estimate. There have been experimental studies to investigate this and other aspects of this problem.[23,24] But the recent observation of metal-insulator transition in high mobility 2DES in the absence of the field has led to a great surge of interest in the problem. These observations made on metal-oxide-semiconductor field-effect transistors, show that an insulator to metal transition occurs on increasing the electron density.[25,26] The behavior of the system in fields perpendicular and in the plane of the film also shows rather unexpected features. Since here there is a clear violation of the scaling theory, strong arguments have been given to assign a primary role to electron-electron interactions in understanding this transition.[27,28]

Motivated by the above discussion, we feel that it is important to resolve the delocalization question for noninteracting electrons in the presence of the magnetic field for two-dimensional layers as well as films of finite thickness. This is continuation of our earlier studies, where we studied the problem without any magnetic field.[29,30] Specifically we examined the role of thickness in inducing delocalization of

 

electronic states and studied the crossover from two dimensions to three dimensions at weak disorders. We found that numerically there is strong evidence of insulator to metal transition with increasing thickness, and one can draw in disorder versus thickness plane a phase diagram delineating the metallic and insulating regimes.

The numerical method has the inevitable limitation due to system size, which is particularly severe at weak disorders, where the localization lengths become larger or comparable to system size. So we also studied the problem analytically by extending the self-consistent theory of Vollhardt and Wölfle[31,32] (VW) to finite-thickness films. This also allows us to check whether the weak-scattering corrections can give a good description of the 2D to 3D crossover in films with finite thickness. We found that the theory predicts only localized states, though localization lengths increase very rapidly across a boundary in thickness-disorder plane. Continuing these studies with magnetic field which also has the potential to delocalize states, seems to us a further step to understand the robustness of the interference phenomenon that leads to 2D localization.

In this paper, we present both numerical and analytical results using magnetic field and film thickness as parameters. The coupling of magnetic field to spins is ignored in these calculations, as the spin polarization plays little role in the absence of the interactions. For analytical results we use the self-consistent theory as was generalized by Yoshioka, Ono, and Fukuyama[33] to situations lacking time-reversal invariance. We use two self-consistent procedures to calculate the metal-insulator transition boundary in the parameter space of field, thickness and temperature. These two self-consistent procedures lead to different predictions in two dimensions, so it is worth examining them in a more general situation. The paper is organized as follows. In Sec. II, we present numerical results. In Sec. III, following the work of Vollhardt and Wölfle,[32] and Yoshioka *et al.*,[33] we set up the self-consistent equation for diffusion in thin films with perpendicular magnetic field. In Sec. III, we present the solutions for two-dimensional layers. This extends the work of Yoshioka *et al.* in some ways, like calculation of metal-insulator boundary when phase-coherence length is finite and expressions for conductivity in the self-consistent theory. In Sec. IV, the above set of results are presented for films of nonzero thickness. Finally, in Sec. V, we summarize all the results, and present a comparison of numerical results with analytical results obtained in two procedures.

## II. NUMERICAL RESULTS

We first report numerical studies on disorder-induced localization in two-dimensional layers and in films with few layers in presence of a uniform weak magnetic field. These studies are done on the Anderson model with nearest-neighbor hopping and site disorder. The disorder in on-site energies is described by a rectangular probability distribution of width $W$. The magnetic field $B$ is incorporated by putting a phase factor in the hopping matrix elements leading to the Hamiltonian

$$H=\sum_{j} \epsilon_j |\vec{j}\rangle\langle\vec{j}| + \sum_{j,j'} V_{j,j'}(B)|\vec{j}\rangle\langle\vec{j}'|, \tag{1}$$

where

$$V_{j,j'}(B)=V\exp[-2\pi i\alpha x(y-y')], \tag{2}$$

where $\vec{j}=(x,y)$ and $\alpha=Ba^2/(hc/e)$, which is the magnetic flux through a single plaquette measured in units of flux quantum.

Our numerical results have been obtained using the well-known technique proposed by McKinnon and Kramer[34] and Pichard and Sarma.[35] This procedure allows the calculation of the localization length at any given energy of a quasi-one-dimensional system in the shape of a long bar of thickness $b$ and width $m$. To extract information about the electron localization in a film of thickness $b$ and disorder $W$, one studies the localization length $\xi(W,\alpha,b,m)$ for states at the band center, as a function of the increasing width $m$. If states are localized then as $m$ increases, $\xi(W,\alpha,b,m)$ saturates to a value $\xi_\infty(W,\alpha,b)$, which may be regarded as the localization length for the band center state of the film. On the other hand, if states are extended $\xi(W,\alpha,b,m)$ grows faster than linear with $m$. The finite-size scaling arguments show that the linear growth marks the transition between localized and extended regimes. Further, the curves on either side of the transition for different values of $W$, $\alpha$ and $b$ can be collapsed on to a single set of curves by scaling $\xi$ and $m$ with suitably chosen values of $\xi_\infty(W,\alpha,b)$. On the metal side, $\xi_\infty(W,\alpha,b)$ is a length proportional to the resistivity.[34] Our calculations have been done for thicknesses of one to five layers. The highest value of the width $m$ is taken to be 30, while the length of the system is taken to be 10 000 layers.

To settle the localization question at weak disorders, where the localization lengths become very large, the use of scaling ideas to analyze the numerical results has been a very successful method. It has provided strong evidence to support the conclusions of the scaling theory in two dimensions in the absence of the field.[34] In the presence of the field, single parameter scaling theory does not apply, but Huckenstein and Kramer[14] have demonstrated that the scaling procedure employed earlier can still be used effectively. In the presence of the field, a new method based on the calculation of Chern numbers has been devised, which gives a much sharper delineation between extended and localized states.[16,36]

Since our interest is in the metal-insulator transition, we have limited studies only to the states at the band center, for which the scaling procedure of McKinnon and Kramer seems adequate. We first present results for a two-dimensional system ($b=1$). Figure 1 shows these results for seven values of disorder at a field corresponding to $\alpha=0.01$. One sees here a discernible trend toward delocalization at disorder values $W=3$ and 2. In Fig. 2 we show results at other values of $\alpha$, but now both axes of $\xi$ and $m$ have been scaled by appropriately chosen values of $\xi_\infty(W,\alpha,1)$. We see here that 12 curves for different values of $W$ and $\alpha$ can be collapsed into three curves by scaling. The full lines in the figure are best possible fits to the scaled data. One sees here the evidence

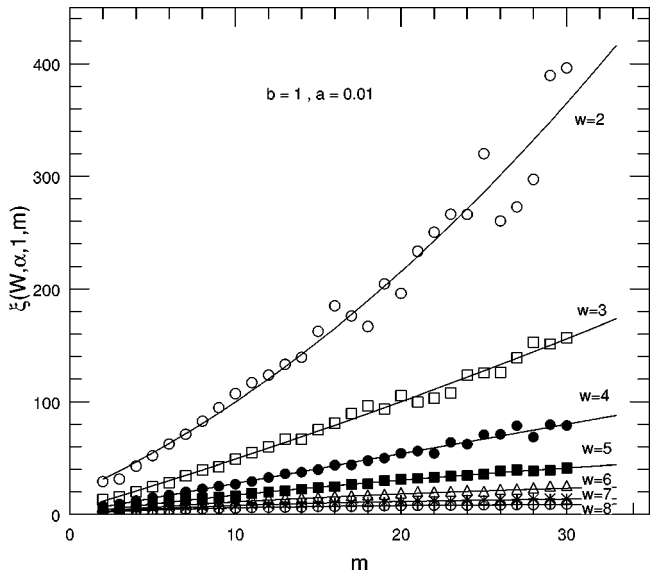


FIG. 1. Plots of $\xi(W,\alpha,1,m)$ with $m$ for two-dimensional layers at $\alpha=0.01$ for seven different values of $W$.

for a transition from saturating curves at large values of $W$ to curves rising faster than linear at smaller values of $W$. In order to further confirm the existence of both extended and localized regimes, we carry out the scaling analysis used by McKinnon and Kramer[34] by drawing plots of $\xi/m$ versus $\xi_\infty/m$. These are shown in Fig. 3. Though the data show some fluctuation, the evidence for two kinds of behaviors is quite apparent. The inset of Fig. 3 shows the phase boundary between localized and extended regimes in the $\alpha$-$W$ plane.

Next we show results for thin films. Figure 4 show plots of $\xi(W,\alpha,b,m)$ with $m$ for 1 to 5 number of layers for $W=5$ and $\alpha=0.01$. Here one sees delocalization with thickness. These results telly with our earlier results of similar

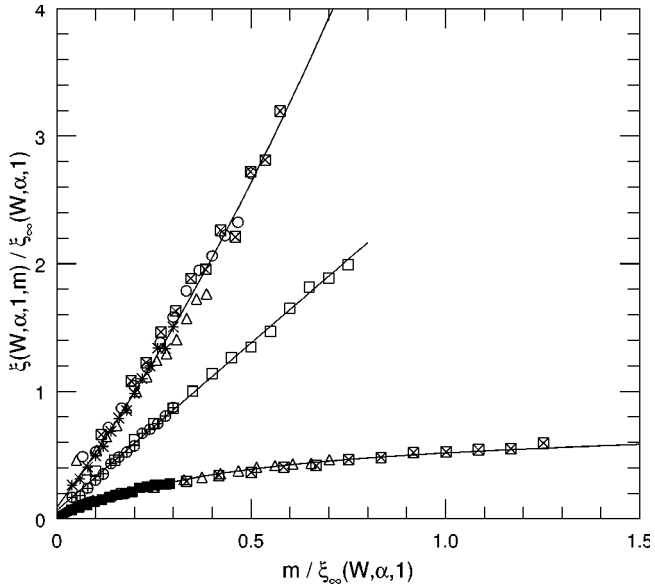


FIG. 2. Scaling plots of $\xi(W,\alpha,1,m)/\xi_\infty(W,\alpha,1)$ with $m/\xi_\infty(W,\alpha,1)$ in two dimensions. The values $\xi_\infty(W,\alpha,1)$ are chosen by trial to achieve scaling. Solid lines are the best algebraic fits.

delocalization in the absence of the magnetic field.[29,30] Again to confirm the existence of localized and extended regimes, we replot in Figs. 5 and 6 data of 14 curves at different values of $W$ and $\alpha$ in two scaled plots mentioned above. Here we see a fairly clear evidence of delocalization induced by thickness as well as field. In view of the limitation of the numerical method at weak disorders, we examine the problem analytically in the following section.

## III. SELF-CONSISTENT EQUATIONS FOR DIFFUSION

At the microscopic level, the scaling ideas have been supported by the self-consistent theory of localization proposed by Vollhardt and Wolfle.[31,32] This theory provides an account of both the metallic and insulating regimes, and is particularly suitable to deal with weak disorders. VW theory was extended by Yoshioka, Ono, and Fukuyama[33] to situations when the time-reversal invariance is absent.

In VW theory one calculates the density response function, which is related to particle-hole pair propagator. The density fluctuations propagate diffusively due to particle number conservation, with a diffusion constant $D(\vec{q},\omega)$, which gets strong size and dimension dependent quantum corrections from the vertex involving particle-particle channel. These corrections formally account for the enhanced interference between the time reversed paths and also have the same diffusive character when time-reversal invariance is present. Yoshioka *et al.* argued that when the time-reversal invariance is not present the particle-hole and particle-particle channels are not related and one needs two diffusion constants. The self-consistency relation of VW gets replaced by a set of two relations between the diffusion constants of the two channels. Applying these ideas to the two-dimensional system with a magnetic field, Yoshioka *et al.* found that there is no metal-insulator transition induced by the field at any disorder, i.e., the states remain localized in the presence of the field at the weakest disorder.

To set up the self-consistent equations for frequency-dependent diffusion constant, one first deals with the situation in the absence of the field, but without assuming time-reversal invariance. As mentioned above, this requires calculation of the density response function, which in turn involves evaluation of irreducible vertex function involving particle-particle channel. This irreducible vertex in turn involves scattering ladder in the particle-hole channel. Both these vertices have diffusive character. The self-consistent equations are obtained by replacing the bare diffusion constant that occurs in the perturbative treatment of these vertices by true frequency-dependent diffusion constants, $D^{ph}(\omega)$ and $D^{pp}(\omega)$ corresponding to the two channels. With a minor reformulation of Yoshioka *et al.*[33] one is led to the following equations.

$$\frac{D_0}{D^{ph}(\omega)}=1+\frac{1}{\pi\hbar N_F(d)}\frac{1}{L^d}\sum_q \frac{1}{-i\omega+D^{pp}(\omega)q^2}, \quad (3)$$

$$\frac{D_0}{D^{pp}(\omega)}=1+\frac{1}{\pi\hbar N_F(d)}\frac{1}{L^d}\sum_q \frac{1}{-i\omega+D^{ph}(\omega)q^2}, \quad (4)$$

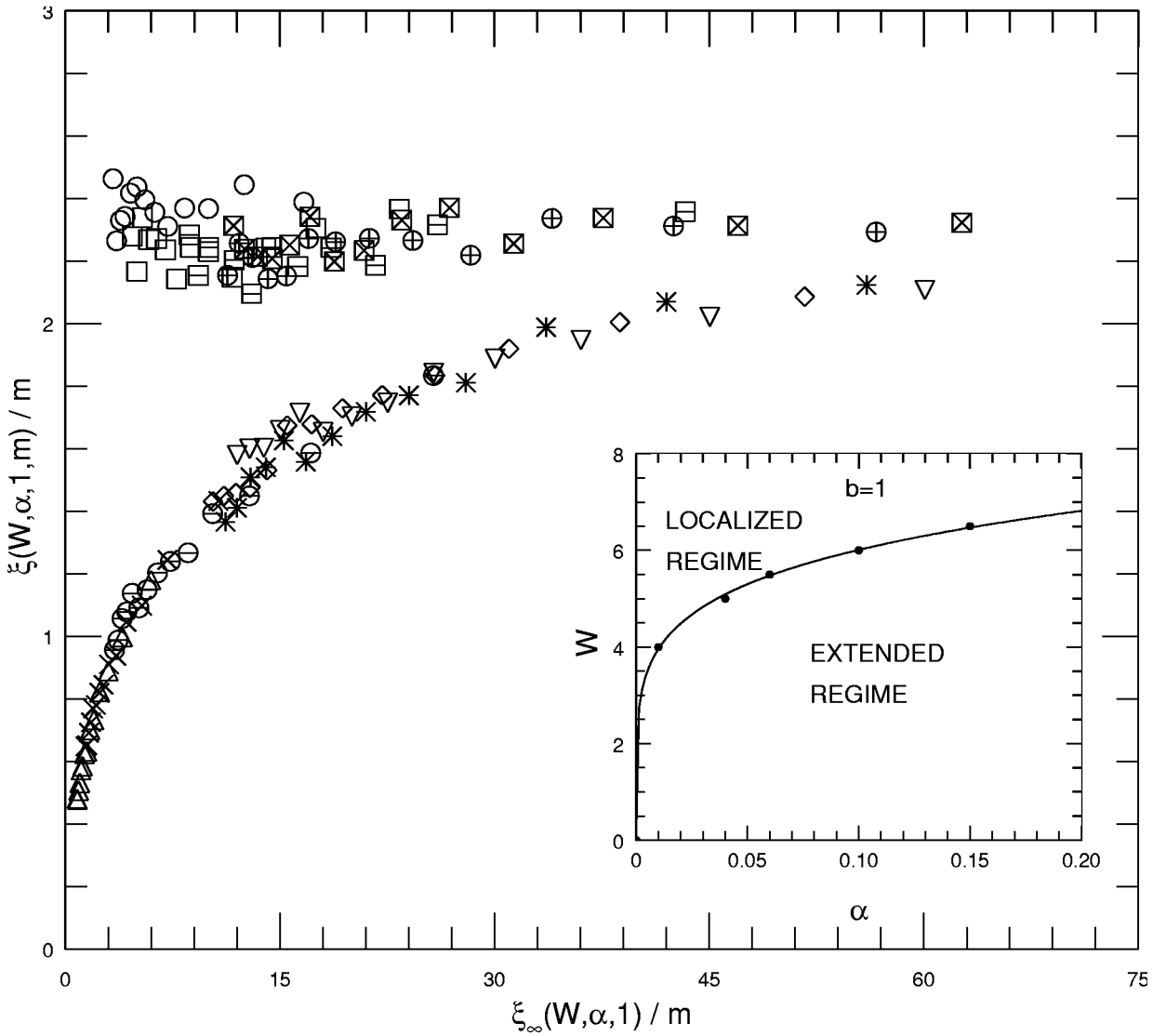


FIG. 3. Scaling plots of $\xi(W,\alpha,1,m)/m$ with $\xi_\infty(W,\alpha,1)/m$ in two dimensions. The values of $\xi_\infty(W,\alpha,1)$ are chosen to achieve the best scaling and they match with those of Fig. 2. The inset graph shows the phase boundary between localized and extended regimes in $W$-$\alpha$ plane.

where $D_0$ denotes the bare diffusion constant, $N_F(d)$ the density of states at the Fermi level in $d$ dimensions, and $L$ is the linear system size. When time reversal invariance is present, $D^{ph}(\omega)$ will be equal to $D^{pp}(\omega)$ and the above two equations reduce to a single equation recovering the VW self-consistent equation.

The conductivity $\sigma(\omega)$ and localization length $\xi(\omega)$ are related to $D^{ph}(\omega)$ by the following relations:

$$\sigma(\omega)=e^2N_FD^{ph}(\omega), \tag{5}$$

$$\xi^{-2}(\omega)=-\frac{i\omega}{D^{ph}(\omega)}, \tag{6}$$

$$\eta^{-2}(\omega)=-\frac{i\omega}{D^{pp}(\omega)}, \tag{7}$$

where we have also included another length, $\eta(\omega)$, corresponding to $D^{pp}(\omega)$. In the limit $\omega\rightarrow 0$, Eqs. (3) and (4) will give an insulating solution if $\xi^2(\omega)$ and $\eta^2(\omega)$ tend to limits that are real positive numbers, whereas Eqs. (3) and (4) give a metallic solution if $D^{ph}(\omega)$ and $D^{pp}(\omega)$ approach real positive limits as $\omega\rightarrow 0$. Thus we can solve for the two kinds of solutions by taking the zero frequency limit appropriately.

Now we extend these considerations to films of nonzero thickness $b$,[30] in the presence of a perpendicular magnetic field. A simple procedure to calculate backscattering corrections in thin films is the path-integral method. The key assumption of the method is that allowed paths in the presence of impurities can be taken to be random walks.[7,6] This allows the evaluation of the relevant propagator by solving the diffusion equation in the desired geometry. In our case this is just the probability for a particle to return to the origin after a time $t$. The presence of magnetic field modifies the diffusion equation by changing $\vec{\nabla}$ to $[\vec{\nabla}-(2ie/\hbar c)\vec{A}]$, where $\vec{A}$

is magnetic vector potential. This gives results equivalent to the semiclassical treatment of the magnetic field.[10] If we take $(\vec{r},z)$ to be the coordinates in the thin film, where $\vec{r}$ is a vector along the plane and $z$ is the coordinate along thickness, the probability of return, $P(\vec{r},z,t)$, to the starting point $(\vec{r},z)$ after time $t$ is

$$P(\vec{r},z,t)=\frac{1}{\pi b l_B^2}\sum_{n,m}\cos^2\left(\frac{\pi m z}{b}\right)\exp\left[-D_0\left(\frac{\pi^2 m^2}{b^2}+2\epsilon_n\right)t\right], \tag{8}$$

where $l_B$ is the magnetic length which is given by $l_B^2=\hbar c/2eB$. $\epsilon_n=(1/l_B^2)(n+\frac{1}{2})$, $n=0,1,2,3,\ldots$, are the discrete Landau-like diffusion eigenvalues. The boundary condition is that the current normal to the surfaces of the film vanishes.[7] Using this probability, extension of the self-consistent equations of localization takes the form

$$\frac{D_0}{D^{ph}(\omega)}=1+\frac{1}{D^{pp}(\omega)}\frac{1}{\pi^2\hbar l_B^2 N_F(3)b}\times\sum_{n=0}^{n_{max}}\sum_m\frac{1}{-\frac{i\omega}{D^{pp}(\omega)}+\frac{2n+1}{l_B^2}+\frac{\pi^2m^2}{b^2}+\frac{1}{L_\phi^2}}, \tag{9}$$

$$\frac{D_0}{D^{pp}(\omega)}=1+\frac{1}{D^{ph}(\omega)}\frac{2}{\pi\hbar N_F(3)bL^2}\times\sum_{q,m}\frac{1}{-\frac{i\omega}{D^{ph}(\omega)}+q^2+\frac{\pi^2m^2}{b^2}+\frac{1}{L_\phi^2}}. \tag{10}$$

Note that the diffusion constants, $D^{ph}(\omega)$ and $D^{pp}(\omega)$, depend on parameters $\lambda$ and $b$, but this dependence is not being explicitly written in the above and the following equations for reasons of brevity. The upper cutoff limit on the summation over $n$ in Eq. (9) is $n_{max}\approx\pi^2 l_B^2/2l^2$, where $l$ is the elastic mean-free path. In writing these equations we have also accounted for a finite phase coherence length $L_\phi$ which is important when we discuss transport at nonzero temperatures.[37] At low temperatures $L_\phi\propto T^{-p}$, where the index $p$ depends on the mechanism of inelastic scattering.

The important point to note is that the magnetic field affects only the first equation, which involves the vertex of the particle-particle channel. The momenta entering the calculation of particle-hole channel are such that the modification of $\vec{\nabla}$ to $[\vec{\nabla}-(2ie/\hbar c)\vec{A}]$ does not affect it.[33] These arguments are plausible only in the small field limit $l_B\gg l$. In view of the additional approximations that go into self-consistent theory in the presence of the field, we have also explored a simpler procedure in which the two diffusion constants have been set equal in Eq. (9). The merit of this procedure is that its conclusions regarding field delocalization agree with some earlier theoretical work[21,38] and our numerical work.

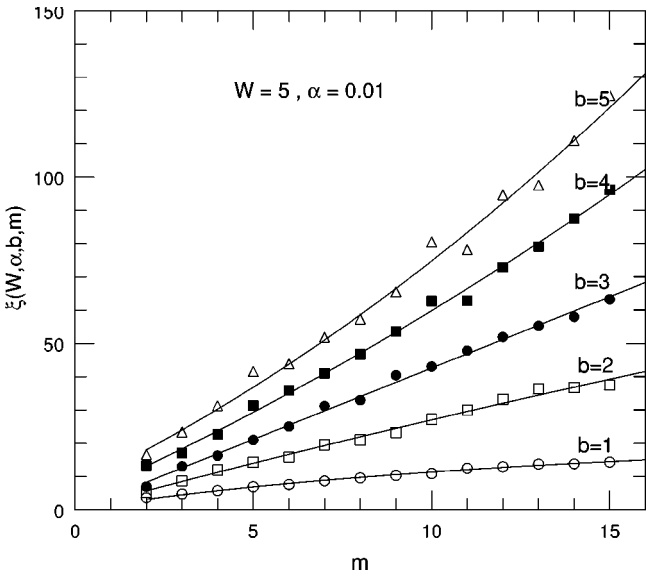


FIG. 4. The variation of $\xi(W,\alpha,b,m)$ with $m$ for films with five thicknesses, at fixed values of $W=5$ and $\alpha=0.01$.

It is convenient to write these equations in terms of dimensionless disorder parameter $\lambda=\hbar/2\pi E_F\tau=1/\pi k_F l$, where $\tau$ is the elastic mean-free time. To consider the crossover from two to three dimensions we record that $\hbar N_F(2)D_0=\lambda^{-1}$ and $\hbar N_F(3)D_0=4/(3\pi l\lambda^2)$. It is easily checked that Eqs. (9) and (10) have correct two-dimensional

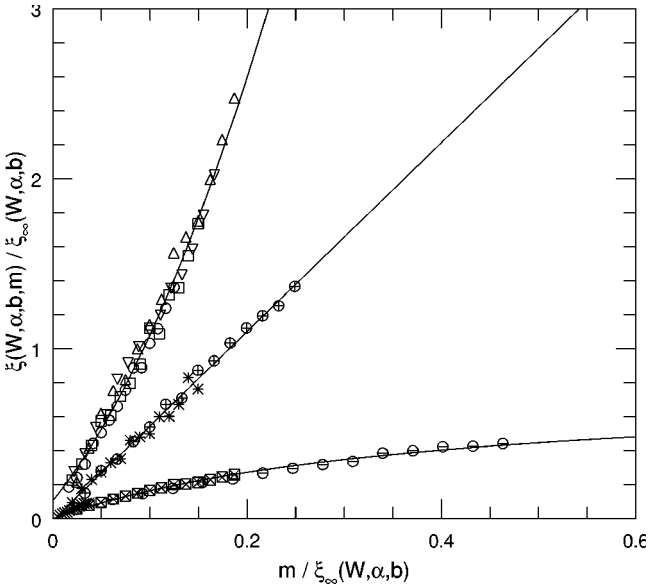


FIG. 5. Scaling plots of $\xi(W,\alpha,b,m)/\xi_\infty(W,\alpha,b)$ vs $m/\xi_\infty(W,\alpha,b)$ in films for different values of $b$, $W$, and $\alpha$. Solid lines are the best algebraic fits to the scaled data.

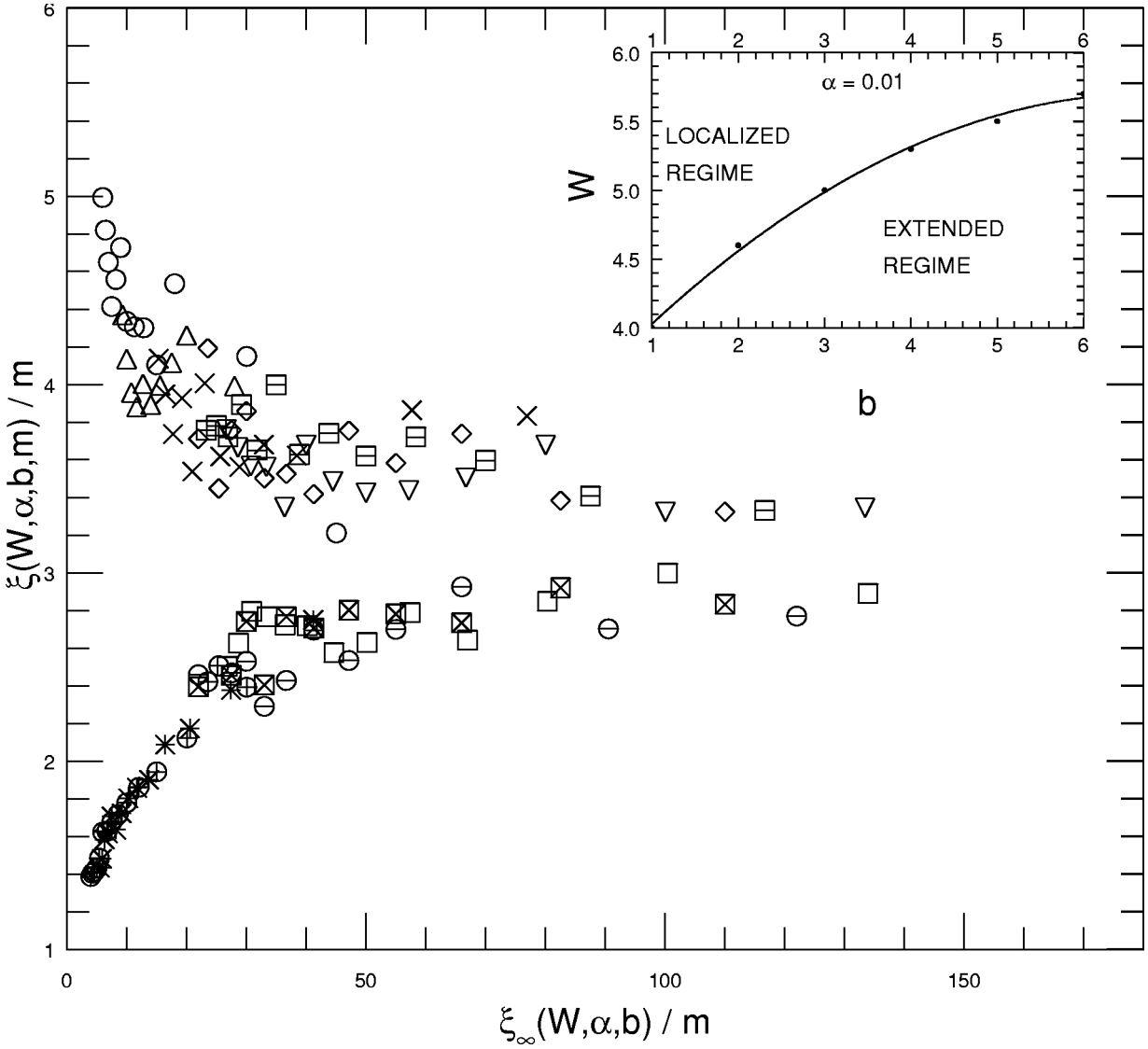


FIG. 6. Scaling plots of $\xi(W,\alpha,b,m)/m$ with $\xi_\infty(W,\alpha,b)/m$ in films for different values of $b$, $W$, and $\alpha$. The inset curve shows the phase diagram in $W$-$b$ plane showing the localized and extended regimes at $\alpha=0,01$.

limit as $b\to 0$, and three-dimensional limit as $b\to L$. In the following section we consider the 2D limit of the equations and their solutions.

### IV. SOLUTIONS IN TWO DIMENSIONS

The 2D limit is obtained by keeping only $m=0$ term and letting $bN_F(3)\to N_F(2)$. The main results were obtained by Yoshioka *et al.*[33] Here we summarize these and consider some more situations giving explicit results. We also give a comparison of these results with those obtained by setting diffusion constants in two channels equal, as discussed above. For the time being we drop the $L_\phi$ term in both the equations. We first look for insulating solutions by taking the limit $\omega\to 0$ and allowing for finite positive limits for $\xi(\omega)$ and $\eta(\omega)$. One reaches the following equations:

$$\left(\frac{\tilde{\eta}}{\tilde{\xi}}\right)^2=\frac{\lambda}{\pi^2\tilde{l}_B^2}\sum_{n=0}^{n_{max}}\frac{1}{\tilde{\eta}^{-2}+\dfrac{2n+1}{l_B^2}}, \tag{11}$$

$$\left(\frac{\tilde{\xi}}{\tilde{\eta}}\right)^2=\frac{2\lambda}{\pi L^2}\sum_q\ \frac{1}{\xi^{-2}+q^2}, \tag{12}$$

where $\tilde{\xi}=\xi/l$, $\tilde{\eta}=\eta/l$, $\tilde{l}_B=l_B/l$, $\tilde{b}=b/l$, $\tilde{L}=L/l$. Now eliminating $\tilde{\eta}$ from Eqs. (11) and (12), we obtain the following equation for $\tilde{\xi}$:

$$\left(\frac{1}{\lambda}\right)^2=\frac{f(\tilde{L},\tilde{\xi})}{4\pi^4}\sum_{n=0}^{n_{max}}\frac{1}{n+\dfrac{1}{2}+\dfrac{\lambda\tilde{l}_B^2}{4\pi^2\tilde{\xi}^2}f(\tilde{L},\tilde{\xi})} \tag{13}$$

$$\approx \frac{f(\tilde{L},\tilde{\xi})}{4\pi^4}\left[\psi\left(\frac{\pi^2\tilde{l}_B^2}{2}\right)-\psi\left(\frac{1}{2}+\frac{\lambda\tilde{l}_B^2}{4\pi^2\tilde{\xi}^2}f(\tilde{L},\tilde{\xi})\right)\right] \quad (14)$$

with

$$f(\tilde{L},\tilde{\xi})=\ln\left(\frac{\pi^2+\tilde{\xi}^{-2}}{\pi^2\tilde{L}^{-2}+\tilde{\xi}^{-2}}\right). \quad (15)$$

Here $\psi$ is the digamma function and we have used the fact that for small values of the field, $n_{max}$ is a large number.

First we note that in the limit $\tilde{L}\to\infty$, the right-hand side of Eq. (13) increases unboundedly with $\xi$, allowing for a solution for $\xi$ at all values of $\lambda$. This, as Yoshioka *et al.* had concluded, implies that even in the presence of the field the electronic states remain localized at the weakest disorder. Next we consider the situation at finite temperatures, where $L_\phi$ is finite. This is done by replacing $\tilde{L}$ by $\tilde{L}_\phi$ in Eq. (13). Now the right-hand side of Eq. (13) has a finite value as $\tilde{\xi}\to\infty$, which means that localized solutions can exist only for $\lambda>\lambda_2^c(B,L_\phi)$ where $\lambda_2^c(B,L_\phi)$ is the value at which $\tilde{\xi}=\infty$ and is given by

$$\lambda_2^c(B,L_\phi)=\frac{2\pi^2}{\sqrt{\ln(\tilde{L}_\phi)[\psi(n_{max})-\psi(\frac{1}{2})]}}$$

$$\approx\frac{2\pi^2}{\sqrt{\ln(\tilde{L}_\phi)[\ln(2\pi^2\tilde{l}_B^2)+\gamma]}}, \quad (16)$$

where $\gamma$ is the Euler's constant. Thus for finite $L_\phi$, the magnetic field does cause a delocalization transition in two dimensions.

As discussed above, we also compare these results with another procedure in which both the diffusion constants are set equal. We label this procedure as II and the earlier one as I to present the results. Now we obtain the following equation for the localization length ($L_\phi=\infty$).

$$\frac{1}{\lambda}=\frac{1}{\pi^2}\sum_{n=0}^{n_{max}}\frac{1}{2n+1+\frac{l_B^2}{\xi^2}}. \quad (17)$$

Since the right-hand side of Eq. (17) has a finite limit as $\xi\to\infty$, we clearly have a threshold disorder below which the states are extended even with $L_\phi=\infty$. This result is consistent with the zero-temperature delocalization implied in Khmelnitskii's work[21] and the global phase diagram.[38] In Fig. 7(a), we have plotted the threshold values of disorder as a function of the field $B$ obtained in two procedures. For procedure I, we have taken $\tilde{L}_\phi=100$ for comparison. The qualitative behavior is similar to the numerical results shown in the inset of Fig. 3.

Now let us consider metallic solution in 2D system for $\lambda<\lambda_2^c(B,L_\phi)$. We first consider procedure I. Taking $\omega\to0$ in Eqs. (9) and (10) and allowing finite values to $D_{ph}$ and $D_{pp}$, we reach the following results:

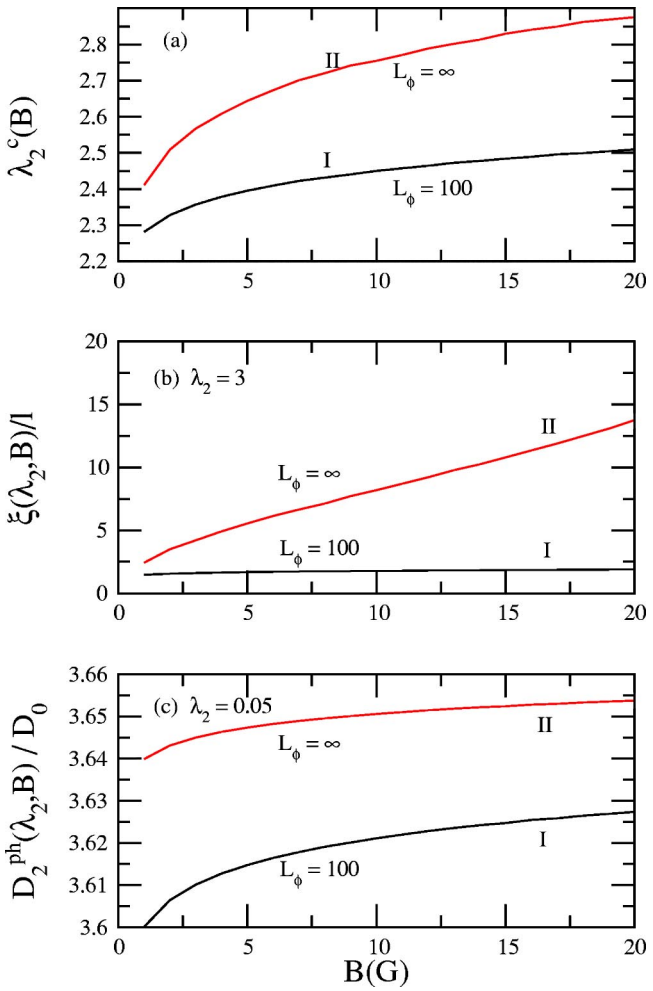


FIG. 7. (a) Plots of $\lambda_2^c(B)$, the critical value of disorder at which MIT occurs in two dimensions, with $B$ in the two procedures discussed in the text. (b) Variation of $\xi(\lambda_2,B)/l$ with $B$ for a fixed value of disorder $\lambda_2=3$ for a two-dimensional layer using the two procedures. (c) Plots of diffusion constant $D_2^{ph}(\lambda_2,B)/D_0$ for a two-dimensional layer with $B$ for a fixed value of disorder $\lambda_2=0.05$ (less than the $\lambda_2^c$) in the two procedures.

$$\frac{D_0}{D_2^{ph}}=1+\frac{D_0}{D_2^{pp}}\frac{\lambda}{2\pi^2}\sum_{n=0}^{n_{max}}\frac{1}{n+\frac{1}{2}}, \quad (18)$$

$$\frac{D_0}{D_2^{pp}}=1+\frac{D_0}{D_2^{ph}}\frac{\lambda}{2\pi^2}\ln(\tilde{L}_\phi). \quad (19)$$

From these equations the physical diffusion constant, $D^{ph}(\lambda,B,L_\phi)$, is easily extracted and one obtains for the conductivity

$$\sigma_2(\lambda,B,L_\phi)=e^2N_F(2)D_0\frac{1-\left(\frac{\lambda}{\lambda_2^c(B,L_\phi)}\right)^2}{1+\frac{\lambda}{2\pi^2}[\ln(2\pi^2\tilde{l}_B^2)+\gamma]}. \quad (20)$$

This result is a generalization of the perturbation formula for magnetoresistance.[10,11] From this expression the temperature and field dependence of the conductivity are easily seen. In Figs. (7b) and (7c), we have given the plots (marked I) of the variation of localization length and diffusion constant with magnetic field. For comparison we also plot these quantities as obtained by procedure II. The expression for conductivity in this procedure is given by

$$\sigma_2(\lambda,B)=e^2N_F(2)D_0\left[1-\frac{\lambda}{\lambda_2^c(B)}\right]. \tag{21}$$

The magnitudes in the two procedures are of the same order at this value of $L_\phi$, but procedure I gives smaller localization lengths. The notable point is that the two procedures have distinct predictions regarding metal-insulator transition. According to procedure I, only at nonzero temperatures a metal-insulator transition can be driven by field or disorder. This MIT should be observable as a change in temperature variation from logarithmic at higher temperature to a possibly Mott's variable-range-hopping form, $\exp[(T_m/T)^{1/4}]$. On the other hand procedure II suggests that MIT would be driven by magnetic field alone at zero temperature or a temperature range where $L_\phi$ is comparable to the system size. A straightforward extension of the formula allows us to include the temperature effect due to $L_\phi$ in procedure II also.

From these expressions it is straightforward to derive the critical behavior of localization length and conductivity around MIT line in $(B,\lambda)$ plane. We write these results when the transition line is approached along the field axis:

$$\xi^{-2}(\lambda,B)=C\left[\psi\left(\frac{B_0}{B}\right)-\psi\left(\frac{B_0}{B_c}\right)\right], \quad B<B_c, \tag{22}$$

$$\frac{\sigma_2(\lambda,B)}{\sigma_2^0}=G\left[\psi\left(\frac{B_0}{B_c}\right)-\psi\left(\frac{B_0}{B}\right)\right], \quad B>B_c, \tag{23}$$

where $B_0=hc/4el^2$, and $C$ and $G$ depend on the self-consistent procedure. Their values are

$$C=\frac{2\pi^2}{\lambda\tilde{l}_B^2\ln(\tilde{L}_\phi)\psi'\left(\frac{1}{2}\right)} \quad \text{I}, \tag{24}$$

$$=\frac{2}{\tilde{l}_B^2\psi'\left(\frac{1}{2}\right)} \quad \text{II}, \tag{25}$$

and

$$G=\frac{\lambda^2\ln(\tilde{L}_\phi)}{4\pi^2} \quad \text{I},$$

$$=\frac{\lambda}{2\pi^2} \quad \text{II}. \tag{26}$$

These expressions show that the critical exponent for conductivity and $\xi^{-2}$ with respect to the approach to the MIT boundary in the $\lambda$-$B$ plane is unity, but its amplitude being proportional to $B_c^{-1}$ increases with decreasing disorder.

We feel that the experimental investigation of this phenomenon is nicely extended and sharpened by use of film thickness also as a parameter. The following section is devoted to such considerations.

## V. RESULTS FOR FINITE-THICKNESS FILMS

In this section we give results for finite thickness films. Now Eqs. (9) and (10) are used as such. For the insulating regime, the $\omega\to 0$ gives

$$\left(\frac{\tilde{\eta}}{\tilde{\xi}}\right)^2=\frac{3\lambda^2}{8\pi\tilde{b}}\ g(\tilde{\eta},\tilde{b},B), \tag{27}$$

$$\left(\frac{\tilde{\xi}}{\tilde{\eta}}\right)^2=\frac{3\lambda^2}{8\pi\tilde{b}}\ h(\tilde{\xi},\tilde{b},\tilde{L}_\phi), \tag{28}$$

where, the functions $g$ and $h$ are given by

$$g(\tilde{\eta},\tilde{b},B)=\sum_n\left[\frac{\tilde{b}\coth\left(\tilde{b}\sqrt{\frac{2n+1}{\tilde{l}_B^2}+\tilde{\eta}^{-2}}\right)}{\sqrt{\frac{2n+1}{\tilde{l}_B^2}+\tilde{\eta}^{-2}}}+\frac{1}{\frac{2n+1}{\tilde{l}_B^2}+\tilde{\eta}^{-2}}\right], \tag{29}$$

$$h(\tilde{\xi},\tilde{b},\tilde{L}_\phi)=\ln\left[\sqrt{\frac{\pi^2+\tilde{\xi}^{-2}}{\pi^2\tilde{L}_\phi^{-2}+\tilde{\xi}^{-2}}}\ \frac{\sinh(\tilde{b}\sqrt{\pi^2+\tilde{\xi}^{-2}})}{\sinh(\tilde{b}\sqrt{\pi^2\tilde{L}_\phi^{-2}+\tilde{\xi}^{-2}})}\right]. \tag{30}$$

Then for $\tilde{\xi}$ one obtains

$$\left(\frac{1}{\lambda}\right)^4=\frac{9}{64\pi^2\tilde{l}_B^2\tilde{b}^2}\ h(\tilde{\xi},\tilde{b},\tilde{L}_\phi)\ \sum_{n=0}^{n_{max}}\left[\frac{\tilde{b}\ \coth(\tilde{b}\sqrt{A})}{\sqrt{A}}+\frac{1}{A}\right], \tag{31}$$

where

$$A=\frac{2n+1}{\tilde{l}_B^2}+\frac{3\lambda^2}{8\pi\tilde{b}\tilde{\xi}^2}\ h(\tilde{\xi},\tilde{b},\tilde{L}_\phi). \tag{32}$$

Considerations similar to those given above again show that in the limit $L_\phi\to\infty$, the solution for $\xi$ can be found for any values of magnetic field and thickness. This means that at zero temperature, neither field nor thickness can induce an

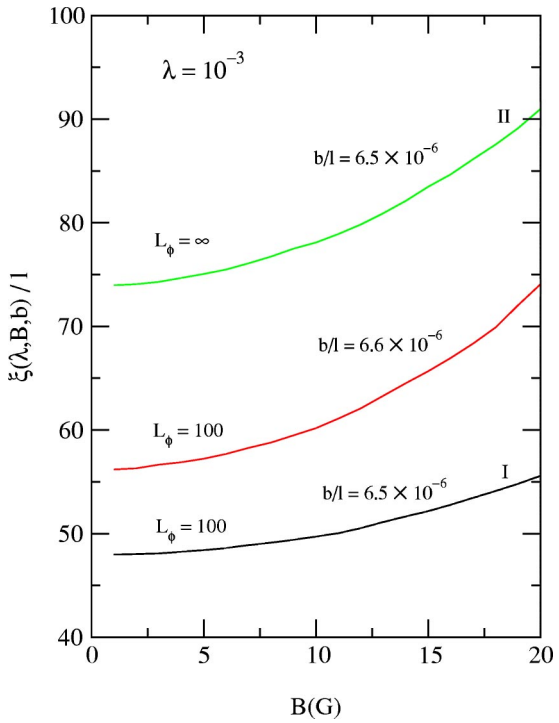


FIG. 8. Plots of $\xi(\lambda,B,b)$ with $B$ in films for two different values of $b$ and for disorder $\lambda=10^{-3}$ and $L_\phi=100$ in procedure I. Plot marked II is obtained using procedure II with $L_\phi=\infty$ for comparison.

insulator to metal transition in thin films, a conclusion which is at variance with numerical results. The solutions for $\xi$ as a function of magnetic field at two thicknesses are shown in Fig. 8. The values of $\xi$ increase very rapidly with thickness, indicating the tendency toward delocalization. These results are very similar to our earlier work on thickness dependence of localization in the absence of the field. When $L_\phi$ is finite, then just as in the two-dimensional case, one can find a threshold disorder $\lambda^c(B,L_\phi,b)$, below which a metallic phase exists. This is given as

$$\lambda^c(B,L_\phi,b)=\sqrt{\frac{8\pi\tilde{b}\tilde{l}_B}{3}}\left[\frac{1}{h^c g^c}\right]^{1/4}, \tag{33}$$

where

$$h^c=\ln\left[\tilde{L}_\phi\,\frac{\sinh(\pi\tilde{b})}{\sinh\left(\frac{\pi\tilde{b}}{\tilde{L}_\phi}\right)}\right], \tag{34}$$

$$g^c=\sum_{n=0}^{n_{max}}\left[\tilde{b}\tilde{l}_B\frac{\coth\left(\frac{\tilde{b}}{\tilde{l}_B}\sqrt{2n+1}\right)}{\sqrt{2n+1}}+\frac{\tilde{l}_B^2}{(2n+1)}\right]. \tag{35}$$

Since the threshold disorder $\lambda^c$ depends on $B$, $\tilde{b}$, and $\tilde{L}$, metal-insulator transition would be induced by thickness, temperature, and magnetic field. We show typical variation of $\lambda^c(B,L_\phi,b)$ with magnetic field in Fig. 9(a) and with

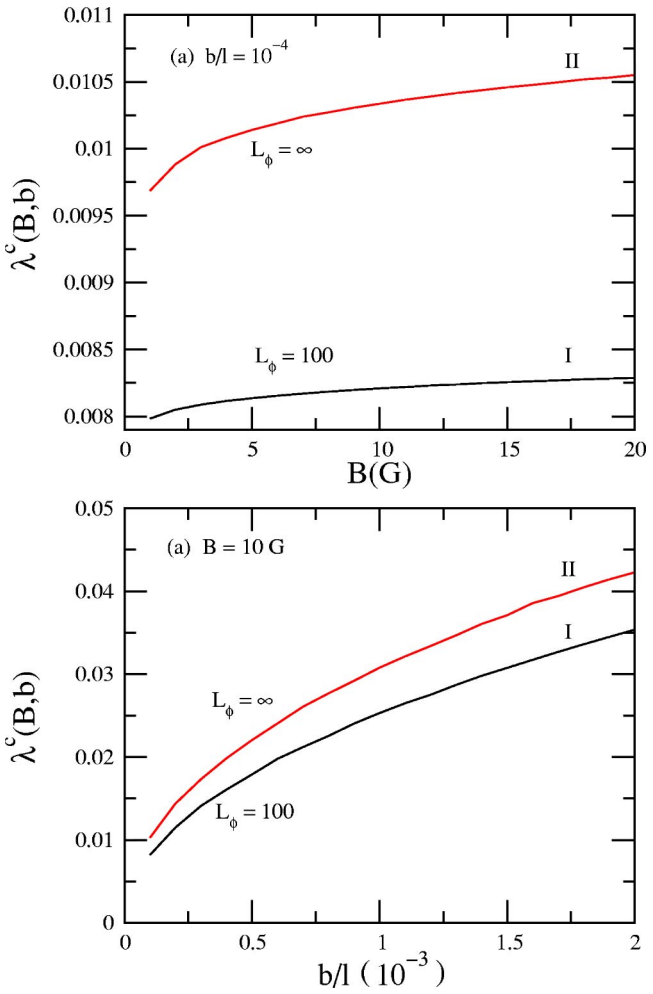


FIG. 9. (a) Plots of critical disorder $\lambda^c(B,b)$ in films as a function of $B$ at a fixed $b=10^{-4}$, for $L_\phi=100$ in procedure I and $L_\phi=\infty$ in procedure II. (b) Plots of $\lambda^c(B,b)$ with $b$ at fixed $B=10G$ in procedure I for $L_\phi=100$ and in procedure II for $L_\phi=\infty$, respectively.

thickness in Fig. 9(b) labeled as curves I. Next we look for metallic solution in thin films. Following an exercise similar to the two-dimensional case, we obtain the conductivity below the threshold disorder to be

$$\sigma(\lambda,B,L_\phi,b)=e^2N_F(3)D_0\,\frac{1-\left(\frac{\lambda}{\lambda^c(B,L_\phi,b)}\right)^4}{1+\frac{3\lambda^2}{8\pi\tilde{b}\tilde{l}_B^2}g^c}. \tag{36}$$

The variation of the diffusion constant at a disorder value below threshold with magnetic field and thickness are shown in Figs. 10(a) and 10(b), respectively, at $\tilde{L}_\phi=100$. For the sake of completeness we also report the results according to self-consistent procedure II. The insulating solution for the localization length $\xi$ assumes the form

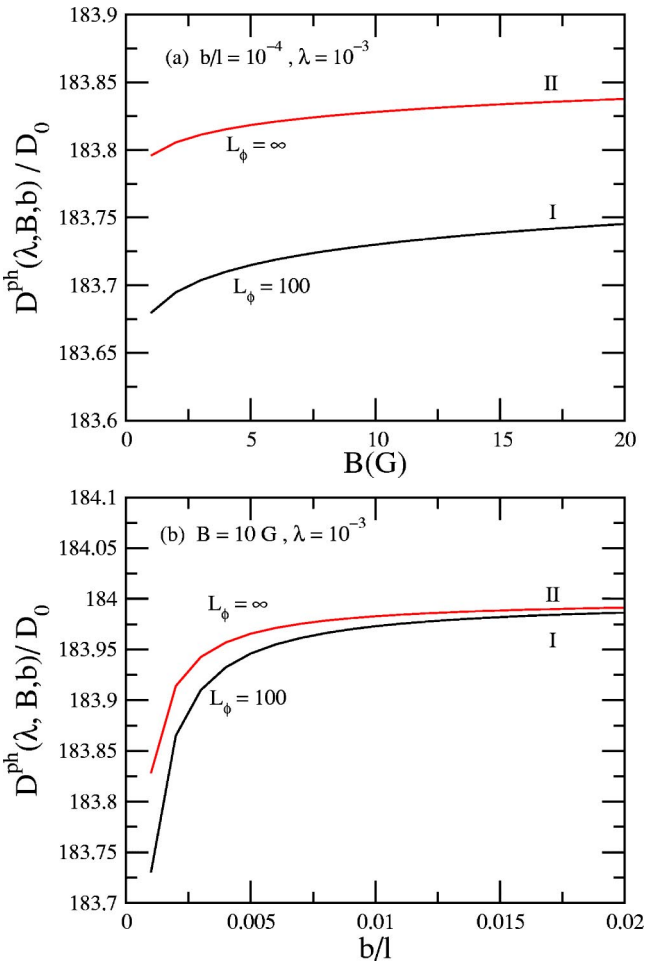


FIG. 10. Plots of diffusion constant $D^{ph}(\lambda,B,b)/D_0$ (a) with $B$ at $\lambda=10^{-3}$ and $\tilde{b}=10^{-4}$; (b) with $\tilde{b}$ at $\lambda=10^{-3}$ and $B=10G$ obtained in procedures I and II.

$$\frac{1}{\lambda}=\frac{1}{2\pi^2 l_B^2}\sum_n\left[\frac{\tilde{b}\coth\left(\tilde{b}\sqrt{\frac{2n+1}{\tilde{l}_B^2}+\tilde{\xi}^{-2}}\right)}{\sqrt{\frac{2n+1}{\tilde{l}_B^2}+\tilde{\xi}^{-2}}}+\frac{1}{\frac{2n+1}{\tilde{l}_B^2}+\tilde{\xi}^{-2}}\right]. \tag{37}$$

Here we have set $L_\phi$ to infinity. Now as before we obtain a threshold value of $\lambda$ below which insulating solution is not found. This threshold obtained by setting $\xi^{-1}=0$ is shown in Fig. 9. Here curves labeled as II show variations of threshold disorder as a function of field and thickness. Thus we find that in this procedure thickness induces MIT in the presence of the field but not without it. At all events localization lengths increase very rapidly with the thickness. Below the threshold one can calculate the conductivity as function of the field and thickness. This is given by the following formula in procedure II:

$$\sigma(B,b,\lambda,)=e^2N_F(3)D_0\left[1-\frac{\lambda}{\lambda^c(B,b)}\right]. \tag{38}$$

The corresponding diffusion constant is shown in Figs. 10(a) and 10(b) marked as curves II.

## VI. SUMMARY AND DISCUSSION

In this section we discuss and compare various results presented in this paper, in two-dimensional layers and for films with few layers.

### A. Results in two dimensions

(a) Numerical calculations for two-dimensional Anderson model give strong evidence of delocalization of states at band center with magnetic field in the weak disorder regime. The results on localization lengths for quasi-one-dimensional layers can be scaled by a field and disorder dependent length and show two distinct behaviors with width $m$, corresponding to extended and localized solutions in the presence of the field.

(b) We used self-consistent theory,[31] which is valid in the weak field limit ($l_B \ll l$), by adopting two procedures. The procedure I (Ref. 33) involves self-consistent equations between two diffusion constants corresponding to particle-hole channel and particle-particle channel. With this procedure at zero temperature ($L_\phi=\infty$), one finds only localized solutions at any disorder and any field. So no transition is seen, which is at variance with the numerical results. However, when $L_\phi$ is finite, one finds a field-dependent threshold disorder below which diffusion is nonzero. Accordingly, at nonzero temperatures the theory predicts an insulator to metal transition driven by the field, where the temperature dependence of conductivity changes from activated form (VRH) to logarithmic. We give explicit results on the dependence of localization length and conductivity on various parameters in the two regimes.

(c) With the self-consistent procedure II, in which both the diffusion constants are set equal, one obtains a different result. Here one finds that the magnetic field can drive a transition to a metallic state even at zero temperature. Thus at low temperatures ($L_\phi \approx \infty$), the predictions of the two procedures can be experimentally distinguished. Procedure II is in qualitative agreement with numerical results, but quantitative comparison is not possible due to different nature of models and their parameters.

We now comment on the difference between the numerical results and the self-consistent theory. This could possibly be related to the difference between the behaviors of tight-binding model (TBM) and continuum model (CM) in the presence of the field. The work of Yang and Bhatt[16] brings out the difference between the manner in which extended states float out on decreasing the field in two models. In TBM with decreasing field, the extended states at the edges are removed first and MIT happens when the states at the

band center get annihilated. Whereas in the CM, extended states float up to infinite energy. This point needs investigation.

### B. Thin films: role of thickness

(d) Numerical results for thin films with few layers (two to five) presented here and in our earlier papers[29,30] show that the localization length increases rapidly with film thickness and there is a transition to extended regime. The small magnetic field enhances the tendency toward delocalization.

(e) The self-consistent theory with procedure I does not give conducting solution at any width and any magnetic field when temperature is zero. This result is at variance with numerical results, though localization length does increase rather rapidly with thickness in these solutions. This result is in line with our earlier zero-field result. However, when $L_\phi$ is taken to be finite, one can have a metal-insulator transition which should be observable at finite temperatures by varying the field.

(f) With procedure II, one does get an MIT at zero temperature with magnetic field. The role of thickness is to decrease the field threshold at which the system becomes conducting. The numerical values for localization length and conductivity are of same order as in procedure I in most of the range of parameters.

In conclusion, we note that there seems to be a genuine difference between numerical results and self-consistent theory as regards thickness. The self-consistent theory is physically based on the dominance of back-scattering quantum corrections. They give correct trends in the more general situations arising due to the presence of the magnetic field and finite thickness of films, but they are not adequate to give rise to thickness induced MIT.

For comparison to real systems, one has to consider the role of interaction among electrons, but the analysis given here is a useful input for such considerations.

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