Bohr and Dirac*

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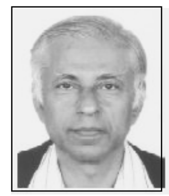
We present an account of the work of Niels Bohr and Paul Dirac, their interactions and personalities.



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1. Introduction

In this essay I would like to convey to my readers something about the personalities and work of Niels Bohr and Paul Dirac, juxtaposed against one another. Let me hope that the portraits I will paint of these two great figures from the world of physics will be faithful to the originals. The year 1985 was celebrated as the centenary of Bohr's birth, while Dirac passed away in October of the previous year. There was a gap of almost a generation between them. Let us also recall that Einstein's life spanned the period 1879 to 1955; so he was just six years older than Bohr.

For Bohr and Dirac, the most important work of their lives was bound up with the strange story of the quantum–the struggle to adapt and alter the fabric of classical physics to accommodate Planck's quantum of action. This called for an overhauling of all three components of the classical scheme–matter, motion and radiation. Naturally Bohr appeared on the scene at an earlier phase of the struggle than did Dirac, and several others were also involved, but here our focus will be on these two.

2. Planck's Interpolation

Some of you may remember that Planck made his momentous discovery sometime in the evening of Sunday, October 7, 1900 (incidentally, Bohr's fifteenth birthday). The experimental physicist Heinrich Rubens and his wife had visited the Plancks for tea that afternoon.

Rubens told Planck of his and Kurlbaum's measurements of the black-body radiation spectrum in the far infrared limit, where he had found definite deviations from the Wien radiation law. This law was a theoretical one which had been proposed in 1896 by Wien, and which Planck had believed to be exactly valid. Soon after the Rubens left, Planck set to work to find an interpolation between Wien's Law, known to be valid at high frequencies, and the low frequency measurements just reported to him by Rubens, which incidentally agreed with the theoretical results of Rayleigh and Jeans. It was thus that Planck arrived at his celebrated radiation law. It is somewhat staggering to realise that quantum theory was born or discovered in this way in the space of a few hours!

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3. Bohr at Cambridge and Manchester

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During his doctoral work on the electron theory of metals, completed in 1911, Bohr had realised very clearly that there was a need for a radical departure from the laws of classical electrodynamics in the atomic domain. It was extremely fortunate for him that in April 1912 he went to work briefly with Rutherford at Manchester, after a disappointing stint with J J Thomson at Cambridge. At Manchester he came to know of Rutherford 's model of the atom in which the positively charged core of the atom, the nucleus, containing practically all the mass, occupied a negligible volume at the centre of the atom. This was in contrast to Thomson's model, in which the positive charge was spread out uniformly over a finite volume of atomic dimensions. Many problems and possibilities immediately became clear to Bohr. On the one hand, in order to produce in this model a length scale of the order of the atomic size, and also to ensure stability of the electron orbits, it was essential to bring in Planck's constant. On the other hand, it now appeared that all the chemical properties of an element should depend only on one datum, namely the number of peripheral electrons, i.e., the atomic number rather than the mass number. In fact, Bohr saw that while chemistry was determined by the outermost electrons of the atom, all radioactive processes like α and β emission originated from the nucleus, deep inside the atom. It appears that at this stage Bohr took Rutherford's model more seriously than Rutherford himself did.

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Turning to the structure of the atom, Bohr assumed that the electrons moved in concentric circular rings around the nucleus. Classical electrodynamics could never explain the stability of such an arrangement; but Bohr had already anticipated the need for a fundamental departure from classical ideas in this realm. He was familiar with Planck's method of quantizing the motion and the energy of simple harmonic motion, and he now adapted it to the motion of an electron in the Coulomb field of the nucleus. As much by inspiration as by deduction he was able to arrive at the right order of magnitude for atomic sizes, and at the expression $E_n = -A/n^2$ for the allowed energies of an electron bound in an atom. Here the integer n takes values $1, 2, 3, \cdots$. For all this of course, Planck's constant was essential, but at that time the exact form of the quantum condition was beyond him.

At this stage another important event occurred – he was called upon to investigate the passage of α-particles through matter and to analyze the processes by which they ionized the atoms of matter, losing energy and slowing down as they did so. This was a matter of practical importance in Rutherford's laboratory. The fact that he could give a satisfactory classical account of this process, whereas classical ideas failed completely within the atom, led him to the following truth: however deep the break with classical ideas might be, the new theory would have to agree with the old one in the limit of low frequencies or large quantum numbers. This was the origin of the famous Correspondence Principle, which played such a major part in subsequent developments.

4. Connection to Rydberg's Formula

At this point in his thinking, Bohr had dealt only with the structure and stability of the atom, and had not yet connected up with atomic spectroscopy or radiation phenomena. He returned from Manchester to Copenhagen in July 1912, married Margrethe Norlund in August 1912, and set about writing up the ideas conceived in Manchester. It was only in early 1913 that his mind suddenly turned to problems of atomic radiation. Atomic spectroscopy was a well-developed field with a lot of data on the characteristic spectral lines and frequencies associated with various elements. There also existed several empirical formulae, giving simple expressions for many series of spectral lines. H M Hansen, a colleague of Bohr's at the University of Copenhagen, asked him in early 1913 if he knew of Rydberg's formula which expressed every frequency as the difference of two terms, and which for hydrogen took the simple form

$$\nu_{mn} = R\left(\frac{1}{n^2} - \frac{1}{m^2}\right)$$

where both n and m were integers. Bohr had not known this even though it had been around since 1890, and Rydberg worked at the nearby University of Lund in southern Sweden. So this query and information from

Hansen came as a complete surprise to Bohr. But at the same time he saw that it gave the missing clue to the problem of quantization in the atom. He compared his own formula $E_n = -A/n^2$ for quantized electron energies in an atom with individual terms in Rydberg's expression and immediately realized that each spectral line corresponded to a transition of an electron from one allowed state to another, accompanied by the emission of a quantum of radiation. In the Planck–Einstein spirit, it was Bohr who first saw the Rydberg law as an expression of the conservation of energy,

$$h\nu_{mn} = E_m - E_n, E_m = -hR/m^2 \ .$$

By demanding agreement with classical theory for large n, Bohr was able to completely pin down the quantization condition as well as to calculate the value of Rydberg's constant. The break with classical physics came with the fact that none of the spectral frequencies ν_{mn} coincided with any of the classical orbital frequencies, but such a break was essential to explain the stability of the atom, as anticipated by Bohr. In fact, he said that Rydberg's formula gave him such a transparent clue that he immediately saw the quantum picture of the emission of radiation. He was sure he was on the right track inspite of the total breakdown of classical physics; at the same time the Correspondence Principle was obeyed.

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In 1913 he published his three famous papers on the constitution of atoms and molecules, where he stated his two fundamental postulates: (1) the electron could only be in one of a special set of stationary states which had to be chosen out of all possible classical motions by imposing quantum conditions; (2) the transition of the electron from one such state to another is a non-classical and non-visualizable process, during which a single quantum of radiation is emitted or absorbed according to the Rydberg–Bohr frequency condition.

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5. Reactions to Bohr's Work, Later Developments

Many predictions of Bohr's theory were checked in Rutherford's laboratory, but the English physicists, in particular Fowler and Jeans, were skeptical and accepted his ideas only reluctantly. It seems that in Göttingen there was a sense of scandal and bewilderment. But both Einstein and Sommerfeld saw immediately the significance of Bohr's ideas.

I have devoted a considerable amount of space to recounting this early phase of Bohr's work, because it was the foundation of all else that followed. Indeed, though the quantum of action was discovered in the properties of radiation, the route to the new quantum mechanics was via the mechanics of the atom. And the application of Planck's ideas to the dynamics of matter, which Dirac was to later describe as the most difficult first step, was taken by Bohr.

Bohr was fully aware of the limitations of his theory. It was necessary to generalize the quantum condition from the circular motions of a single particle to the motions of general mechanical systems; to analyze the relationship between classical and quantum aspects of atomic phenomena; and to explore the many applications of his theory. To do all this, he gradually built up a school around himself in Copenhagen. One of his earliest collaborators was Kramers from Holland, who joined him in 1916. By 1919, he had an Institute of his own. Meanwhile his programme had also been taken up by the groups at Göttingen and Munich, led respectively by Max Born and Sommerfeld. The three centers worked in an atmosphere of friendly cooperation with frequent exchanges of ideas, and sharing of successes, hopes and people. Pauli and Heisenberg, among others, travelled frequently from one of these centers to another. In 1915, Sommerfeld found the general form of the quantum conditions for any so-called multiply-periodic system, and soon Bohr adopted Sommerfeld's mathematical methods. Instead of a picture of electrons moving in concentric circular orbits in a plane, Bohr could now deal with shells of electron orbits, tackle complex atoms and their spectra, and go on to elucidate the structure of the periodic table. This was of course, a great shot in the arm for chemistry. One must remember that Bohr did all this even before the Pauli exclusion principle and the electron spin had been discovered. In all this work the Correspondence Principle was the constant guide, being used both brilliantly and judiciously. In 1921 the Correspondence Principle was extended to dispersion by Ladenburg, and Kramers followed this up in Copenhagen. In this work he was joined by Heisenberg. (Along the way, Bohr collected the Nobel Prize for 1922.) But not all the data could be satisfactorily explained by the theory. Bohr remained acutely aware how far he was from a logically consistent framework which was able to explain his two postulates and at the same time be in harmony with the Correspondence Principle. In fact, the period 1923–1925 witnessed a crisis in the old quantum theory. To this period belongs a famous paper of Bohr, Kramers and Slater. In this, Bohr tried to give an overall picture of radiative processes taking place in the atom, and the authors suggested that classical causality had to be replaced by a purely statistical description. This paper had a deep influence on Heisenberg, as it showed even more clearly the inadequacy of the classical picture of atomic processes.

As is well known, the resolution of the crisis came with Heisenberg's discovery of matrix mechanics in June–July 1925. This was a direct outgrowth of his work with Kramers in Copenhagen on dispersion, and of the influence on him of the Bohr–Kramers–Slater work. But all that is another story.

6. Dirac at Cambridge

Meanwhile, back at the ranch in Cambridge, a young Paul Dirac had joined R H Fowler as a research student in 1923, after getting a degree in electrical engineering. For two years he worked on applying Hamiltonian methods to multiply–periodic systems in the framework of the Rutherford–Bohr model, but that did not lead to any significant success. Then in September 1925, his lucky break came when, by a somewhat roundabout route, he learnt of Heisenberg's discovery of matrix mechanics. This was the spark that ignited him. He soon elaborated, practically in isolation, his own version of quantum mechanics, giving it a particularly abstract and elegant structure. One might remember here that Heisenberg's achievement had been aided by continuous contact and exchange of ideas with Bohr, Born, Pauli, Kramers and Sommerfeld. In any case, once the key step had been taken by Heisenberg, progress towards the establishment of a mathematically satisfactory quantum mechanics was extremely rapid and was essentially finished by early 1927. Schrödinger's discovery of wave mechanics had come in early 1926, and its equivalence to Heisenberg's version soon after. One of Dirac's key contributions in this phase was the exposure of the link between classical and quantum mechanics. This was the most beautiful expression of the Correspondence Principle and, said Dirac, it had given him the most pleasure of all his discoveries.

7. Interpreting Quantum Mechanics

From 1925 to 1927, the most important advances were being made by Dirac in Cambridge, Heisenberg, Born and Jordan in Göttingen, and Schrödinger in Zurich. During this period, Bohr was in a sense watching from a distance, with a critical but approving attitude. He had inspired and oriented the work of the others; and the new theory had attained the goals he had set himself all along. The departure from classical physics he had sensed and foreseen for so long was now explicitly expressed; relations among physical quantities could no longer be maintained in the classical numerical sense, but only in a more abstract algebraic sense. Every physical attribute of a system could not always be reduced to a number. When the stage was set to find the physical meaning of the mathematical structure, Bohr re-entered the scene. The deeper understanding of the situation needed Bohr and his philosophical bent of mind. Indeed Heisenberg said of him:

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In early 1927, between the two of them, Bohr and Heisenberg developed what we now call the 'Copenhagen interpretation of quantum mechanics'. In this, they were greatly aided by the transformation theory of quantum mechanics, which had just been developed by Dirac and Jordan. Heisenberg's contribution was the uncertainty relations. Bohr's was the complementarity idea. According to the latter, every classical concept retains its usefulness in quantum mechanics, but not necessarily simultaneously. According to Bohr, this was the greatest lesson of quantum mechanics – that the classical concepts, each individually valid, might be mutually exclusive. In later years he would say that physics had by its simplicity shown the way to this profound idea, but that the idea itself was applicable to much more complex situations, such as the relation between physics and life.

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Einstein critically attacked the Copenhagen interpretation at the two Solvay Congresses of 1927 and 1930, and it was Bohr who answered him each time and proved the logical consistency of quantum mechanics. Finally Einstein had to concede, saying only that he still felt there was an unreasonableness about it all. Of Bohr himself he said:

> His is a first-rate mind, extremely critical and far-seeing, which never loses track of the grand design,

and

> He is truly a man of genius, it is fortunate to have someone like that.

8. Dirac's Later Work

Turning our attention now to Dirac for a while, I have already recounted how he burst on to the scene in late 1925. Thereafter, he kept going like a house on fire, with a steady and staggering profusion of fundamental ideas and discoveries. One of his most important papers, on the quantum theory of the emission and absorption of radiation, was written at Bohr's Institute in Copenhagen; so he too had been drawn into Bohr's circle. By applying the principles of quantum mechanics to the electromagnetic field, Dirac brought to a successful conclusion the work begun by Planck in 1900, and also inaugurated quantum field theory. Then there was the discovery of the new statistics named after him and Fermi, the relativistic theory of the electron, the prediction of the positron and the general concept of antimatter, the idea of the magnetic monopole, and many more. In the midst of all this, he wrote the classic book *The Principles of Quantum Mechanics*, often compared with Newton's *Principia*. It would take a great deal of space to do justice to all that Dirac accomplished in this period. Just as Bohr had made the preceding era a heroic one, Dirac turned this one into the 'Golden Age of Theoretical Physics'.

There is a charming anecdote from the Solvay Congress of 1927, which is worth recalling. In the interval between two sessions, Bohr asked Dirac what he was working on, to which Dirac replied that he was looking for a satisfactory relativistic wave equation for the electron, which would combine special relativity and quantum mechanics properly. Bohr then told him that such an equation had already been found by Klein and Gordon, but before Dirac could explain why he was not satisfied with it, the bell rang and they had to go back to the next session. Dirac later said:

> ··· It rather opened my eyes to the fact that so many physicists were quite complacent with a theory which involved a radical departure from some of the basic laws of quantum mechanics, and they did not feel the necessity of keeping to these basic laws in the way that I felt.

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– Dirac

9. Dirac's Style

Dirac's style is essentially mathematical, and he turned out to be a master craftsman in the art of theoretical physics. He created with ease the mathematical tools that he needed. Bohr on the other hand was somewhat like Faraday. As Heisenberg said,

> ··· his insight into the structure of the theory was not a result of a mathematical analysis of the basic assumptions, but rather of an intense occupation with the actual phenomena, such that it was possible for him to sense the relationships intuitively rather than derive them formally.

For Dirac, considerations of mathematical beauty and symmetry were of the highest importance, and he was

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– Dirac

matchless in the art of manipulating and working with the abstract. Bohr, on the other hand, was much more concerned with the problems of interpretation and communication, the difficulties and ambiguities inherent in language, and other such philosophical questions.

Dirac's writings have a characteristic and unmistakeable directness, simplicity and beauty. Bohr, on the other hand, is much harder to read because each long sentence of his contains a great deal of thought in a highly compressed form. He spent a lot of effort in the choice of each important word. Bohr's style of work was to have a junior collaborator sit at a desk and take down notes while he himself kept pacing up and down the room, forming and changing and reforming his phrases and sentences. Watching him at one such session, Dirac apparently said something to the following effect:

> Professor Bohr, when we were young we were taught never to start a sentence until we knew how to finish it.

Bohr's speech and handwriting were, respectively, inaudible and illegible. On both counts, Dirac was far superior. As Bohr himself said:

> Whenever Dirac sends me a manuscript, the writing is so neat and free of corrections that merely looking at it is an aesthetic pleasure. If I suggest even minor changes, Paul becomes unhappy and generally changes nothing at all.

As I recalled earlier, Bohr was very deeply interested in the problems of biology, which he saw as a fertile field of application for his Principle of Complementarity. In fact, for him physics was a far simpler problem. In Dirac's writings I have been able to find a reference to biology. In his paper of 1931 concerned with the magnetic monopole, he says,

> There are at present fundamental problems in theoretical physics awaiting solution, e.g., the relativistic formulation of quantum mechanics and the nature of atomic nuclei (to be followed by more difficult ones such as the problem of life)$\cdots$

At another time he is supposed to have said that his equation for the electron explained all of chemistry and most of physics. Presumably for him, the problem of life was just one more of the things that theoretical physics would deal with in good time!

Bohr created and inspired an international school of theoretical physics; and his influence upon others was as much by direct contact and involvement in their struggles as through his writings. Dirac, on the other hand, worked largely on his own. He did not create a school of any kind, although his influence on others through his writings and ideas has been enormous.

10. Bohr's Later Work

In the years following the creation and completion of quantum mechanics, Bohr turned to the problems of nuclear physics while Dirac was more concerned with relativistic quantum field theory and later on with gravitation and cosmology as well. However, there is a classic contribution by Bohr along with Rosenfeld in 1933 to quantum field theory. They analyzed the consistency of applying the principles of quantization to the electromagnetic field – something which Dirac had done in 1927 – and demonstrated the logical necessity of doing this if the quantum mechanics of particles and, in particular, Heisenberg's uncertainty relations were to be maintained.

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11. Two Personalities

As human beings, there is a great deal worthy of admiration in both Bohr and Dirac, and a touching simplicity and sincere modesty in their dealings with others. Dirac was always most ready to acknowledge his debt to others. And in seminars, it seems that Bohr would always preface his questions with the statement that he only wished to better understand the speaker's point of view. Bohr concerned himself with political matters and spoke a great deal on philosophical issues as well, while Dirac seems to have avoided both these areas. Bohr was quite categorical that quantum mechanics was complete; and the most valuable lesson it had taught us was that of complementarity. He was anxious to extend its application to other fields such as reason and instinct, heredity and environment, physics and biology. His debate with Einstein, begun in the 1927 Solvay Congress, continued for more than two decades, and he adhered to his point of view. In the 70's however, Dirac had this to say,

> ··· the present form of quantum mechanics should not be considered as the final form. It is the best that one can do up till now. But one should not suppose that it will survive indefinitely into the future. And I think that it is quite likely that at some future time we may get an improved quantum mechanics in which there will be a return to determinism and which will, therefore, justify the Einstein point of view.

One is left speculating on what Dirac actually had in mind.

Physicists are familiar with many lovely sayings and stories about and by Bohr and Dirac. And they are all really a reflection of their greatness as human beings. Bohr was always a synthesizer of conflicting points of view. On one occasion he said,

> The opposite of a correct statement is a false statement. But the opposite of a profound truth may well be another profound truth.

On another occasion he is quoted as saying,

> There are things that are so serious that you can only joke about them.

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– Bohr

One of Dirac's most celebrated statements was about the value of mathematical beauty in physics. He said,

> ··· it is more important to have beauty in one's equations than to have them fit experiment ··· It seems that if one is working from the point of view of getting beauty in one's equations, and if one has really a sound insight, one is on a sure line of progress.

This reminds us of the poet John Keats saying, "What the imagination seizes as beauty must be truth – whether it existed before or not."

Bohr paved the way from the world of classical physics to the world of the quantum, guiding everybody through the most difficult period with his unerring instinct and intuition. And when the great victory had been won, it was he who most comprehensively assessed the impact it had for the nature and goals of science. Dirac was one of the chief architects of the victory, and he went on to raise theoretical physics to unparalleled heights of imagination and beauty. As much for their heroic labours as for their great human qualities, Bohr and Dirac will always rank among the greatest scientists of all time.

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