Sediment Entrainment Probability and Threshold of Sediment Suspension: An Exponential Based Approach

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Abstract: This study examines the probability for sediment entrainment to bed-load and the probability for the threshold condition of sediment to be in suspension. The theoretical analysis is based on a simple one-sided exponential distribution of probability function. The probability distributions are derived from a truncated universal Gram-Charlier series expansion based on the exponential or Laplace type distributions for turbulent velocity fluctuations, as established earlier by the authors. The key criterion of sediment entrainment is the hydrodynamic lift acting on a solitary particle to exceed submerged weight of the particle, as was considered by H. A. Einstein, M. S. Yalin and others. In this way, a simple probability function for sediment entrainment to bed-load in terms of Shields parameter containing the lift coefficient is obtained. It was found that the value of lift coefficient as 0.15 satisfactorily fitted the probability function versus Shields parameter curve with the experimental data. On the other hand, the key criterion of the threshold of sediment suspension is the fluctuations of vertical velocity component to exceed terminal fall velocity of the particle. The probability function for the threshold of a sediment particle to be in suspension is obtained in terms of Shields parameter as a function of shear Reynolds number. Curves for different values of probabilities are drawn in respect of Shields diagram. For the value of probability 0.05, the threshold of sediment suspension is indicated. The prediction curves for the threshold of sediment suspension are proposed in terms of Rouse number versus Shields parameter and also Shields parameter versus shear Reynolds number.

CE Database subject headings: Bed loads; Fluvial hydraulics; Open channel flow; Probability density functions; Sediment transport; Streamflow; Suspended loads; Turbulent flow

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Introduction

Probabilistic theories of sediment transport by flowing streams as bed-load and suspended-load have been developed by different researchers. They are mainly based on the hypothesis that the velocity fluctuations in turbulent flows contribute to sediment entrainment not only in the bed-load motion but also to bring the sediment in suspension. In *bed-load transport*, the particles may slide or roll or perform brief jumps, termed *saltation*, but remain close to the bed. On the other hand, in *suspended-load transport*, the particles perform much higher jumps remaining appreciable period of time in the main stream, but only occasionally return to the bed and again go up. The processes of bed-load and suspended-load are highly intermittent in nature. Thus, the analyses require the determination of the probabilities of bed particles to entrain as bed-load and/or to be in suspension. Despite the initial attempt that was made during 1930s, a handful of researches focus on these issues. However, there leaves a scope to explore the problems further, because Bose and Dey (2010) [also see Dey et al. (2012)] showed that the strong prevalence of turbulent bursting in the near-bed flows provoke the non-Gaussian type of distributions of probability densities of the turbulence quantities. A brief state-of-the-art of the researches on the probabilities of bed-load and suspended-load transports is outlined below:

Lane and Kalinske (1939) and Einstein (1942) laid the foundation of the applicability of probabilistic concepts to study the bed-load transport. They introduced an entrainment probability function for the sediment entrainment to bed-load. Subsequent investigations by various researchers viewed the probability of sediment entrainment in different ways and put forward formulations for probability in terms of entrainment or pickup probability function. The entrainment probability function is a function of the nondimensional bed shear stress, termed *Shields parameter*. Previously, the most innovative contribution was due to Einstein (1950), who developed a formula for the entrainment function based on the Gaussian probability distribution of the fluctuating hydrodynamic lift acting on a particle to exceed its submerged weight. The entrainment probability function P is

$$P = 1 - \frac{1}{\pi^{0.5}} \int_{-0.143\Theta^{-1}-2}^{0.143\Theta^{-1}-2} \exp(-t^2)\,dt \tag{1}$$

where Θ = Shields parameter, $u_*^2/(\Delta g d)$; u_* = shear velocity; Δ = submerged relative density, s – 1; s = relative density of sediment, that is ρ_s/ρ; ρ_s = mass density of sediment; ρ = mass density of water; g = acceleration due to gravity; and d = representative sediment size, that is the median or weighted mean diameter. Engelund and Fredsøe (1976) gave an empirical formula for the entrainment probability function by using experimental data of Guy et al. (1966) and Luque (1974). The formula was subsequently modified by Fredsøe and Deigaard (1992) in the form

$$P = \left[1 + \left(\frac{\mu_d \pi / 6}{\Theta - \Theta_c}\right)^4\right]^{-0.25} \tag{2}$$

where Θ_c = threshold Shields parameter, $u_{*c}^2/(\Delta gd)$; u_{*c} = threshold shear velocity; and μ_d = coefficient of dynamic friction. However, using the same methodology, Cheng and Chiew (1998) obtained an approximate expression for the entrainment probability function based on the assumption of a normal probability distribution for the streamwise velocity fluctuations. They obtained the following expression for the entrainment probability:

$$P = 1 - 0.5\frac{0.21-\sqrt{\Theta C_L}}{|0.21-\sqrt{\Theta C_L}|}\sqrt{1-\exp\left[-\left(\frac{0.46}{\sqrt{\Theta C_L}}-2.2\right)^2\right]} - 0.5\sqrt{1-\exp\left[-\left(\frac{0.46}{\sqrt{\Theta C_L}}+2.2\right)^2\right]} \qquad (3)$$

where C_L = lift coefficient. Later, Wu and Lin (2002) noted that since only positive fluctuations of the streamwise velocity can cause an entrainment of bed particles, a log-normal distribution could be better suited to derive an expression for the entrainment probability. They gave

$$P = 0.5 - 0.5\frac{\ln(0.044\Theta^{-1}C_L^{-1})}{|\ln(0.044\Theta^{-1}C_L^{-1})|}\sqrt{1-\exp\left\{-\frac{2}{\pi}\left[\frac{\ln(0.044\Theta^{-1}C_L^{-1})}{0.724}\right]^2\right\}} \qquad (4)$$

Wu and Chou (2003) further refined the theory by excluding the small fraction of the lower portion of a solitary particle, that can rest on the top of two bed particles of equal size, lying in the dead flow zone. They considered both the lifting and rolling modes of entrainment threshold.

The suspended-load transports above the bed-load zone, termed *bed-layer* [see Einstein et al. (1940)]. The mechanism of the particle motion from the bed-layer to the suspension state is not yet well understood. The reason is attributed to the intricacy of near-bed turbulence characteristics along with intermittent particle exchange at the interface of the bed-layer. Based on the experimental results, Bagnold (1966) and Xie (1981) simply set the upper limit for the threshold of sediment suspension as u_*/w_s = 0.8 and $0.2\kappa^{-1}$, respectively, where w_s = terminal fall velocity of a particle; and κ = von Kármán constant. Then, van Rijn (1984b) gave the upper limit in terms of the particle parameter d_* [$= (\Delta g/\upsilon^2)^{1/3}$, where υ = kinematic viscosity of water]:

$$\frac{u_*}{w_s}(1 < d_* \le 10) = \frac{4}{d_*} \tag{5a}$$

$$\frac{u_*}{w_s}(d_* > 10) = 0.4 \tag{5b}$$

Sumer (1986) and Celik and Rodi (1991), on the other hand, gave the threshold criterion for sediment suspension in terms of Shields parameter Θ as a function of shear Reynolds number R_* ($= u_*d/\upsilon$). They considered sediment particles to be in suspension from the sediment bed with no motion rather than from the top of the bed-layer. According to Sumer (1986),

$$\Theta(R_* \le 70) = \frac{17}{R_*} \tag{6a}$$

$$\Theta(R_* > 70) = 0.27 \tag{6b}$$

On the other hand, Celik and Rodi (1991) gave it as

$$\Theta(R_* \le 0.6) = \frac{0.15}{R_*} \tag{7a}$$

$$\Theta(R_* > 0.6) = 0.25 \tag{7b}$$

Note that the above expressions defining the threshold criterion for sediment suspension are empirical and devoid of probabilistic considerations. Cheng and Chiew (1999) however brought the probabilistic consideration for the first time by noting that a particle to bring in suspension, when the vertical velocity fluctuations v' in a turbulent flow exceed the terminal fall velocity w_s of the particle. It is an event at the top of the bed-layer. Again, by using the Gaussian distribution for the vertical velocity fluctuations v', they determined the total probability P_s of a particle to be in suspension. Giving trials with suitable values of the total probability, they obtained the curves of Θ versus R_*, in the form of the Shields diagram, for the probability criterion for the threshold of sediment suspension from the bed-layer. Their equation of total probability P_s is

$$P_s = 0.5 - 0.5\left[1 - \exp\left(-\frac{2}{\pi}\cdot\frac{w_s^2}{\sigma_2^2}\right)\right]^{0.5} \tag{8}$$

where σ_2 = root-mean square of v', that is $(\overline{v'^2})^{0.5}$, which was obtained for hydraulically smooth flow regime as

$$\frac{\sigma_2}{u_*} = 1 - \exp\left[-0.025\left(-\frac{2.75u_*d}{\upsilon}\right)^{1.3}\right] \tag{9}$$

For hydraulically rough flow regime, they considered $\sigma_2 = u_*$. They concluded that the probability of the threshold of sediment suspension is 0.01.

Previous studies were based on the normal and the log-normal distributions, which primarily occur in case of additive or multiplicative accumulation of errors. This however is not the case of turbulent velocity fluctuations. On the other hand, drawing a similarity with a random signal magnitude of velocity fluctuations $|u'|$ regarded analogous to the service arriving in a queue, Bose and Dey (2010) gave the Gram-Charlier series expansion of the probability densities based on the two-sided exponential or Laplace distribution. In this paper, retaining the principal terms of the series, simple one-sided exponential based series up to quadratic terms of positive values of the variates are considered. From these distributions, simple expressions for the entrainment probability function P for particles to bed-load and the total probability P_s for particles to be in suspension are obtained, in the line of Cheng and Chiew (1998, 1999). The results obtained from the developed theories are compared with those obtained from the earlier studies and the experimental data.

Universal Probability Distribution of Turbulent Velocity Fluctuations

When a unidirectional steady stream flows over a plane bed, which may be smooth or rough or even mobile, the two-dimensional instantaneous velocity components (u, v) at a point (x, y) in the flow can be split by the Reynolds decomposition into the time-averaged part ($\bar{u}$, $\bar{v}$) and the fluctuating part (u', v') in the traditional way:

$$u(x,y,t) = \bar{u}(x,y) + u'(x,y,t)\,,\; v(x,y,t) = \bar{v}(x,y) + v'(x,y,t) \tag{10}$$

Bose and Dey (2010) and Dey et al. (2012) showed that the velocity fluctuations (u', v') obey the Gram-Charlier based two-sided exponential or Laplace distribution. Letting $\hat{u} = u'/\sigma_1$ and $\hat{v} = v'/\sigma_2$, where σ_1 = root-mean square of u', Bose and Dey (2010) explained that the probability density function $p_{\hat{u}}(\hat{u})$ (henceforth pdf) for the streamwise velocity fluctuations can be given by

$$p_{\hat{u}}(\hat{u}) = \frac{1}{2}\left[1 + \frac{1}{2}C_{10}\hat{u} - \frac{1}{8}C_{20}(1 + |\hat{u}| - \hat{u}^2) - \frac{1}{48}C_{30}\hat{u}(3 + 3|\hat{u}| - \hat{u}^2)\right.$$

$$\left. + \frac{1}{384}C_{40}(9 + 9|\hat{u}| - 3\hat{u}^2 - 6|\hat{u}|^3 + \hat{u}^4) + \cdots\right]\exp(-|\hat{u}|) \tag{11}$$

where $C_{10} = m_{10}$; $C_{20} = -1 + (m_{20}/2)$; $C_{30} = -2m_{10} + (m_{30}/6)$; $C_{40} = 2 - (3m_{20}/2) + (m_{40}/24)$ and $m_{jk} = \overline{\hat{u}^j\hat{v}^k}$ (Bose and Dey 2010).

A similar expression holds for the pdf $p_{\hat{v}}(\hat{v})$ for the vertical velocity fluctuations, in which the coefficients C_{10}, C_{20}, C_{30} and C_{40} are to be replaced by another set of coefficients C_{01}, C_{02}, C_{03} and C_{04}, respectively. Estimated from the experimental results, their values depend on the flow velocity, the location with respect to the bed and the sediment size forming the roughness if the bed is erodible. It is noteworthy in these data that the coefficients C_{10}, C_{01}, C_{30} and C_{03} are of the order of 0.001; while C_{20} and $C_{02} \approx -0.5$ and C_{40} and $C_{04} \approx 0.6$. Thus, in order to keep the study independent of the experimental data particularly that of sediment size, it is assumed that

C_{20} and $C_{02} \approx -0.5$ and the rest of the coefficients are effectively negligible due to their smallness or division by a large number, such as 384. Thus, Eq. (11) reduces to

$$p_{\hat{u}}(\hat{u}) = \frac{1}{32}(17 + |\hat{u}| - \hat{u}^2)\exp(-|\hat{u}|) \tag{12}$$

Likewise, the pdf $p_{\hat{v}}(\hat{v})$ for the vertical velocity fluctuations can be expressed.

Probability of Sediment Entrainment

Fig. 1 shows a schematic of a solitary particle, whose submerged weight is F_G, resting on a horizontal bed formed by the sediment particles. The solitary particle is subjected to hydrodynamic drag F_D and lift F_L induced by the flow. Here, as we are concerned about the turbulent flow, F_D and F_L represent their instantaneous values. The instantaneous near-bed streamwise velocity u_b, which can be decomposed as $u_b = \bar{u}_b + u'$, is the main agent of an entrainment of similarly lying solitary particles on the bed surface. Wu and Lin (2002), following Nelson et al. (1995), appropriately argued that the streamwise entrainment is only possible when the streamwise velocity fluctuations $u' > 0$, for which the pdf follows Eq. (12), becomes the one-sided exponential based Gram-Charlier series. Therefore,

$$\left.\begin{aligned} p_{u'}(u' \geq 0) &= \frac{1}{16\sigma_1}(17 + \hat{u} - \hat{u}^2)\exp(-\hat{u}) \\ p_{u'}(u' < 0) &= 0 \end{aligned}\right\} \tag{13}$$

It can be verified that $\int_{-\infty}^{\infty} p_{u'}(u')du' = \int_{0}^{\infty} p_{u'}(u')du' = 1$. The pdf given by Eq. (13) is plotted in Fig. 2 for different values of $\sigma_1 = 0.5$, 1 and 1.5.

The probability for sediment entrainment was modeled in various ways by different researchers. Following Einstein (1950) and Yalin (1977), Cheng and Chiew (1998) considered an unstable particle placed on the bed, such that it is likely to be lifted by the flow provided $F_L > F_G$. Importantly, the instantaneous lift force F_L acting on a particle fluctuates in accordance with the velocity fluctuations u' of the near-bed velocity u_b; while the submerged weight F_G of a particle is a constant for a given particle size. Thus, one can consider the total entrainment probability P as F_L exceeds F_G. Now, in a turbulent flow, F_L can be expressed as

$$F_L = \frac{1}{2} C_L \rho u_b^2 \frac{\pi d^2}{4} \tag{14}$$

The C_L in the near-bed flow region of a fully developed flow is approximately a constant. On the other hand, F_G is given by

$$F_G = \Delta \rho g \frac{\pi d^3}{6} \tag{15}$$

Therefore, $F_L > F_G$ implies that $u_b > B$ or $u' > B - \bar{u}_b$, where

$$B = \sqrt{\frac{4\Delta g d}{3C_L}} \tag{16}$$

Thus, using Eq. (13), we can write

$$P = \int_{B-\bar{u}_b}^{\infty} p_{u'}(u')du' = \frac{1}{16}\int_{a}^{\infty}(17 + \hat{u} - \hat{u}^2)\exp(-\hat{u})d\hat{u} = \frac{1}{16}(16 - a - a^2)\exp(-a) \tag{17}$$

where $a = (B - \bar{u}_b)/\sigma_1$.

Cheng and Chiew (1998) estimated the time-averaged near-bed velocity $\bar{u}_b$, using the logarithmic law and fixing zero-displacement level at $0.25d$ and zero-velocity level at y_0 (= $k_s/30$, where k_s = Nikuredse equivalent roughness height considered as $2d$) below the top of the closely packed bed particles (Hinze 1975; van Rijn 1984a). They assumed that a particle placed in an interstice between two bed particles is about to move. In this way, they estimated $\bar{u}_b = 5.52u_*$ acting on the particle in an initial position at $y = 0.6d$. Recently, Dey et al. (2012) found that when the bed particles move, the von Kármán constant κ diminishes from its universal value 0.41, and the zero-displacement level and the zero-velocity level move up as compared to their values in immobile beds [also see Dey and Raikar (2007), Gaudio et al. (2010), Dey et al. (2011) and Gaudio and Dey (2012)]. These modify the estimation of near-bed velocity from the logarithmic law as $\bar{u}_b$ = 6.4u_*, which is used here. Quoting Kironoto and Graf (1994), Cheng and Chiew (1998) estimated $\sigma_1 = 2u_*$. Using these results, a can be expressed as

$$a = \frac{B - \bar{u}_b}{\sigma_1} = \frac{1}{2\sqrt{\Delta g d \Theta}} \left(\sqrt{\frac{4\Delta g d}{3C_L}} - 6.4\sqrt{\Delta g d \Theta} \right) = \frac{1}{\sqrt{3C_L \Theta}} - 3.2 \tag{18}$$

Thus, using Eq. (18), Eq. (17) yields the entrainment probability function P in terms of Shields parameter Θ.

The lift coefficient C_L is an important parameter required to evaluate a. Unfortunately, there is no consensus, as widely varying values of C_L were reported in literature. Einstein and El-Samni (1949) measured the lift force directly as a static pressure difference between the top and the bottom points of hemispheres. They obtained lift coefficient C_L as 0.178. They also studied the effect of turbulent velocity fluctuations on lift. The experiments revealed a constant average lift force with superimposed random fluctuations that follow the normal-error law. Their results were used by the Task Committee (1966) of the Journal of Hydraulics Division to estimate the lift force per unit area equaling 2.5τ_{0c}; where τ_{0c} is the threshold bed shear stress. It suggested that the lift force is an important mechanism towards the sediment entrainment. However, Chepil (1961) pointed out that once the particle moves, the lift tends to diminish; while the drag increases. Then, several attempts were made to estimate the lift relative to drag. Chepil (1961) measured the lift to drag ratio as about 0.85 for $47 < Ud/\upsilon < 5\times10^3$ in a wind stream U on hemispherical roughness having diameter d; while Brayshaw et al. (1983) measured the ratio as

1.8 for the same type of roughness at $R_* = 5.2\times10^4$. Aksoy (1973) and Bagnold (1974) found the lift to drag ratio on a sphere as about 0.1 and 0.5 at $R_* = 300$ and 800, respectively. Apperley (1968) studied a sphere laid on gravels and found the lift to drag ratio as 0.5 at $R_* = 70$. Further, Patnaik et al. (1994) estimated C_L ranging from 0.1 to 0.4, which is adopted here for testing the probability model.

Fig. 3 depicts the theoretical curve Θ versus P for $C_L = 0.15$ obtained by solving Eq. (17) using Eq. (18). The theoretical curve matches well with the experimental data of Guy et al. (1966) and Luque (1974) for the trial value of $C_L = 0.15$, which is within the range of C_L obtained in aforementioned studies. The data of Guy et al. (1966) that correspond to dunes have less agreement, because the present analysis does not include the flow resistance due to bedforms. According to Cheng and Chiew (1998), the curve Θ versus P for $C_L = 0.25$ corresponds to the experimental data and is also superimposed for the comparison with the present curve. However, the present curve corresponds closely with the curves of Fredsøe and Deigaard (1992) and Cheng and Chiew (1998) for $P < 0.2$, where the experimental data also collapse satisfactorily on these curves. The Shields parameter Θ for rough flow regime ($R_* > 70$) according to Yalin and Karahan's (1979) diagram is 0.046, for which the probability of entrainment is 0.1% as obtained from Fig. 3. It implies that 0.1% of all the particles on a given bed area are in motion under the threshold condition of sediment entrainment. In fact, the concept of sediment threshold refers to a short range of bed shear stress (or the Shields parameter) over which a transition takes place from an immobile bed to become a mobile bed (surface particles in motion) (Mantz 1977; Dey and Raikar 2007). Fig. 4 shows the curves P versus Θ for $C_L = 0.12$ and 0.2 forming an envelope of the experimental data. Note that Dey et al. (1999) showed that C_L varies with shear Reynolds number R_*, which makes an understanding of the range of C_L = 0.12 to 0.2 as the entrainment threshold takes place within this range.

Threshold of Sediment Suspension

When the bed-load transport takes place, some of the particles may go in suspension in the entire fluid flow zone above the bed-layer. In suspended-load transport, the particles stay occasionally in contact with the bed and are displaced by making more or less large jumps to remain often surrounded by the fluid. The criterion for a particle to bring in suspension is that the vertical velocity fluctuations v' in the flow exceed the terminal fall velocity w_s of the particle, that is $v' > w_s$. Conversely, $w_s > v'$ signifies a termination of suspension unless v' again exceeds w_s at a later time. The vertical velocity fluctuations v' are however random to follow Eq. (11); and their pdf for positive values, as in Eq. (13), can be given by

$$\left.\begin{aligned} p_{v'}(v' \geq 0) &= \frac{1}{16\sigma_2}(17+\hat{v}-\hat{v}^2)\exp(-\hat{v}) \\ p_{v'}(v' < 0) &= 0 \end{aligned}\right\} \tag{19}$$

It satisfies the condition $\int_{-\infty}^{\infty} p_{v'}(v')dv' = 1$. The plots of Eq. (19) for σ_2 = 0.5, 1 and 1.5 would be like those given in Fig. 2.

The total probability P_s of a particle to remain in suspension is thus given by an expression analogous to that of Eq. (17). It is

$$P_s = \int_{w_s}^{\infty} p_{v'}(v')dv' = \frac{1}{16}(16-b-b^2)\exp(-b) \tag{20}$$

where $b = w_s/\sigma_2$. Given w_s and if σ_2 is estimated by the rms value $(\overline{v'^2})^{0.5}$, the total probability P_s depends on the value of σ_2 at a given point in the flow, as w_s is a constant for a given particle size.

Near the bed, if the bed-layer is very thin, the bed is regarded as a rough. According to the experimental studies by Grass (1971), Nezu (1977), Kironoto and Graff (1994) and Dey and Raikar (2007), the rms value $(\overline{v'^2})^{0.5}$ is approximately equal to the shear velocity u_*. Thus, in this case, one can take $\sigma_2 \approx u_*$; and the P_s is given by

$$P_s = \frac{1}{16}\left(16 - \frac{w_s}{u_*} - \frac{w_s^2}{u_*^2}\right)\exp\left(-\frac{w_s}{u_*}\right) \tag{21}$$

For a comparatively thicker bed-layer, the bed is regarded as a smooth. The hydraulically smooth flow regime was thoroughly examined by Grass (1971), concluding to an empirical formula, that is

$$\frac{\sigma_2}{u_*} = 1 - \exp(-0.093R_*^{1.3}) \tag{22}$$

In this case, the P_s is rewritten as

$$P_s = \frac{1}{16}\left(16 - \frac{u_*}{\sigma_2}\cdot\frac{w_s}{u_*} - \frac{u_*^2}{\sigma_2^2}\cdot\frac{w_s^2}{u_*^2}\right)\exp\left(-\frac{u_*}{\sigma_2}\cdot\frac{w_s}{u_*}\right) \tag{23}$$

Since the threshold of sediment suspension is studied here, henceforth the notation u_* is replaced by u_{*c}, Θ by Θ_c and R_* by R_{*c}. As in Cheng and Chiew (1999), Eqs. (21) and (23) for hydraulically rough and smooth flow regimes, respectively, can be represented in terms of threshold Shields parameter Θ_c and R_{*c} with the introduction of a particle parameter d_* that gives

$$\Theta_c = \frac{R_{*c}^2}{d_*^3} \tag{24}$$

where $d_* = d(\Delta g/\upsilon^2)^{1/3}$. Therefore, the expression of d_* proposed by Cheng (1997) can be related to w_s/u_{*c} as follows:

$$d_* = \sqrt{\frac{1}{1.2}\left(\frac{R_{*c}w_s}{u_{*c}}\right)^{2/3}\left[\left(\frac{R_{*c}w_s}{u_{*c}}\right)^{2/3}+10\right]} \tag{25}$$

Using Eq. (21), w_s/u_{*c} was first computed from Eqs. (22) and (23) by Newton's method for a given value of P_s and a range of R_{*c} from 0.03 to 10^4. Following this step, d_* was obtained from Eq. (25); and then Θ_c was computed from Eq. (24). The computational results in terms of $\Theta_c(R_{*c})$ for different values of P_s = 0.001, 0.01, 0.05 and 0.1 are presented in Fig. 5. The entrainment threshold curve given by Yalin and Karahan (1979) is also superimposed for the comparison. Note that in this study, Yalin and Karahan's (1979) curve is often used for the comparison, as it is regarded as superior to well-known Shields diagram (Dey 1999; Dey et al. 1999). With an increase in value of the total probability P_s of suspension, the Shields parameter Θ_c for the threshold of sediment suspension is increasingly greater than that for the entrainment threshold obtained from Yalin and Karahan's curve for a given shear Reynolds number R_{*c}. It suggests that the total probability P_s of suspension increases with an increase in bed shear stress for a given sediment size, as the flow with enhanced bed shear stress can bring larger amount of sediment in suspension. As the value of probability P_s reduce further from 0.001, the resulting curve Θ_c-R_{*c} remains very close to that of P_s = 0.001, but never approaches to collapse on the entrainment threshold curve given by Yalin and Karahan. The reason is attributed to the fact that $v' > w_s$ which is the criterion for threshold of suspension cannot be the criterion for an entrainment threshold. Thus, the entrainment threshold obtained by Cheng and Chiew (1999) from the criterion $v' > w_s$ with $P_s = 10^{-7}$ invites uncertainty; and moreover a value of probability 10^{-7} is somewhat ambiguous. Hence, to define the pure bed-load region bounded by the curves of threshold of suspension and entrainment in a Θ_c-R_{*c} diagram, it is appropriate to define entrainment threshold given by another standard curve, such as Yalin and Karahan's curve, that justifies the inclusion of Yalin and Karahan's curve in Fig. 5.

The Rouse number ζ [= $w_s/(\kappa u_*)$] is an essential parameter that provides a measure of the relative effect of the gravity and the turbulence on a sediment particle in suspension. It can therefore be used to examine the condition of suspended sediment concentration. For instance, smaller the values of ζ, more particles are likely to be in suspension. Regarding the computation of Rouse number ζ, the related conversion are made by using Eqs. (24) and (25). In Fig. 6, Rouse number ζ versus particle parameter d_* for probability of suspension P_s = 0.05 are plotted using Eqs. (22) and (23). As P_s = 0.05 produces the curve ζ versus d_* that completely matches with that proposed by Cheng and Chiew (1999), P_s = 0.05 is used as an index for the threshold of sediment suspension in this study. It means that the sediment suspension begins with bringing 5% of particles in suspension from a given area at the top of bed-layer. Note that Cheng and Chiew (1999) who used a Gaussian probability distribution obtained the curve for P_s = 0.1, which is double the value of P_s obtained using the exponential distribution. It implies that the exponential based probability distribution, which has a sharp pick as compared to Gaussian distribution, yields the threshold criterion for suspension at a lower value of probability. Importantly, Bose and Dey (2010) showed that the exponential based probability distributions for the velocity fluctuations are universal, as discussed in introduction. Reverting to Fig. 6, it is evident that ζ increases sharply with d_* up to d_* = 15 and then ζ becomes independent of d_* for d_* > 15. The curves ζ versus d_* drawn from the threshold criterion of suspension given by Bagnold (1966), Xie (1981), van Rijn (1984b), Sumer (1986), Celik and Rodi (1991) and Cheng and Chiew (1999) are superimposed for the comparison. The curves of various investigators yield widely varying results for $d_* < 50$; while the threshold criterion lies in between $\zeta \approx 4.8$ and 6.1 for $d_* \geq 50$. However, Bagnold's (1966) curve provides a much reduced constant value of $\zeta = 3.05$.

Further, in Fig. 7, Rouse number ζ is plotted against Θ_c for $P_s = 0.05$. This curve clearly illustrates that for the given values of ζ and Θ whether the sediment particles of a given size in a flow can be in suspension. In fact, the curve can be used as a predication curve for the determination of threshold criterion for sediment suspension in terms of ζ and Θ_c. For instance, a particle can be in suspension if $\Theta(\zeta = 3.5) > 0.103$, as illustrated in Fig. 7. Fig. 8 illustrates the divisions of suspended-load, bed-load and no motion according to the present study and the entrainment threshold curve of Yalin and Karahan. Therefore, both the diagrams given in Figs. 7 and 8 can be used as for predicting the criterion for sediment suspension.

Conclusions

This study presents the theoretical development of the probability function for sediment entrainment to bed-load and the probability function for the threshold of sediment suspension in free surface flows. The functions are derived from the universal two-sided exponential or Laplace distribution based Gram-Charlier series expansions (Bose and Dey 2010), that are simple and easy to use. The probability for sediment entrainment as a function of Shields parameter for the lift coefficient of 0.15 agrees well with the experimental data. However, the range of lift coefficient 0.12 - 0.2 forms an envelope of all the experimental data of plane bed, indicating that the sediment entrainment takes place within that range. On the other hand, for the value of probability 0.05, the threshold of sediment suspension is indicated. The prediction curves for the criterion of threshold of sediment suspension are proposed in terms of Rouse number as a function of Shields parameter and also Shields parameter as a function of shear Reynolds number.

The present study has couple of important implications. First, it provides a plausible explanation about the probabilistic model on the bed-load transport, where an estimation of entrainment probability plays a key role. Second, following the new universal non-Gaussian probability of turbulence by Bose and Dey (2010), this study sheds some light on the non-Gaussian behavior of entrainment probability and threshold of suspension. Therefore, the findings of the study raise a number of issues that can address how to analyze the sediment entrainment and suspension, as a future scope of research. The most important is how best to include the non-Gaussian behavior of entrainment probability and threshold of suspension into a model of the sediment transport process.

Acknowledges

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Notation

The following symbols are used in this paper:

C_L = lift coefficient;

d = median diameter of particle;

d_* = particle parameter;

F_D = drag force;

F_G = submerged weight of particle;

F_L = lift force;

g = acceleration due to gravity;

k_s = Nikuredse equivalent roughness height;

M_{jk} = $\overline{\hat{u}^j \hat{v}^k}$;

P = total probability function for entrainment;

P_s = total probability function for threshold of suspension;

$p_{\hat{u}}(\hat{u})$ = probability density function for $\hat{u}$;

$p_{u'}(u')$ = probability density function for u';

$p_{\hat{v}}(\hat{v})$ = probability density function for $\hat{v}$;

$p_{v'}(v')$ = probability density function for v';

R_* = shear Reynolds number;

R_{*c} = threshold shear Reynolds number;

s = relative density of sediment;

U = free stream velocity;

u = instantaneous flow velocity in streamwise direction;

$\bar{u}$ = time-averaged u;

$\hat{u}$ = u'/σ_1;

u' = fluctuations of u;

u_* = shear velocity;

u_{*c} = threshold shear velocity;

u_b = instantaneous near-bed velocity;

$\bar{u}_b$ = time-averaged near-bed velocity;

v = instantaneous flow velocity in vertical direction;

$\bar{v}$ = time-averaged v;

$\hat{v}$ = v'/σ_2;

v' = fluctuations of v;

w_s = terminal fall velocity;

x = streamwise distance;

y = vertical distance;

y_0 = zero-velocity level;

Δ = submerged relative density of particle;

Θ = Shields parameter;

Θ_c = threshold Shields parameter;

κ = von Kármán constant;

μ_d = coefficient of dynamic friction;

υ = kinematic viscosity of fluid;

ρ = mass density of fluid;

ρ_s = mass density of sediment;

σ_1 = rms of u';

σ_2 = rms of v';

τ_{0c} = threshold bed shear stress; and

ζ = Rouse number.

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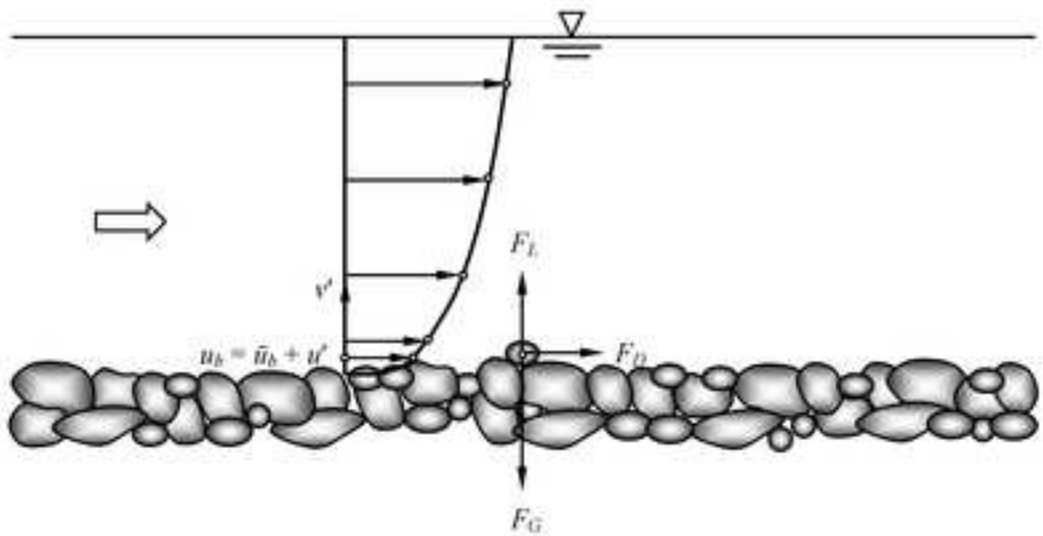

Fig. 1. Schematic of a particle subjected to instantaneous hydrodynamic forces due to near-bed fluctuating velocity components

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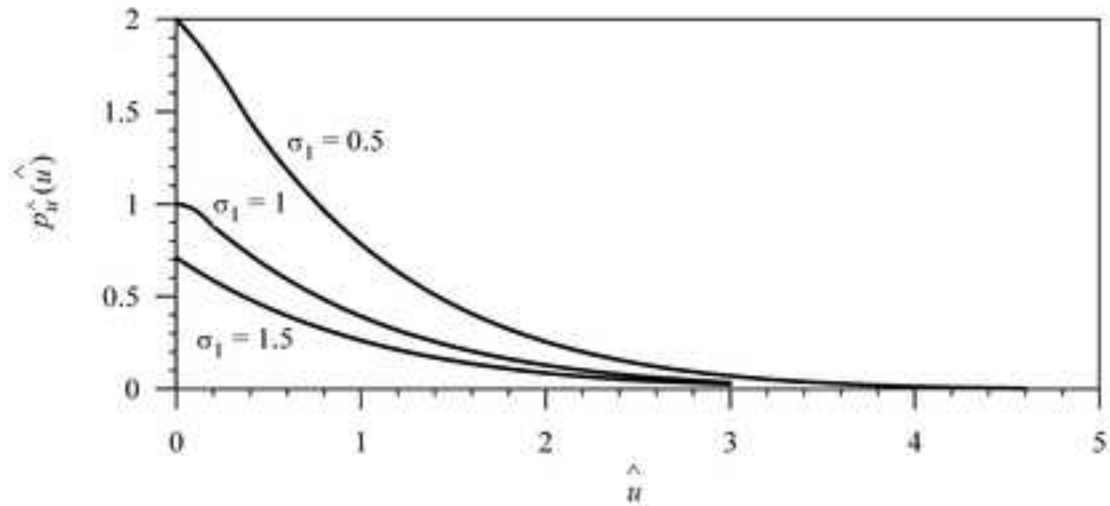

Fig. 2. Probability density $p_{\hat{u}}(\hat{u})$ of normalized streamwise velocity fluctuations $\hat{u}$ for different values of $\sigma_1 = 0.5$, 1 and 1.5 computed from Eq. (12)

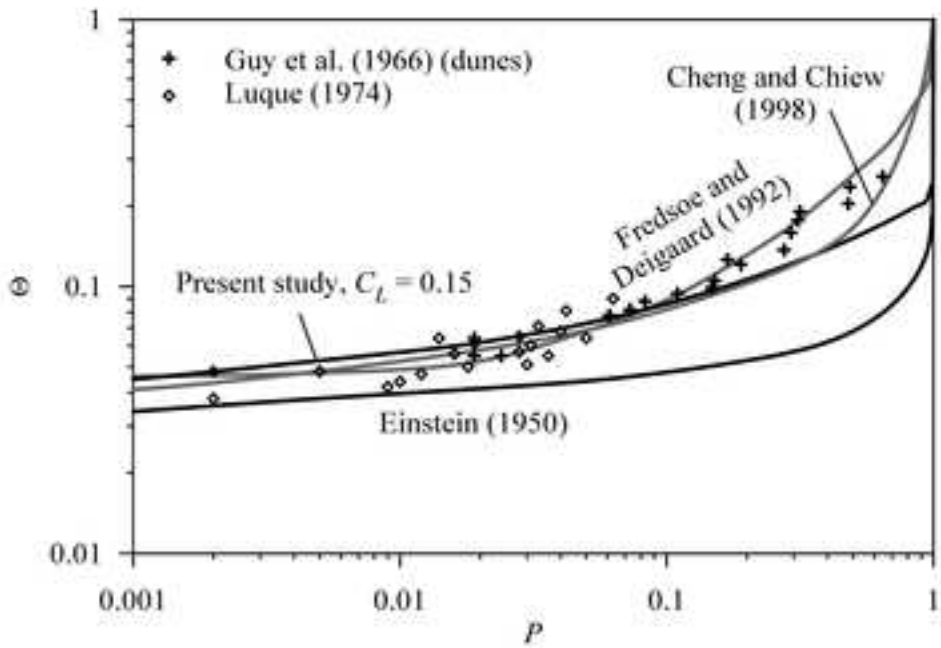

Fig. 3. Variation of Shields parameter Θ with probability P of sediment entrainment. The Θ versus P curves obtained from the entrainment criterion given by Einstein (1950), Fredsoe and Deigaard (1992) and Cheng and Chiew (1998) and the experimental data of Guy et al. (1966) and Luque (1974) are shown for the comparison purpose

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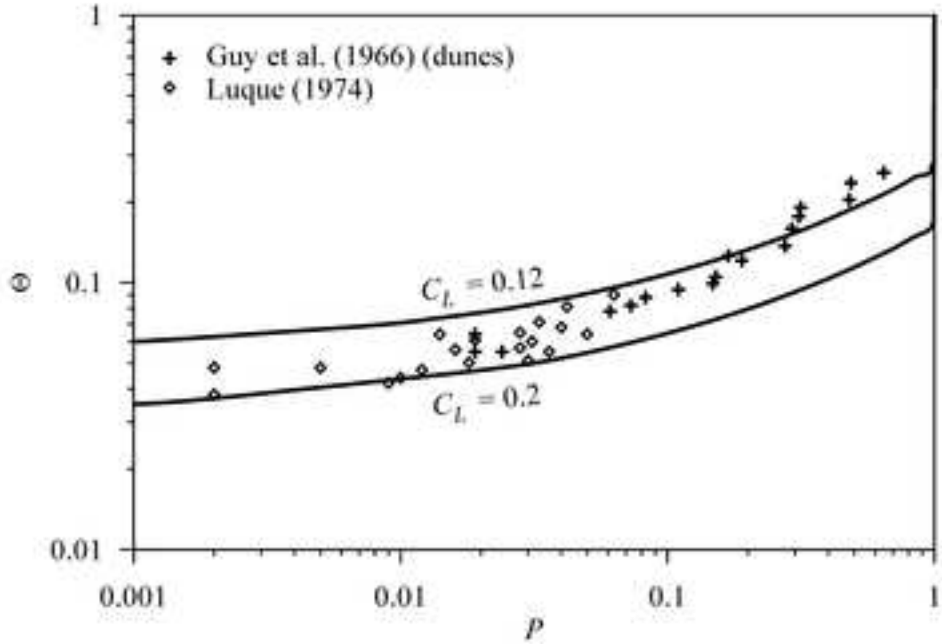

Fig. 4. Curves for Shields parameter Θ versus probability P of sediment entrainment for C_L = 0.12 and 0.2 forming an envelope of the experimental data

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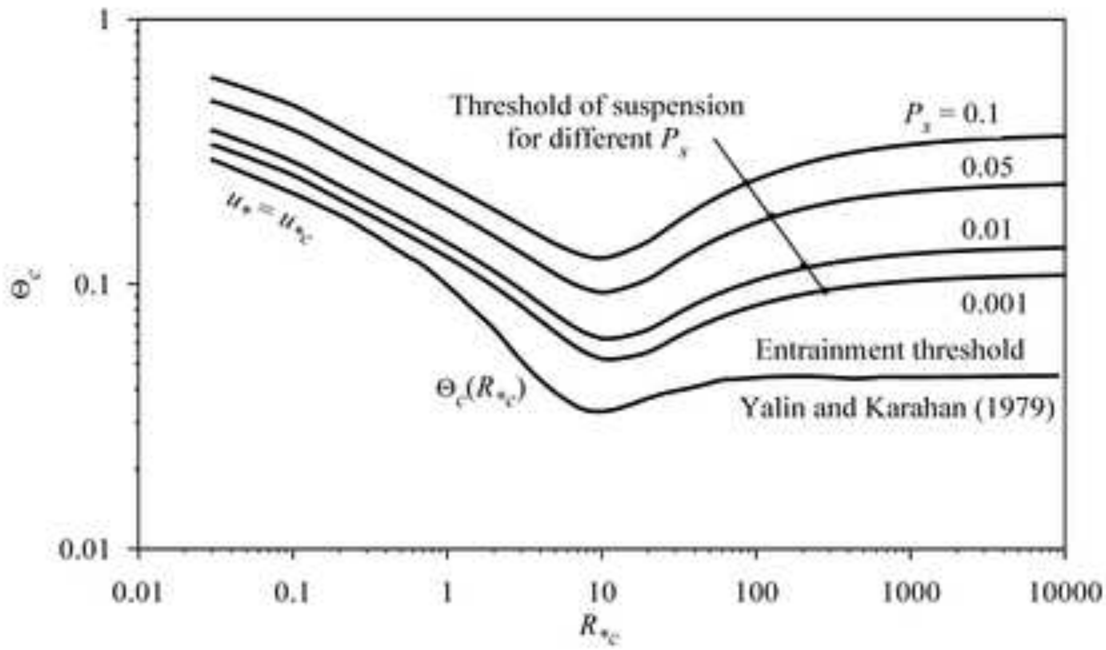

Fig. 5. Curves for the threshold of suspension in terms of $\Theta_c(R_{*c})$ for different values of P_s = 0.001, 0.01, 0.05 and 0.1. Entrainment threshold curve given by Yalin and Karahan (1979) is superimposed for comparison

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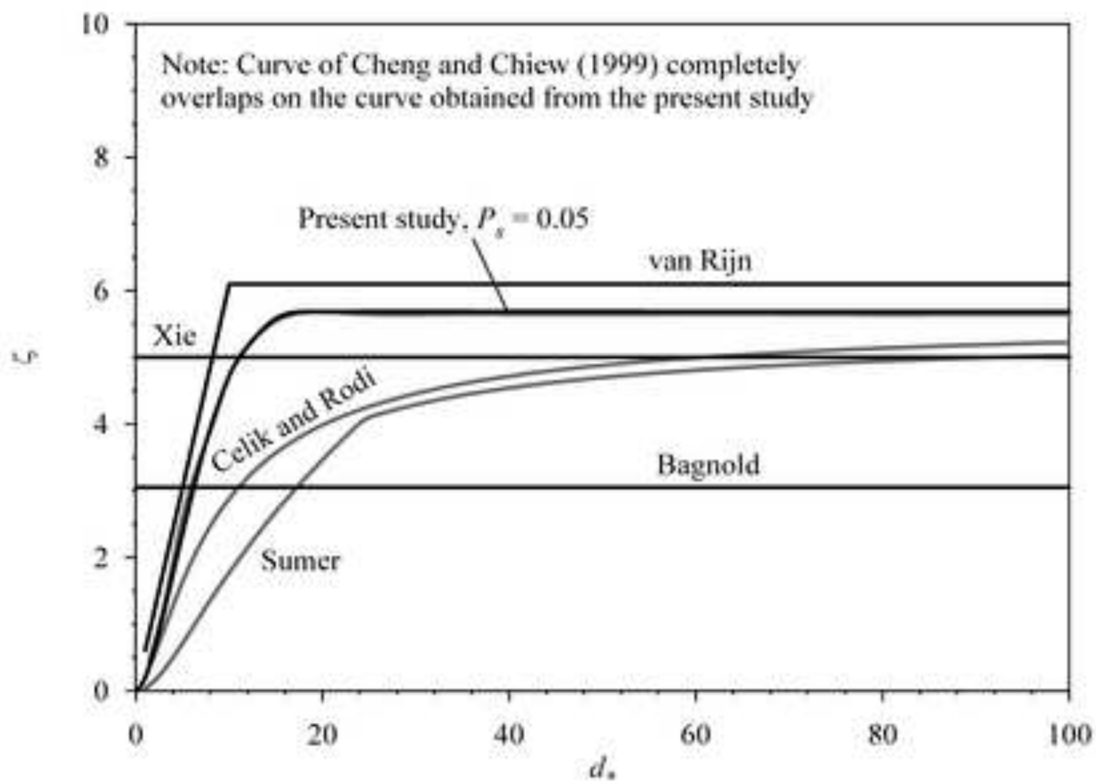

Fig. 6. Variations of Rouse number ζ with particle parameter d_*. The curves of ζ versus d_* drawn from the threshold criterion given by Bagnold (1966), Xie (1981), van Rijn (1984b), Sumer (1986), Celik and Rodi (1991) and Cheng and Chiew (1999) are superimposed for the comparison purpose

Figure

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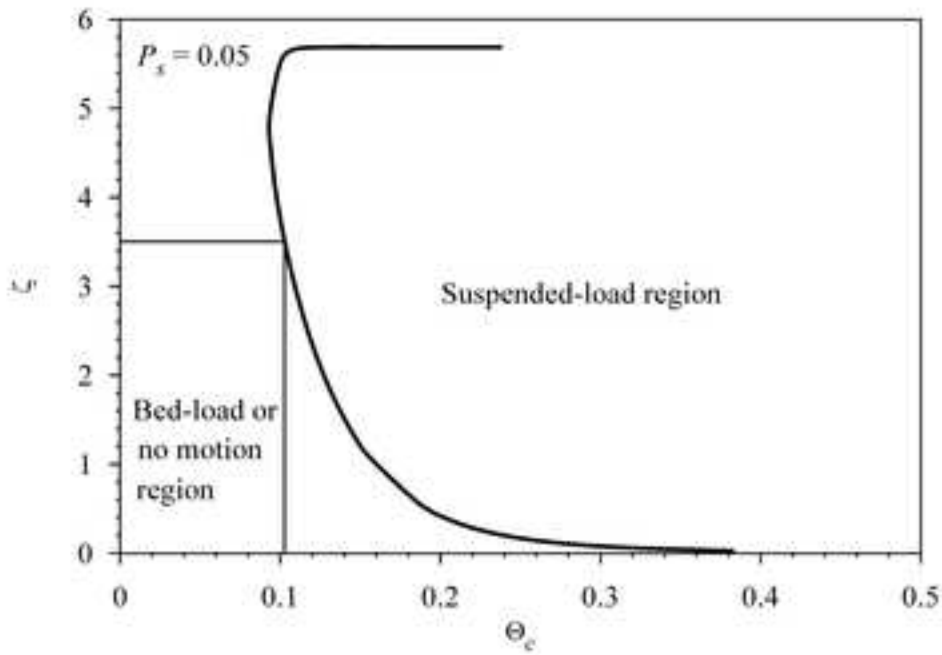

Fig. 7. Diagram for the prediction of threshold of suspended-load from bed-load in terms of Rouse number ζ as a function of Θ_c