Planetary motions and the birth of classical mechanics*

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The subject matter of this article straddles the birth of modern science and involves several larger-than-life figures – Nicolas Copernicus (1473–1543), Tycho Brahe (1546–1601), Galileo Galilei (1564–1642), Johannes Kepler (1571–1630) and Isaac Newton (1642–1727). This account will be woven in a connected and coherent way around their lives and work. Copernicus came first in this group; then Brahe, Galileo and Kepler overlapped and were in contact with one another; and after they were all gone came Newton.

MATHEMATICS, medicine and astronomy are the oldest areas of knowledge in all civilizations. In each culture there has evolved some model of the universe, some 'system of the world', as a framework within which to fit astronomical observations. For our present purpose we are concerned with the so-called medieval model. Before describing it, let us briefly look at some of the essential details about planets and their orbits (Table 1).

From ancient times till near the end of the 18th century – and this includes the period from Copernicus to Newton – only the earth and five other planets were known.

The medieval model was a combination of Greek science going back to Aristotle and Ptolemy, and later Christian theology. Figure 1 is a picture of the universe as presented by Dante Alighieri in the 13th century in his *Divine Comedy*[1]. The earth is a globe at the centre. Then there are nine solid translucent crystal spheres, each centred on the earth. The first seven revolve and transport respectively the Moon, Mercury, Venus, Sun, Mars, Jupiter and Saturn in sequence. The eighth sphere carries the fixed stars. Last of all comes the ninth sphere, the Primum Mobile, which like an engine drives all the others. Beyond that is heaven, the home of God. In Aristotle's version, there were actual spheres, while later Ptolemy replaced them by circular orbits. This emphasis on spheres and circles reflected the feeling that these were the most perfect geometrical figures. There was also a sharp distinction between terrestrial matter and phenomena, and celestial ones. In this picture the heavens were perfect and unchanging.

The medieval model of the universe slowly gave way under advances in astronomy and mathematics, over the 16th and 17th centuries. The principal blows were struck by Copernicus, Tycho Brahe, Galileo and Kepler. Let us at this point recall briefly the life and work of each of them[2].

Copernicus was born in 1473 in Torun in Poland. He stands at the boundary – the transition point – between medieval and modern science. For a ten-year period, from age 24 to 34, he studied in Italy at Bologna, Padua and Ferrara. He was very close to his teacher, Domenico Maria da Novara, the relationship between them being as between a guru and a shishya. Copernicus studied mathematics, astronomy, philosophy, law, medicine and theology – the times seem long gone since one could even contemplate such an endeavour. He also had direct access to Greek sources. During his sojourn in Italy, Copernicus saw the need to replace the Ptolemaic system by a simpler one in which calculations would be easier. His ideas were elaborated much later and led to his book *De Revolutionibus Orbium Celestium* (*On the Revolution of the Celestial Spheres*). He retained the notion that celestial bodies travel on circles lying on uniformly rotating spheres; in any case the data available to him was not accurate enough to show up departures from circular motions. Figure 2 shows a picture of his model of the universe[3]. The sun, rather than the earth, was at the centre. Then followed in sequence the spheres carrying Mercury, Venus, the earth, Mars, Jupiter, Saturn and the fixed stars. This heliocentric model had been suggested by Aristarchus in 300 BC, and Copernicus rediscovered it. In his words, as the sun is the visible form of God, 'in the centre of everything the sun must reside; in the most beautiful temple created by God, there is the place which awaits him where he can give light to all the planets'[4].

Copernicus showed mathematically that this model could work and was simpler than Ptolemy's. However for fear of censure he presented it cautiously as a hypothesis, useful only for calculations. He also spoke of the concept of relative motions, with respect to the sun or the earth regarded as stationary; and showed that the earth both rotates on its own axis and revolves

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Table 1. Planets and orbits

Planet	Mean distance from sun (million miles)	Revolution period	Number of satellites	Remarks
Mercury	36	88 days	0	Known since antiquity
Venus	67	225 days	0	"
Earth	93	1 year	1	–
Mars	142	1 year 322 days	2	"
Jupiter	483	11.9 years	16	"
Saturn	866	29.5 years	18	"
Uranus	1783	84 years	15	Discovered 1781 by Herschel in England
Neptune	2794	165 years	8	Predicted 1843–46 by Adams/ Le Verrier; discovered 1846 in Berlin
Pluto	3666	249 years	1	Discovered 1930 by C. W. Tombaugh in USA

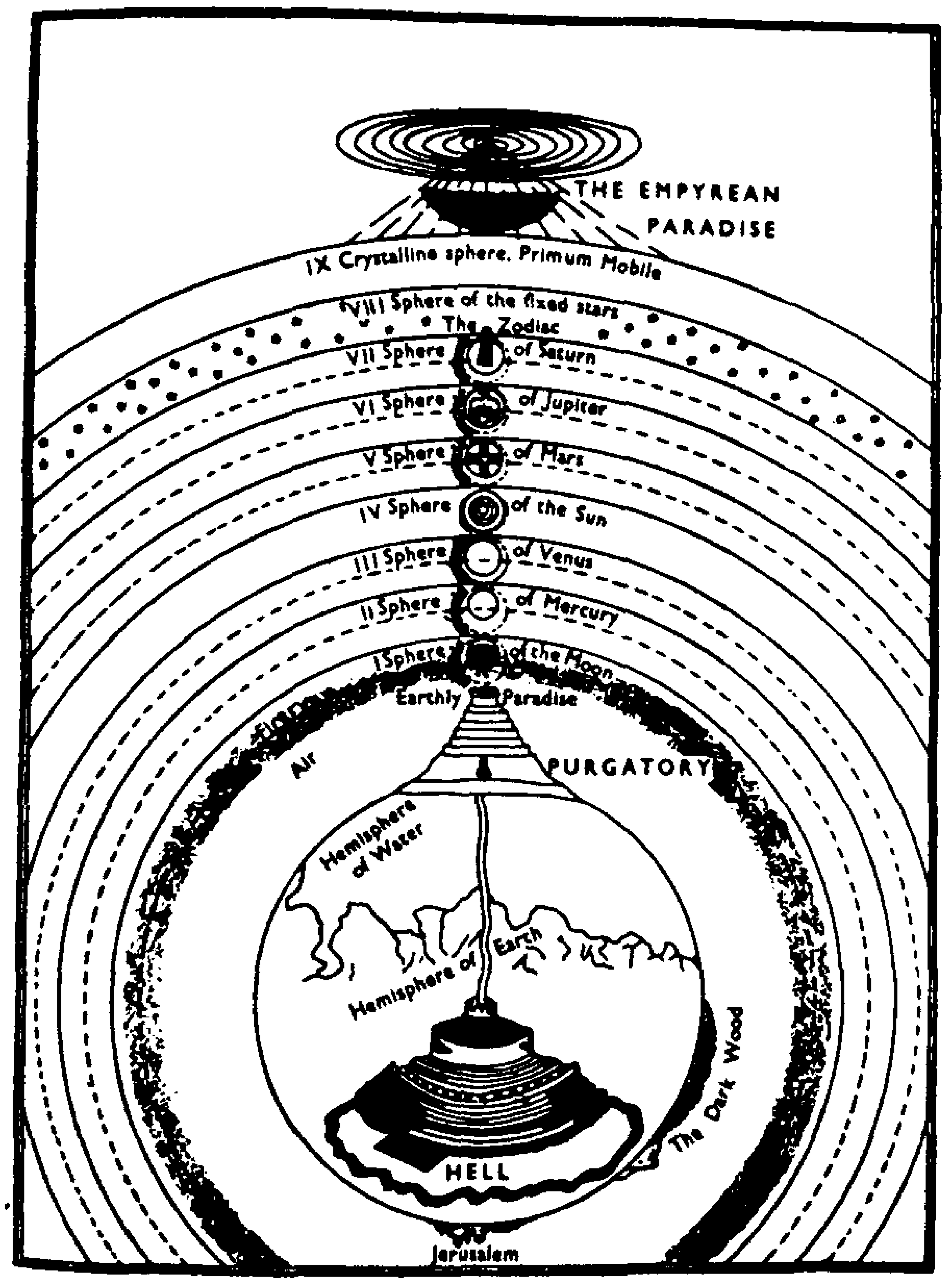


Figure 1. The medieval model of the universe, from Dante's *Divine Comedy*, 13th century.

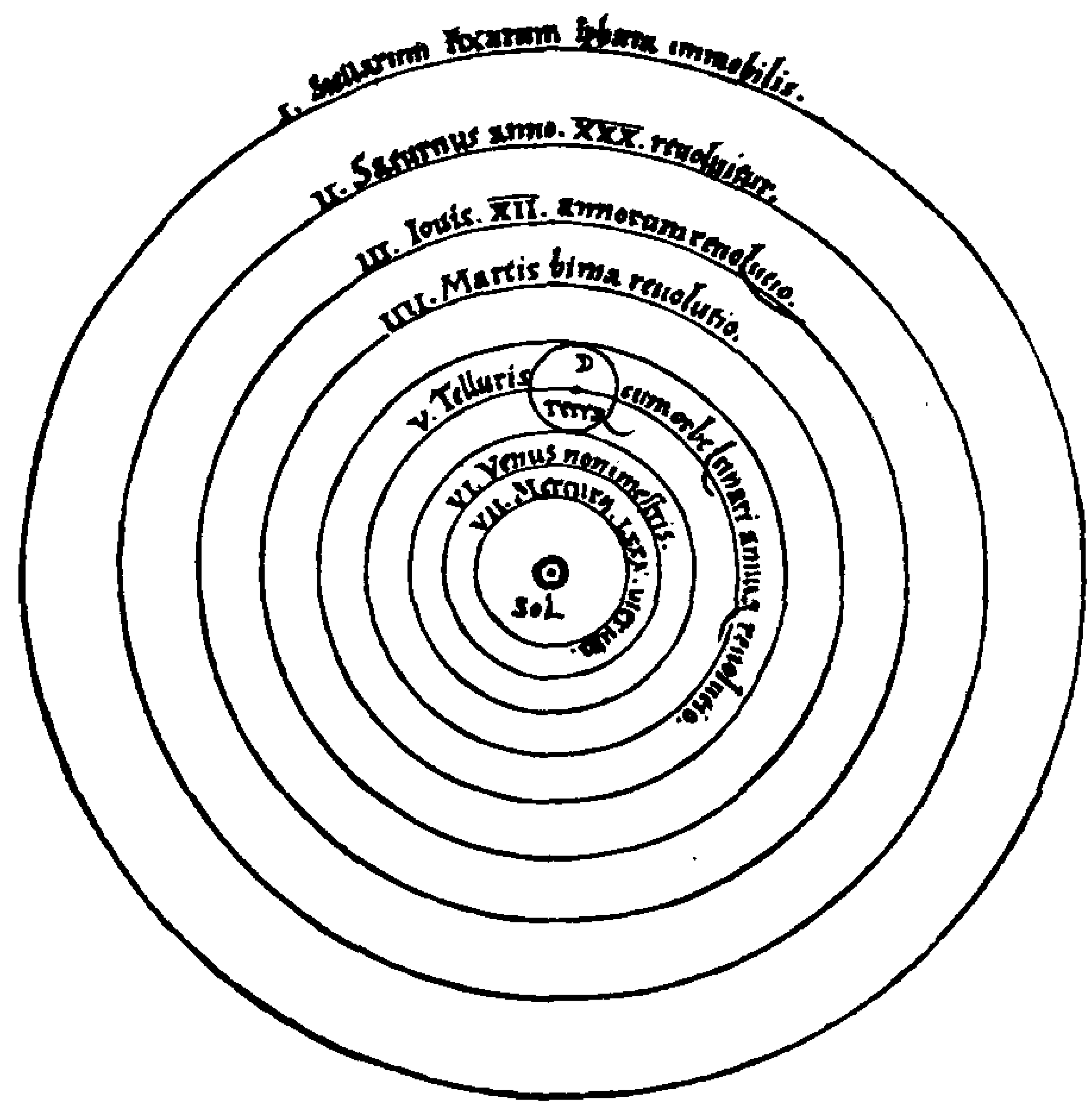


Figure 2. The Copernican model of the universe, 1543 AD.

about the sun. His book was technical and difficult for the average reader; it took some fifty years for his ideas to spread. Giordano Bruno, Galileo and Kepler were among the few who understood and accepted Copernicus' ideas. Bruno paid with his life in 1600; while Galileo and Kepler reinforced each other's beliefs privately, before each publicly acknowledged the new system.

Brahe came from the Swedish nobility, and was interested in astronomy and astrology. He was also a very gifted instrument maker and observational astronomer. The King of Denmark supported him handsomely to set up a fine observatory on the island of Hveen. Here he designed his own new instruments, and kept daily astronomical records meticulously for many years. His figures are accurate to within 1′ or 2′ of arc, about one-sixtieth of a degree. (Recall that these were the days before optical telescopes.) This later became a treasure for Kepler to work on.

Two major discoveries of his made it clear to Tycho that the heavens are not immutable, that celestial bodies are changeable: one was the appearance and disappearance of a star in the years 1572–1574, another the passage of a comet beyond the moon in 1577. However, he did not accept the Copernican model but had a hybrid of his own – the sun and the moon circled the earth, while the other five planets circled the sun. For ease in calculations, this stood midway between the Ptolemaic and the Copernican systems.

Around 1599, due to changed fortunes, Brahe left Denmark and moved to Prague as 'Imperial Mathematician' to the Emperor Rudolph II. Kepler joined him there in January 1600, but Brahe died soon after in November 1601. Galileo admired Brahe's instruments and data but not his theories. Ultimately Brahe's data together with the Copernican model percolated to others and set the stage for major advances by Kepler and Galileo.

The lives and stories of Kepler and Galileo overlap a great deal. For convenience let us look first at Kepler and then turn to Galileo, making some comparisons as we go along. Kepler was a student of theology in Tubingen, gifted in mathematics and possessing mystical tendencies. In contrast, Galileo was led much more strongly by the power of reason and experiment. From his teacher Michael Maestlin at Tubingen, Kepler learned privately of the Copernican model. In 1594 he was obliged to leave for Graz as a high school mathematics teacher. Here while in class he had a flash of insight revealing to him a new geometrical model for planetary orbits with the sun at the centre. There were precisely six planets because there were exactly five perfect solids, and these were nested in successive spheres. The solids in sequence are the cube, the tetrahedron, the dodecahedron, the icosahedron and the octahedron. The innermost sphere lay inside the cube, touching its faces; the next one passed through its vertices, but touched the faces of the tetrahedron; and so on. The fit to the radii of planetary orbits was almost, but not quite, perfect. He wrote out his ideas, mixed with much speculation, in his book *Mysterium Cosmographicum* published in 1596, and sent copies to Galileo, Brahe and others. In this book he publicly declared support to the Copernican model – Galileo did so much later, in 1613.

The agreement of his model with the data was almost perfect. Kepler saw that to make progress much more accurate data was needed, and only Tycho had it. Kepler wrote to Tycho for help and, as it happened, simultaneously the latter invited the former to join him at Prague as his assistant. This was in 1600. Tycho had been much impressed by Kepler's work, and wanted him to help vindicate his own hybrid model. Soon after, when Tycho died in November 1601, Kepler ended up inheriting all the precious data (notwithstanding problems with Tycho's son-in-law Tengnagel).

Now Kepler set to work at a feverish pace; he did enormous calculations to improve his model, working particularly on the troublesome orbit of Mars. The period 1601 to 1619 – over which his three laws of planetary motions were discovered – was one of high drama. In brief the main steps he took were: (a) he created the true heliocentric model by placing the sun at the 'centre' of planetary orbits, modifying Copernicus' procedure of placing the geometric centre of the earth's orbit there; (b) he found that each planetary orbit lies in a plane containing the sun; (c) the orbit of Mars, which proved the most difficult, almost fitted a circle except for two data points; (d) as a result, with Tycho's accurate data, he saw that a circular orbit for Mars was not possible; (e) he also gave up the assumption of uniform speed for Mars as it traverses its orbit, allowing a speeding up or slowing down when close to or far away from the sun; (f) from the data he then discovered the Law of Areas: the radius vector from the sun to Mars swept out equal areas in equal intervals of time; (g) and again from the data he found at last that Mars moved on an ellipse with the sun at a focus. These two laws discovered in the order mentioned during 1602 were announced seven years later in the 'Astronomia Nova' but in the opposite order: the Law of the Ellipse became the First Law, and the Law of Areas the Second. Imagine waiting for seven years today before announcing laws of such magnitude!

After dealing with Mars, Kepler showed that these two Laws hold for the earth too, and then extrapolated them to all the other planets. The Third Law, which links different planets to one another, took much more work; a true search for harmony in the heavens, it was announced in 1619 in the 'Harmonice Mundi'. It states that the periods of revolution are proportional to the 3/2 powers of the mean distances from the sun. This became the key to the discovery of the inverse square law of gravitation by Newton and others. Added to all this, Kepler came very close to the law of gravity. He saw the sun – and not angels with flapping wings – as the agent keeping the planets on their courses. But he unfortunately died before Galileo's delayed publication, in 1632, of his experimental studies on the laws of motion, especially the laws of inertia and of uniformly accelerated motion.

Galileo chose physical science in preference to medicine which his father had wanted him to pursue. He was highly respected as a mathematician at the University of Padua. Whereas in his correspondence with Kepler he supported the Copernican model, till as late as 1606 in his lectures he taught the medieval model. Only in

1613, as mentioned earlier, did he publicly express support for the Copernican view. His 'change of heart' came from his discoveries in astronomy.

During the period 1589–1591 at Pisa, his researches were mainly concerned with the description of motion: falling bodies, inclined planes, uniform acceleration. (Here we may parenthetically remark that it took Galileo several years to arrive at the concept of acceleration which to us today seems so obvious and natural!) Thanks to his work, 'for the first time, all the important features of motion – distance covered, speed, and acceleration – were expressed in terms of time. Galileo had realized that time is the independent variable in the description of motion; indeed, he was the first to time physical events. This was an extremely fruitful idea... it implied an entirely new conceptual view of the world, codified by Isaac Newton forty eight years later'. Then, in 1609, he heard that a telescope had been invented in Holland. He promptly reinvented it, and achieved even greater power. Then he turned his telescope to the heavens – and changed astronomy for ever!

Galileo saw that the moon was scarred, with mountains and valleys, hardly a perfect sphere; he found sunspots which moved. In 1610 he found four satellites circling Jupiter, all coplanar and the nearer ones circling faster. So everything in the sky did not circle the earth! And just as Jupiter with its circling moons went round the sun, so too our earth with our moon could orbit the sun. Again in 1610 he discovered the phases of Venus, just like our moon. So Venus like the earth was dark, went round the sun, and reflected the sun's light. He found moons around Saturn too, and almost saw the rings. Here was telling evidence for the Copernican theory, and the immutability of the heavens had been destroyed.

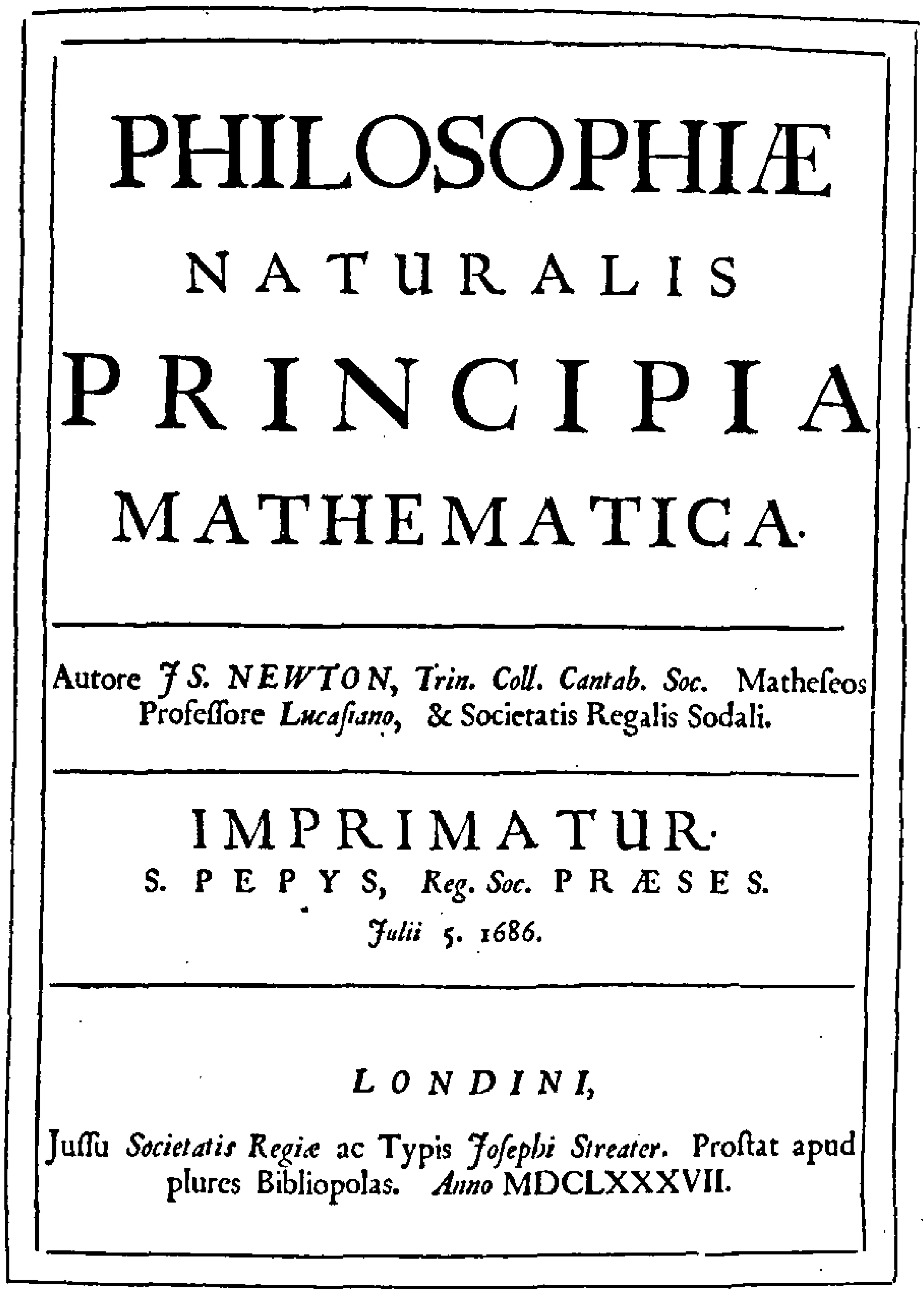
We all know the trouble Galileo got into with the Church upon expressing his views in his 1632 treatise *Dialogue on the two Chief World Systems, the Ptolemaic and the Copernican*. After this historic controversy and confrontation, he returned to his earlier studies on the science of motion, and published his results in *Discourses Concerning two New Sciences*. This is the first great modern work on the kinematics of motion. Smuggled out of Italy in 1638 and published in Holland, the message spread. To some degree Galileo was continuing an existing tradition of study of mechanics, created by the Schoolmen of the Universities of Oxford and Paris in the fourteenth century. John Buridan – probably better known for his ass – had already created the 'impetus' concept. But much he made himself, especially in showing that Aristotle's ideas about motion were untenable.

As precursors to Newton, and as the giants on whose shoulders he stood, Kepler and Galileo left him a splendid legacy – the laws of planetary motion, and the laws of mechanics. Newton synthesized and systematized all this, discovered the Law of Universal Gravitation which unified terrestrial and celestial phenomena, and went far ahead in many ways. He accomplished the most decisive transition from description to explanation of natural phenomena.

Already around 1665 at age twenty three, Newton had conceived of the universal inverse square law of gravity, and seen that it could explain both the fall of the apple on the earth, and the moon's going round the earth. As he recalled many years later: 'I deduced that the forces w^{ch} keep the Planets in their Orbs must be reciprocally as the squares of their distances from the centers about w^{ch} they revolve: & thereby compared the force requisite to keep the Moon in her Orb with the force of gravity at the surface of the earth, & found them answer pretty nearly'[5]. But he could not proceed further for two reasons. One was the rather inaccurate data available to him at that time. The other was a major mathematical problem which he solved only in 1685: the proof that with an inverse square law of force, and no other, a uniform spherical mass acts as though all its mass were concentrated at its centre.

Now we come to the famous 1684 visit of Edmund Halley to Newton in Cambridge[6]. In January of that year, at a discussion among Halley, Christopher Wren and Robert Hooke at the Royal Society, the question arose of deriving *all* of Kepler's Laws from the principles of dynamics. Many had realized that this was the key problem whose solution was crucial for further progress. By combining Kepler's Third Law with a recent formula of Huyghens for the centripetal force, Halley had concluded that the force of attraction between sun and planet must be proportional to the inverse square of the distance between them. And he was not the only one to have found this – so had Wren and Hooke too. But Kepler had anticipated it, and Newton had used it in 1665. At the Royal Society discussion, Hooke claimed he could conversely derive all of Kepler's Laws from the inverse square law of gravity; but he would not divulge the details till the others had tried and failed. While Wren doubted Hooke's claim, Halley admitted he was unable to do this. All in all the stage was set for Halley's visit to Newton in August 1684. What transpired is best told in Newton's own reflections recorded by de Moivre:

'In 1684 Dr Halley came to visit him in Cambridge, after they had been some time together, the Dr asked him what he thought the Curve would be that would be described by the Planets supposing the force of attraction towards the sun to be reciprocal to the square of their distance from it. Sir Isaac replied immediately that it would be an Ellipsis, the Doctor struck with joy and amazement asked him how he knew it, why saith he I have calculated it, whereupon Dr Halley asked him for his calculation without any farther delay, Sir Isaac


PHILOSOPHIÆ
NATURALIS
PRINCIPIA
MATHEMATICA.

Autore *J S. NEWTON*, *Trin. Coll. Cantab. Soc.* Mathefeos Profeffore *Lucafiano*, & Societatis Regalis Sodali.

IMPRIMATUR.
S. PEPYS, *Reg. Soc.* PRÆSES.
Julii 5. 1686.

LONDINI,

Juffu *Societatis Regiæ* ac Typis *Jofephi Streater*. Proftat apud plures Bibliopolas. *Anno* MDCLXXXVII.

Figure 3. Title page of Newton's *Principia*, 1686 AD.

looked among his papers but could not find it, but he promised him to renew it, & then to send it him . . . .'

As Westfall comments, Newton was being cagey in saying he had misplaced his work – he too wanted to be careful and not let others see his calculations! The missing papers have since been found. Anyway this was enough indication for Halley to persuade Newton to publish his findings so that all the world could see what he had accomplished. This became a herculean effort lasting three years and resulting in the publication, over 1686–1687, of the '*Philosophiae Naturalis Principia Mathematica*' – *Mathematical Principles of Natural Philosophy* – the greatest book on science ever written[7]. The title page is shown in Figure 3. It was like creating a new language with its own grammar, and writing an epic poem in it, all at once! In the *Principia*, Newton laid the foundations of classical mechanics in an axiomatic and deductive form a' la Euclid, and created the framework for progress in the succeeding centuries. He picked up from Galileo's work on mechanics, and extended, systematized and completed it, all at once. Then he applied it to his law of universal gravitation and showed how all of Kepler's Laws followed. He demonstrated that a mathematical model of natural phenomena, involving observed and measured quantities and possessing predictive power, could be built.

Newton's First Law of Motion was a restatement of Galileo's principle of inertia. Even Descartes had said that an isolated body moves in a straight line. The Second Law of Motion had been grasped by Galileo. The Third Law – the equality of action and reaction – was Newton's own and unique contribution. It became the basis and set the pattern for Conservation Principles. All this was presented against a background of clearly stated views on the natures of space and time.

The Law of Gravity as we saw had been foreseen by Kepler, but the unification of the terrestrial with the celestial was Newton's achievement. He showed that the moons of Jupiter and of Saturn going around their parent planets obey Kepler's Laws; the planets going around the sun do so; the moon round the earth does likewise. Everywhere throughout the solar system his principles of mechanics and gravity held sway. His theory for cometary orbits went beyond Kepler, and he explained the tides and many another phenomenon.

That is the story of planetary motions and the birth of classical mechanics. Newton's methods were propagated mainly in Europe by Euler, Lagrange, Laplace and others. And success followed success. One stunning episode from the nineteenth century is worth brief mention, and that is the story of Neptune. Around 1820 there remained some disagreements between theory and observations on the orbit of Uranus, and Bessel suggested there might be another planet causing disturbances. (Of course in Kepler's time the mutual influences of the planets upon one another were not accessible, but much water had flowed under the bridge since then.) In 1843 a student at St. John's College in Cambridge, John Couch Adams, completed a dissertation on this problem and calculated the orbit of the suspected new planet in detail. In 1845 he alerted the Greenwich Observatory to look for it. Independently, Urbain Jean Le Verrier of Paris showed in June 1846 that the data on Uranus could not be explained without the presence of another planet about twice as far away. He sent his prediction to the Berlin Observatory. Finally success came at Berlin: on two successive nights, September 23rd and 24th of 1846, the planet Neptune was spotted just where the theorists had predicted it would be!

Later in the century, discrepancies arose between theory and observations for the precession of the perihelion of mercury. After accounting for all known perturbing effects, there remained an unexplained precession amounting to 43" of arc per century. Faith in Newton's programme was so strong that it was assumed that yet another planet was at work – and it was named Vulcan!

But there is no Vulcan, and that problem was solved in an entirely unexpected way by Einstein's general theory of relativity. That theory gave a language to talk about a much larger world than that of the Ancients or of Newton's World System – but that, as the bartender said to Irma, is another story.

Suggestions for further reading, references and footnotes

The theme of this presentation has been so well studied for so long that excellent sources are available in plenty. Only a very select set of possible references will therefore be presented here.

1. This figure is taken from Hanbury-Brown, R., *The Wisdom of Science – its relevance to Culture and Religion*, Cambridge University Press, 1986, p. 43.
2. Giorgio Abetti, *The History of Astronomy*, Sidgwick and Jackson, London, 1954; Arthur Berry, *A Short History of Astronomy*, Dover, New York, 1961; Mathews, P. M., 'Galileo and Kepler – Precursors of Newton', in Proceedings of the Seminar on '300 Years of Newton's *Principia*' (eds Shrivastava, S. K. and Mukunda, N.), ISRO–IISc Space Technology Cell, Indian Institute of Science, Bangalore, 1986.
3. Taken from Brown–Hanbury, R. ref. 1 above, p. 46.
4. Abetti, G. ref. 2 above, p. 70.
5. Richard S. Westfall, *The Life of Isaac Newton*, Cambridge University Press, 1993, p. 39.
6. Richard S. Westfall, ref. 5 above, ch. 8.
7. In this connection, see the highly interesting account given by D. Speiser, '*Newton's Principia*', CERN Report 80–02, 25th February 1980, CERN, Geneva, Switzerland.

Origin of the highest energy cosmic rays*

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After briefly recalling the history of discovery of cosmic rays, the present status of the subject is discussed. Cosmic-ray particles with energy exceeding 10^{20} eV have been detected. The origin of cosmic rays remains an unsolved problem in physics and astrophysics. The nature of the source(s) as well as the physical mechanism(s) responsible for endowing the cosmic-ray particles with extremely high energies are not known with certainty. We discuss some recent ideas in this context with special emphasis on the problem of origin of the highest-energy cosmic rays.

1. A brief history of cosmic rays

The story of the discovery of cosmic rays forms one of the most fascinating episodes of the history of science in the very early part of the current century. It is hard to associate the discovery of cosmic rays entirely with any one single experiment. Indeed, a number of remarkable experiments performed by a number of adventurous physicists, many of whom were tantalizingly close to the 'discovery', preceded the actual announcement of the discovery. A nice account of this history is given in the book by Pomerantz[1] which forms the basis of the historical aspects of the subject described in this section.

The Austrian physicist Victor Hess announced his discovery of 'an extra-terrestrial source of penetrating radiation' in 1912 after a series of heroic balloon-borne experiments performed by him over the previous one year. The 'penetrating radiation' was later christened 'cosmic rays' by Robert Millikan in 1926. The word *extra terrestrial* is important here, for it was well known at the time of Hess's experiments that our own Earth is also a source of 'penetrating radiation' due to natural radioactivity of soil and various rocks.

Natural radioactivity was the 'in' physics in the closing years of the nineteenth century and the early years of the twentieth century. Within a year of the discovery of X-rays by Röntgen in 1895, Becquerel discovered natural radioactivity (in 1896). Even before the discovery of radioactivity, experiments with gold-leaf electroscope, an instrument that could measure the presence of free electric charges (i.e. ionization) in a medium, and which played a major role in the history of discovery of cosmic rays, showed presence of leakage currents associated with ionization in sealed containers even in apparent absence of any obvious ionizing source; the leakage current seemed to correspond to an average rate of formation of ions of ~ 10 ions/cm^3/sec. This ionization was naturally attributed to the presence of 'radioactive material' in air and in soil. As revealed by later experiments, this conclusion was largely correct, but not entirely so!

In 1898 Rutherford established two different kinds of radioactive emissions: (i) α-rays and (ii) β-rays. The α-rays were found to be of high ionizing capacity, easily absorbable in media (range ~ few cm), and were later identified as the nuclei of ^{4}He atoms. The β-rays, in

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