Similaritons in nonlinear optical systems

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Abstract: By using the lens-type transformation, exact soliton and quasi-soliton similaritons are found in (1+1), (2+1) and (3+1)-dimensional nonlinear Schrödinger equations in the context of nonlinear optical fiber amplifiers and graded-index waveguide amplifiers. The novel analytical and numerical results show that, in addition to the exact solitonic optical waves, quasi-solitonic optical waves with Gaussian, parabolic, vortex and ring soliton profiles can evolve exact self-similarly without any radiation.

OCIS codes: (190.4370) Nonlinear optics, fibers; (230.7370) Waveguides; (190.3270) Kerr effect.

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#92333 - $15.00 USD Received 1 Feb 2008; revised 12 Mar 2008; accepted 28 Mar 2008; published 21 Apr 2008

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1. Introduction

The nonlinear Schrödinger equation (NLSE) and its stationary solutions, such as solitons and vortices, have been extensively studied in various fields such as nonlinear optical systems, plasmas, fluid dynamics, Bose-Einstein condensation and condensed matter physics. Fiber optics and waveguide optics are used in most of the important applications, and studies of optical similaritons, which are the self-similar waves that maintain their overall shapes but with their parameters such as amplitudes and widths changing with the modulation of system parameters, have recently attracted much attention [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14].

Generally speaking, optical similaritons can be divided into two categories. The first category is the asymptotic optical similaritons, which are mainly described by the compact parabolic, Hermite-Gaussian and hybrid functions [1, 2, 3, 4] in the context of nonlinear optical fiber amplifiers. Later, the concept of self-similar evolution of parabolic pulses was transplanted to the context of nonlinear planar waveguide amplifiers where the asymptotic parabolic similaritons were found not only for (1+1)-dimensional NLSE but also for (2+1)-dimensional NLSE [5]. The second category is the exact optical similaritons, which are mainly described by exact soliton solutions [6, 7, 8, 9, 10, 11, 12, 13, 14].

Among the two categories, the exact solitonic similaritons are more intriguing because their stability is guaranteed [12]. There are many ways to find the exact solitonic similaritons, and it must be noted that, the process of finding the exact solitonic similariton is in essence the reduction of original NLSE, which is generally inhomogeneous, into standard, homogeneous NLSE, and all of the methods can be unified by the powerful generalized inverse scattering technique

#92333 - $15.00 USD Received 1 Feb 2008; revised 12 Mar 2008; accepted 28 Mar 2008; published 21 Apr 2008

with variable spectral parameter [6, 8, 14]. Extensive research work has been carried out in the context of obtaining exact solitonic similaritons having sech or tanh-type profiles pertaining to (1+1)-dimensional NLSE. Some of them are as follows: In [7], soliton management regimes for nonlinear optical applications were considered for the first time. A similarity transformation to the autonomous NLSE was constructed for the first time and general applications for the dispersion-managed optical systems were studied in details in [10]. Moreover, the dynamics and interaction of bright and dark solitons were studied using the NLSE model with an external nonstationary harmonic potential in [15] and [16] respectively. In addition, the method of similarity solutions have been used in detail in nonlinear wave theory. Similarity solutions for the Korteweg de Vries, modified Korteweg de Vries, NLSE and sine-Gordon equations have been studied extensively in literature [17, 18, 19, 20]. However, only a few results have been reported about exact optical similaritons described by the quasi-soliton solutions in dispersion-managed optical fibers [21, 22]. Such kind of exact quasi-soliton similariton has more attractive properties than those of the soliton because of its reduced interaction and smaller peak power than the soliton [21] and allows a possible pedestal-free pulse compression [22].

However, in nonlinear fiber optics, it is well known that in addition to dispersion management, there exist the nonlinearity and amplification management. Thus, the prime aspect of this paper is to seek the exact optical similaritons, other than those with sech or tanh-type profile, under the dispersion, nonlinearity and amplification management. On the other hand, since all the exact optical similaritons are found in (1+1)-dimensional NLSE, then one would ask whether the exact optical similaritons exist in (2+1) or even in (3+1)-dimensional NLSE or not. The other important aspect of this paper is to report the existence of exact optical similaritons in these higher dimensional NLSEs.

2. Model equations

Here we outline the NLSEs under investigation as below. For (1+1)-dimensional systems, there are two different types of NLSE. The first one is

$$iu_z + \frac{1}{2}u_{xx} + \sigma|u|^2u + f(z)\frac{x^2}{2}u = i\frac{g(z)}{2}u, \tag{1}$$

which describes the propagation of optical beam in a planar graded-index waveguide amplifier with the refractive index $n = n_0 + n_1 f(z)x^2 + n_2|I|^2$ [11, 13]. Here, the beam envelope u, propagation distance z, spatial coordinate x and amplification parameter $g(z)$ are respectively normalized by $(k_0|n_2|L_D)^{-1/2}$, L_D, w_0 and L_D^{-1}, with the wavenumber $k_0 = 2\pi n_0/\lambda$ at the input wavelength λ, the diffraction length $L_D = k_0 w_0^2$ and the characteristic transverse scale $w_0 = (2k_0^2 n_1)^{-1/4}$. The nonlinear coefficient is $\sigma = \mathrm{sgn}(n_2) = \pm 1$ and the inhomogeneous parameter is $f(z)$, which describes the inhomogeneity of waveguide. Note that when the amplification parameter vanishes, the dynamics and interaction of bright and dark solitons have been studied [15, 16]. While the second one is

$$i\psi_Z + \frac{\beta(Z)}{2}\psi_{TT} + \gamma(Z)|\psi|^2\psi = i\frac{G(Z)}{2}\psi, \tag{2}$$

which describes the propagation of picosecond optical pulses through dispersion and nonlinearity management fiber. [6, 9, 12]. Here $\psi(Z,T)$ is the pulse envelope in comoving coordinates, $\beta(Z)$ is the group velocity dispersion parameter, $\gamma(Z)$ is the nonlinearity parameter, and $G(Z)$ is the amplification parameter. In fact, the above two equations are almost identical: if we introduce the transformations $u(z,t) \to \sqrt{|\gamma(Z)/\beta(Z)|}\psi$, $z \to \int_0^Z \beta(z')dz'$, $x \to T$ and $g \to G/\beta - (\beta_Z\gamma - \gamma_Z\beta)/\beta^2\gamma$, then Eq. (2) can be transformed to Eq. (1) with $\sigma = \mathrm{sgn}(\beta\gamma)$ and $f = 0$.

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The (2+1)-dimensional NLSE under consideration is

$$iu_z+\frac{1}{2}(u_{xx}+u_{yy})+\sigma|u|^2u+f(z)\frac{x^2+y^2}{2}u=i\frac{g(z)}{2}u, \tag{3}$$

which describes the propagation of optical beam inside the two-dimensional graded-index waveguide amplifier with refractive index $n=n_0+n_1 f(z)(x^2+y^2)+n_2|I|^2$. The normalizations are the same with that used in Eq. (1). Finally we consider the (3+1)-dimensional NLSE of the form

$$iu_z+\frac{1}{2}(u_{xx}+u_{yy})+\frac{\delta}{2}u_{\tau\tau}+\sigma|u|^2u+f(z)\frac{x^2+y^2}{2}u=i\frac{g(z)}{2}u, \tag{4}$$

which describes the propagation of optical bullet inside the two-dimensional graded-index waveguide amplifier with refractive index $n=n_0+n_1 f(z)(x^2+y^2)+n_2|I|^2$ [23]. Here $\delta=$sgn(β) with β being the group velocity dispersion parameter, τ the retarded time, while other normalizations are the same as used in Eq. (1).

3. Similaritons in (1+1)-dimensional systems

3.1. Spatial similaritons

To investigate the exact self-similar evolution of optical beam, the establishment of scaling properties of optical beam parameters such as amplitude and width is of significant importance. Here we first assume that the beam width varies as $\ell(z)$. Then the scaling property of the beam power and the invariability of the function form of the beam envelope determine the beam amplitude to be proportional to $\exp[\int_0^z g(z')/2dz']/\sqrt{\ell(z)}$. Secondly, as the beam width changes, the "expansion velocity" of beam envelope is given as $\ell_z x/\ell$, hence the beam phase contains the quadratic term $\ell_z x^2/2\ell$ [24]. Note that the quadratic phase is a natural choice due to the variation of beam width, not just an assumption. Lastly, the propagation distance z should also be scaled by $\ell(z)$ accordingly. Thus the beam envelope u can be expressed as follows

$$u(z,x)=\frac{\exp[\int_0^z g(z')/2dz']}{\sqrt{\ell(z)}}U(Z,X)\exp[i\frac{\ell_z}{2\ell}x^2], \tag{5}$$

where $Z=Z(\ell)$ and $X=x/\ell$. Further calculations show that when $Z_z=\ell^{-2}$ and $\ell_z=-g\ell$, the governing equation for $U(Z,X)$ is obtained as

$$iU_Z+\frac{1}{2}U_{XX}+\sigma|U|^2U=K\frac{X^2}{2}U, \tag{6}$$

where $K=(g^2-g_z-f)\ell^4$. The transformation utilized in Eq. (5) is known as lens-type transformation [25], which has been widely used in BEC and plasma problems [10, 26, 27, 28, 29, 30, 31]. Such transformation enables us to obtain the information of original equation (1) via the investigation of Eq. (6). Specially, when K vanishes, Eq. (6) turns to the well-known standard, homogeneous NLSE which possesses soliton solutions [32]. Therefore, when parameters $g(z)$ and $f(z)$ satisfy the self-similarity condition $f=g^2-g_z$, namely, $K=0$, the exact solitonic similariton solution for Eq. (1) can be recovered by lens-type transformation (5) with $\ell=\exp[-\int_0^z g(z')dz']$ and $Z=\int_0^z \exp[2\int_0^{z'} g(z'')dz'']dz$. The exact soliton-like similaritons obtained by this way is equivalent to those presented in [11, 12, 13].

Other than the exact solitonic similaritons, lens-type transformation can lead us to a more general type of exact optical similaritons. We find that when K is constant, Eq. (6) may possess stationary state solution $U(Z,X)=S(X)\exp(i\mu Z)$, where μ is the propagation constant (Note that here the word "may" means that for negative K, it is difficult to say whether the stationary

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state solution exists or not, while for positive K the stationary state solution always exists and it is localized). Then, the corresponding exact similariton solution for Eq. (1) can be written as

$$u(z,x) = \frac{1}{\ell(z)} S\left[\frac{x}{\ell(z)}\right] \exp\left[i\mu \int_0^z \frac{dz'}{\ell(z')^2} - i\frac{g(z)}{2}x^2\right]. \tag{7}$$

Thus we conclude that, i) when the optical beam $S(x)\exp\left[-i\frac{g(0)}{2}x^2\right]$ is injected into a graded-index waveguide amplifier, this beam will evolve exact self-similarly if the amplification parameter $g(z)$ and inhomogeneous parameter $f(z)$ satisfy the following self-similarity condition

$$f = g^2 - g_z - K\exp[4\int_0^z g(z')dz']. \tag{8}$$

ii) the amplitude and width of exact optical similaritons depend totally on the amplification parameter $g(z)$, and iii) the exact solitonic similaritons are just a subclass of the general similaritons (7), since when $K = 0$, nontrivial stationary state solution $U(Z,X)$ takes the exact soliton profile.

We hereafter assume K to be positive constant. In this context, we recall that for positive K, Eq. (6) becomes equivalent to the NLSE with variable dispersion and zero gain term, derived for the first time by Shiva Kumar and Akira Hasegawa, and whose stationary solutions are the well known quasi-solitons [21]. Note that the exact optical similaritons (7) are conceptually equivalent to the quasi-solitons in dispersion-managed system, where the exact quasi-soliton similaritons were found to have relatively lower power and reduced interaction than exact solitons [21, 22]. We here further investigate their potential applications.

We recall that in experiments, in order to obtain exact solitonic similariton, one should first produce soliton by injecting optical beam (usually Gaussian beam) into nonlinear waveguide, and then propagate the produced solitons to the graded-index waveguide amplifier. Therefore, two waveguides are needed to propagate exact solitonic similaritons, and wave radiation always exists during the evolution of original Gaussian beam to soliton beam. However, if $S(X)$ is chosen to have the same profile with the injected Gaussian beam, only one graded-index waveguide amplifier is needed for the exact self-similar propagation of such beam, which means that there will be no radiation. In practice, we can properly choose the parameters $f(z)$ and $g(z)$ according to self-similarity condition (8) to make $S(X)$ asymptotically approach to the Gaussian profile $A\exp(-X^2/2W^2)$. We find that when A and W are sufficiently small, then the Kerr nonlinearity can be neglected and when $K \approx W^{-4}$, $S(X)$ do closely approach to Gaussian profile. Numerical calculations of Eq. (6) reveal that even when $A = 1$ and $W = 0.5$, the stationary state solutions $S(X)$ are well described by Gaussian function when $K = 13.2$ for $\sigma = 1$ and $K = 18.8$ for $\sigma = -1$, respectively. The self-similar evolution of this Gaussian beam has been confirmed by numerical simulations of Eq. (1) both for focusing and defocusing nonlinearity, respectively. Specifically, here we choose the gain parameter $g(z) = \tanh(z)$. Thus, $\ell = \text{sech}(z)$, which implies that the amplitudes of Gaussian similaritons increase as $A/\text{sech}(z)$ while their widths decrease as $W\text{sech}(z)$. Figure 1 depicts the excellent agreement between the theoretical predictions and numerical simulations. Further numerical simulations with other amplification parameters and corresponding inhomogeneous parameters lead to similar results.

Contrary to the fact that small initial power of optical beam $\int_{-\infty}^{\infty}|u(x,0)|^2dx$ leads to the negligible Kerr nonlinearity, we find that large initial power results in the negligible diffraction term U_{XX} when $\sigma = -1$. In this case, the stationary state solution of Eq. (6) takes the compact parabolic form: $U = A\sqrt{1 - X^2/W^2}\exp(-iA^2Z)$ when $|X| < W$, and $U = 0$ when $|X| > W$ with $K = 2A^2/W^2$. We recall that such parabolic beams can be generated in the planar waveguide [4], where the governing equation is similar to Eq. (1) but with $f(z) = 0$ and $g(z)$ equals to positive constant g_1. Observe that under such configuration the output beam has a quadratic

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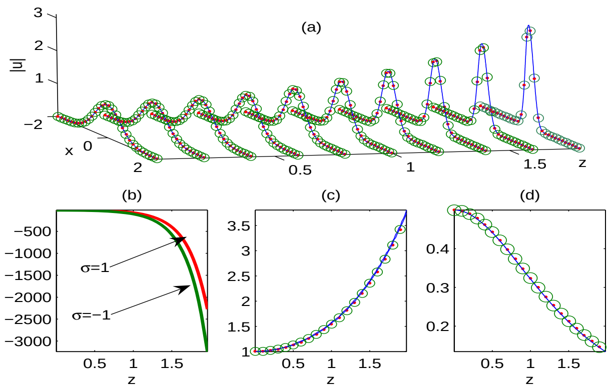

Fig. 1. (a) Self-similar evolution of initial Gaussian beams, (b) inhomogeneous parameters $f(z)$, (c) the amplitudes and (d) the widths (defined as $\sqrt{\int_{\infty}^{\infty} 2x^2|u|^2dx/\int_{\infty}^{\infty}|u|^2dx}$) of Gaussian similaritons as functions of propagation distance z. The solid lines are theoretical predictions, dots and open circles are numerical simulations with $K = 13.2$ for $\sigma = 1$ and $K = 18.8$ for $\sigma = -1$, respectively.

phase $-g_1/6x^2$, and its amplitude and width satisfying $W = \sqrt{18}A/g_1$. From transformation (5) or Eq. (7) we know that, for the self-similar propagation of such parabolic beam inside the graded-index waveguide amplifier, the gain parameter g should be equal to $g_1/3$ for the continuity of quadratic phase, meanwhile, f and g should satisfy the similarity condition (8) with $K = g_1^2/9$. This theoretical prediction is confirmed by the numerical simulation, as shown in Fig. 2.

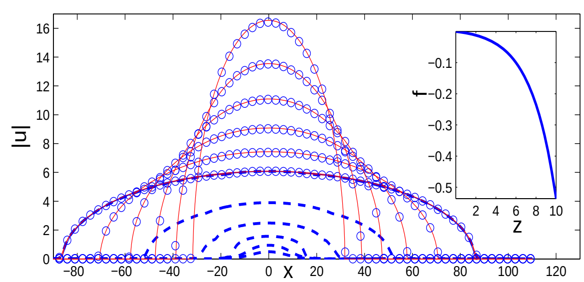

Fig. 2. The theoretical prediction of the propagation of parabolic similaritons (solid lines) inside the graded-index waveguide amplifier with gain parameter $g = g_1/3$ and inhomogeneous parameter f is confirmed by numerical simulations (open circles), where the parabolic beam (at $z = 24$) is generated by injecting a Gaussian beam ($z = 0$) into the homogeneous planar waveguide with gain parameter $g_1 = 0.3$ (dashed-lines show its evolution into parabolic beam). From bottom to top, the propagation distance z is 0, 4.8, 9.6, 14.4, 19.2, 24, 26, 28, 30, 32 and 34, respectively.

3.2. temporal similaritons

As stated in Section 2 that Eq. (2) can be transformed to Eq. (1), we immediately know that Eq. (2) possesses general type of optical similaritons when g satisfies the following equation

$$g^2 - g_z = K\exp[4\int_0^z g(z')dz']. \tag{9}$$

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However, it must be noted that in this equation, $g = G/\beta - (\beta_Z\gamma - \gamma_Z\beta)/\beta^2\gamma$ is a function of Z instead of z, so strictly speaking, Eq. (9) should take the form

$$g^2 - \frac{g_Z}{\beta} = K\exp[4\int_0^Z g(z')\beta(z')dz']. \tag{10}$$

Thus, when $K = 0$, the exact optical similaritons are solitonic similaritons. One can check that the exact solitonic similariton is equivalent to those found in [6, 9, 12]. While $K \neq 0$, the exact optical similaritons are quasi-soliton similaritons [21, 22]. However, it seems that the self-similarity condition is quite complicated when the dispersion, nonlinearity and amplification management appear together. Specifically, when there exists the dispersion management alone, we obtain the self-similarity condition for exact quasi-soliton similaritons from Eq. (10)

$$\beta\beta_{ZZ} - \beta_Z^2 = K, \tag{11}$$

which is identical with the result reported in [21]; when the dispersion coefficient is constant, for example, $\beta = 1$, the self-similarity condition for exact quasi-soliton similaritons is

$$G + \frac{\gamma_Z}{\gamma} = -\frac{Z + c_2}{(Z + c_2)^2 + c_1}, \tag{12}$$

where c_1 and c_2 are constant with $K = \frac{c_1}{(c_2^2 + c_1)^2}$.

4. Similaritons in (2+1)-dimensional systems

For (2+1)-dimensional NLSE (3), we find that only when the amplification parameter is absent, it can be transformed to the following equation under the self-similarity condition (8)

$$iU_Z + \frac{1}{2}(U_{XX} + U_{YY}) + \sigma|U|^2U = K\frac{X^2 + Y^2}{2}U, \tag{13}$$

via the lens-type transformation

$$u(z,x,y) = \frac{1}{\ell}U(Z,X,Y)\exp[i\frac{\ell_z(x^2 + y^2)}{2\ell}], \tag{14}$$

where $(X,Y) = (x,y)/\ell$, $Z_z = \ell^{-2}$, $\ell_z = -g\ell$.

It is remarkable that, i) although we continue to use g in Eq. (8), it has nothing to do with the amplification parameter anymore: now it is just a sign to identify $-\ell_z/\ell$ and this means the power of optical similariton is constant, and ii) the change of the amplitude and width of optical beam is determined by ℓ, and ℓ is controlled by $f(z)$. Similar to the discussion in (1+1)-dimensional case, we can choose inhomogeneous parameter $f(z)$ to let the Gaussian or parabolic beam propagate self-similarly in two-dimensional graded-index waveguide. Furthermore, we find that such waveguides also support the exact self-similar evolution of other soliton structures such as ring solitons and vortex rings. Figures 3 and 4 show the examples of the exact self-similar evolution of an initial optical vortex with topological charge 1 and an initial ring soliton in two-dimensional graded-index waveguides. In both cases, the numerical simulations (solid lines) agree well with theoretical predications (marked by cross) which imply that the amplitudes of the vortex and ring solitons are inversely proportional to ℓ while their widths are proportional to ℓ.

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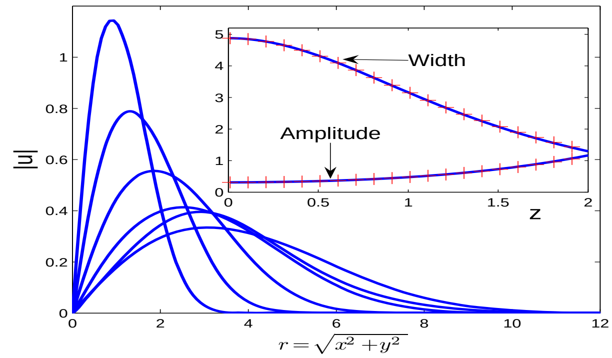

Fig. 3. Exact self-similar evolution of vortex beam in two-dimensional graded-index waveguide with $\sigma=-1$ and $f(z)=1-2\text{sech}^2(z)-K/\text{sech}^4(z)$ such that $\ell=\text{sech}(z)$. From bottom to top, the six radial profiles correspond to the propagation distance $z=0,0.4,0.8,1.2,1.6,2.0$, respectively. Here $K=0.01$.

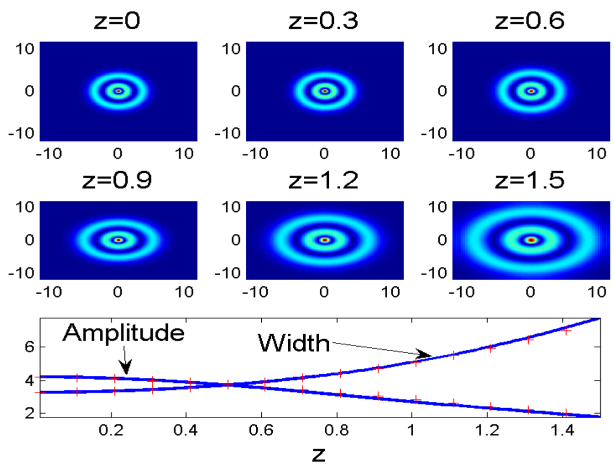

Fig. 4. Exact self-similar evolution of ring soliton in two-dimensional grade-index waveguide with $\sigma=1$ and $f(z)=1-K\text{sech}^4(z)$ such that $\ell=1/\text{sech}(z)$. Here $K=0.01$.

5. Similaritons in (3+1)-dimensional systems

In the above sections, we have discussed the construction of exact self-similar solutions via lens-type transformation for (1+1) and (2+1)-dimensional NLSEs. Now, in this section, we plan to investigate physically important self-similar solutions in (3+1)-dimensional NLSE, which describes the evolution of optical bullet in two-dimensional graded-index optical waveguides. For this purpose, we use the following lens-type transformation

$$u(z,x,y,\tau)=\frac{1}{\ell}U(Z,X,Y,T)\exp\left[i\frac{g}{2}(x^2+y^2+\frac{\tau^2}{\delta})\right], \tag{15}$$

where $(X,Y,T)=(x,y,\tau)/\ell$, $\ell_z=g\ell$ and $Z_z=\ell^{-2}$. Substituting this in Eq. (4) and performing some manipulations, it can be converted to

$$iU_Z+\frac{1}{2}(U_{XX}+U_{YY}+\delta U_{TT})+\sigma|U|^2U=\frac{K_{XY}}{2}(X^2+Y^2)+\frac{K_T}{2}T^2, \tag{16}$$

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where the parabolic potential parameters

$$K_{XY} = (g_z + g^2 - f)\ell^4, \quad K_T = (g_z + g^2)\ell^4. \tag{17}$$

Obviously, when K_{XY} and K_T are non-negative constants, Eq. (16) possesses stationary state solutions. This implies that the optical bullets described by Eq. (4) can evolve exact self-similarly under the following exact self-similarity condition, which is obtained by solving Eq. (17)

$$g = \frac{z + c_2}{(z + c_2)^2 + c_1}, \quad f = \frac{K_T - K_{XY}}{2\ln[(z + c_2)^2 + c_1]}, \tag{18}$$

where c_1 and c_2 are constants satisfying $K_T = \frac{c_1}{(c_2^2 + c_1)^2}$.

It is rather interesting that, in (3+1)-dimensional NLSE, the amplitude of the optical bullet is proportional to $\exp(\int_0^z -g(z')dz')$. This means that when the amplification parameter $g(t)$ is positive/negative, the amplitude of exact self-similar optical bullet decreases/increases and its width increases/decreases, which is in contrast to optical similaritons in (1+1)-dimensional NLSE.

6. Conclusion

In summary, we have found a more general type of exact optical similariton solutions in the context of nonlinear optical fiber amplifiers and graded-index waveguide amplifiers. By using the lens-type transformation, we reduced the (1+1), (2+1) and (3+1)-dimensional NLSEs to standard NLSEs with additional constant parabolic potential possessing stationary state solution. Under certain conditions, we found that optical waves with Gaussian, parabolic, vortex and ring soliton profiles can propagate exact self-similarly without any radiation. It should be emphasized that, when the parabolic potential is absent, we obtained the exact spatial and temporal soliton similaritons of (1+1)-dimensional NLSEs.

Acknowledgments

This work has been supported by the National Natural Science Foundation of China (No. 10672147), and Provincial Natural Science Foundation of Shanxi (No. 2007011007). KP wishes to thank DST-Ramanna Fellowship, IFCPAR (Ref.: IFC/3504-F/2005/2064), DAE-BRNS, CSIR, Government of India, for financial support through research projects.

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