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Switching dynamics of a two-dimensional nonlinear couplers in a photopolymer – A variational approach

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Abstract. We study the optical switching of the two-dimensional nonlinear coupler in a doped photopolymer. The coupled nonlinear Schrödinger equations (CNLSEs) describing our coupler system are analysed using Lagrangian variational method. From the Lagrangian, a set of coupled ordinary differential equations (ODEs) describing the system dynamics is obtained. This set of ODE's is further reduced to single coupled equation and an analytical solution is obtained using the cnoidal functions and the system dynamics is studied. The key factor for switching mechanism of our coupler system is the metal-induced surface plasmon resonance (SPR). This SPR-induced local nonlinear effects results in self-focussing of the optical beam through the launched core. A description of a particle in a well is also made to study the photon switching through the coupler system.

Keywords. Optical switching; variational approach; Jacobi's elliptic function; surface plasmon resonance; coupled nonlinear Schrödinger equation.

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1. Introduction

The seed for the nonlinear directional coupler (NLDC) was first made by Jensen in the year 1982 [1]. After its introduction, nonlinear optics was grown rapidly and it is greatly attractive for its potential applications in thrust areas of optics such as optical bistability, power splitters, couplers and logic systems [2,3]. NLDCs are four-port devices that can split an optical field into two coherent but physically separated parts and vice versa. By suitably adjusting the intensity of the input field, the optical beam can be switched between the two ports [4]. Interest in manufacturing optical waveguides and other integrated optical devices using polymers has grown because of low cost and easy fabrication. Compared to semiconductor materials and dielectric materials, polymers are inexpensive and can be used to fabricate any optical devices and integrated circuits [5]. There are a multitude of different polymers with a desired set of optical, electrical and mechanical properties. The prime advantages of polymers are the refraction index and the attenuation. Both as function of wavelengths, play vital role in the light-controlled devices. Important properties of the polymers used for integrated optics including acrylates, polymides and olefins are summarized by Eldada and Shaklette [6].

In this work, we study the switching characteristics of a two-dimensional NLDC formed in a photopolymer doped with nanometals. Metal-doped photopolymer received great attention for its excitation of the localized surface plasmon [7,8]. Plasmon is the quantum of collective longitudinal oscillation of valence electron clouds in metals. In typical noble metals, volume plasmon results in energy excitation in the range of 5–10 eV. They found many applications in optoelectronics, optical switches and in optical computers [9]. The experimental study of optical properties of metal-doped materials was made by Ganeev and Ryasnyansky [10]. When the wavelength of the incident electromagnetic field coincides with the resonance wavelength of the noble metal particles, the surface plasmon will be excited. This SPR-induced effects in nanometals leads to a significant enhancement of the local field intensity, which results in decreased radiative decay, large nonlinear optical response, strong enhanced Raman scattering and also rapid photo-induced enhancement in isomeration of azopolymer molecules [11,12].

Here, we investigate our system by means of a set of coupled nonlinear Schrödinger equations. This set of equations is theoretically analysed by the variational method developed by Anderson [13]. Following the variational principles a set of first-order equations for each optical parameter describing the system dynamics is derived and finally an analytical solution is obtained for the system. A potential well description is also made to study the condition for optical bistability.

The paper is organized as follows. Section 2 deals with the necessary theoretical model describing the nonlinear directional coupler. The system is studied using the variational approach in §3 and an analytical solution is derived for the energy difference through the system using the Jacobian elliptic functions. A potential well formulation is presented in §4 for the analysis of the NLDC. Section 5 concludes the paper.

2. Theoretical model

Coupler system acts as a passive device for low-intensity input optical field. When such an optical beam is introduced into the through-port of a coupler, it suffers diffraction and a part of the input field spreads to the neighbouring core. But, when a sufficiently high-intensity optical field is introduced through the through-port, the optical beam gets self-focussed because of the balance between diffraction and nonlinearity. As a result of classic nonlinear Kerr effect, input field travels through the launched core itself. Here, we deal with the coupler made of photopolymers such as PMMA and polystyrene which shows weak nonlinearity. To enhance the nonlinear absorption property, dielectric host is uniformly doped with metal particles such as silver, gold and copper particles which show enhanced SPR [14,15].

The third-order nonlinear optical properties of metal particles-doped dielectric materials depend significantly on many factors such as the materials themselves (kind of metal and host medium, metal concentration, particle size, shapes and spatial arrangement) and the excitation laser (wavelength, intensity, beamwidth) [16]. The localized surface plasmon resonance (SPR) excited in noble metal clusters exhibits selective photoabsorption, scattering and local electromagnetic field enhancement. The refractive index induced by the high-intensity optical field is given by [17]

$$\frac{\partial \Delta n}{\partial t} = A_P I^P \left(1 - \frac{\Delta n}{\Delta n_s}\right), \tag{1}$$

where A_P is a real coefficient which depends on the doped material, intensity $I = |\Psi|^2$ and P is one (two) for one (two) photon process [18]. Ψ is the amplitude of the wave envelope.

Light propagation through an NLDC, formed using a metal particle-doped photopolymer, is governed by the following normalized CNLSEs [19]:

$$i\frac{\partial A_j}{\partial z} + \frac{1}{2}\nabla_\perp^2 A_j + N A_j + \gamma |A_j|^2 A_j + \kappa A_{3-j} = 0, \tag{2}$$

where $A_j(r,z)$ is the optical field through the medium with $j = 1, 2$ for wave guides 1 and 2, respectively. $\nabla_\perp^2$ is the Laplacian operator along the transverse region which can be written as follows:

$$\nabla_\perp^2 = \frac{1}{r^{D-1}}\frac{\partial}{\partial r}\left(r^{D-1}\frac{\partial}{\partial r}\right). \tag{3}$$

For the two-dimensional case, $D = 2$ and $r = \sqrt{x^2 + y^2}$. $\gamma = n_2\omega_\mathrm{p}/cA_\mathrm{eff}$ is the nonlinearity parameter where n_2 is the nonlinear refractive index, ω_p is the plasmon frequency, c is the velocity of light and A_eff is the effective area of the core. Also, the waveguiding term $N = a_0^2 k^2 n_0 \Delta n$, where a_0 is the initial beamwidth, n_0 is the initial refractive index and $k = 2\pi/\lambda_\mathrm{p}$ is the propagation constant with λ_p the plasmon wavelength. The coupling coefficient κ is defined as $\pi/2L_\mathrm{c}$ where L_c is the coupler length. The beam propagation is along the longitudinal direction (z-axis) and diffraction along the transverse direction (x–y axes). The above equation describes a system of NLDC formed in a photopolymer whose nonlinearity is enhanced by the doping of nanometal particles. A number of studies was done about the SPR enhancement in the host medium doped with metal particles. The wavelength at which extinction is minimum can be tuned by adjusting the metal particle's size, shape, volume fraction, interparticle distance and the dielectric properties of the metals as well as medium [14,15]. Here, we study the dynamics and switching characteristics of the coupler by varying nonlinearity parameter.

3. Variational approach

Variational method is widely applied to obtain approximate solutions of problems related to the optical beam propagation using the nonlinear Schrödinger equation (NLSE). The main advantage of the variational approach is that it provides an explicit approximate analytical expression for different parameters of a propagating pulse governed by the NLSE. The importance of variational principles in physics has long been appreciated. The whole physics of the problem is expressed in terms of a single function through which the equations of motion are obtained by taking the functional derivatives. The basis of the approximate solutions of different physical problems is formulated using this idea. The potential application of this principle can be viewed in different physical systems like fluid dynamics, thermodynamics, magnetism, plasma physics and electromagnetic theory. Historically, the variational principle was introduced to describe conservative physical systems. The success of the system depends on the choice of a suitable input profile for the corresponding system [13]. We proceed with writing the Lagrangian for the system of eq. (2) which is given by

$$L = \sum_{j=1}^{2} L_j + L_{12} \tag{4}$$

with

$$L_j = \frac{ir^{D-1}}{2}\left[A_j \frac{\partial A_j^*}{\partial z} - A_j^* \frac{\partial A_j}{\partial z}\right] + \frac{r^{D-1}}{2}\left|\frac{\partial A_j}{\partial r}\right|^2 \\ -r^{D-1}N|A_j|^2 - \gamma r^{D-1}\left(\frac{|A_j|^4}{2}\right), \tag{5}$$

and

$$L_{12} = -r^{D-1}\kappa(A_1^* A_2 + A_2^* A_1), \tag{6}$$

where L_1 and L_2 represent the single Lagrangian and L_{12} indicates the interaction Lagrangian. Assuming the Gaussian input wave profile, which is suitable to describe the diffraction of the optical field

$$A_j(r,z) = F_j(z)\exp(-\rho r^2 + i\theta_j(z)), \tag{7}$$

where $F_j(z)$ is the amplitude of the field, $\theta_j(z)$ is the phase variable and ρ is a constant. The reduced Lagrangian is given by

$$\langle L\rangle = \int_0^\infty L r \mathrm{d}r. \tag{8}$$

Substituting eq. (7) in eqs (5) and (6), and applying eq. (8) we get

$$\langle L\rangle = \frac{15\pi^{3/2}\sqrt{\rho}F_j(z)^2}{256\sqrt{2}} + \frac{\pi^{3/2}}{\rho^{3/2}}\Bigg[-\frac{NF_j(z)^2}{16\sqrt{2}} - \frac{\gamma F_j(z)^4}{128} \\ -\frac{\exp(i\theta_1(z) - i\theta_2(z))\kappa F_j(z)F_{3-j}(z)}{16\sqrt{2}} \\ -\frac{\exp(-i\theta_1(z) + i\theta_2(z))\kappa F_j(z)F_{3-j}(z)}{16\sqrt{2}} + \frac{F_j(z)^2 \frac{\partial\theta_j(z)}{\partial z}}{16\sqrt{2}}\Bigg]. \tag{9}$$

Now varying eq. (9) with respect to the variational parameters $F_1(z)$, $\theta_1(z)$, $F_2(z)$ and $\theta_2(z)$, we get the following set of four coupled ordinary differential equations of the form

$$\frac{\partial \theta_1(z)}{\partial z} = \frac{\kappa \cos(\theta_1(z) - \theta_2(z)) F_2(z)}{F_1(z)} + \frac{\gamma F_1(z)^2}{2\sqrt{2}} - \frac{15\rho^2}{16} + N, \tag{10}$$

$$\frac{\partial \theta_2(z)}{\partial z} = \frac{\kappa \cos(\theta_1(z) - \theta_2(z)) F_1(z)}{F_2(z)} + \frac{\gamma F_2(z)^2}{2\sqrt{2}} - \frac{15\rho^2}{16} + N, \tag{11}$$

$$\frac{\partial F_1(z)}{\partial z} = \kappa \sin(\theta_1(z) - \theta_2(z)) F_2(z), \tag{12}$$

$$\frac{\partial F_2(z)}{\partial z} = -\kappa \sin(\theta_1(z) - \theta_2(z)) F_1(z). \tag{13}$$

After defining the new phase variable as $\theta(z) = \theta_1(z) - \theta_2(z)$, variable for the energy difference as $U(z) = F_1(z)^2 - F_2(z)^2$, the total energy is given by $F_1(z)^2 + F_2(z)^2 = E$ (constant). Using the above chosen variables, we arrive at the following set of two coupled ordinary differential equations:

$$\frac{\mathrm{d}\theta}{\mathrm{d}z} = \frac{-2\kappa U \cos\theta}{\sqrt{E^2 - U^2}} + \frac{\gamma U}{2\sqrt{2}}, \tag{14}$$

$$\frac{\mathrm{d}U}{\mathrm{d}z} = 2\kappa \sqrt{E^2 - U^2} \sin\theta. \tag{15}$$

We find that the above two equations can be reduced to a single equation by finding a constant of motion [19]. The constant of motion for our coupler system is obtained as

$$G = \left[-2\kappa \sqrt{E^2 - U^2} \cos\theta + \frac{\gamma U^2}{4\sqrt{2}} \right]. \tag{16}$$

By using the constant of motion, a single equation for U can be written as

$$\frac{\mathrm{d}U}{\mathrm{d}z} = \pm \left[(2\kappa)^2 (E^2 - U^2) - \left(G - \frac{\gamma U^2}{4\sqrt{2}} \right)^2 \right]^{1/2}. \tag{17}$$

The periodic solutions for eq. (17) can be obtained by using the Jacobian elliptic function. We consider the cnoidal periodic solution for $U(z)$ which takes the form

$$U(z) = \varrho cn[\Omega z, m], \tag{18}$$

where ϱ and Ω are arbitrary constants and m is the modulus parameter of the elliptic function which takes the value $0 < m < 1$. Cnoidal waves are periodic waves with sharp crests separated by wide flat troughs. Here, the wave characteristics are described in the parametric form using modulus parameter m, over the range 0 and 1, of the elliptic integrals. Thus, there are two known limits to the cnoidal waves. The first one is the solitary wave theory which occurs when the period of the Jacobian elliptic function is infinite ($m = 1$). The second limit is the linear wave theory which occurs for $m = 0$ where the cnoidal wave approaches the sinusoidal wave [21]. Substituting the above solution in eq. (17) and after simplifying and equating the coefficient of cn function from the above equation, we find the values for ϱ and Ω as $\varrho = 4\sqrt{2}m\Omega/\gamma$ and $\Omega = \sqrt{\sqrt{2}G\gamma - 16\kappa^2}/2\sqrt{(1-2m^2)}$.

Substituting the values of ϱ and Ω in eq. (18) we arrive at

$$U(z) = \frac{4\sqrt{2}m\Omega}{\gamma} cn\left[\frac{\sqrt{\sqrt{2}G\gamma - 16\kappa^2}}{2\sqrt{(1-2m^2)}} z, m\right] \tag{19}$$

and similarly the cnoidal sn solution for $U(z)$ takes the form

$$U(z) = \varrho sn[\Omega z, m]. \tag{20}$$

Substituting the above ansatz in eq. (17), after simplifications, we obtain the values for ϱ and Ω as $\varrho = \sqrt{-32m^2\Omega^2}/\gamma^2$ and $\Omega = \sqrt{2G\gamma\kappa^2}/\sqrt{2}\sqrt{2m^2+1}$. Substituting the values of ϱ and Ω in eq. (20), we arrive at

$$U(z) = \frac{\sqrt{-32m^2\Omega^2}}{\gamma^2} sn\left[\frac{\sqrt{2G\gamma\kappa^2}}{\sqrt{2}\sqrt{2m^2+1}} z, m\right]. \tag{21}$$

Energy difference of the input optical field between the two cores for various nonlinearity values is plotted using the obtained cnoidal solution. When $\gamma = 0.3$ the energy difference between the cores is very high, shown by solid line of figure 1a. Initially, when $z = 0$ most of the energy is propagated through the launched core itself. Energy sharing between the two cores occurs, when the input field reaches coupling length. For this coupling length a part of the field is periodically coupled to the neighbouring port as an evanescent wave. For further increase in $\gamma = 0.5$, the energy difference between the cores decreases, indicating almost equal energy sharing between the input and cross core as shown by the dashed line of figure 1a. This energy transmission to the neighbouring core is due to overlapping modes of evanescent wave guided through the throughput port with the cross port. With increase in nonlinearity value, the periodicity of the system also changes. The equal energy exchange between the phase-matched modes occurs if the coupling constant $\kappa = \pi/2Lc$. The periodic change in phase with the oscillation of energy coupled between the two cores can be seen clearly through both figures figure 1a and figure 1b obtained for cn and sn solution respectively. When the nonlinearity parameter value reaches $\gamma = 0.7$ shown by the dot–dashed line, the energy difference between the cores increases and results in detuning of the system. The change in the periodicity further detunes the system. This increase in energy difference is due to local index change of the photopolymer as a result of Kerr effect induced by the resonance local excitation of the doped metal particles. This resonance excitation of doped particles is very large and its magnitutde is greater than that of the diffraction. As a result, the diffraction of the optical beam is overcome by the nonlinear intensity-dependent refractive index. At this stage, the coupler becomes active with decrease in energy sharing to the cross core. When γ increases further, the energy difference between the cores increases due to SPR-induced self-focussing.

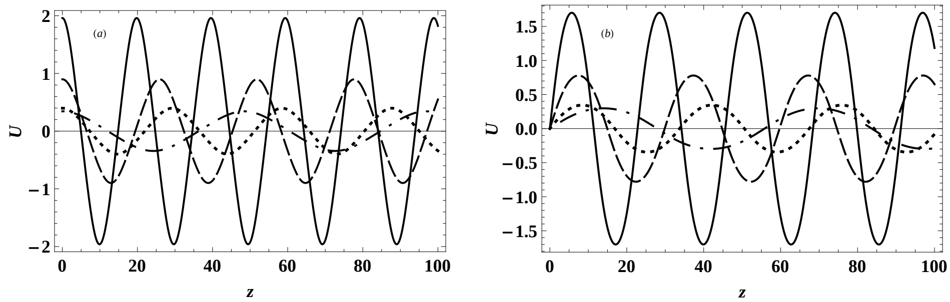

Figure 1. Plots show the variation of energy difference (U) of the input optical field between the two cores of the coupler with distance (z) of eq. (17) for various values of nonlinearity parameter γ. Figures 1a and 1b are *cn* solution and *sn* solution respectively. Solid line indicates $\gamma = 0.3$, dashed line indicates $\gamma = 0.5$, dot–dashed line indicates $\gamma = 0.7$ and dotted line indicates $\gamma = 1$. Other physical parameter are $\kappa = 0.2$, $G = 0.58$ and $m = 0.3$.

This results in the decrease in energy sharing to the cross core and energy in the cross core becomes minimum, shown by the dotted line of figure 1a for the value corresponding to $\gamma = 1$. Thus, the coupler system gets detuned and becomes an active device. Similarly figure 1b gives the result of the snoidal counterpart of the periodic solution.

4. Potential well description

The switching dynamics of energy transmission through the nonlinear fibre coupler is studied by energy evolution in each port accompanied with time-varying particle position in potential well and in general, such a study has been made by Anderson [13,20] and for an all-optical coupler by Paré and Florjańczyk [19]. The potential well description of the photon transition in a potential well is described by

$$\frac{1}{2}\left(\frac{\mathrm{d}U}{\mathrm{d}z}\right)^2 + \Pi(U) = 0. \tag{22}$$

From eq. (17), the potential $\Pi(U)$ for our system is obtained as

$$\Pi(U) = \left[\frac{(2\kappa)^2}{2} - \frac{G\gamma}{4\sqrt{2}}\right]U^2 + \frac{\gamma^2 U^4}{64} \tag{23}$$

and the above equation can be rewritten in a simplified form for our convenience as

$$\Pi(U) = \alpha_1 U^2 + \alpha_2 U^4, \tag{24}$$

where $\alpha_1 = \left[\frac{(2\kappa)^2}{2} - \frac{G\gamma}{4\sqrt{2}}\right]$ and $\alpha_2 = \frac{\gamma^2}{64}$. Setting the arbitrary initial condition for the switching of the photon using eq. (17)

$$U(z = 0) = \pm 1, \tag{25}$$

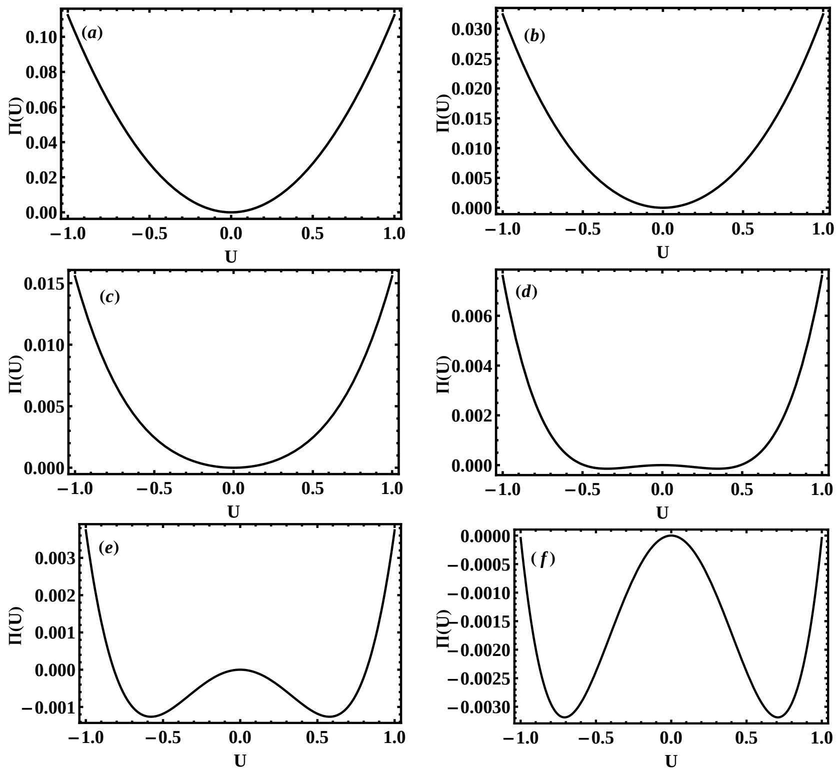

Figure 2. Photon transformation with increase in nonlinearity for various nonlinear paramater values of γ $(0 > \gamma < 1)$. (**a**) $\gamma = -0.3$, (**b**) $\gamma = 0.5$, (**c**) $\gamma = 0.75$, (**d**) $\gamma = 0.8$, (**e**) $\gamma = 0.85$ and (**f**) $\gamma = 0.9$.

$$\left.\frac{\mathrm{d}U}{\mathrm{d}z}\right|_{U=0} = 0, \tag{26}$$

and the potential function is given by

$$\Pi(U = 0) = 0, \tag{27}$$

$$\left.\frac{\mathrm{d}\Pi}{\mathrm{d}U}\right|_{U=0} = 0, \tag{28}$$

$$\Pi(U = \pm 1) = \alpha_1 + \alpha_2, \tag{29}$$

$$\left.\frac{\mathrm{d}\Pi}{\mathrm{d}U}\right|_{U=\pm 1} = \pm(2\alpha_1 U + 4\alpha_2 U^3). \tag{30}$$

Photon transition along the potential well described by eq. (24) for various nonlinear parameter values is plotted for nonlinear parameter values $\gamma < 0$ to $\gamma > 1$. For the initial condition of the particle $U(z = 0) = +1$, particle is at rest and the potential is independent of the nonlinear parameter γ and height of the well is high. By increasing the optical input power, the height of the particle inside the well is lowered. The total energy transfer is achieved, when the particle reaches $U = -1$ and matches with the one analysed by Paré and Florjańczyk [19]. The switching phenomenon of the system depends on the value of coupling constant also. Initially, when $\gamma = -0.5$ the system shows single well potential and the photon is subjected to travel in this potential shown by figure 2a. System shows similar behaviour till the γ value reaches 0.5, and the time taken by the particle to reach the position $U = -1$ is very short. Thus, there is an approximately equal energy exchange between the cores at this potential shown by figures 2a and b. Thus, the system acts as a tuned coupler.

For further increase in the input power, the anharmonicity increases and the flattening of potential well is initiated (shown by figure 2c) for $\gamma = 0.7$. Again increasing the nonlinearity value, there is an abrupt change in the periodic power exchange between the cores. Further increase in nonlinearity to $\gamma = 0.8$ system has a symmetric double well potential as shown in figure 2d. For $\gamma = 0.85$, increase in the depth of the potential well is viewed well through figure 2e and at this potential with negative energy, particle has to travel for a long time in a double well potential to reach $U = -1$. As the γ value reaches ≈ 1 at $\gamma = 0.9$ shown by figure 2f the depth of the potential is further increased. As the nonlinear parameter is increased from negative to positive values, it induces change in the nonlinear refractive index of the system due to excitation of SPR of the doped metal particles. Due to this change in refractive index, phase shift is introduced in the system and the coupler is detuned. Because of the increase in the negative potential well, it takes a long time for the particle to reach $U = -1$. Thus, the optical field propagating through the coupler is self-trapped inside the minimum energy state and the propagation is restricted to the launching core itself. This increase in nonlinearity modifies the energy transmission to the neighbouring core and it gradually decreases and reaches the minimum. This describes the switching phenomenon of the optical field through the coupler using photon transition.

5. Conclusion

In this paper, we have studied the dynamics and switching characteristics of a two-dimensional NLDC formed in a doped photopolymer. From the CNLSEs, we arrive at a set of first-order ordinary differential equations describing the dynamics of NLDC using the Lagrangian variational method. This set of ordinary differential equations is further reduced to a set of coupled equations by defining constant of motion and periodic solutions in terms of the Jacobi's elliptic function is obtained. The periodicity of the coupler system for through- and cross-port is shown clearly. From the obtained results it is shown clearly that the detuning of the coupler system due to excitation of surface plasmon is the manifestation of nonlinearity. Increase in nonlinearity concentrates optical beam in the local region as a result of optical Kerr effect. As a result of these effects, the input beam through the launching core is self-trapped and the energy shared to the cross core becomes minimum. We have also studied the switching dynamics of the coupler using the potential well description of photon transition. The switching is described by photon transition from higher to lower state through fibre core is observed clearly in the potential plots. As a result of the increase in nonlinearity the photon is subjected to travel in a double well potential with negative energy. Due to this negative potential the time taken by the photon to reach $U = -1$ is very long. This results in switching mechanism of the active coupler.

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References

[1] S M Jensen, *IEEE J. Quant. Electron.* **18**, 1580 (1982)
[2] S Trillo, S Wabnitz, E M Wright and G I Stegeman, *Opt. Lett.* **13**, 672 (1988)
[3] K Kitayama and S Wang, *Appl. Phys. Lett.* **43**, 17 (1983)
[4] G P Agrawal, *Applications of nonlinear fiber optics* (Academic Press, New York, 2006)
[5] R G Hunsperger, *Integrated optics: Theory and technology* (Springer-Verlag, Berlin, 2002)
[6] L Eldada and L W Shaklette, *IEEE J. Quant. Electron.* **6**, 54 (2000)
[7] K Ishkawa and T Okudo, *J. Appl. Phys.* **98**, 043502 (2005)
[8] S K Lim, K J Chung and C K Kim, *J. Appl. Phys.* **98**, 084309 (2005)
[9] U Hohenester and J Krenn, *Phys. Rev.* **B72**, 195425 (2005)
[10] R A Ganeev and A I Ryasnyansky, *Appl. Phys.* **B84**, 295 (2006)
[11] A V Zayats and I I Smolyaninov, *J. Opt.* **A5**, S16 (2003)
[12] G Laurent, N Felidj and J Aubard, *J. Chem. Phys.* **122**, 011102 (2005)
[13] D Anderson, *Phys. Rev.* **A27**, 3135 (1983)
[14] A A Scalisi, G Compagnini, L D'Urso and O Pugilisi, *Appl. Surf. Sci.* **226**, 237 (2004)
[15] K Uchida, S Kaneko, S Omi, C Hata, H Tanji, Y Asahara, A J Ikushima, T Tokizaki and K Nakamura, *J. Opt. Soc. Am.* **B11**, 1236 (1994)
[16] M G Papadopoulos, J Leszczynski and A J Sadlej, *Non-linear optical properties of matter: From molecules to condensed phases* (Springer, Netherland, 2006)
[17] D P Hand and P S Russell, *Opt. Lett.* **15**, 102 (1990)
[18] A A Sukhorkov, S Shoji, Y S Kivshar and S Kawata, *J. Non. Opt. Phys. Mater.* **11**, 391 (2002)
[19] C Paré and M Florjańczyk, *Phys. Rev.* **A41**, 6287 (1990)
[20] D Anderson, M Lisak and T Reichel, *Phys. Rev.* **A38**, 1618 (1988)
[21] P F Byrd and M D Friedman, *Handbook of elliptic integrals for engineers and scientists* (Springer-Verlag, New York, 1971)