A New Era of Exotic Electromagnetism

K Porsezian and Ancemma Joseph

Nature has its own limits and naturally-occurring materials exhibit restricted electromagnetic properties bound within those limits. However, modern day researchers have developed artificially structured material composites, called 'metamaterials', that possess significant potential to provide electromagnetic properties that are quite unusual and are not found in nature. This article is aimed at giving an introductory review of this class of 'designer materials' and their superior properties that are not present in their constituent components.

(left) K Porsezian works at the Department of Physics, Pondicherry Central University, Puducherry, India. He is investigating solitons and modulation instability in nonlinear optics, Bose-Einstein condensates, metamaterials, photo-refractive materials, photonic crystal fibers and also integrability aspects of nonlinear partial differential equations.

(right) Ancemma is working towards her doctoral degree in Pondicherry University, Puducherry, India. Her primary research interests are based on studying the nonlinear pulse propagation through nonlinear periodic structures and negative index materials.

1. Metamaterial – Advent

Metamaterials are artificially constructed structures exhibiting unconventional characteristics that have never been observed in nature before. This fact gives them their name 'metamaterials' – materials beyond (natural) materials. Unlike natural media, these are materials which derive their properties from their structure rather than directly from their composition as shown in *Figure* 1. Though the notion of metamaterials was originally proposed in regard to the idea of a negative refractive index, or negative-index materials (NIMs), today the various interpretations of the term have led to their versatile terminologies like double negative material(DNG), left-handed material (LHM), backward phase material(BPM), negative phase velocity (NPV)material, etc.

The vital decisive factor responsible for the unusual behavior of these materials is the effective negative refractive index resulting from the negative real part of the

Keywords
Metamaterial, negative refraction, left-handedness.

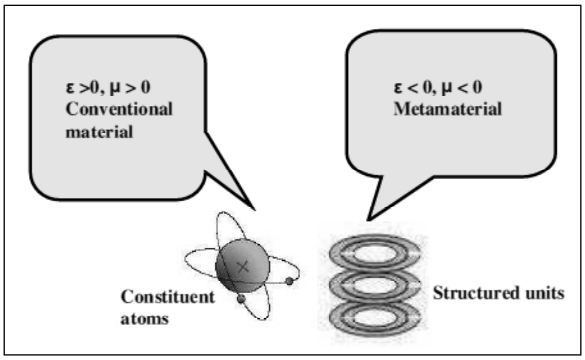

Figure 1. Conventional and metamaterial composition.

magnetic permeability and negative real part of electric permittivity. The dielectric permittivity and magnetic permeability are the key factors responsible for all the electric and magnetic characteristics exposed by materials. While some materials possess negative permittivity (like for metals below their plasma frequency), no natural materials are seen with negative permeability. Tracing back the history of metamaterial concept, it was the Soviet physicist Victor Veselago's genius which postulated the physical permissibility of materials possessing simultaneous negative permittivity and permeability back in the late 1960s. He had envisioned the unusual electrodynamics of double negative materials in his famous seminal paper in *Soviet Physics* Uspekhi. This putative concept which lay latent for almost three decades got resurrected with the pioneering effort of John Pendry of Imperial College, London who proposed a blueprint structure exhibiting simulataneous negative permeability and negative permittivity.

It was the Soviet physicist Victor Veselago's genius which postulated the physical permissibility of materials possessing simultaneous negative permittivity and permeability back in the late 1960s.

These materials, which are still in their infancy, have exposed a variety of fascinating effects such as reversal of Snell's law, reversed Doppler effect, Cerenkov radiation, sub-diffraction imaging, photon tunneling, backward wave antennas, phase combination, electrically small resonators, and are expected to satisfy a multitude of other requirements in complex environments. In the next section, we peep into each of the intriguing and exciting traits of metamaterials.

2. Metamaterial – Unusual Traits

2.1 *Left-handedness*

From the foundations of macroscopic electrodynamics given by J C Maxwell (1831–1879), it is well known that the propagation of electromagnetic waves in a medium is determined by dielectric permittivitiy and magnetic permeability of the medium. Accordingly, the effective refractive index of a medium can be written as $n = \sqrt{\epsilon\mu}$. Now, as per Veselago's notion, if both ϵ and μ take up negative values in a given wavelength range, we can write $\mu = |\mu|e^{i\pi}$ and $\epsilon = |\epsilon|e^{i\pi}$. Then one can easily see that the refractive index

$$n = \sqrt{|\epsilon||\mu|e^{2i\pi}} = \sqrt{|\epsilon||\mu|}\sqrt{e^{2i\pi}} = -\sqrt{|\epsilon||\mu|}. \qquad (1)$$

From the foundations of macroscopic electrodynamics given by J C Maxwell (1831–1879), it is well known that the propagation of electromagnetic waves in a medium is determined by dielectric permittivitiy and magnetic permeability of the medium.

This implies that the refractive index of the medium with simultaneous negative ϵ and μ must be also negative. Now what could be the consequences of this fact? For a monochromatic plane wave, the Maxwell's equations can be written as $\vec{k} \times \vec{E} = \frac{\omega}{c}\mu\vec{H}$ and $\vec{k} \times \vec{H} = -\frac{\omega}{c}\epsilon\vec{E}$.

Interestingly, one can see that, while for positive values of ϵ and μ, $\vec{E}$, $\vec{H}$ and $\vec{k}$ form a right-handed triplet, but for Veselago's case, they form the left-handed triplet. While Poynting vector $\vec{S} = \frac{c}{4\pi}\vec{E} \times \vec{B}$ is always directed away from the source for both left and right-handed cases, the wave vector surprisingly takes up two directions, 'away from the source' for the right-handed and 'towards the source' for the left-handed case. One can see this fact well depicted in *Figure* 2.

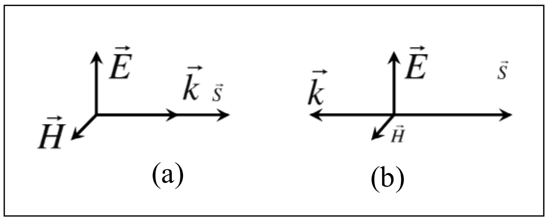

Figure 2. (a) Right-handed media with $\varepsilon > 0$, $\mu > 0$.
(b) Left-handed media with $\varepsilon < 0$, $\mu < 0$.

2.2 *Negative Refraction*

A striking feature of the negative index material is the reversal of one of the fundamental principle of optics, the Snell's law. Light that enters from an ordinary medium to the metamaterial medium will undergo refraction opposite to that occuring in the conventional ordinary medium. How could this be explained? Consider the refraction phenomena occurring when a beam travels from one medium into another.

Light that enters from an ordinary medium to the metamaterial medium will undergo refraction opposite to that occuring in the conventional ordinary medium.

Suppose if the two media are of positive index with $\epsilon > 0$ and $\mu > 0$, then we will have an ordinary refraction case. But for the case when the second medium is of negative index with $\epsilon < 0$ and $\mu < 0$, we have a beam going from an ordinary medium into a negative medium. One should remember here that, for any case, the boundary conditions require tangential components of $\vec{E}$ and $\vec{H}$, and normal components of $\vec{D}$ and $\vec{B}$ to be continuous at the interface, i.e.,

$$\begin{aligned} E_{t1} &= E_{t2}, \;\; H_{t1} = H_{t2}, \\ \text{and } \epsilon_1 E_{n1} &= \epsilon_2 E_{n2}, \;\; \mu_1 H_{n1} = \mu_2 H_{n2}. \end{aligned} \tag{2}$$

Now, it is clear that x and y components of the field are not changed at transition from medium 1 to 2, regardless of the signs of ϵ and μ. As for the normal z components of the field, they preserve their directions if ϵ and μ preserve their signs in both media. Otherwise it is obvious that they change their directions and hence the negative refraction as shown in *Figure* 3.

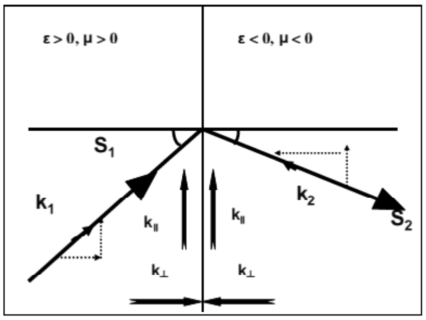

Figure 3. Negative refraction.

2.3 *Reversed Doppler Shift*

Thinking of oneself standing at the crossroads and hearing an approaching ambulance will immediately bring to our mind the 'Doppler effect'. As the vehicle races towards us, the pitch of the siren becomes higher, and as it recedes, it gets lowered. This well-known fact of an apparent change in the frequency of a wave when there is a relative motion between the observer and the source of waves is termed as Doppler effect. As the object emitting the wave approaches the observer, the intervals between the waves diminish; in other words, the radiation gets squeezed, resulting in increased frequency and a blue shift. As the object recedes away from the observer, the wave gets stretched leading to a red shift.

An apparent change in the frequency of a wave when there is a relative motion between the observer and the source of waves is termed as Doppler effect.

Now how could this effect get reversed in metamaterials? The apparent frequency measured by the observer when the source is moving inside the medium at velocity v is

$$\omega' = \gamma(\omega + \vec{k}.v) \quad \text{with} \quad |\vec{k}| = n\omega/c. \tag{3}$$

Taking up the relativistic factor $\gamma = (1 - v^2/c^2)^{-1/2}$ and considering emission along the direction of the motion of the source in the ordinary ($n = 1$) or metamaterial medium ($n = -1$), we get the apparent frequency to be

$$\begin{aligned} \omega' &= \sqrt{\frac{c+v}{c-v}}\ \omega \quad \text{for right-handed medium,} \\ \text{and } \omega' &= \sqrt{\frac{c-v}{c+v}}\ \omega, \quad \text{for left-handed medium.} \end{aligned} \tag{4}$$

Thus one can see that the Doppler effects gets exactly reversed in a metamaterial medium due to the reversed phase vector.

2.4 *Reversal of Fermat's Principle*

The original principle of Fermat states that a beam of light chooses such an actual path between two points which is the one traversed in the least time. It was further reformulated as the route chosen by the light to be of minimum optical length. However, in the case of metamaterial the term 'minimum' (both for path or time) is not apt.

The original principle of Fermat states that a beam of light chooses such an actual path between two points which is the one traversed in the least time.

Let us consider the case of light propagating between two points say A and B. First, as in Case 1 of *Figure* 4(a), choose point A to be in a medium of positive refractive index n_1, and point B to be in a medium of positive refractive index n_2. In this case of positive n_1 and n_2, the ray will travel along AO_1B with the incident and refractive angles (φ and ψ respectively) obeying Snell's law. Note that for Snell's law to be valid, the optical path length must vanish and here the variation of the optical length $n_1\ AO_1 + n_2\ O_1B$ will be minimum through AO_1B. If both the mediums n_1 and n_2 are considered to be negative, the propagation will be along the same path, but the wave vector will be opposite to the

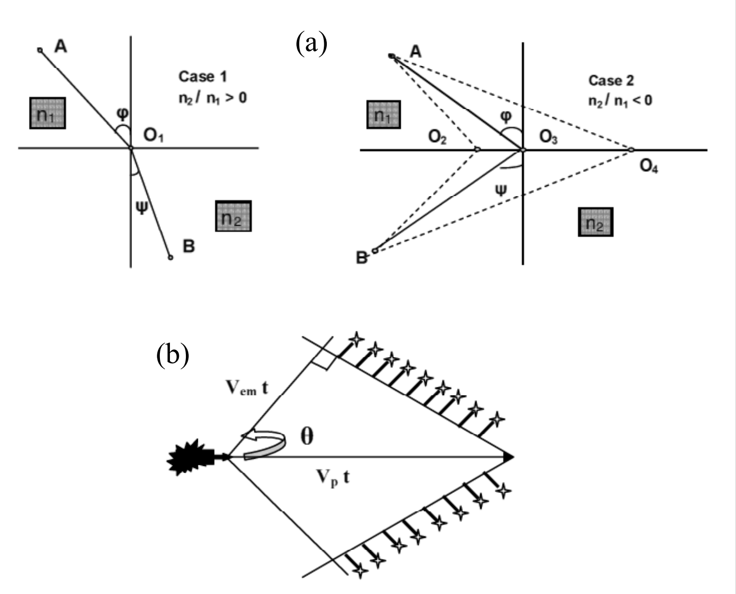

Figure 4. (a) The possible trajectories for the beam to pass between two points A and B between two different mediums (b) Cerenkov radiation.

direction of rays leading to negative optical length and it turns out to be maximum for the actual path $\mathrm{AO_1B}$.

Consider a situation when $n = n_2/n_1$ is negative, i.e., light is propagating from an ordinary medium to a metamaterial medium as shown in Case 2 of *Figure* 4a. Here there is an ambiguity in the actual path being a minimum or a maximum. The actual path traversed by the light will be along $\mathrm{AO_3B}$, with incident and refracting angles obeying the Snell's law, but this time for the negative value of ψ. The virtual paths $\mathrm{AO_2B}$ or $\mathrm{AO_4B}$ will not be chosen by the light even though they are shorter or longer in terms of the time of travel.

The proper formulation would be that light choses to travel along the path that corresponds to a local extremum in the spatial derivative of the total travel time through all possible paths.

Hence the proper formulation would be that light choses to travel along the path that corresponds to a local extremum in the spatial derivative of the total travel time through all possible paths. An extremum could be defined as variation in the optical path becoming zero, i.e., $\int_B^A n(r)\mathrm{d}s = 0$. Thus for the case of a beam of light traveling from right- to left-handed metamaterial medium, the variation in the path $\mathrm{AO_3B}$ is minimum. Unlike the Fermat's version of the term 'minimum optical pathway', it is an extremum which could be minimum, maximum or a point of inflection.

2.5 *Reversed Cerenkov Radiation*

The phenomenon of Cerenkov radiation discovered by Soviet Physicist Pavel A Cerenkov is the emission of a cone of electromagnetic radiation when a charged particle (usually electrons) passes through a medium at a speed greater than the speed of light in that medium. One can explain the physics behind this as the response of the medium to the transit of charged particle through it. As the charged particle travels, it disrupts the local electromagnetic field, displacing and polarising the electrons in the medium. After the transit, the electrons restore themselves to equilibrium with the emission of photons which would constructively interfere to give the

The phenomenon of Cerenkov radiation discovered by Soviet Physicist Pavel A Cerenkov is the emission of a cone of electromagnetic radiation when a charged particle (usually electrons) passes through a medium at a speed greater than the speed of light in that medium.

observed intensity. Now the angle of the cone of radiation in a normal medium is acute while that in a metamaterial medium is obtuse.

For a metamaterial medium, the refractive index *n* has a negative value, and therefore this angle becomes obtuse, i.e., the particle will radiate from a cone behind itself, rather than in front of it.

To explain this reversal, consider *Figure* 4b, where the charged particle situated at the left corner at time $t = 0$, traverses to the right corner with velocity V_p equal to γc in time t. The distance traversed will be equal to $V_p\ t$. The refractive index of the medium being n, the cone of radiation would have traversed a distance $\frac{c}{n}t$. Hence the acute angle of this cone will be

$$\cos\theta = \frac{V_p}{V_{em}} = \frac{1}{n\gamma}. \qquad (5)$$

Now for a metamaterial medium, the refractive index n has a negative value, and therefore this angle becomes obtuse, i.e., the particle will radiate from a cone behind itself, rather than in front of it.

2.6 *Reversed Goos–Hänchen Shift*

Consider a collimated beam incident on a rarer medium from a denser medium at an angle greater than the critical angle; the phenomenon one would expect to emerge is 'total internal reflection'. Here the incident beam reflects back into the incident medium, but it does so from a point which is displaced from the point at which it struck the plane. This lateral shift was discovered by F Goos and H Hänchen and hence the name of this effect.

Consider a collimated beam incident on a rarer medium from a denser medium at an angle greater than the critical angle; the phenomenon one would expect to emerge is 'total internal reflection'.

To have a mathematical picture of this phenomenon, consider a wave impinging onto a rarer medium at an angle greater than the critical angle. The reflection coefficient is a complex number with unit amplitude and phase ϕ, i.e.,

$$\text{Reflection coefficient } R = \exp(2I\phi),$$

$$\text{where } \phi = -\tan^{-1}\left(\frac{\mu_0\sqrt{k_x^1 - k_t^2}}{\mu_t k_z}\right). \qquad (6)$$

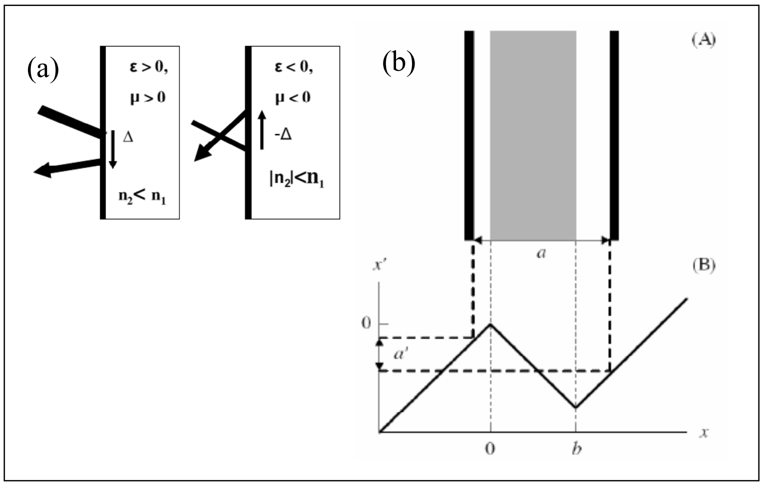

Figure 5. (a) Goos–Hänchen shift, (b) Casimir effect in metamaterial.

Note that k_t denotes the transmitted wavenumber and k_x, k_z are the components of the incident wave-vector. The phase ϕ introduces a spatial shift in the reflection of a finite beam, which is termed the Goos–Hänchen shift.

Considering the case of a beam incident from a positive index medium with refractive index n_1 on an ordinary positive index medium with refractive index $n_2 > 0$ and when the angles of incidence are sufficiently away from the critical angle and the grazing angle, the displacement of the beam was shown to be $\triangle = -\frac{\partial \phi}{\partial k_x}$ as depicted in first case of *Figure* 5a. But as the incident beam encounters a metamaterial medium with negative index of refraction, i.e., with $n_2 < 0$, the phase ϕ takes up the value opposite in sign to that of positive index materials as μ_t takes up negative value. This in turn leads to the Goos-Hänchen shift in the direction opposite to that of materials with positive indices of refraction as depicted in second figure of *Figure* 5a.

The Goos-Hänchen shift is in the direction opposite to that of materials with positive indices of refraction.

2.7 *Reversal of Casimir Force of Attraction*

In 1948, HBG Casimir predicted the attraction between a pair of neutral, parallel conducting plates, the effect being purely due to the effect of quantum mechanical vacuum modes. An amazing feature is that this fundamental attraction can be turned into repulsion if a metamaterial is sandwiched between the parallel plates. It can be explained from *Figure* 5b as the transformation of Casimir cavity of size a in physical x space into a cavity in x' of size a'. The cavity in physical space increases when the transformed cavity decreases. Consequently, the attractive Casimir force in transformed space turns into a repulsive force in physical space. This could lead to the achievement of quantum levitation.

The attractive Casimir force in transformed space turns into a repulsive force in physical space.

3. Metamaterial – Realization

While Veselago had hypothesized the concept of metamaterials with $\epsilon < 0$ and $\mu < 0$, this notion remained unappreciated for decades as no metamaterial seem- ed to exist naturally. One may easily find natural materials with $\epsilon < 0$ (e.g., gold, silver and other metals) up to the visible frequencies; but no magnetic materials with $\mu < 0$ are known to exist at optical frequencies till today. The boom in the metamaterial research came with the exciting suggestion made by Pendry *et al.* to utilize split-ring resonators to achieve negative magnetic permeability.

The boom in the metamaterial research came with the exciting suggestion made by Pendry *et al* to utilize split-ring resonators to achieve negative magnetic permeability.

The negative permittivity of a metamaterial structure can be realized by considering an array of long, parallel, thin metal wires embedded in a dielectric medium (*Figure* 6a). With such an array of unit cell length a and

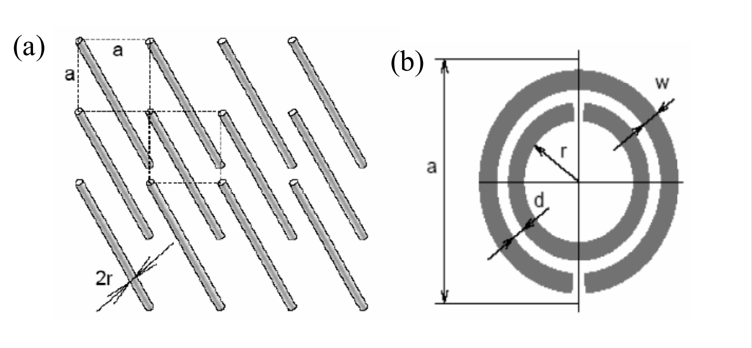

Figure 6. Constituents of metamaterial: (a) An array of metallic wires; (b) A split-ring resonator.

the radius of single wire r, the plasma frequency for the longitudinal plasma mode is

$$\omega_p = \sqrt{\frac{2\pi c^2}{a^2 \ln a/r}}, \text{ with the effective dielectric permittivity } \epsilon = 1 - \frac{\omega_p^2}{\omega^2}. \tag{7}$$

At frequencies less than plasma frequency ω_p, one achieves negative permittivity.

A highly conductive split-ring resonator structure (*Figure* 6b) with the capacitance between the rings balancing the inductance provides the desired negative magnetic permeability. As one applies a time-varying magnetic field perpendicular to the ring's surface, the magnetic field produced by the currents induced in dependence on the resonant properties of the structure, either oppose or enhance the incident field, thus resulting in positive or negative effective μ. For the circular double split-ring resonator, the effective permeability is

$$\mu = 1 - \frac{\pi r^2/a}{1 + (2\sigma i/\omega r \mu_0) - (3d/(\pi^2 \mu_0 \omega^2 \epsilon_0 \epsilon r^3))}. \tag{8}$$

As one applies a time-varying magnetic field perpendicular to the ring's surface, the magnetic field produced by the currents induced in dependence on the resonant properties of the structure, either oppose or enhance the incident field, thus resulting in positive or negative effective μ.

The thin wire–split-ring resonator structure is the vital building block for achieving negative effective ϵ and μ and negative refractive index but only in THz frequencies. Many different structures like 'cut-wires separated by dielectric spaces', 'fishnet structures', 'paired nanorods' are fabricated these days to realize metamaterials in the optical regime.

4. Metamaterial – Exciting Future

An exciting application of metamaterial achieved practically is super, hyper lenses.

Out of the exotic electromagnetic applications which bring forth the concept of metamaterial into the forefront of the research arena, the prominent exciting application of metamaterial achieved practically is super, hyper lenses.These lenses could be better than conventional lenses in the sense that, an ordinary lens cannot

The evanescent field gets enhanced inside the metamaterial lens and gets restored to give sub-diffraction resolution.

focus light down to more than about half its wavelength, the diffraction limit. What does this mean? As one considers a source and a lens placed along the z axis, the electromagnetic field emanating from the source could be written as a superposition of plane waves which could be written as

$$E(x,y,z,t)=\sum_{K_xK_y}A(K_xK_y)\exp i(K_zz+K_xx+K_yy-\omega t),\ \text{where } K_z=\pm\sqrt{\frac{\omega^2}{c^2}-(K_x^2+K_y^2)},\quad (9)$$

with ± corresponding to ordinary and metamaterial medium respectively. In ordinary materials, when $(K_x^2+K_y^2)<\frac{\omega^2}{c^2}$, then it corresponds to propagating modes with long-range information, and when $(K_x^2+K_y^2)>\frac{\omega^2}{c^2}$, then K_z becomes imaginary, leading to evanescent fields decaying exponentially with z. These modes actually contain the information about the high frequency (minute scale) features of the source being imaged.The maximum possible resolution that can be obtained from such a case would be $\frac{\omega}{c}=\frac{2\pi}{\lambda}$. In other words, if the lens is at a distance larger than the operating wavelength $\simeq\lambda$, then K_z component will not be seen; thus λ remains the basic resolution limit in far-field approximation. Now in the case of metamaterial, with $K_z=-\sqrt{\frac{\omega^2}{c^2}-(K_x^2+K_y^2)}$, the evanescent modes containing high frequency features will grow instead of decaying exponentially. Thus the evanescent field gets enhanced inside the metamaterial lens and gets restored to give sub-diffraction resolution.

Shielding an object with a metamaterial cloak would render light to curve around the object and return to its original path on the far side.

Another novel application of negative refraction which is like magic could be 'invisibility cloaking'. Shielding an object with a metamaterial cloak would render light to curve around the object and return to its original path on the far side. The detector or the observers of the object would be completely unaware of the presence of such an object – even its shadow will not be visible.

Apart from all these important and exciting linear effects and their consequences, there is yet another very important window, the nonlinear effects of metamaterials, that has remained vastly unexplored. One can achieve the prospect of nonlinear electromagnetic responses, like cubic or quadratic nonlinear responses, in metamaterial structures by insertion of nonlinear elements within them. It could be done by embedding the split-ring resonators in a Kerr-type dielectric or by including certain nonlinear elements (e.g., diodes) in the split-ring resonators' paths.

The nonlinear effects of metamaterials has remained vastly unexplored.

A few groups have started exploring this avenue with results like exhibition of second harmonic generation, optical bistability, parametric amplification, soliton formation, etc., in metamaterials. The surface of nonlinear metamaterial medium acts as a mirror reflecting off energy in the form of second harmonics when there is an exact phase matching between a backward-propagating wave of the fundamental frequency and the forward-propagating wave. The phenomenon of bistability which enables a system to exhibit two steady transmission states for the same input intensity has also been predicted of a metamaterial when embedded in a Kerr-nonlinear medium leading to effective nonlinear medium switching on and off of the magnetic permeability. A mechanism of nonlinear coupler with positive index and negative index channel could result in the bistability due to the backward coupling between the modes propagating in the two channels. Periodic metamaterial structure with metal-dielectric (Kerr-type nonlinear) slabs show evidence of self focusing and discrete soliton formation due to three-fold interplay between periodicity, nonlinearity and surface plasmon tunnelling. Parametric amplification and four-wave mixing kind of nonlinear effects have also been predicted. Even a novel amalgamation of metamaterial and fiber optics, a 'meta-fiber', is foreseen where the duo – 'conventional and surface plasmon

A mechanism of nonlinear coupler with positive index and negative index channel could result in the bistability due to the backward coupling between the modes propagating in the two channels.

waveguiding' – is expected to emerge with many potential applications.

The bottomline is that these exotic metamaterials may not be specifically oriented towards one or a few areas of physics, but their unusual properties will offer benefits over a wide range of applications.

Acknowledgements

The author wishes to thank the Government of India agencies, (CSIR, IFCPAR, DST-DFG and DST-Ramanna Fellowship), for financial support. The authors also thank Dr. S Sivaprakasam, Pondicherry University, for critical reading of the manuscript.

Suggested Reading

[1] G Veselago, The electrodynamics of substances with simultaneously negative values of ε and μ, *Sov Phys Usp.*, Vol.10, pp.509–514, 1968.
[2] Breakthrough of the year: The runners-up, *Science* Vol.302, No.5653, pp.2039–2045, 2003.
[3] J Valentine, S Zhang, T Zentgraf, E Ulin-Avila, D A Genov, G Bartal tal and X Zhang, Three-dimensional optical metamaterial with a negative refractive index, *Nature*, Vol.455, pp.376–379, 2008.
[4] D R Smith, J B Pendry and M C K Wiltshire, Metamaterials and negative refractive index, *Science*, Vol.305, pp.788–792, 2004.
[5] A K Sarychev and V M Shalaev, *Electrodynamics of Metamaterials*, World Scientific, Singapore, 2007.
[6] S A Ramakrishna and T M Grzegorczyk, *Physics and Applications of Negative Refractive Index Materials*, CRC Press, New York, 2009.

Address for Correspondence
K Porsezian
Department of Physics
Pondicherry University
Pudhucherry 605 014, India.
Email: ponzsol@gmail.com

Ancemma Joseph
Email:
ancyjoseph_phypu@yahoo.co.in