LETTERS TO THE EDITOR

	PAGE
On Linear Estimation and Testing of Hypothesis. BY C. RADHAKRISHNA RAO	154
Activation of Nitrogen at Liquid Air Temperature. BY S. S. JOSHI AND A. PURUSHOTHAM	155
A Short Note on the Infra Traps of Kotah State. BY V. S. DUBEY AND Y. K. AGRAWAL	155
X-Ray Topographs of Diamond. BY G. N. RAMACHANDRAN	156
Rupture of Water Drops Falling on Liquid Surfaces. BY B. N. SINGH AND S. P. SINHA	157
Ethyl Alcohol as a Solvent for Modified Romonowsky Stains. BY B. S. ROY	157
Rearrangement of Santenone-Quinone. BY RAM NARAYAN CHAKRAVARTI	158
The Occurrence and Inheritance of a New Type of Hairiness in Asiatic Cottons. BY K. RAMIAH AND V. N. PARANJPE	158
Studies on the Physiology of Mustard. BY J. C. SEN GUPTA AND NIRAD KUMAR SEN	160
Trichodina *sp. from the Capsular Gland of* Ariophanta ligulata (Fer.). BY P. N. RAJAKRISHNA MENON	161
Burnt Paddy-Husk for the Control of Insects in Stored Food Grains. BY M. J. NARASIMHAN AND B. KRISHNA MURTHY	162
Relief for Science Teachers. BY S. S. BHATNAGAR	162

ON LINEAR ESTIMATION AND TESTING OF HYPOTHESIS

1. THE object of the present note is to consider the most general problem in the theory of linear estimation and get suitable generalisations of Markoff's theorem and to derive suitable tests of significance connected with estimated functions.

2. There are n stochastic variates $T_1, T_2, \ldots T_n$ with the variance and covariance matrix $\Lambda = ((\lambda_{ij}))$ such that

$$E(T_i) = \theta_i = a_{i1}\tau_1 + a_{i2}\tau_2 + \ldots + a_{im}\tau_m \quad (2\cdot1)$$

and m not necessarily less than n. Some of the τ's may be regression constants in which case their coefficients will be functions of concomitant variates. We want a linear function $L(T) = b_1T_1 + b_2T_2 + \ldots\ldots + b_nT_n$ such that $E\{L(T)\} = \theta = l_1\tau_1 + l_2\tau_2 + \ldots\ldots + l_m\tau_m$, a given parametric function and $V\{L(T)\}$ is minimum. If $L(T)$ can be found to possess such properties then it is called the best estimate of θ.

3. It has been shown that the set of equations

$$Q_i = L_i\Lambda^{-1}y' = \sum_{j=1}^{m} L_i\Lambda^{-1}L_j'\tau_i = h_i \quad (3\cdot1)$$

$(i = 1, 2, \ldots m)$, where L_i and y are the row matrices $(a_{1i}, a_{2i}, \ldots a_{ni})$ and $(T_1, T_2, \ldots T_n)$ and the dash represents their transpose obtained by minimising

$$L = \Sigma\Sigma\,\lambda^{ij}\,(T_i - \theta_i)\,(T_j - \theta_j), \quad (3\cdot2)$$

where λ^{ij} are the elements of Λ^{-1}, are such that Q_i is the best estimate of h_i and the best estimate of θ is given by $\Sigma\, c_i Q_i$ where c's are such that $\theta \equiv \Sigma\, c_i h_i$. If the rank of the matrix of the equations (3·1) is less than m, then all parametric functions are not estimable

4. The set of equations (3·1) possesses the following properties.

(i) Variance of Q_i is the coefficient of τ in the ith equation;

(ii) Covariance of Q_i and Q_j is the coefficient of τ_j in the ith equation;

(iii) If $\Sigma\, c_i Q_i \equiv 0$, then the parametric function $\Sigma\, c_i\tau_i$ is not estimable;

(iv) If $\Sigma\, a_i Q_i$ and $\Sigma\, b_i Q_i$ are the estimates of $\Sigma\, l_i\tau_i$ and $\Sigma\, m_i\tau_i$, then their variances and covariances are given by

$$\begin{pmatrix} \Sigma\, al & \Sigma\, bl \\ \Sigma\, am & \Sigma\, bm \end{pmatrix}$$

(v) The number of estimable parametric functions is equal to the number of functionally independent Q's.

(vi) An intrinsic property is that all the above five properties hold good even with a subset of the equations obtained by eliminating one or more τ's.

(vii) The best estimate of any estimable parametric function $\Sigma\, l\tau$ is unique and is obtained by substituting any particular solution for τ's obtained by solving the equations (3·1) or any set of equations derived from it; and so also the expressions for the variances and covariances.

5. If we want to test the hypothesis $\Sigma\, l\tau = \xi$ then we construct the statistic

$$w = \frac{\Sigma\, c\,Q - \xi}{\sqrt{\Sigma\, cl}},$$

where $\Sigma\, c\,Q$ is the estimate of $\Sigma\, l\,\tau$ and refer the normal tables for tests of significance. If we want to test the composite hypothesis $\Theta_i = \Sigma\, l_{ij}\tau_j = \xi_i, i = 1, 2, \ldots k$ we take a linear compound of these k relations

$$\Sigma\,\lambda_i\,(\Theta_i - \xi_i)$$

with the corresponding estimate

$$\Sigma\,\lambda_i\,(\Sigma\, c_{ij}Q_j - \xi_i) = \Sigma\,\lambda_i P_i$$

and variance $\Sigma \Sigma \lambda_i \lambda_j V_{ij}$ where $V_{ij} = \text{cov}(P_i P_j)$ and construct the above statistic

$$V = \frac{\Sigma \lambda_i P_i}{\sqrt{\Sigma \Sigma \lambda_i \lambda_j V_{ij}}}.$$

We choose the compounding coefficients such that V^2 is maximun. This leads to the determinental equation

$$|P_i P_j - V^2 V_{ij}| = 0.$$

The distribution of V^2 on the non-null hypothesis is obtained as

$$\text{Const}\cdot e^{-\frac{V^2}{2}} (V^2)^{\frac{k-1}{2}} I_{\frac{k}{2}} (V\phi)\, d V^2.$$

on the assumption that the y's form a multivariate normal system. The necessary statistics, when the variances and co-variances are not known are obtained by studentising the above statistics. Some of the important distributions will be discussed in a paper to be published in full elsewhere.

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April 4th, 1944.

ACTIVATION OF NITROGEN AT LIQUID AIR TEMPERATURE

The behaviour of active nitrogen is remarkable towards heat. The disappearance of its after-glow on heating, first noticed by Lord Rayleigh[1] has been found by us to be subject to the pressure conditions of the activated gas:[2] the quenching temperature is increased by an increase of the gas pressure.[2] Rayleigh also observed that if a stream of active nitrogen is led through a tube cooled by liquid air, the after-glow is quenched.[1] Our recent results show that the occurrence of this quenching also depends upon the magnitude of the gas pressure; when this last was raised above 40 mm., the glowing nitrogen escaped the *Rayleigh quenching* produced, at say 15 mm. under the same conditions.

Attempts were next made to minimise a possible insufficiency in the cooling of the gas at high pressures. The nitrogen was streamed through spirals, in all about 3 yards long, cooled by liquid air. The last spiral surrounded the Crooke's tube used for the nitrogen activation by electrical discharge. With the discharge tube under liquid air, it was remarkable to observe that the Rayleigh quenching of the nitrogen after-glow did not occur, at even such small pressures of about 1 mm., at which, under normal conditions, it was easily produced.

An interesting result was now observed when the gas pressure was reduced beyond 0·1 mm. An after-glow of 15-17 minutes' duration was produced; platinum and aluminium electrodes were used. Starting with fresh electrodes, both the intensity and duration of the after-glow increased markedly with repeated discharges. Results were in essence the same with pure nitrogen and with an admixture of 0·8 per cent of oxygen, except for the fact that with the latter a greater number of discharges were necessary than with pure nitrogen, to produce this long after-glow; it was not observed if the gas pressure was reduced to that due to a Topler. This long after-glow could not be ascribed to activated vapour of mercury or that of the pump oil, since there were liquid air traps on either side of the discharge tube. Observations with a direct vision spectroscope showed that this long after-glow was a nitrogen emission; besides, it was not observed when the nitrogen was replaced by air, hydrogen or carbon dioxide, other conditions being unaltered Cooling by liquid air during activation is more favourable than an elaborate pre-cooling before exposure to discharge, for the production of this long after-glow. That on the cessation of the discharge and after allowing the temperature of the system to rise, there ensues a sensible rise of pressure, suggests that a 'clean up' or adsorption of the nitrogen on the electrodes is a possible factor in the occurrence of this phenomenon.

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May 10, 1944. A. Purushotham.

1. Strutt, R J., *Proc. Roy Soc.*, 1911, **A 85**, 219-29.
2. Joshi and Purushotham, *Proc. Ind. Acad. Sci*, 1944, **A 19**, 159-62.

A SHORT NOTE ON THE INFRA TRAPS OF KOTAH STATE

Recently the writers had been to Kotah State in connection with the mineral survey of the State There they studied a formation which has been referred to and mapped as Infra Trap in the old G.S.I. map. This lies over the Vindhyans and is below the trap Its thickness is much variable being from a few feet to about 100 feet. The formation consists of a hard cherty silicious rock probably formed by the deposition of the volcanic ash in the lakes existing at that time. It appears that the rocks of this formation were deposited just before the outpouring of the lava. In this formation the authors have found fossils, both animal and plant. The animal fossil probably seems to be the jaw of some reptilian group and the plant fossil showing sporo-carps, spores, etc., are probably of the Marsiliaceæ group. As far as the authors are aware this is the first locality where the fossils have been found just below the trap in Rajputana. The fauna and flora revealed are quite interesting. Detailed work is going on and the results will be published later.

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April 27, 1944. Y. K. Agrawal.