computers
S
fluids

Computers & Fluids 30 (2001) 621-641

Least squares kinetic upwind method on moving grids for unsteady Ruler computations

V. Ramesh [a], S.M. Deshpande [b,*]

[a] *Computational and Theoretical Fluid Dynamics Division, National Aerospace Laboratories, Bangalore 560017, India*
[b] *AR and DB centre for Excellence in Aerospace CFD, Department of Aerospace Engineering, Indian Institute of Science, Bangalore 560012, India*

Accepted 10 November 2000

Abstract

The present paper describes the extension of least squares kinetic upwind method for moving grids (LSKUM-MG). LSKUM is a kinetic theory based upwind Euler solver. LSKUM is a node based solver and can operate on any type of mesh or even on an arbitrary distribution of points. LSKUM-MG also has the capability to work on arbitrary meshes with arbitrary grid velocities. Results are presented for a moving piston problem and flow past an airfoil oscillating in pitch.

Keywords: Euler; Moving grids; Kinetic scheme; Upwind; Least squares kinetic upwind method

1. Introduction

Computation of unsteady flows is an important problem in the field of aerodynamics. Prediction of aeroelastic behaviour in industrial applications needs an accurate prediction of the unsteady pressure loads. Helicopter rotor load aerodynamics is one example where unsteady flow computation is essential. Flow through turbine blades involving stators and rotors is another typical case where unsteady flow computation is needed. All such applications in general involve moving boundaries. Computation of flows involving moving boundaries in general involves moving grids. Some of the approaches [1] involving moving grids, transform the equations of motion to a moving frame. Also the boundary condition is treated in the moving frame itself. But these methods generally require interpolation of solution from the moving grids to background static grids.

In the present approach using least squares kinetic upwind method (LSKUM) we address these issues in an innovative way where many of the difficulties faced by the conventional methods are circumvented. LSKUM is a kinetic theory based upwind Euler solver. This method has the capability to work on any type of mesh or even on an arbitrary distribution of points [2]. The motivation for the present work was to extend the method to moving grids without involving any transformation of the equations of motion to a non-inertial frame. Also general formulation is required in the sense if there is no grid movement, it must automatically reduce to that for stationary grids. Such a type of formulation could then work on any grid in which each grid point could have arbitrary grid velocity. The method can be applied to multi moving surfaces and also does not need any interpolation of the solution from moving grids to background stationary grids. The boundary condition has been implemented in the present approach in a very elegant way using specular reflection model of kinetic theory of gases. We first present detailed mathematical formulation of 1D LSKUM on moving grids with some results. Then the extension of LSKUM to moving grids for 2D problems along with results for flow past an airfoil oscillating in pitch are presented.

2. 1D LSKUM on moving grids

2.1. Formulation

Consider the 1D Boltzmann equation

~~
at ~ ~ x =J (1)

where *f is* the velocity distribution function, *v* is the molecular velocity. [1] J represents a collision term which vanishes in the Euler limit, when *f is* a Maxwellian distribution. The Maxwellian distribution, F, in one dimension is given by

~

F = #~~~~~~~~ ~ ~~~ ~ ~~~~~ ~~~

o

where /3 = *1/(2RT),* and I_0 is the internal energy due to non-translational degrees of freedom, 1_0 = ~~~~i~ RT and *u* is the fluid velocity, R is the gas constant and *T* is the absolute temperature of the fluid. Therefore in the Euler limit we get

OF ~~
at ~ ~ aax = 0 (3)

Now let w represent the grid velocity of any grid point. Then we can rewrite Eq. (3) equivalently as

[1] Here we use the word ""molecular" " not in the sense of physical molecule we generally understand, but a general term to mean particles or pseudo-molecules as understood in particle methods.

$$\frac{OF}{Ot} \sim \sim \underset{x}{\overset{\sim\sim}{8}} \sim\sim\sim \sim\sim \underset{8x}{\sim\sim} \sim \sim \quad (4)$$

Introduce a special derivative

$$\sim \sim\sim \sim \sim\sim\sim\sim\sim\sim \sim \sim \sim\sim \sim \sim \sim\sim \sim \quad \sim\sim\sim$$

This special derivative is a derivative following the grid point and is very similar to the usual derivative following a fluid particle used often in fluid dynamics. Further let $v = v - w$ be the particle velocity relative to the grid point. Then Eq. (4) can be compactly written as

$$\overset{\sim\ \sim}{dF} \quad \sim\sim \quad \sim\sim\sim\sim\sim\sim \quad \sim \sim \sim\sim \sim \sim \quad \sim\sim \quad \sim\sim\sim$$

This equation also has some very interesting characteristics. Consider the case when the grid velocity w becomes equal to fluid velocity u. In this case we have $v = v - u =$ c, where c is known as the peculiar velocity in kinetic theory of gases. We should also note that c is a normally distributed random variable around zero mean velocity, and can take all possible values varying from -cx to +00. The special time derivative of F along the grid point path now becomes the total derivative along the particle path u = dx/dt. Thus Eq. (5) can then be written as

$$\overset{\sim\sim}{(} \sim\sim \sim \sim\sim\sim\sim\sim\sim \quad \sim\sim \sim \sim at \quad \sim \sim \sim\sim \sim \quad \sim\sim\sim$$

The Boltzmann equation for the above moving grid (moving with u) now can be written as

$$\frac{dF}{dt} + c\frac{OF}{ax} \sim 0 \quad (8)$$

The above equation is the starting point of the Lagrangian description of the fluid flow through the kinetic model. In fact this is the basic equation around which Manoj et al. [3] have developed the kinetic smooth particle hydrodynamics (KSPH) method. When the grid velocity becomes zero then the above formulation reduces to that for a static grid automatically. The development of 2D KSPH method starts with

$$\frac{dF}{at} \sim \sim\sim \frac{\sim\sim}{ax} \sim \sim\sim\ a \sim \sim \quad \sim\sim\sim$$

and then uses least squares discretisation of the spatial derivatives in Eq. (9). The streamline upwind version of KSPH is based on locally rotated frame *(s, n)* where s is along a local streamline and n is the normal coordinate. Then Eq. (9) in this rotated frame becomes

$$\frac{dF}{} \sim 6 \frac{\sim\sim}{\sim} \sim 6 \overset{\sim\sim}{\sim} \text{ at } t = 0 \quad (10)$$

where ccl and 66_2 represent the components of the peculiar velocity along the local streamline direction and its normal. Upwind stencil (US) can be used to discretise *aF/as* while full stencil (FS) can be used to discretise OF/an. OF/an. OF/an. OF/an. OF/an. OF/an. Such a discretisation is perfectly reasonable as fluid is advected along s-direction.

Let us come back to the case when grid velocity w is arbitrary. Splitting vv into positive and negative parts and discretising the time derivative to first-order in the Eq. (6), we get the update scheme for the distribution function F as

$$F^{n+1} = F'' - At \; \frac{\sim}{\sim} \; \frac{OF}{\sim\sim\sim} \; \frac{\sim}{2} \; \frac{\sim\sim}{Ox}$$

where $F''+^{1}$ represents the updated distribution at the new position of the grid, At represents the time step. Following the moment method strategy [4] we now define a moment function vector, V', as

$$\sim\sim\sim \; Z_{J}^{\sim 1 \sim} \qquad \sim\sim\sim\sim$$

and the P moment of F by,

$$\sim\sim\sim \quad 10\sim \quad \sim\sim \sim \sim\sim \sim\sim$$

~ The updating of the velocity distribution is now mapped [4] to the updating of the state vector U at the Euler level by taking Y' moments of the Eq. (11). Thus we have at the Euler level the state update formula

$$\sim\sim\sim\sim \sim \sim\sim \sim \sim\sim \sim \frac{\sim\sim\sim}{} \sim \frac{\sim\sim\sim}{\sim\sim m} \qquad \sim\sim\sim\sim$$

where

$$\sim \sim\sim \sim\sim \sim \sim$$

$$\sim\sim \; \sim 2\sim\sim\sim \; \sim \sim \sim\sim m$$

$$\left\langle \Psi, \frac{\bar{v} - |\bar{v}|}{2} F \right\rangle \equiv GX_{\mathrm{m}}^{-}$$

U is the state vector given by $U = (\rho \;\; \rho u \;\; \rho e)^{T'}$ $GXm\,m$ represent the split fluxes for a moving grid. The split fluxes $GXm\,m$ are related to the usual KFVS [4] fluxes, and are given by

$$GXm\,m = \sim\sim\sim\sim X^{\sim} \qquad \sim\sim\sim\sim$$

where $GXm\,m$ represent the split fluxes for a moving grid, $\sim X^{\sim}$ represent the split fluxes which are similar to those for a static grid except that the fluid velocity components in this case are relative to the grid velocity and matrix [A] transforms the fluxes to those on the moving grid. The fluxes $\sim X^{\sim}$ and the transformation matrix $[A]$ are given by

$$\begin{bmatrix} GX_s^{\pm}(1) \\ GX_s^{\pm}(2) \\ GX_s^{\pm}(3) \end{bmatrix} = \rho\left\{\tilde{u}\tilde{A}^{\pm} \pm \tilde{B}\right\}$$

~ ~ ~ ~~ ~ ~~~~~~ ~ ~~~~

~

~~ ~ ~ ~ ~ ~ ~ ~ ~2 ~ ~~~~~

u e

1 0 0

~w 1 O ~ (17)

where

$u = u - w,$ A~ ~ ~ ~ ~~~~ 2

~ ~~~ ~ ~~ ~ u~ $B =$ e -ss2

$\frac{1}{2RT}$, S ~ ~~~ w = grid velocity

2.2. Least squares evaluation of the spatial derivatives for moving grid

Consider 1D grid as shown in Fig 1. Let o be any node at which we want to update the solution. Further w_0 is the grid velocity of node o and w_i is the grid point velocity of any point i in the neighbourhood of the point o. We can observe from Eq. (11) that in order to obtain a first-order update to the solution at point o we need to evaluate ~ ~~ ~~~~~~~~ ~~~~ v_o is the molecular velocity relative to grid point velocity w_0. For 1D grid, the least squares approximation to FF_0 is given by,

~ ~~~~~ ~ ~ ~~~ ~~~~~ ~~~ ~ ~~ ~ ~~~ ~ ~~ ~ ~~~ ~~~~

We easily observe from the above formula, evaluation of ~ ~~ v~~~~~ ~~ ~~~~ ~~~~~~~~ ~~~ ~~~~~ ~~~~~~ of the moments of F at the neighbourhood points i of the node o. We refer to these neighbouring points as secondary nodes and the neighbouring points of the neighbourhood of node o is referred to as tertiary points. At the secondary nodes we need to evaluate ~ ~~ ~~~~~~ ~~~ therefore FF must be expressed in terms of v_o, that is,

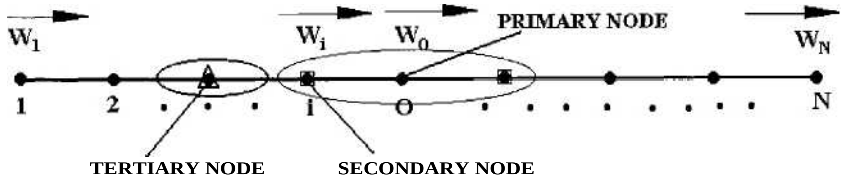

Fig. 1. 1D Moving grid.

∼

~~ ~ ~~ ~~ ~~~ ~ ~ ~~~~~ ~ ~~~~~~~~~ ~ ~~~~]

~~

The relative fluid velocity at node i with respect to the grid velocity at node o is denoted by $u_i(w_o)$ and must not be confused with $u_i(w_i)$ which is the relative fluid velocity at node i with respect to w_i. The important point being made here is that i represents a secondary node and the primary node being o at which spatial discretisation is done and hence all computations must be done with respect to velocity relative to primary node.

Let us also consider the two-step defect correction procedure for second order approximation to the derivative FF_0. In the first step, the first order approximation is evaluated using Eq. (18). In the second step, we just replace AFF *by* $A\sim F_i$ to get the second-order approximation to the derivative, that is,

F~~~ ~~ a~~ ~~~~~ ~~~ ~ ~~~ ~ ~~~~~~~~~ ~~~~ ~ ~~~ ~ ~~~ ~~~~

~ 2

From the above expressions we can see that in order to get a second-order accurate solution for a moving grid, we need to obtain first-order derivatives at the secondary points. This in turn requires moving fluxes at the tertiary points which as mentioned before have to be evaluated with respect to molecular velocity relative to the primary node o.

2.3. *Kinetic treatment of boundary condition for a moving solid wall*

In this section we describe the updating of the velocity distribution on moving boundaries. Consider a point P which lies on the piston moving with a velocity u_p as shown in Fig. 2. In order to update the distribution F at a body point P we follow the principle of specular reflection used for static boundaries, except that now for a moving boundary, we consider the velocity of the molecules relative to the moving piston.

The Maxwellian at P is now split into two parts as

$$F_P = F_I \cup F_R \tag{20}$$

where F_I is the Maxwellian corresponding to the incident particles and F_R is the Maxwellian corresponding to the reflected particles. In the present case of a moving piston we can easily see that all the particles with velocity $v < 0$ relative to the piston will hit the wall. Thus we have

$$F_I = F_1(v) \quad \text{for } v<0$$

The reflected part is then constructed from the incident distribution using the specular reflection principle as

$$F_R = F_I(\sim u) \quad \text{for } v>0 \text{ where } u = -v, \quad v = v - u_p.$$

Fig. 2. Moving piston problem.

Therefore the update for the distribution F and hence for U can be written as

~ ~ ~~~ = ~~~ ~ ~ ~ ~ R R ~

where

~ l ~~~~~ = Fl l - ~~~F for v < o

~ R~~ (w) = ~n~~ (117); u~ ~v for ee>o

~~n+1 n+1 — ~ ~ ~~ ~ ~~~~

2.4. Results for moving piston problem

The LSKUM on moving grid has been applied to 1D piston problem. We have considered both compression as well as expansion cases (corresponding to piston moving in and out).

For the compression case ($u_p > 0$), given the pressure ratio p_2/p_1, the piston velocity and the density ratio across the shock are given by the following relations [5]

~P ~ ~ ~ ~~ ~ ~ ~~ ~ aV v+' P2 ~ ~~ Y+1 ~~ l ~~~~

~~ ~ ~ Y-1~~ ~~ / ~~ +1 ~ ~~ ~~~ P1 (22)

where subscript 2 represents higher pressure side and subscript 1 represents lower pressure side, p pressure, a speed of sound and p density. Similarly for the expansion case $u_p < 0$ we have

~p ~ ~~~ / y-1 ~ ~~ ~~ ~ ~ Pl (23)

~~~ ~~ ~ ~ ~ ~~ ~~~ ~~ ~ ~~ ~ (24)

where subscript 2 represents lower pressure side and subscript 1 represents higher pressure side.

In the present computations for a given pressure ratio we calculate the piston velocity using above expressions. In the code we specify this piston velocity as input parameter and obtain the pressure jump as a part of the solution. For the compression as well as the expansion cases, the grid velocity of the first grid point is equal to the piston velocity. The grid velocity of the last grid point is equal to the shock speed (C_S) for the compression case and for the expansion case it is equal to the speed of the sound (a_{te}). The grid velocity for the interior points is then linearly interpolated between the end values. This is pictorially shown in Fig. 3.

Figs. 4 and 5 show results for the compression case with the piston velocity u_p = 469.8 m/s which corresponds to a pressure jump of 5. Figs. 4 and 5 show the density and pressure jump propagation at different time instants obtained by using first order LSKUM on a grid with 1001
~~~

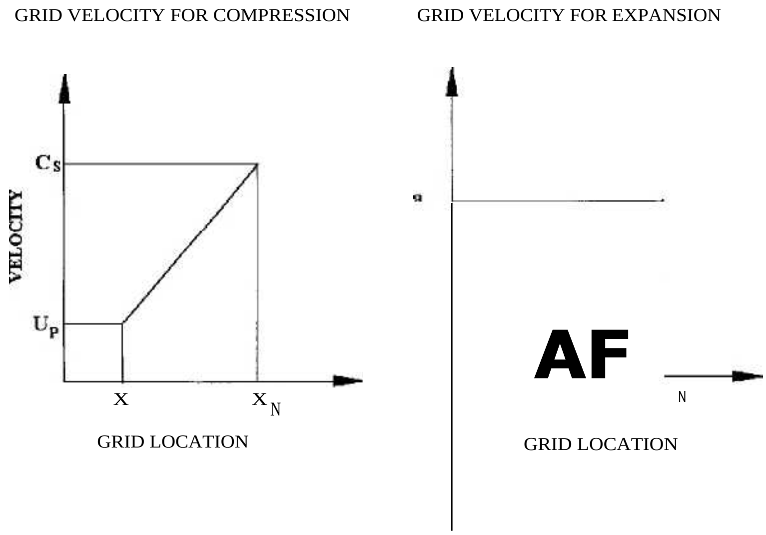

Fig. 3. Grid velocity ty interpolation for 1D piston problem.

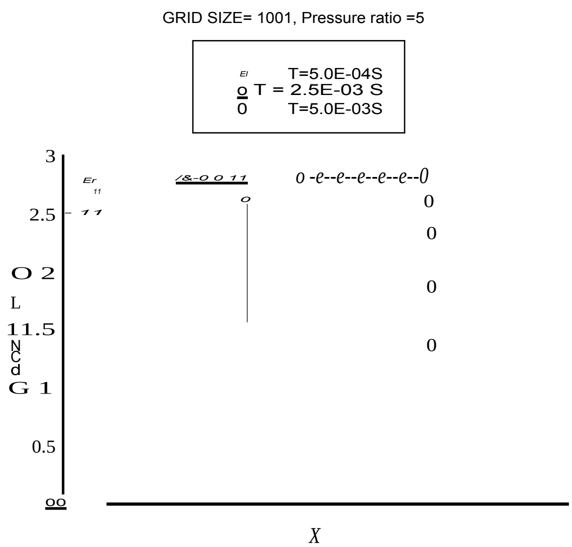

Fig. 4. Density plot for pressure ratio 5.

points. It can be seen from these plots that the moving shock has been captured very well. For this case the exact density ratio is 2.818 which is exactly reproduced by computations.

Figs. 6 and 7 show the plots for the shock propagation in terms of density and pressure jumps. These plots are for u_p = 2640.78 m/s which corresponds to a pressure jump of 100. Again we can see that LSKUM solver has successfully captured the shock propagation even for this high pressure ratio condition. The grid for this case contains 1001 points.

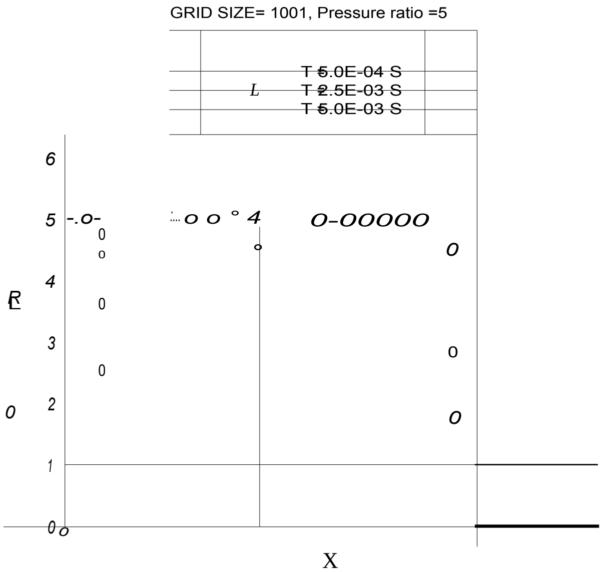

Fig. 5. Pressure plot for pressure ratio 5.

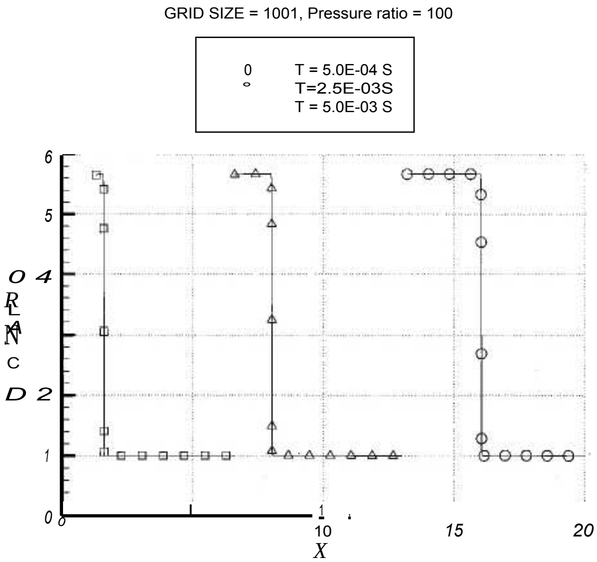

Fig. 6. Density plot for pressure ratio 100.

Fig. 8 shows a comparison of the first and second order calculations for a compression case with pressure ratio of 5, using 501 points in the grid. Obviously the second order solution captures the discontinuity more sharply. Figs. 9 and 10 show the results for the expansion case with pressure ratio of 0.02. The grid used in this case has 1001 points. It can be observed that the smooth variation of density has been captured very well.

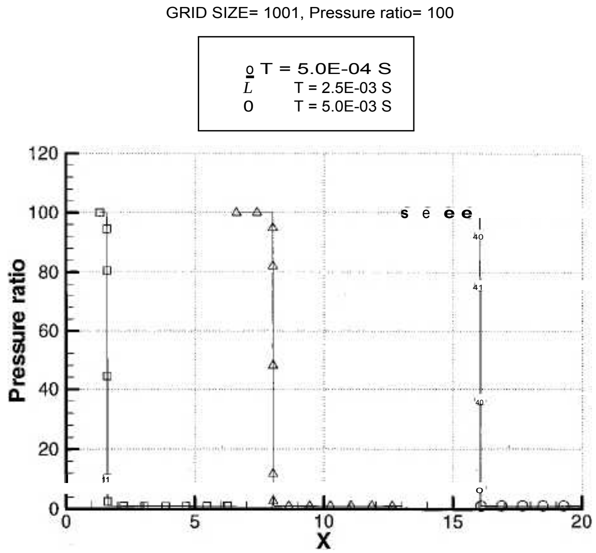

Fig. 7. Pressure plot for pressure ratio 100.

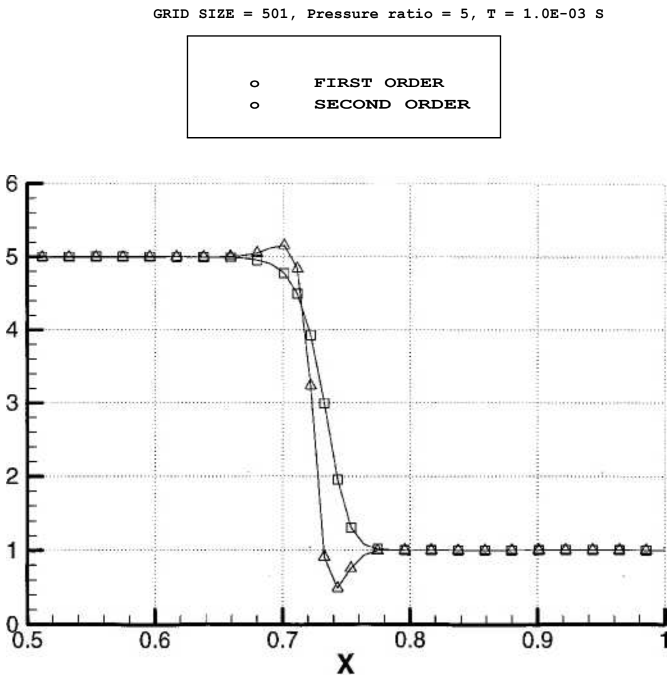

Fig. 8. Comparison of first- and second-order solution: pressure plot.

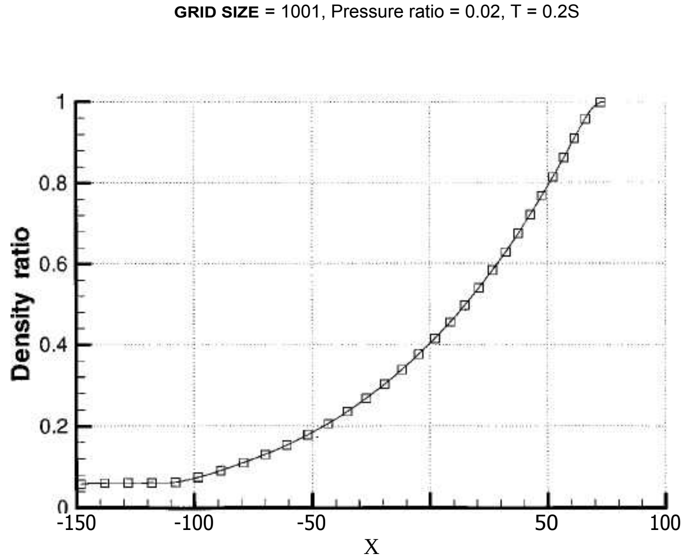

Fig. 9. Density plot for expansion case.

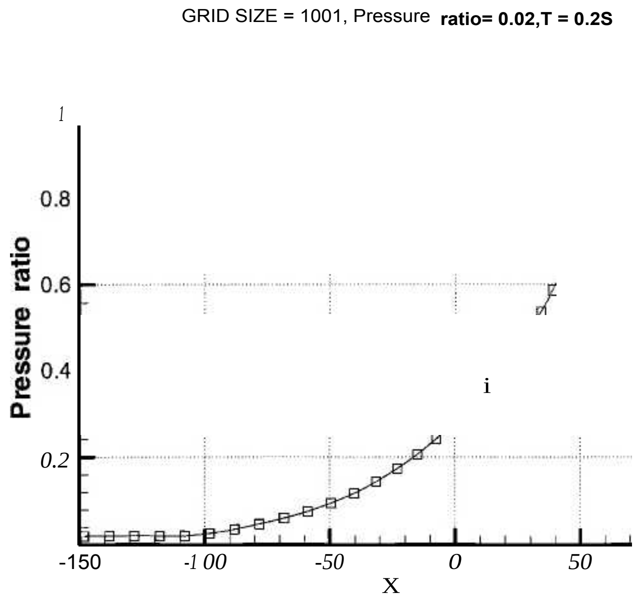

Fig. 10. Pressure plot for expansion case.

Tables 1 and 2 give a quantitative comparisons of the computations with the exact values for both compression and expansion cases. It can be observed clearly from these tables that there is excellent agreement with the exact values.

Table 1
Compression case: comparison with exact values

Pressure ratio	u_p (m/s)	$(P2/P1)_{exact}$	$(P2/P1)_{eomp.}$
5	469.8	2.818	2.818
10	2640.8	5.669	5.669

Table 2
Expansion case: comparison with exact values

Pressure ratio	u_p (m/s)	$(P2/P1)_{exact}$	$(P2/P1)_{eomp.}$
0.2	-355.4	0.316	0.316
0.02	-740.74	0.061	0.061

3. LSKUM on moving grids 2D

3.1. Formulation

Consider the 2D Boltzmann equation

Of ~~ + v, ax + v2 of ~ ~ ~~~~

where f is the velocity distribution function, v_1 and v_2 are the cartesian components of the molecular velocity. J represents a collision term which vanishes in the Euler limit, when f is a Maxwellian distribution. The Maxwellian distribution, F, in two dimension is given by

∾ ∾ () ▪ ∾∾∾ ∾ ∾ ∾∾∾∾ ∾ ∾∾∾∾ ∾ ∾∾∾∾ ∾ ∾∾∾∾ ∾ ∾∾∾∾∾ ∾∾∾∾

where /3 = 1/(2RT), and I_o is the internal energy due to non-translational degrees of freedom, 1_0 = ∾¶i~ RT and ul and u2 are the cartesian components of the fluid velocity, R is the gas constant and T is the absolute temperature of the fluid. Therefore in the Euler limit we get

OF ~~ + v1 OF ~~ + v2 ∾ ∾ ∾ a~ ∾∾∾∾

Now let w_1 and w_2 represent the cartesian components of the grid velocity of any grid point. Then we can rewrite Eq. (27) equivalently as

~ at ∾ ∾∾ ∾∾ ∾ ∾∾ ay ∾ ∾ ∾∾∾ ∾ ∾∾∾ O~~ ∾ ∾∾∾ ∾ ∾∾∾ ∾∾ ∾ ∾ ∾∾∾∾

Introduce a special derivative

~ ~ (IF at moving ~ ~ ~~ at ~ ~~ ~~ ax ~ ~~ ~~ ay

This is a special derivative following a grid point and further let $w_1 = v_1 - w_1, vv_2 = v_2 - w_2$ be the components of the particle velocity relative to the grid point. Then Eq. (28) can be written as

$$\frac{\tilde{d}F}{\tilde{\ }} \sim \sim\sim \sim\sim \sim \sim\sim \sim\sim \sim \sim$$ ~~~~

This is the 2D Boltzmann equation for a moving grid. Splitting $\tilde{v}_1$ and $\tilde{v}_2$ into positive and negative parts and discretising the time derivative to first order in the above equation, we get the update scheme for the distribution F as

$$F^{n+1} - F^{n} - \Delta t \left[\ldots \frac{\partial F}{\partial x} + \frac{v_1 - \ldots}{\sim} \frac{\partial F}{\partial x} \sim \ldots \right]^{n}$$ ~~~~

where F^{n+1} represents the updated value of F at the new position of the grid, Δt represents the time step. The updating of F is now mapped [5] to the updating of the state vector U at the Euler level by taking Ψ moments of the Eq. (30). We then obtain

$$\sim\sim\sim\sim \sim \sim\sim \sim \sim\sim \frac{\sim\sim\sim m}{\sim\sim} \sim \frac{\sim\sim\sim\sim}{\sim\sim} \sim \frac{\sim\sim\sim m}{\sim\sim} \sim \frac{\sim\sim\sim \sim}{\partial y}$$ ~~~~

where

$$\left\langle \Psi, \frac{\tilde{v}_1 + |\tilde{v}_1|}{2} F \right\rangle = GX_m^{+}$$

$$\left\langle \Psi, \frac{\tilde{v}_1 - |\tilde{v}_1|}{2} F \right\rangle \equiv GX_m^{-}$$

$$\left\langle \Psi, \frac{\tilde{v}_2 + |\tilde{v}_2|}{2} F \right\rangle = GY_m^{+}$$

$$\left\langle \Psi, \frac{\tilde{v}_2 - |\tilde{v}_2|}{2} F \right\rangle \sim \sim \sim\sim \sim$$

U is the state vector given by $U = (\rho\ \rho u_1\ \rho u_2\ \rho e)^T$, GXm and GYm represents the split fluxes for a moving grid. The moving split fluxes are given by the following expressions,

$$\sim\sim\sim = [A]G_{\sim}^{+} \tag{32}$$

where Gm represent the split fluxes for a moving grid, GS represent the split fluxes which are similar to those for a static grid except that the velocity components in this case are relative to the grid velocity and matrix $[A]$ transforms the fluxes to moving grid. The transformation matrix $[A]$ is given by

$$\begin{bmatrix} 1 & 0 & 0 \\ W_2 \;\; 0 & 1 & 0 \\ \frac{w_2^2+w_2^2}{2} \;\; w_1 & W_2 & 1 \end{bmatrix} \qquad (33)$$

The expressions for the split fluxes GS S are as follows. The x-components of GS S are given by

$$GX^{+}(3) \quad \ldots \quad i \pm uu2Bl\} \quad \ldots \quad 1\,p \quad \ldots \quad +i\,p \qquad (34)$$

The y-components of GS S are given by

$$\begin{matrix} GY^{+}(1) \\ GY^{+}(2) \\ GY^{+}(3) \\ GY^{+}(4) \end{matrix} \quad \begin{matrix} p\{uu2A2\,2 \pm B_2\} \\ p\{uuluu2A2\,2 \pm uuiB2\} \\ \ldots \\ \ldots + I\,p \ldots 2 \ldots (y+1 \ldots] \end{matrix} \qquad (35)$$

2v/1t#

2RT

3.2. Kinetic treatment of moving boundary condition for solid wall 2D

Consider a boundary point b lying on a moving surface as shown in Fig. 11. Let $w_b = w_1 i + w_2 j$ be the grid point velocity along the tangential and the normal direction x -y as shown in the figure. Following the specular reflection principle [4], the velocity distribution function at b is constructed as the union of two half Maxwellians corresponding to the incident and reflected particles, that is,

$$f_b = F_I \cup F_R \qquad (36)$$

The incident part corresponds to the particles with the relative velocity normal to the wall $w_2 < 0$. The reflected part is constructed from the incident part as shown below.

$$F_I = F_I(v_1, v_2) \quad \text{for } v_2 < 0$$

$$F_R = F_I(v_1, -v_2) \quad \text{for } w_2 > 0$$

The update formula at a boundary point b is then given by

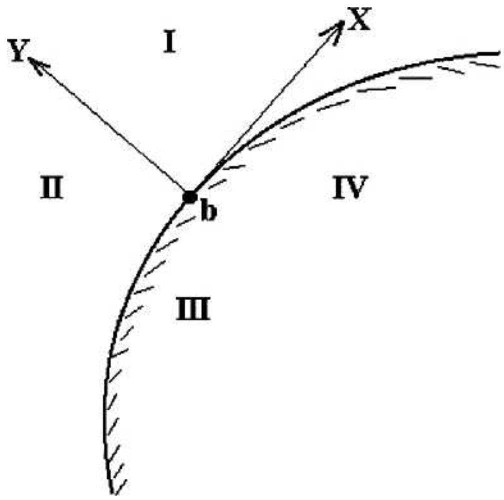

Fig. 11. A typical moving boundary.

~~~~ ~ ~~~~ ~ ~ ~~ ~ ~ ~~~ (37)

where

~ ~~~~ ~ ~~ ~ ~~ ~~~ ~~~~~ ~~ ~~~ ~ :~~ ~~ ~~ ~~ ~ v~ ~~~ ~~ ~ ~

~ ~~ = ~ ~ ~~~ (~v1 -~v2) for $v_2 > 0$

~b~~ ~~~ ~ ~b~~~ ~ ~~~ ~ ~~.+ ~~~ ~ ~~ aX ~ ~~ aY ~ ~~ a ~~~ ~ ~ ~~ ~~ ~

=0 for l=3

where Δt is the time step, ~~m~ and ~~m~ represent the x and y quadrant split moving fluxes respectively in the first and second quadrants. For example ~~~ ~ represents x quadrant split fluxes in the first quadrant, i.e. for $v_1 < 0$, $v_2 < 0$. Similarly ~~m~~ ~~m~ and ~~~ ~ represents x quadrant split fluxes in the second quadrant, third and fourth quadrant respectively.

The moving boundary fluxes which are expressed in terms of the quadrant wise split fluxes are given by the following expressions,

~m~ = ~~~~~S~ ~~~~

where ~m~ represents the quadrant wise split fluxes for a moving grid, ~S " represents the quadrant wise split fluxes which are similar to that of a static grid except that the velocity components in this case are relative to the grid velocity and matrix $[A_b]$ converts the fluxes to moving grid. The small difference in $[A_b]$ compared to the $[A]$ for interior points is, the element $a_b(3, 3)$ of the matrix $[A_b]$ is zero. The transformation matrix $[A_b]$ is given by
~~~~

$$\begin{bmatrix} \sim & \sim & \sim & \sim \\ \sim & 1 & 0 & 0 \\ w_2 & 0 & 0 & 0 \\ \sim & w_1 & w_2 & 1 \end{bmatrix} \tag{39}$$

The x-component of the quadrant split fluxes are given by,

$$\ldots u \ldots 2 \ldots \gamma+1 \; p \ldots \tag{40}$$

The y-component of the quadrant split fluxes are given by,

$$\ldots 2 \; p \ldots 2 \ldots u \ldots 2 \ldots \gamma+1 \; p \ldots \tag{41}$$

3.3. Kinetic treatment of boundary condition on moving outer boundary

The treatment of the boundary condition for moving outer boundary [6] is very much similar to the above analysis, which in turn is very much similar to the kinetic outer boundary condition developed by Ramesh et al. [7] for stationary grids. As before, the velocity distribution at any point b on the moving outer boundary is constructed as the union of two half Maxwellians corresponding to incoming and outgoing particles. The velocity distribution corresponding to the incoming particles is constructed from the freestream conditions and that for the outgoing particles is obtained from the interior points. Therefore for the union of the two velocity distributions we have,

$$f^{n+1} \ldots_{\text{out}} \tag{42}$$

where

$$\ldots = F^{n+1} \quad \text{for } v_2 < 0$$

$$\ldots \quad V \ldots$$

Taking Y' moments of Eq. (42) we get the update formula for the outer boundary points as,

~~~~ ~ ~~ ~ ~~ ~~~m~ ~ ~~~m~ ~ ~~~m ~ ~~~m ~ aY ~~~~

where Δt is the time step, ~~~,~ and ~~ ~ ~ represent the quadrant split moving fluxes in the third and fourth quadrants respectively. The expressions are already given in previous section. The term U;;, in Eq. (43) is function of $U_{\sim}$ and U_{bb} given by the expression,

~~~~ ~ ~~~~~ ~ ~ ~~ 2 ~~~~~~ ~ ~ ~ ~~~~~ z ~~~~ ~ ~ ~ ~~~~~ ~ ~~~ ~ ~ ~ ~ ~ ~~~~~~ ~ ~~~ ~ ~~~~ ~ ~~ ~ ~ 2 ~ ~ ~ ~~ z ~ 2 ~~~~ ~~~~

~~~~~ ~~~~~~~~~ ~ ~~~~~~ ~~ ~~~~~~~~~~ ~~~~~~~~~ ~~~ ~ ~ ~~~~~ ~ ~~ ~ ~~ ~~~~~ ~ ~~~~~~

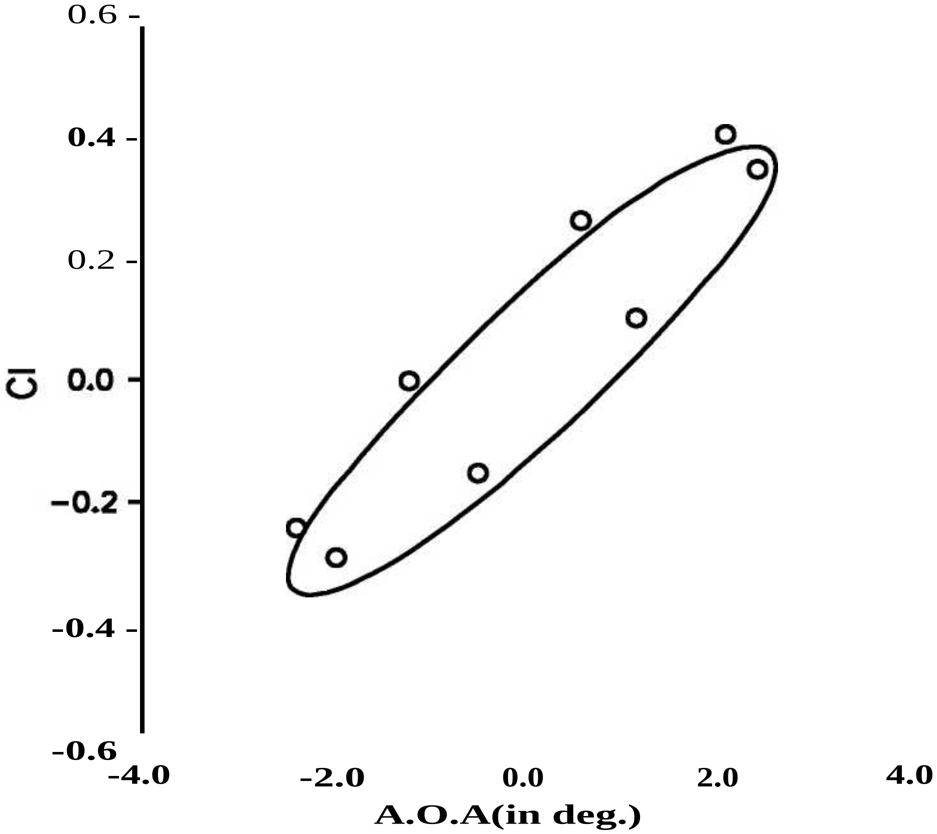

Fig. 12. Instantaneous lift coefficient versus angle of attack - comparison with experiment.
~~~~~

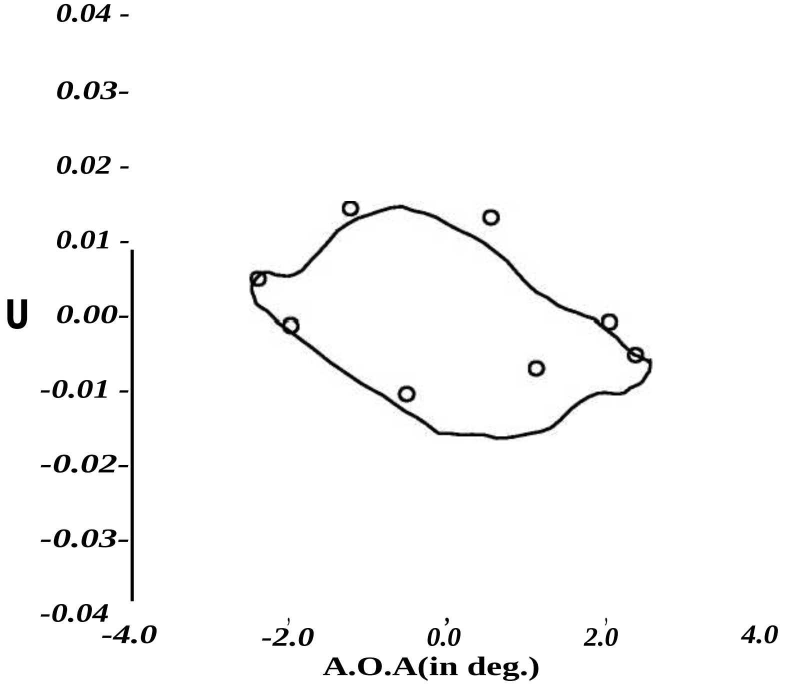

Fig. 13. Instantaneous pitching moment coefficient versus angle of attack - comparison with experiment.

3.4. Results and discussions for 2D problem

The present method, that is, LSKUM on moving grid has been applied to computation of unsteady flow past an airfoil undergoing pitching oscillations with the hinge at quarter chord point of the airfoil. The airfoil chosen is NACA 0012. This is a standard AGARD [8] test case and has been used by many investigators for checking their numerical algorithms. The oscillation cycle is defined by,

$$a = a_m + oa_o \sin(wt) \quad \text{where } a_m = 0.016^\circ \; oa_o = 2.51^\circ.$$

Reduced frequency based on chord length c of the airfoil is $k = we/2U_{,,,,}$ = 0.0814, where w is the circular pitch frequency and U,,,,, is free stream fluid velocity. The freestream Mach number for this test case is 0.755. An unstructured grid with 4074 points has been used. It has 160 points on the airfoil and 40 points on the farfield boundary. Farfield boundary is at 10 chords distance. The necessary connectivity information was generated using quad-tree based search algorithm [9]. There are many ways of moving the grid. One simple method employed in the present work is pitching up and down of the whole grid along with the airfoil. For such a grid movement it is very easy to calculate the grid velocity. Figs. 12 and 13 shows the comparison of computations with experiment of AGARD test case [6], the lift coefficient C C and pitching moment coefficient C_m (about quarter chord) versus instantaneous angle of attack a have been considered for such a comparison. Fig. 14 shows instantaneous C_p plots for various angles of attack. Fig. 15 shows

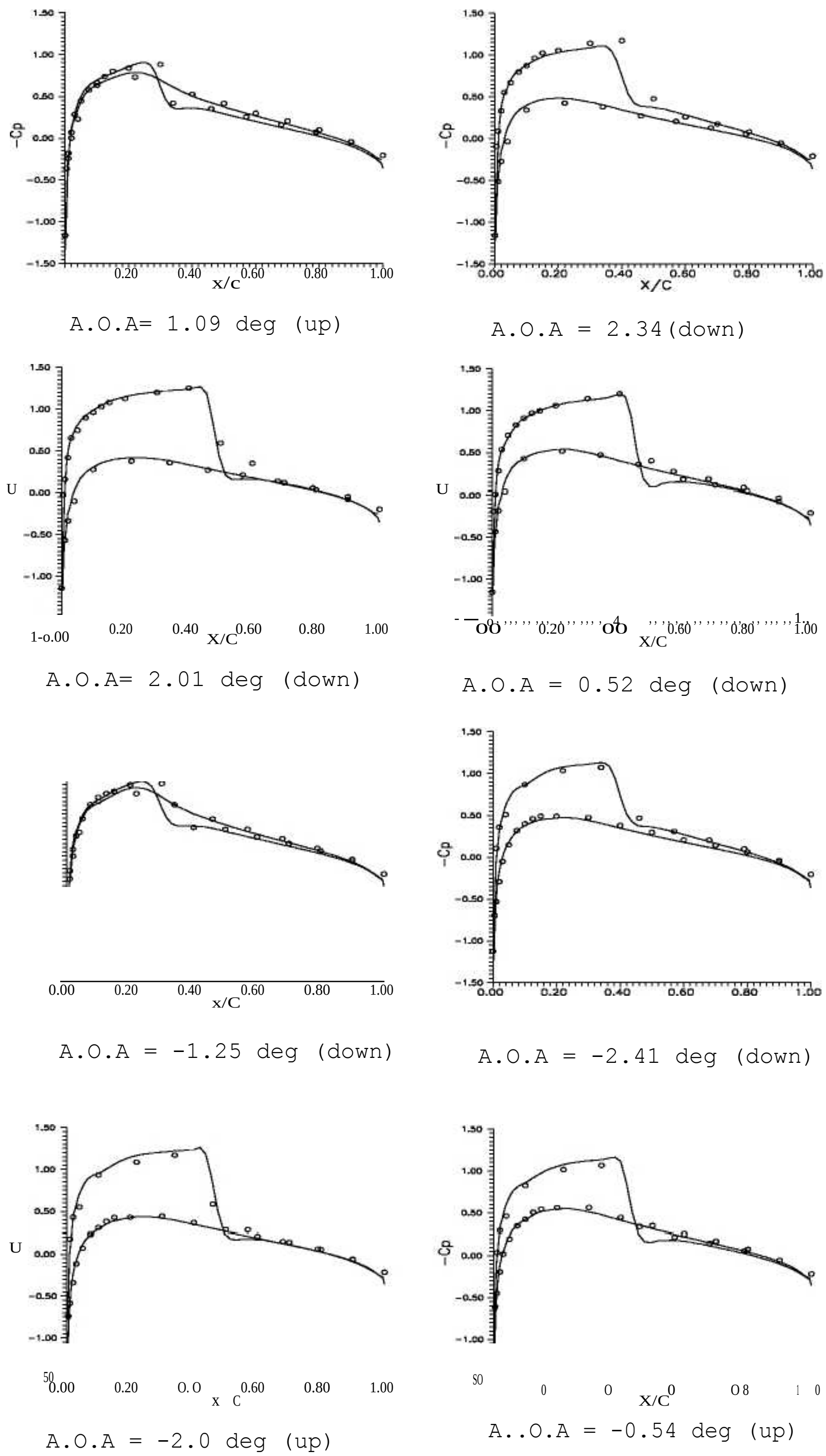

Fig. 14. Instantaneous pressure coefficient plots.

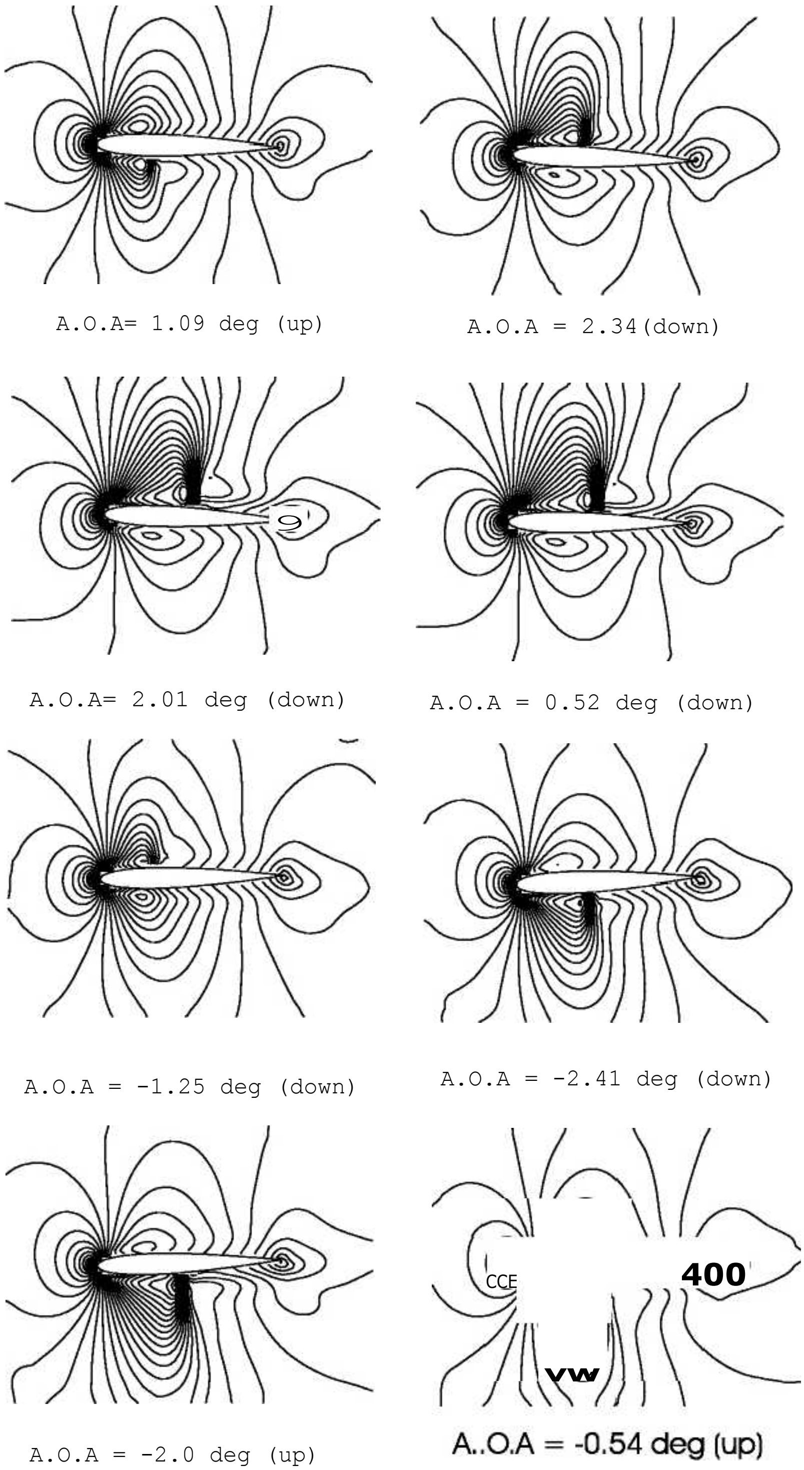

Fig. 15. Instantaneous Mach contour plots.

instantaneous Mach contours for various angles of attack clearly showing the movement of the shock at different time instants characterising the unsteady behaviour of the flow.

4. Conclusions

The LSKUM first developed by Ghosh and Deshpande [2] and later on further developed by Ramesh [9] has been extended in a novel way to problems involving moving grid. This method called LSKUM-MG is based on introduction of special derivative following a grid point and therefore reduces to the usual Lagrangian formulation when grid point velocity equals fluid velocity. The KSPH method developed by Manoj et al. [3] can be considered as a special case of LSKUM-MG. The present LSKUM-MG solver has been applied to the 1D moving piston problem and accurate results have been obtained. Also the 2D LSKUM-MG has been used successfully to compute the unsteady flow past airfoil oscillating in pitch. Treatment of boundary conditions on moving walls has also been developed within the kinetic framework. The computed results compare quite well with the experimental results of the AGARD [8] test case. Application of the LSKUM-MG developed in the present work to store separation problem involving time dependent chimera meshes is a very challenging problem with a lot of potential.

References

[1] Boniface J-C, Guillen Ph, Le Pape M-C, Darracq D, Beaumier P. Development of a chimera unsteady method for the numerical simulation of rotorcraft flowfields. ONERA TP 1998-4, also AIAA-98-0421.

[2] Ghosh AK, Deshpande SM. Least squares kinetic upwind method for inviscid compressible flows. AIAA Paper no. 95-1735, 1995.

[3] Manoj Kumar B, Raghurama Rao SV, Deshpande SM. Kinetic smooth particle hydrodynamics method using peculiar velocity based upwinding and least squares. Fluid Mechanics Report 98 FM 2, Centre of Excellence in Aerospace CFD, Department of Aerospace Engineering, Indian Institute of Science, Bangalore, 1998.

[4] Mandal JC, Deshpande SM. Kinetic flux vector splitting for Euler equations. Computers Fluids 1994;23:447-78.

[5] Liepmann HW, Roshko A. Elements of gas dynamics. Wiley, London, 1956.

[6] Jain Sachin B. Least squares kinetic upwind method on moving grids for unsteady Euler computation. ME Project Thesis, Department of Aerospace Engineering, IISc, Bangalore, India, January 2000.

[7] Ramesh V, Mathur JS, Deshpande SM. Kinetic treatment of the far-field boundary condition. Fluid Mechanics Report 97 FM 2, Centre of Excellence in Aerospace CFD, Department of Aerospace Engineering, Indian Institute of Science, Bangalore, April 1997.

[8] AGARD-R-702. Compendium of unsteady aerodynamic measurements, August 1982.

[9] Ramesh V. PhD Thesis under preparation, Department of Aerospace Engineering, IISc, Bangalore, India.