Design of Truss-Structures for Minimum Weight using Genetic Algorithms

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Abstract

Optimization of truss-structures for finding optimal cross-sectional size, topology, and configuration of 2-D and 3-D trusses to achieve minimum weight is carried out using real-coded genetic algorithms (GAs). All the above three optimization techniques have been made possible by using a novel representation scheme. Although the proposed GA uses a fixed-length vector of design variables representing member areas and change in nodal coordinates, a simple member exclusion principle is introduced to obtain differing topologies. Moreover, practical considerations, such as inclusion of important nodes in the optimized structure is taken care of by using a concept of basic and non-basic nodes. Stress, deflection, and kinematic stability considerations are also handled using constraints. In a number of 2-D and 3-D trusses, the proposed technique finds intuitively optimal or near-optimal trusses, which are also found to have smaller weight than those that are reported in the literature.

1 Introduction

Optimal design of truss-structures has always been an active area of research in the field of search and optimization. Various techniques based on classical optimization methods have been developed to find optimal truss-structures (Dorn, Gomory, and Greenberg, 1964; Haug and Arora, 1989; Krish, 1989; Ringertz, 1985; Topping, 1983; Vanderplaat and Moses, 1972). However, most of these techniques can be classified into three main categories: (i) Sizing, (ii) Configuration, and (iii) Topology optimization.

In the sizing optimization of trusses, cross-sectional areas of members are considered as design variables and the coordinates of the nodes and connectivity among various members are considered to be fixed (Goldberg and Samtani, 1986). The resulting optimization problem is a nonlinear programming (NLP) problem. The sizing optimization problem is extended and made practically useful by restricting the member cross-sectional areas to take only certain pre-specified discrete values (Rajeev and Krishnamoorthy, 1992).

In the configuration optimization of trusses, the change in nodal coordinates are kept as design variables (Imai and Schmit, 1981). In most studies, simultaneous optimization of sizing and configuration has been used. The resulting problem is also a NLP problem with member area and change in nodal coordinates as variables.

In the topology optimization, the connectivity of members in a truss is to be determined (Krish, 1989; Ringertz, 1985). Classical optimization methods have not been used adequately in topology optimization, simply because they lack efficient ways to represent connectivity of members.

Although the above three optimization problems are discussed separately, the most efficient way to design truss-structures optimally is to consider all three optimization methods simultaneously. In most attempts, multi-level optimization methods have been used (Dobbs and Felton, 1969; Ringertz, 1985). In such a method, when topology optimization is performed, member areas and the truss configuration are assumed to be fixed. Once an optimized topology is found, the member areas and/or configuration of the obtained topology are optimized. It is obvious that such a multi-level optimization technique may not always provide the globally best design, since both these problems are not linearly separable.

Genetic algorithms (GAs) have also been in all the above three optimization problems. Goldberg and Samtani (1986) and Rajeev and Krishnamoorthy (1992) have used only size optimization, whereas Hejela, Lee, and Lin (1993) have used a two-level optimization scheme of first finding multiple optimal topologies and then finding the optimal member areas for each of the truss topologies. Rajan (1995) has used all three optimization methods to only two 2-D truss design problems. Member connectivity is coded by using boolean variables (`1` for presence and `0` for absence). Member areas and change in nodal displacements are used separately as variables. Since a separate binary string is used to denote presence and absence of all members, the representation scheme is highly sensitive to this binary string, thereby introducing artificial nonlinearity in the optimization problem.

In this paper, we have used a representation scheme which naturally allows all three optimizations to be used simultaneously. Moreover, the representation scheme also allows a canonical real-coded genetic algorithm (GA) to be used directly. In order to make the solutions practically useful, a concept of basic and non-basic nodes is introduced, which emphasizes important nodes to be present in the optimized solution. Stress, deflection, and kinematic stability considerations are also added as constraints to find functionally useful trusses. The proposed technique is applied to a number of truss-structure design problems. The optimized trusses are compared with that reported in the literature.

2 Proposed Methodology

In a truss-structure design, certain nodes are important and must exist in any feasible design. And certain nodes are added for load sharing and are optional. The important nodes are usually the ones which carry a load (a force) or which support the truss. These information are usually specified by the user (or designer) and must be honored in the design process of a truss-structure. These important nodes are named as the *basic* nodes, in this paper. On the other hand, the optional nodes are sometimes used in a truss to help distribute the stresses better on individual members. These nodes are named as *non-basic* nodes. Thus, the objective in a truss-structure design, (with all three design optimization methods) is to find which optional nodes are necessary in a truss, what coordinates of these optional nodes and which members must be present so that a goal (often, the weight of the truss) is optimized by satisfying certain constraints (often, the stresses in members and displacements of nodes).

The proposed algorithm assumes a *ground* structure, which is a complete truss with all possible member connections among all nodes (basic or non-basic) in the structure. Thus, in a truss having n nodes, there are a total of $m = \binom{n}{2}$ different members possible. A ground structure[1] is a collection of all these members.

A truss in the proposed GA is represented by specifying a cross-sectional area for each member in the ground structure. Thus, a solution represented in the GA population is a vector of m real numbers within two specified limits. Although every solution in a GA population will have m cross-sectional areas, its phenotype (the truss itself) may not have all m members. The presence or absence of a member in the ground structure is determined by comparing the cross-sectional area of the member with a user-defined small critical cross-sectional area, ϵ. If an area is smaller than ϵ, that member is assumed to be

[1] It is absolutely not necessary to have all $\binom{n}{2}$ members in a ground structure. Problem knowledge can be used to discard some members in the ground structure. We shall show later how a good choice of a ground structure aids in finding better trusses using a GA.

absent in the realized truss. This is how trusses with differing topologies can be obtained with a fixed-length representation of the truss member areas. This representation scheme has another advantage. Since member areas are directly used, the values higher than ϵ specify the actual member cross-sectional area. It is interesting to note that the critical area ϵ and lower and upper bounds on the cross-sectional areas must be so selected that, although working in the range $(\boldsymbol{A}^{\min}, \boldsymbol{A}^{\max})$, there is an adequate probability of making a unwanted member absent in a solution. We have chosen $\boldsymbol{A}^{\min} = -\boldsymbol{A}^{\max}$ and a small positive value for ϵ, so that there is almost an equal probability of any member being present or absent in a truss. If a member is absolutely essential in a truss, the genetic operators quickly make the corresponding member area in all solutions in a population positive. This reduces the chance of making the member absent in children trusses.

In subsequent discussions, we denote m as the number of members present in a realized truss and not the total number of members in the ground structure, for clarity.

With the above discussion, we now present the formulation of the truss-structure optimization problem as a nonlinear programming (NLP) problem:

$$
\begin{array}{rlll}
\text{Minimize} & f(\boldsymbol{A}) = \sum_{j=1}^{m} \rho_j \ell_j \boldsymbol{A}_j & & \\
\text{Subject to} & G1 \equiv \text{Truss is acceptable to the user} & & \\
 & G2 \equiv \text{Truss is kinematically stable} & & \\
 & G3 \equiv S_j - \sigma_j(\boldsymbol{A}, \boldsymbol{\xi}) \geq 0 & j = 1, 2, \ldots, m & \\
 & G4 \equiv \delta_k^{\max} - \delta_k(\boldsymbol{A}, \boldsymbol{\xi}) \geq 0 & k = 1, 2, \ldots, n & \\
 & G5 \equiv \boldsymbol{A}_i^{\min} \leq \boldsymbol{A}_i \leq \boldsymbol{A}_i^{\max} & i = 1, 2, \ldots, m & \\
 & G6 \equiv \boldsymbol{\xi}_i^{\min} \leq \boldsymbol{\xi}_i \leq \boldsymbol{\xi}_i^{\max} & i = 1, 2, \ldots, n' &
\end{array}
\tag{1}
$$

In the above NLP problem, the design variables are the cross-sectional areas of members present in a truss (denoted as $\boldsymbol{A}$) and the coordinates of all n' non-basic nodes (denoted as $\boldsymbol{\xi}$). The parameters S_j and $\delta_k^{\max}$ are the allowable strength of the j-th member and the allowable deflection of the k-th node, respectively. We describe each of the above terms in the following:

Objective function: In this paper, we have considered the weight of the overall truss as the objective function, whereas other criteria such as reliability and dynamic characteristics can also be considered. The parameter ρ_j and ℓ_j are the material density and length of j-th member, respectively.

Constraint G1: The user specifies the location and the number of basic nodes for supports and loads. Thus, a feasible truss must have all the basic nodes. This constraint is checked first. If any one of the basic nodes is absent in the truss, a large constant penalty is assigned to the solution and no further calculation of objective function or constraints is done.

Constraints G2: Since trusses with different topologies are created by genetic operators, trusses which are not kinematically stable can also be generated. Trusses must be kinematically stable so that it does not generate into a mechanism. One of the ways to check the kinematic stability of a truss is to check the positive-definiteness of the stiffness matrix created from the member connectivities. If the matrix is positive-definite, the truss is kinematically stable. However, the computation of the positive-definiteness of a matrix of a reasonable size is enormous. We reduce the frequency of such computations by first checking the Grubler's criterion (Ghosh and Mallik, 1988):

$$\text{Degree-of-freedom (D-O-F)} = 2n - m - n_l, \tag{2}$$

where n_l is the number of degrees-of-freedom lost at the support nodes. If the D-O-F is non-positive, the corresponding truss is a not a mechanism. Since a truss has to be a non-mechanism, we first check the above simple criterion. If the truss is a mechanism, we penalize the solution by assigning a large value which is proportional to the D-O-F obtained by the above equation. Thereafter, the corresponding truss is not sent to FEM routine for further calculations such as stiffness matrix, stresses and displacements. If the truss is not a mechanism, we then sent the truss to the FEM routine and check the positive-definiteness of the stiffness matrix. If the matrix is not positive-definite, a large penalty proportional to the violation of positive-definiteness is assigned to the solution and no further calculation of stress or deflection is made.

Constraints G3: In a feasible truss, all members must have stresses within the allowable strength of the material. Since, usually a truss is subjected to a number of different loading conditions applied separately, these constraints must be used for each loading condition. Since the trusses of various topologies are created on the fly, some of them may be statically determinate and some of them may be statically indeterminate. Thus, we have used a finite element method (FEM) to calculate the stresses and deflection in a truss. It is also noteworthy that since each truss is different in its topology, the members and nodes of the truss is needed to be automatically numbered before calling a FEM routine. FEM procedure is developed for 2D as well as 3D trusses. A suitable automatic node numbering scheme is also developed.

In order to have significant effect of all constraints, we normalize all constraints shown above in the following manner so that all constraint violations get equal importance:

$$G1 \equiv \frac{S_j}{\sigma_j(\boldsymbol{A}, \boldsymbol{\xi})} - 1 \geq 0. \tag{3}$$

In the case of any constraint violation (that is, if $S_j \leq \sigma_j(\boldsymbol{A}, \boldsymbol{\xi})$), a bracket-operator penalty term (Deb, 1995; Rao, 1984) is added to the objective function.

Constraints G4: Like in Constraints G1, all nodes (basic or non-basic) in the truss must not deflect more than the allowable limit due to the application of loads. Like Constraints G1, these constraints are also normalized and the constraint violation (if any) is added to the objective function by using a bracket-operator penalty term.

Constraints G5 and G6: Since real-coded GA allows the variables to be bounded within specified limits, these constraints will be automatically satisfied.

The fitness of a solution is dependent on the constraint violations and thus calculated as follows:

$$F(\boldsymbol{A}, \boldsymbol{\xi}) = \begin{cases} 10^9, \quad \text{if G1 is violated}, \\ 10^8(\text{constraint violation}), \\ \qquad \text{if G2 is violated with D-O-F constraint}, \\ 10^7(\text{constraint violation}), \\ \qquad \text{if G2 is violated with} \\ \qquad \text{positive-definiteness constraint}, \\ f(\boldsymbol{A}, \boldsymbol{\xi}) + 10^5 \sum_{j=1}^{m} |\langle \text{G3}_j \rangle| \\ \qquad +10^5 \sum_{k=1}^{n} |\langle \text{G4}_k \rangle| \\ \qquad \text{otherwise}. \end{cases} \tag{4}$$

In the above expression, the operator $\langle \ \rangle$ is the bracket-operator penalty term.

3 Proposed Optimization Algorithm

Since the design variables take any real number, a real-coded genetic algorithm is used in this study. In a real-coded GA, variables are not coded in binary strings (Goldberg, 1989), instead GA operators are directly applied on real numbers. Although any selection operator can be used, specialized crossover and mutation operators must be used to effectively create children solutions from parent solutions. In the study here, we use simulated binary crossover (SBX) and a parameter-based mutation operator (Deb and Agrawal, 1995). We describe these operators in the following subsections.

3.1 Simulated Binary Crossover (SBX)

A probability distribution is used around parent solutions to create two children solutions. In the proposed SBX operator, this probability distribution is not chosen arbitrarily. Instead, such a probability distribution is first calculated for single-point crossover operator in binary-coded GAs and then adapted for real-parameter GAs. The detailed analysis can be found elsewhere (Deb and Agrawal, 1995). To make this distribution independent of parent solutions, we derived the probability distribution as a function of a non-dimensionalized parameter:

$$\beta = \left| \frac{y^{(2)} - y^{(1)}}{x^{(2)} - x^{(1)}} \right|, \tag{5}$$

where $y^{(1)}$ and $y^{(2)}$ are children solutions and $x^{(2)}$ and $x^{(1)}$ are parent solutions. The chosen probability distribution is as follows:

$$P(\beta) = \begin{cases} 0.5(\eta_c + 1)\beta^{\eta_c}, & \text{if } \beta \leq 1; \\ 0.5(\eta_c + 1)/\beta^{\eta_c + 2}, & \text{otherwise,} \end{cases} \tag{6}$$

where η_c is a parameter which controls the extent of spread in children solutions. A small value of η_c allows solutions far away from parents to be created as children solutions and a large value restricts only near-parent solutions to be created as children solutions. The following two observations are found in crossover operators used in binary-coded GAs:

1. The mean decoded parameter value of two parent strings is invariant among resulting children strings, and

2. If the crossover is applied between two children strings at the same cross site as was used to create the children strings, the same parent strings will result.

The above probability distribution preserves both these observations by keeping the average of the parent and children solutions the same and by assigning equal overall probability for creating solutions inside and outside the region enclosed by parent solutions. In short, the implication of this crossover is that near-parent solutions are more likely to be created than solutions far away from parents. Although this property in a crossover operator is intuitively a good property, it has been shown elsewhere (Deb and Agrawal, 1995) that this operator respects *interval* schema processing, an important matter in the successful working of any real-parameter GA.

The procedure of computing children solutions $y^{(1)}$ and $y^{(2)}$ from two parent solutions $x^{(1)}$ and $x^{(2)}$ are as follows:

1. Create a random number u between 0 and 1.

2. Find a parameter $\bar{\beta}$ using the polynomial probability distribution (equation 6), developed in Deb and Agrawal (1995) from a schema processing point of view, as follows:

$$\bar{\beta} = \begin{cases} (2u)^{\frac{1}{\eta_c+1}}, & \text{if } u \leq 0.5, \\ \left(\frac{1}{2(1-u)}\right)^{\frac{1}{\eta_c+1}}, & \text{otherwise.} \end{cases} \tag{7}$$

3. The children solutions are then calculated as follows:

$$
\begin{aligned}
y^{(1)} &= 0.5\left[(x^{(1)}+x^{(2)}) - \bar{\beta}|x^{(2)}-x^{(1)}|\right], \\
y^{(2)} &= 0.5\left[(x^{(1)}+x^{(2)}) + \bar{\beta}|x^{(2)}-x^{(1)}|\right].
\end{aligned}
$$

The above procedure is used for variables where no lower and upper bounds are specified. Thus, the children solutions can lie anywhere in the real space $[-\infty, \infty]$ with varying probability. For calculating the children solutions where lower and upper bounds (x^l and x^u) of a variable are specified, equation 7 needs to be changed as follows:

$$
\bar{\beta} = \begin{cases} (\alpha u)^{\frac{1}{\eta_c+1}}, & \text{if } u \le \frac{1}{\alpha}, \\ \left(\frac{1}{2-\alpha u}\right)^{\frac{1}{\eta_c+1}}, & \text{otherwise,} \end{cases} \tag{8}
$$

where $\alpha = 2 - \beta^{-(\eta_c+1)}$ and β is calculated as follows:

$$
\beta = 1 + \frac{2}{y^{(2)} - y^{(1)}} \min[(x^{(1)} - x^l),\ (x^u - x^{(2)})].
$$

It is assumed here that $x^{(1)} < x^{(2)}$. A simple modification to the above equation can be made for $x^{(1)} > x^{(2)}$. The above procedure allows a zero probability of creating any children solution outside the prescribed range $[x^l, x^u]$. It is intuitive that equation 8 reduces to equation 7 for $x^l = -\infty$ and $x^u = \infty$.

For handling multiple variables, each variable is chosen with a probability 0.5 in this study and the above SBX operator is applied variable-by-variable. This way about half of the variables get crossed over under the SBX operator. SBX operator can also be applied once on a line joining the two parents. In all simulation results here, we have used $\eta_c = 2$.

3.2 Parameter-based Mutation Operator

A polynomial probability distribution is used to create a solution y in the vicinity of a parent solution x (Deb, 1997). The following procedure is used for variables where lower and upper boundaries are not specified:

1. Create a random number u between 0 and 1.

2. Calculate the parameter $\bar{\delta}$ as follows:

$$
\bar{\delta} = \begin{cases} (2u)^{\frac{1}{\eta_m+1}} - 1, & \text{if } u \le 0.5, \\ 1 - [2(1-u)]^{\frac{1}{\eta_m+1}}, & \text{otherwise,} \end{cases} \tag{9}
$$

   where η_m is the distribution index for mutation and takes any non-negative value.

3. Calculate the mutated child as follows:

$$
y = x + \bar{\delta}\Delta_{\max},
$$

   where $\Delta_{\max}$ is the maximum perturbance allowed in the parent solution.

For variables where lower and upper boundaries (x^l and x^u) are specified, above equation may be changed as follows:

$$
\bar{\delta} = \begin{cases} \left[2u + (1-2u)(1-\delta)^{\eta_m+1}\right]^{\frac{1}{\eta_m+1}} - 1, & \text{if } u \le 0.5, \\ 1 - \left[2(1-u) + 2(u-0.5)(1-\delta)^{\eta_m+1}\right]^{\frac{1}{\eta_m+1}}, & \text{otherwise,} \end{cases} \tag{10}
$$

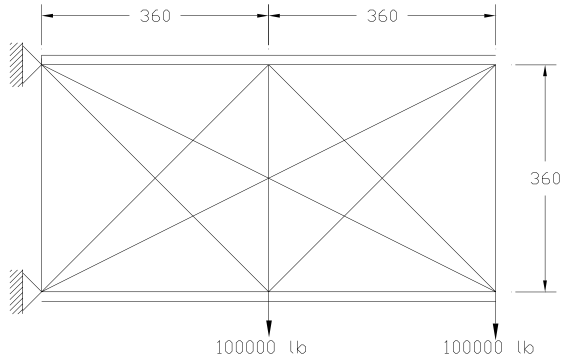

Figure 1: The 15-member, six-node ground structure.

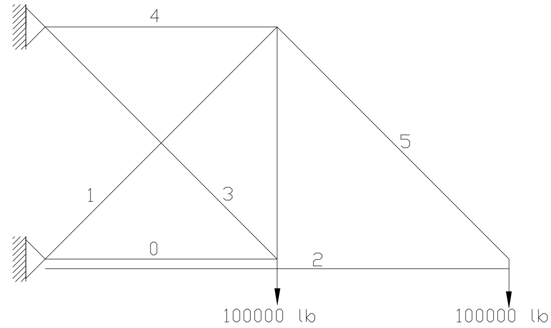

Figure 2: Optimized truss obtained from 15-member, six-node ground structure using population sizes 300 and 450.

where $\delta = \min[(x - x^l),\ (x^u - x)]/(x^u - x^l)$. This ensures that no solution would be created outside the range [x^l, x^u]. In this case, we set $\Delta_{\max} = x^u - x^l$. Equation 10 reduces to equation 9 for $x^l = -\infty$ and $x^u = \infty$.

Using above equations, we can calculate the expected normalized perturbance ($(y - x)/(x^u - x^l)$) of the mutated solutions in both positive and negative sides separately. We observe that this value is $O(1/\eta_m)$. Thus, in order to get a mutation effect of 1% perturbance in solutions, we should set $\eta_m \approx 100$.

We terminate a GA simulation when a pre-specified number of generations is elapsed.

4 Results

In all simulations presented in this section, we have used a crossover probability of 0.9 and a mutation probability of 0.1. The population size used in a simulation is dependent on the number of members in the ground structure. Since an initial random population is always used, it is expected that the required population size would depend on the problem complexity. It is intuitive that in truss-structure design problems, as the number of members in the ground structure increases there exist many different topologies with almost equal overall weight. This suggests that with the increase in members in the ground structure, the resulting NLP problem becomes multi-modal and hence a large population size is necessary to find optimal or near-optimal solutions.

In the following, we discuss the performance of GAs in solving various 2-D and 3-D truss structures and compare the obtained solutions with the best solutions available in the literature. In all figures showing trusses, the dimensions are in inches.

4.1 15-Member, Six-Node Truss

With six nodes, there could be a maximum of $\binom{6}{2}$ or 15 members possible. This 15-member, six-node truss (we called the ground structure) and the loading are shown in Figure 1. For clarity, the overlapping members are shown with a gap in the figure. Following design parameters are used:

Young's modulus	=	10^4 Ksi
Density (ρ)	=	0.1 lb/in^3
Allowable compressive strength	=	25 Ksi
Allowable tensile strength	=	25 Ksi
Allowable displacement	=	2 in
$\boldsymbol{A}^{\min}, \boldsymbol{A}^{\max}$	=	$-35.0, 35.0$ in^2
Critical area (ϵ)	=	0.09 in^2

All 15 cross-sectional areas are used as variables. The corresponding optimized truss obtained using two population sizes of 300 and 450 have the same topology and is shown in Figure 2. The figure shows that only 7 members (out of 15) are necessary in the optimized truss. Although the same topology is obtained for two different GA simulation runs with different population sizes, the cross-sectional areas are a little different (Table 1). The run with larger population size has been able to find a better truss (with an overall weight of 4731.650 lbs.). In this case, the population at 1st generation has the best solution with an overall weight equal to 9285.18 lbs. The above optimized solution of an weight of 4731.650 lbs has been obtained at generation 189.

Table 1: Results of 2D, six-node truss with 15-member ground structure.

Member number (refer Figure 2)	Area of members (in^2)	
	Population Size 300	Population Size 450
0	05.172	05.219
1	20.054	20.310
2	14.845	14.593
3	07.821	07.772
4	28.286	28.187
5	20.446	20.650
Weight of Truss (lb)	4733.443	4731.650

In order to show that the solution obtained using population size 450 is a likely optimal solution, we tabulate the stresses in all members and displacements in all nodes in Table 2. It is clear from the table

Table 2: Stresses and displacements obtained for the optimized solution with population size 450.

Member No.	Stress (Ksi)
0	-19.161
1	-06.962
2	-06.852
3	18.194
4	07.095
5	06.865

Node number	Displacement (in)	
	x	y
2	-0.690	-2.000
3	0.255	-0.757
4	-0.493	-2.000

that although stresses developed in all members are within the allowable strength of (25 Ksi), the solution lies on the intersection of two critical displacement constraints. Since, a maximum absolute displacement of 2 inch is allowed in any node in any direction, both nodes 2 and 4 achieve this limit. In other words, the above calculations suggest that if the weight of the overall truss is attempted to reduce by making member cross-sectional areas any smaller, these displacement limits would have been violated, thereby making this optimized solution a likely candidate for the true optimum solution.

The optimized solution shown in Figure 2 has two members running between the nodes 0 and 2. Although this may not be practical to implement, but GAs have found this solution as optimal. In the next simulation, we avoid such solutions by excluding duplicate members in the ground structure.

4.2 11-Member, Six-Node Truss

The 11-member, six-node ground structure is shown in Figure 3. Topology and size optimization are carried out for this truss using 11 variables corresponding to members areas. The same design parameters as that used in the previous simulations are used here.

After 225 generations, the best truss-structure obtained using GAs with a population size of 220 has an overall weight of 4,899.15 lbs. (Recall that when all 15 member were used in the ground structure, a truss with smaller overall weight was obtained.) The optimized truss satisfies all constraints and is shown in Figure 4. Although there were 11 members and 6 nodes in the ground structure, the GA is able to find a structure with only 6 members and 5 nodes. For the optimized truss, the deflection of the node furthest away from the supports (intersection of members 2 and 5) in x and y directions are -0.562 and 2.000 inches, respectively. Since the deflection in y direction is equal to the maximum allowable deflection, it can be argued that the obtained truss is either optimal or near-optimal.

Figure 5 shows that the best solution in the initial population had a weight over 9,000 lbs and was feasible. The figure also shows how the GA with a population size of 110 finds a truss of weight 4,950.75 lbs, which is a little higher than that obtained using a population size of 220.

The member areas obtained using the proposed GA is compared with the best-known solution available (Ringertz, 1985), which used a multi-level linear and nonlinear programming method, where the same topology with 6 members and 5 nodes was obtained. Table 3 shows that the proposed GA is able to obtain a truss with slightly smaller weight than that reported in Ringertz (1985). It is also interesting to note that although both weights are very similar, the combination of member areas in both trusses is a little different. Comparing to another GA implementation on the same problem (but with discrete member areas) (Rajan, 1995) which found a truss with a weight of 4,962.1 lbs, our solution is much better.

Next, we make the member areas to take only discrete values (in the step of 1 in^2). In this case, we use the discrete version of the SBX and mutation operators (Deb, 1997), thereby allowing only discrete values to be created using crossover and mutation operators. The optimized truss obtained using GA has the same topology as in Figure 4, but now has an overall weight of 4,912.85 lbs. The best known solution in the

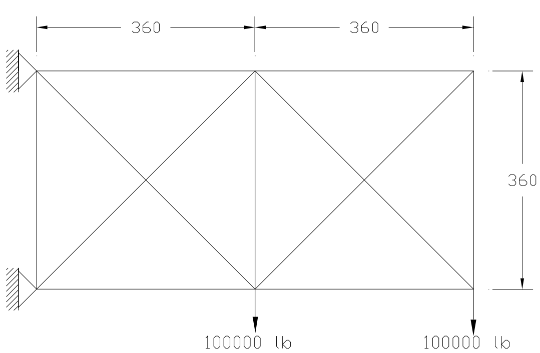

Figure 3: The 11-member, six-node ground structure.

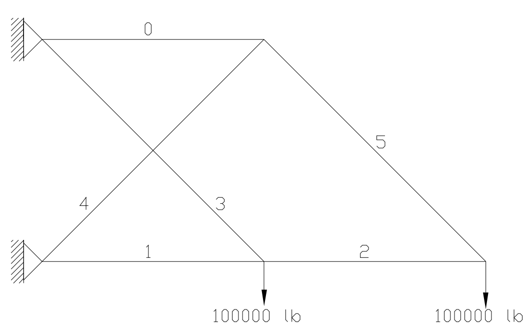

Figure 4: Optimized truss obtained from 11-member, six-node ground structure.

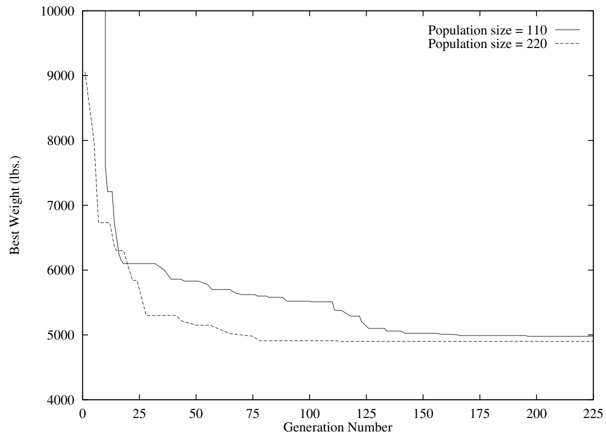

Figure 5: The improvement in the best solution versus generation number.

Table 3: Member areas of the optimized truss for 2-D, 11-member, six-node ground structure.

Member number (refer Figure 4)	Area of members (in^2) Continuous Areas: Proposed	Continuous Areas: Ringertz (1985)	Discrete Areas: Proposed	Discrete Areas: Hajela, Lee, and Lin (1993)
0	29.68	30.10	30	28
1	22.07	22.00	24	24
2	15.30	15.00	16	16
3	06.09	06.08	6	6
4	21.44	21.30	20	21
5	21.29	21.30	21	22
Weight of Truss (lb)	4899.15	4900.00	4912.85	4942.7

literature for the discrete case has the same topology and has a weight of 4,942.70 lbs (Hajela, Lee, and Lin, 1993), which is about 30 lbs more than that obtained by our algorithm. The corresponding member areas are presented in Table 3. Since the proposed GA has found a truss which requires 1 in^2 less area in both members 4 and 5 (which are the largest members in the truss) compared to that in Hajela, Lee, and Lin (1993), the overall weight is smaller. However, to make the truss safer from stress considerations, the cross-sectional area in member 0 had to be increased by 2 in^2.

In order to investigate the effect of the maximum limit of displacement on the optimal design, we redo the simulation runs with three other Δ values. In all cases, the optimized truss has the same configuration as in Figure 4. It is interesting to note that with $\Delta = 4$ and 6 inches, the optimized truss makes one of the displacement constraints active. However, when $\Delta = 8$ inches is used, the stress constraint in member 2 becomes active, and the displacement constraint is not so important. For any further increase in the allowable displacement, the optimized truss is going to be critical in terms of failure due to strength consideration. Figure 6 shows that the optimized weight of the truss reduces polynomially with the allowable displacement. The fitted polynomial allows us to compute the optimal weight W of the truss for smaller deflection limits Δ as $W = 8,695.6\Delta^{-0.816}$. This is because for $\Delta < 8$ inches, the optimal truss makes

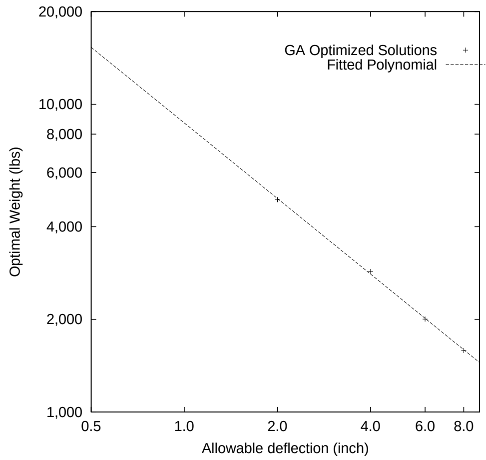

Figure 6: Optimized weight with allowable deflection.

at least one of the deflection constraints active. For example, if $\Delta = 1.0$ is chosen, the optimal weight is extrapolated to be around 8,695.6 lbs.

4.3 Ten-Node, 2D Truss

Next, we apply the proposed GA to a 10-node truss with a ground structure having all pair-wise interconnections (a total of $\binom{10}{2}$ or 45 members). All parameters are the same as before, except that the lower and upper bounds of cross-sectional areas are -1.0 and 1.0 in^2. Although symmetry along the middle nodes could have been used to reduce the number of variables, we have not used this information in this application. With a population size of 1,800, the proposed GA finds the truss shown in Figure 7, which is symmetric. It is seen that out of the 45 members, only 7 members are present in the optimized truss. The

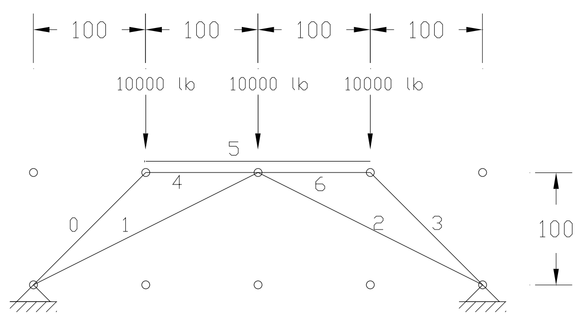

Figure 7: Optimized truss resulting from a ten-node, 45-member ground structure.

overall weight of this truss is 44.033 lbs. The cross-sectional area of members for this optimized truss are listed in Table 4.

4.4 Two-Tier Truss

Next, a two-tier, 39-member, 12-node ground structure (Figure 8) is used for the following optimization studies:

1. Sizing and topology optimization, and

Table 4: Member areas for the optimized truss obtained from ten-node, 45-member ground structure.

Member number (refer Figure 7)	Areas of members (in^2)
0	0.566
1	0.477
2	0.477
3	0.566
4	0.082
5	0.321
6	0.080
Weight of truss (lbs.)	44.033

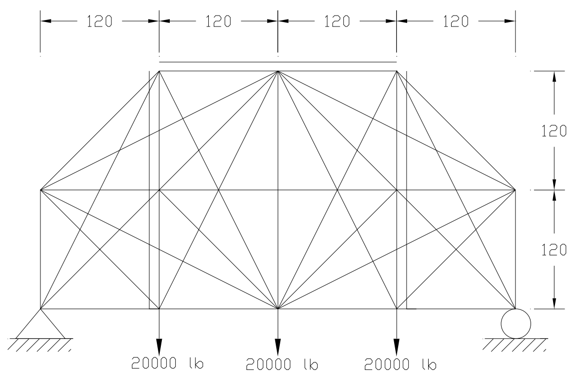

Figure 8: Two-tier, 39-member, 12-node ground structure.

2. Sizing, shape and topology optimization.

The overlapping members are shown with a small gap in the figure for clarity. Symmetry about middle vertical member is assumed, thereby reducing the number of variables to 21. The material properties and maximum allowable deflection are the same as in the previous problem, except the allowable strength is 20 Ksi. Lower and upper bounds of member areas of -2.25 to 2.25 in^2 are used and a critical area of 0.05 in^2 is chosen.

4.4.1 Sizing and Topology Optimization

Simultaneous optimization of sizing and topology is carried out taking 21 continuous variables corresponding to 39 members after considering symmetry. Optimized topology corresponding to a simulation run with population size of 630 is shown in Figure 9 and member areas are listed in Table 5. Of the 39 members and 12 nodes in the ground structure, only 17 members and 10 nodes are retained by the proposed GA. Starting from a weight of 570 lbs found in the best solution in the initial population, the GA has found a truss with a weight of 198 lbs. Since no study of this 2-tier truss is available in the literature, we cannot compare our solution with any other method. Nevertheless, Figure 9 shows that optimized truss is intuitively a much better truss than the ground structure. Moreover, all the critical members carrying large loads have utilized the material maximally so that the stress developed in each of them is almost equal to

Table 5: Member areas for the optimized truss structure in the case of sizing and topology optimization with a population of size 630.

Member	0,1	2,3	4,5	6,7	8,9	10,11	12,13	14,15	16
Area (in^2)	0.050	1.501	0.052	0.050	1.416	1.118	1.001	0.050	1.002

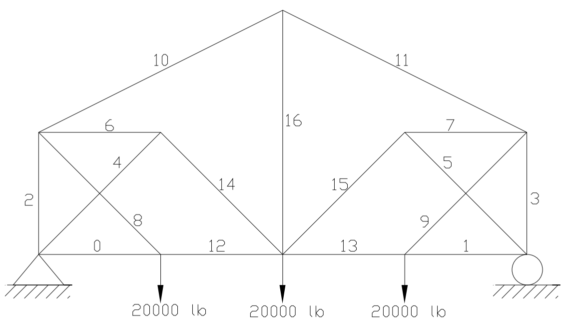

Figure 9: Optimized truss for two-tier, 39-member, 12-node ground structure for sizing and topology consideration with a population of size 630.

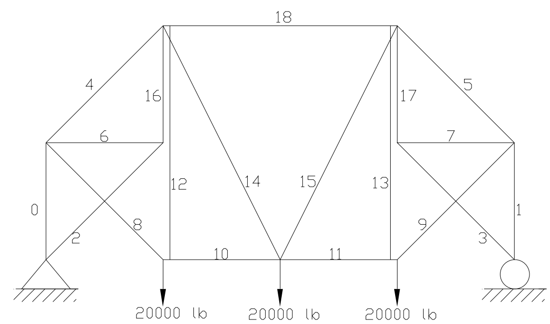

Figure 10: Optimized truss for two-tier, 39-member, 12-node ground structure for sizing and topology consideration with population size 840.

the allowable strength of material (20 Ksi), as shown in below:

Member:	(2,3)	(8,9)	(10,11)	(12,13)	(16)
Stress (Ksi):	19.987	19.972	**20.000**	19.980	19,960

This suggests that the obtained truss is a near-optimal solution.

When we increase the population size to 840, a truss with smaller overall weight emerged (Figure 10). The overall weight of this truss is 196.546 lbs. The member areas are shown in Table 6. It is interesting to note that both this trusses are different in their connectivity and yet have very similar overall weight.

Table 6: Member areas for the optimized truss structure in the case of sizing and topology optimization with population size of 840.

Member	0,1	2,3	4,5	6,7	8,9	10,11	12,13	14,15	16,17	18
Area (in^2)	1.502	0.051	1.063	0.051	1.061	0.751	0.251	0.559	0.052	1.005

4.4.2 Sizing, Topology and Shape Optimization

In simultaneous optimization of sizing, configuration, and topology, cross-sectional area of each member, number of members in the truss, and coordinates of the non-basic nodes (nodes that do not carry a load and nodes that are not support nodes) are kept as decision variables. Seven extra nodal displacement variables, in addition to 21 member area variables discussed previously, are considered here. Nodal displacement of 7 non-basic nodes in x and y directions with respect to their original coordinates in the ground structure are denoted as variables. Using symmetry about the vertical member at the center of the trusses, we reduce the number of these nodal displacement variables to 7 (the top-most node at the center is assumed to have a fixed x coordinate). These extra variables are assumed to vary within $(-120, 120)$ inch.

The optimized nodal configuration and topology corresponding to a GA run with 1,680 population size (up to 300 generations) is shown in Figure 11, and optimized member areas are listed in Table 7. This truss requiring only 15 members and 9 nodes has a weight of 192.19 lbs, which is 3% smaller than that obtained using only sizing and topology optimization. The shape of the truss is also different from that obtained in the previous subsection.

Table 7: Member areas for the optimized truss structure in the case of sizing, topology, and configuration optimization.

Member	0,1	2,3	4,5	6,7	8,9	10,11	12,13	14
Area (in^2)	0.595	1.615	1.293	1.155	0.051	1.166	0.504	1.358

4.5 3-D, 25-member, 10-Node Truss

Here, we consider a couple of applications with three-dimensional trusses. First, a 39-member ground structure as shown in Figure 12 is considered. To make matters simple, we apply two downwards loading of 500 lbs each at both top-most nodes. Members are grouped considering the symmetry on opposite sides and cross-members to be symmetric on all the sides, thus reducing the number of variables to 11. This grouping is done in the following way (refer Figures 12 and 14):

Group:	0	1	2	3
Member:	(0)	(1,2,3,4)	(5,6,7,8)	(9,10,11,12)
Group:	4	5	6	
Member:	(13,14,15,16)	(17,18,19,20)	(21,22,23,24)	
Group:	7	8	9	10
Member:	(25,26,27,28)	(29,30)	(31,32,33,34)	(35,36,37,38)

Young's modulus and density of the material are the same as before. However, an allowable tensile and compressive strength of 40 Ksi is used. An allowable deflection of 0.35 inch is used. The lower and upper

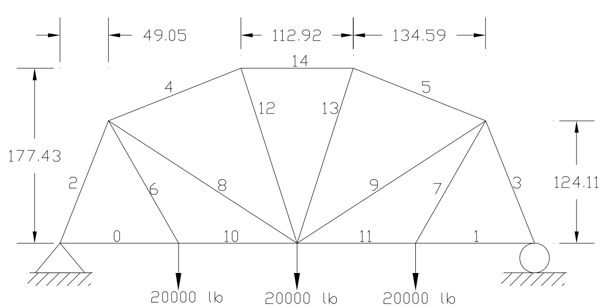

Figure 11: Optimized truss for two-tier, 39-member, 12-node ground structure for sizing, topology, and configuration consideration.

bounds on the member areas are assumed to be -3 to 3 in^2 and a critical area of 0.005 in^2 is chosen. The optimized truss is shown in Figure 13 and the member cross-sectional areas are shown below:

Member (Refer Figure 13)	Area (in^2)
0	0.166
1,2,3,4	0.409
5,6,7,8	0.071
Weight (lbs.)	47.930

Out of 39 members, only 9 members remain in the optimized truss and all middle nodes have been eliminated. This is an intuitive solution for the loading case considered and the proposed GA has been able to clean the undesired members to find the optimized truss-structure.

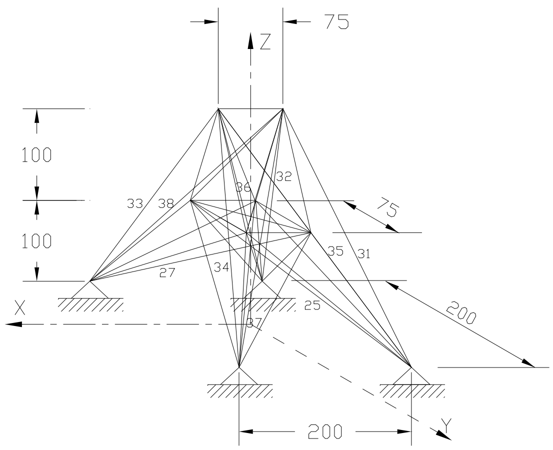

Figure 12: The ground structure for the 3D 39-member, 10-node truss design.

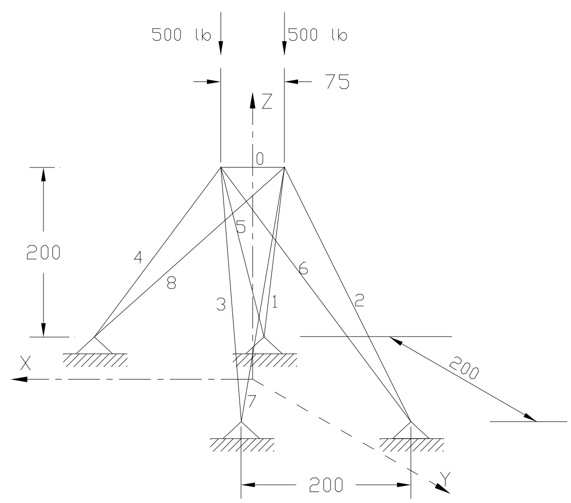

Figure 13: Optimized truss for 3D 39-member, 10-node ground structure.

We now consider the ground structure for the same 10-node, 3-D truss, but containing only 25 members, taken from the literature (Haug and Arora, 1989). The ground structure is shown in Figure 14. Members are grouped as before, except that now there exists only the first 7 groups.

This truss is optimized for two separate loading conditions. In loading case 1, a force vector $(0; 20,000; -5,000)$ lbs is applied on node 1 and a force vector $(0; -20,000; -5,000)$ lbs is applied on node 2. In loading case 2, four force vectors are applied: $(1,000; 10,000; -5,000)$ on node 1, $(0; 10,000; -5,000)$ on node 2, and $(500; 0; 0)$ on nodes 3 and 6. The same loading cases were also used in Haug and Arora (1989). The optimized weight found with the proposed GA (with a population size of 140) has an overall weight of 544.984 lbs, which is smaller than that reported in Haug and Arora (1989). The member areas are compared in Table 8.

Next, we attempt to consider both size and topology optimization. The optimized truss topology obtained using the proposed GA with a population size 280 has a weight of 544.852 lbs. Out of seven groups of member areas used in the ground structure, the optimized truss has only five groups (Groups 0 and 3, totalling five members have been deleted). Member areas corresponding to this solution are listed in Table 9. This truss is also optimal or near-optimal, because the deflection of top two nodes (1 and 2) is equal to the maximum allowable deflection (0.35 inch), which suggests that the truss has adjusted its member areas and member connectivity in such a way which makes the weight of the truss minimum by allowing the deflection to reach the allowable limit.

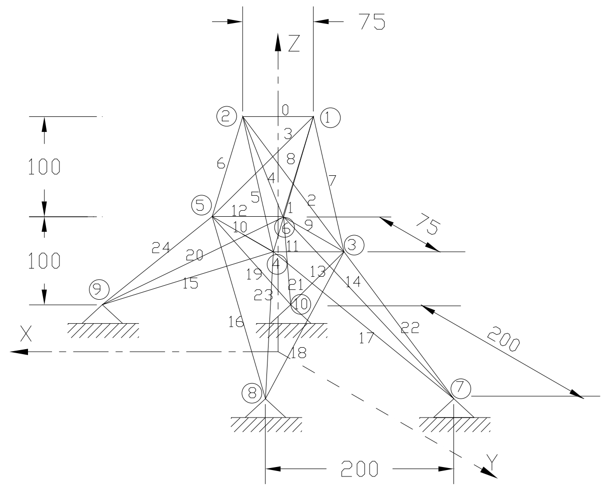

Figure 14: 3-D, 25-member, 10-node ground structure.

5 Conclusions

In this paper, we have developed a GA-based optimization procedure for designing 2-D and 3-D truss structures. Nodes in a truss are classified into two categories: (i) Basic nodes, which are used to support the truss or to apply a load, (ii) Non-basic nodes, which do not support the truss nor they bear any load. The concept of basic and non-basic is introduced to emphasize creation of user-satisfactory trusses and also to reduce the computational time by not performing expensive FEM analysis for unsatisfactory trusses. The trusses of varying topology (connectivity among members) is obtained with a fixed-length vector representation of member areas and with an implicit exclusion of small area members. This way any member having an area smaller than a critical area is considered to be absent in the corresponding solution. This representation scheme allows conventional GA operators to be used directly. Moreover, since the member areas are used as variables, simultaneous sizing as well as topology optimization are achieved. In the case of simultaneous application of all three optimization methods with sizing, topology, and configuration considerations, additional variables corresponding to change in nodal coordinates have been added.

In a number of different truss-structure problems ranging from 2-D, 6-node trusses to two-tier, 39-member truss to 3-D 25-member trusses, the proposed algorithm has been able to find trusses which are better than those reported in the literature and which utilizes material properties or deflection limits optimally. These results suggest the use of the proposed technique in other truss-structure design problems, where a complete optimization with optimal sizing, topology, and configuration is desired.

References

Deb, K. (1997). Mechanical component design using genetic algorithms. *Evolutionary algorithms in engineering applications* (Eds. Dipankar Dasgupta and Z. Michalewicz) New York: Springer-Verlag (pp. 495–512).

Deb, K. (1995). *Optimization for engineering design: Algorithms and examples.* New Delhi: Prentice-Hall.

Deb, K. and Agrawal, R. B. (1995). Simulated binary crossover for continuous search space. *Complex*

Table 8: Sizing optimization results for 3D, 25-member ground structure.

Members number (refer Figure 14)	Areas of member (in^2)	
	Proposed Population size 210	Haug, and Arora, (1989)
0	0.006	0.010
1,2,3,3	2.092	2.048
5,6,7,8	2.884	2.997
9,10,11,12	0.001	0.010
13,14,15,16	0.690	0.685
17,18,19,20	1.640	1.622
21,22,23,24	2.691	2.671
Weight of Truss(in lbs.)	544.984	545.050

Table 9: Member areas for the optimized truss structure.

Group	0	1	2	3	4	5	6
Member area (in^2)	–	2.037	2.969	–	0.699	1.644	2.658

Systems, 9 115–148.

Dobbs, M. W. and Felton, L. P. (1969). Optimization of truss geometry. *ASCE Journal of Structural Division, 95*, (ST10), 2105–2118.

Dorn, W. S., Gomory, R. E., and Greenberg, H. J. (1964). Automatic design of optimal structures. *Journal de Mecanique, 3* (6). 25–52.

Ghosh, A. and Mallik, A. K. (1988). *Theory of mechanisms and machines*. New Delhi: Affiliated East-West Press.

Goldberg, D. E. and Samtani, M. P. (1986). Engineering optimization via genetic algorithms, *Proceedings of the Ninth Conference on Electronic Computations*, ASCE, pp. 471–482.

Hajela, P., Lee, E., and Lin, C. Y. (1993). Genetic algorithms in structural topology optimization. *Topology Optimization of Structures*, pp. 117–133.

Haug, E. J. and Arora J. S. (1989). *Introduction to optimal design.* New York: McGraw Hill.

Imai, K. and Schmith, L. A. (1981). Configuration optimization of trusses. *Journal of Structural Division, ASCE, 107*, (ST5). 745–756.

Krish, U. (1989). Optimal topologies of truss structures. *Computer Methods in Applied Mechanics and Engineering, 72*, 15–28.

Rajan, S. D. (1995). Sizing, shape and topology optimization of trusses using genetic algorithm, *Journal of Structural Engineering, 121*, (10), 1480–1487.

Rajeev, S. and Krishnamoorthy, C. S. (1992). Discrete optimization of structures using genetic algorithms, *Journal of Structural Engineering, 118*, (5), 1233–1250.

Rao, S. S. (1984). *Optimization: Theory and applications*, New York: Wiley.

Ringertz, U. T. (1985). On topology optimization of trusses, *Engineering optimization*, *9*, 209–218.

Topping, B. H. V. (1983). Shape optimization of skeletal structures: A review. *Journal of Structural Engineering, 11*, 1933–1951.

Vanderplaat, G. N. and Moses, F. (1972). Automated design of Trusses for Optimal Geometry, *Journal of Structural Division*, (ST3), 671–690.