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# Numerical experiment with modelled return echo of a satellite altimeter from a rough ocean surface and a simple iterative algorithm for the estimation of significant wave height

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**Abstract.** We have simulated the return echo of a satellite altimeter from a rough ocean surface using an analytical formula and have studied its sensitivity with respect to various oceanic and altimeter parameters. Our numerical experiment shows that for normally observed significant wave heights (SWH) the effect of off-nadir angle (ONA) up to 0·5° on the leading edge is not severe. Also, small surface roughness skewness seems to have little effect on the overall shape of the echo.

Newton's iterative scheme has been used to retrieve SWH from the mean return waveform without noise and with additive Gaussian noise typical of Seasat and Geosat altimeters. It has been observed that SWH can be retrieved in the presence of noise with an accuracy of $\pm 0{\cdot}6$ m for ONA less than 0·5°. For higher ONA, accurate retrieval requires the use of precomputed look-up table along with our scheme.

**Keywords.** Satellite altimeter; return echo; significant wave height.

## 1. Introduction

Among the active microwave instruments used for ocean research, radar altimeter is unique and critical for studies related to ocean circulation, geoid and significant wave height (SWH) and is useful for estimating ocean surface wind speed. Satellite altimetry has evolved through Skylab (McGoogan *et al* 1974), GEOS-3 (Stanley 1979) and Seasat and Geosat missions. Future space missions with improved capability will be TOPEX/Poseidon and ERS-1 altimeter scheduled for launch in 1991. Several special issues of the *Journal of Geophysical Research* (such as Vol. 87, C5, April 30, 1982) describe numerous applications of satellite altimetry.

The basic concepts of the operation and principle of altimeters are fairly simple and straightforward. A short pulse (typically a few nanoseconds) is transmitted towards the sea surface in the nadir direction and after backscattering from the surface the echo is received by the sensor. The accurate time difference between transmission and reception of the pulse gives the height of the instrument above the mean sea level. In addition, the shape of the return pulse contains statistical information of the surface roughness giving SWH. The absolute power measurement gives $\sigma_0$, the backscattering coefficient, which has been shown to be related to ocean surface wind speed (Gairola and Pandey 1986; Pandey 1987).

While the basic concepts are simple, the retrieval of geophysical parameters from the radar altimeter data poses a great challenge due to the accuracy requirement set forth by the oceanographic research community, such as the height measurement accuracy of 5–10 cm, SWH with an accuracy of 0·5 m or 10% whichever is higher and wind speed measurement with an accuracy of 2 m/s. These accuracies coupled with ability to collect these measurements globally over the time interval of a few days will enhance our understanding of many oceanographic processes.

In the present paper we focus our attention on numerical experiment with simulated return echo of a satellite altimeter to determine its sensitivity with respect to oceanic parameters like SWH and surface roughness skewness and also with respect to off-nadir pointing angle. We also present a simple iterative algorithm for estimating SWH from the shape of the return echo. In § 2 we present the theoretical model of the return pulse used in our study. Sections 3 and 4 contain the results of our study and § 5 presents our conclusions. The Appendix contains some mathematical details.

## 2. Theoretical basis

The theoretical model for short pulse scattering from rough surfaces has been described by Brown (1977), with a review of assumptions involved. Hayne (1980) used Brown's model to obtain an analytical expression for the shape of the altimeter return pulse in the case of near-normal incidence. In our presentation here we follow the method of Brown and Hayne. Some further details are given in the Appendix.

We assume the echo to be an average of several individual return pulses after their backscattering from a rough ocean surface. This is the case for actual satellite altimeters. The resulting echo $W(t)$, commonly known as the mean return waveform, is given by the following convolution

$$W(t) = P_{\mathrm{FS}}(t) * q_s(t) * s_r(t), \tag{1}$$

where, $P_{\mathrm{FS}}(t)$ is the average flat surface impulse response, $q_s(t)$ is the radar observed ocean surface elevation probability distribution in the altimeter's time domain and $s_r(t)$ is the radar system point target response. Assuming that $q_s$ is a skewed Gaussian and $s_r$ is Gaussian, one can obtain the following expression for the return echo

$$W(t) = (A/6)\exp[-d(\tau + d/2)] \sum_{n=0} (1/n!)^2 (\beta^2\sigma/4)^n C_n(t), \tag{2}$$

where $A$ is the amplitude and $\tau$ is given by

$$\tau = (t - t_0)/\sigma - d \tag{3}$$

and

$$d = \delta\sigma, \tag{4}$$

$t_0$ being the shift in the time-origin due to a possible tracker bias and $\sigma$ being the composite rise time related to SWH (see Appendix). Further, $\beta$ and $\delta$ are related to the antenna beamwidth and off-nadir angle $\xi$ as elaborated in the Appendix and $C_n(t)$ are the following functions

$$C_n(t) = D_n(\tau)P(\tau) + E_n(\tau)G(\tau), \tag{5}$$

where $G(\tau)$ is a normalized Gaussian and $P(\tau)$ is the probability integral related to the error function. $D_n$ and $E_n$ are low-order polynomials in $\tau$ and involve the ocean skewness parameter. We omit the expressions for $G$, $P$, $E_n$ and $D_n$ for the sake of brevity. Interested readers may consult Hayne (1980) for these expressions.

## 3. Numerical experiment and sensitivity study

Hayne (1980) showed that for normal satellite altimeters only 3 terms need to be kept in the series expansion in equation (2) leaving thereby an error of less than 1% in the simulation of $W(t)$. Accordingly, we have used only a 3-term expansion for our numerical simulation experiment.

For the simulations we have used an idealized altimeter at a height of 800 km above the ocean surface and with an antenna beamwidth of 1·6°. Further $s_r(t)$ is a Gaussian of 3·125 ns full width at 1/2 height. These are nominal Seasat values. The amplitude $A$, being just a scaling constant, does not affect the results of sensitivity study. It has been given a value of 100. The return echo has been calculated for a $q_s(t)$ with zero skewness and for a nadir-pointing altimeter ($\xi = 0$) for SWH ranging from 0 to 15 m at intervals of 3 m. The results are shown in figure 1.

In figure 2 we present the effect of varying $\xi$ on the shape of the return echo for a purely Gaussian $q_s(t)$ with the SWH value of 10 m. Figure 3 shows the effect of non-zero surface skewness and it indicates that a change in slope of the leading edge from the case of zero skewness is noticeable only for a very large absolute value (unity). However, in practice, skewness of the order of 0·2 to 0·3 has been obtained from a variety of data sets (Barrick and Lipa 1985). Thus, as a first approximation, skewness can be ignored for estimating SWH from the slope of the echo.

The return echo in figures 1–3 was computed assuming $t_0 = 0$. In figure 4 we simulate

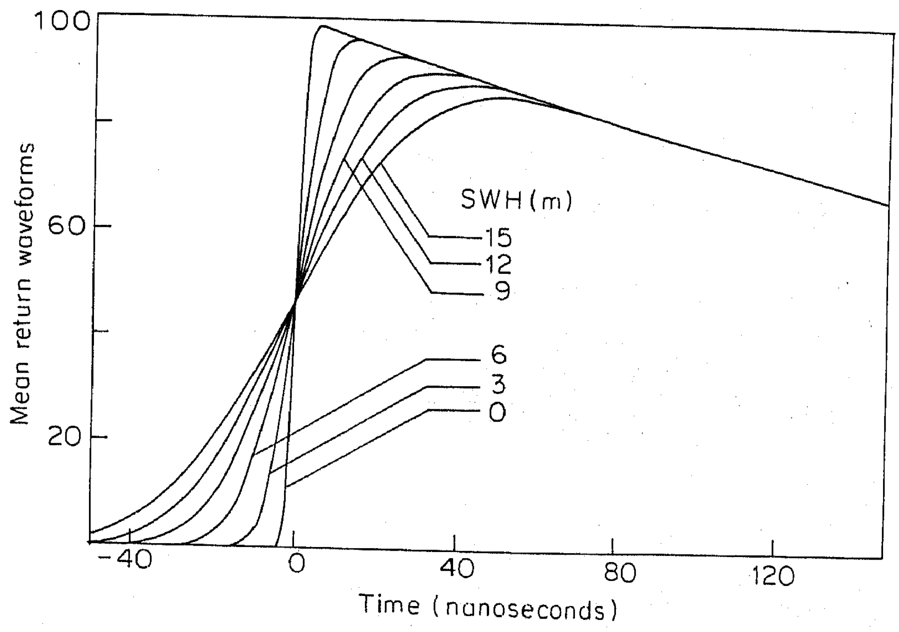


**Figure 1.** Effect of different ocean SWH on idealized mean return waveform of a nadir-pointing radar altimeter for $t_0 = 0$ and $\lambda_s = 0$.

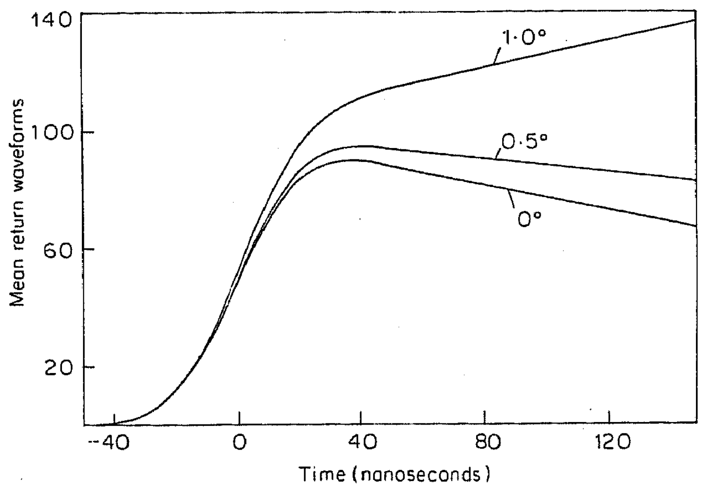


**Figure 2.** Effect of different off-nadir angles on the mean return waveform of an idealized altimeter for $t_0 = 0$, $\lambda_s = 0$ and SWH = 10 m.

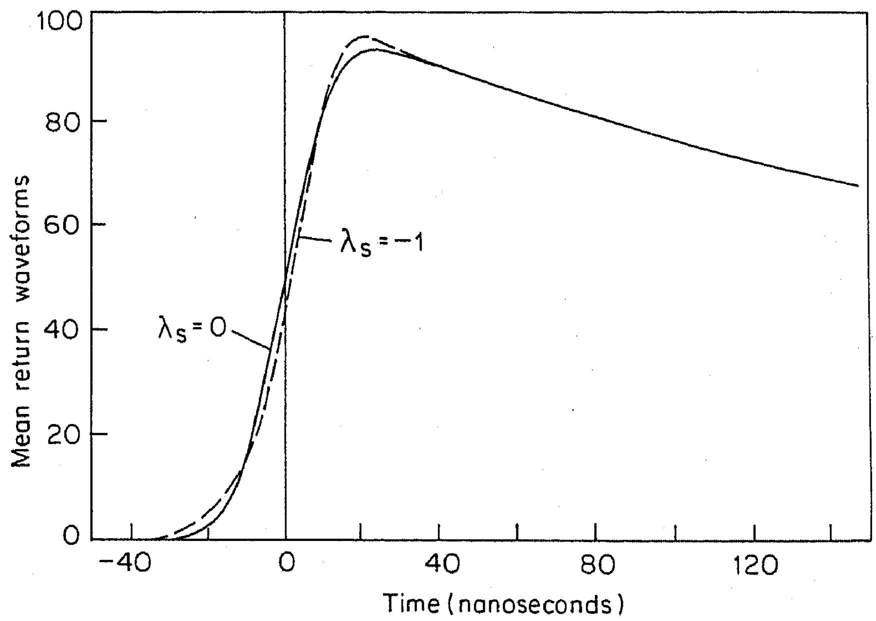


**Figure 3.** Effect of skewness on mean return waveform of an idealized nadir-pointing altimeter for $t_0 = 0$, SWH = 6 m.

the effect of possible tracker bias by taking $t_0 = \pm 5$ ns. As expected, this results in a parallel shift of the echo in the direction of abscissa. This effect will evidently introduce error in finding the height of the mean sea level and has to be taken care of for an accurate height determination. However, it does not affect the overall shape of the echo.

## 4. Estimation of SWH with an iterative technique

Although there are nonlinear least square fitting techniques for the simultaneous estimation of oceanic parameters and off-nadir angle they are quite complicated and costly for computer implementation. Also, a poor initial guess for the parameters being fitted may cause a numerical convergence problem. We present here a simple

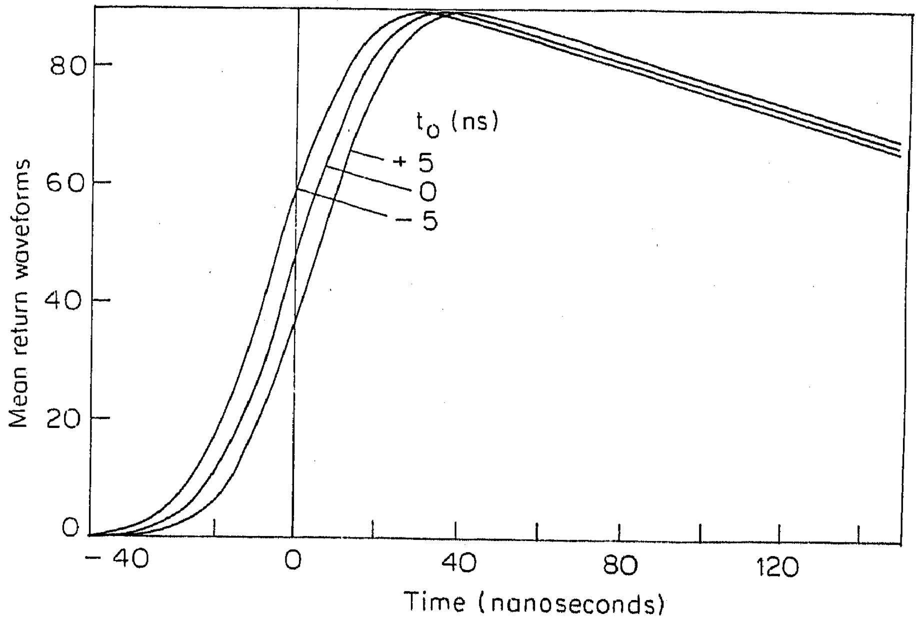


**Figure 4.** Effect of time-origin shift on the mean return waveform of idealized nadir-pointing altimeter for $\lambda_s = 0$, SWH = 10 m.

algorithm for estimating the SWH of an ocean surface with strictly Gaussian elevation probability distribution. As we have seen above, this is a reasonable first approximation since the skewness is quite small in practice. We also assume that there is negligible error in tracking ($t_0 \approx 0$). This error is of the order of 1 to 2 ns and such a shift will not affect much the slope at $t = 0$ on which our retrieval is based. This was clear from figure 4.

Since SWH is defined to be 4 times the RMS waveheight we have

$$\mathrm{SWH} = 4(c/2)\sigma_s, \tag{6}$$

where $\sigma_s$ is the relevant waveheight in units of time (a parameter of $q_s(t)$) and the factor $c/2$, where $c$ is the speed of light, arises due to the conversion from ranging time to surface elevation. Hence, estimation of SWH amounts to an estimation of $\sigma_s$ which is related to $\sigma$ by

$$\sigma_s^2 = \sigma^2 - \sigma_r^2, \tag{7}$$

where $\sigma_r$ is a parameter of the system point target response (see Appendix). In the estimation process we consider that the altimeter is strictly nadir-pointing ($\xi = 0$). The error introduced by such an assumption can be taken care of by means of look-up tables as will be shown later. The correction will require a knowledge of $\xi$ which can be obtained from onboard measurement of roll, yaw and pitch of the satellite and subsequent transmission to the ground. However, $\xi$ can be independently obtained by analysing the plateau region of the echo by a method suggested by Barrick and Lipa (1985).

Under the assumptions made above we have a particularly simple form of the return echo

$$W(t) = A \exp[-d(\tau + d/2)]P(\tau), \tag{8}$$

the slope of which at the origin is

$$S = (\mathrm{d}W/\mathrm{d}t)_{t=0} = (2\pi)^{-1/2}(A/\sigma) - \delta A \exp[0{\cdot}5(\delta\sigma)^2]P(-\delta\sigma). \tag{9}$$

In deriving (9) we have used the fact that $\mathrm{d}P(\tau)/\mathrm{d}\tau = G(\tau)$ and the relation (4) for $d$. Denoting the RHS of (9) by $f(\sigma)$ and rewriting it as

$$F(\sigma) = f(\sigma) - S = 0, \tag{10}$$

we find that the estimation of $\sigma$ is equivalent to finding the zero of the function $F(\sigma)$. Knowing the slope $S$, numerically estimated from the mean return waveform, one can find this zero by Newton's iterative technique (Conte 1965), starting from a reasonable initial guess. In $(k+1)$th iteration we have

$$\sigma_{k+1} = \sigma_k - F(\sigma_k)/F'(\sigma_k). \tag{11}$$

Once $\sigma$ is known, it is easy to find $\sigma_s$ from (7) and consequently SWH from (6). The advantage in using the slope instead of the echo itself is that the slope is unaffected by any additive systematic noise contained in the echo which is frequently the case for real altimeters.

The actual satellite altimeters sample the return echo by means of waveform sampling gates. To test our algorithm, however, we have used time-sampled values of the analytical $W(t)$ for various input SWH and $\xi$. The slope $S$, in each case, was estimated numerically and the corresponding SWH was estimated using the iterative algorithm.

While estimating SWH from real data we can make use of altimeter's backscattering coefficient measurement for improving our initial guess and ensuring convergence. Such a measurement provides the wind speed which could then be used in the wind-wave relation obtained by Pandey *et al* (1986) for the Indian region using Seasat data. This seems to be an illuminating approach for selecting the initial guess.

The estimation procedure was repeated for input SWH ranging from 3 to 15 m at intervals of 1 m and for five different values of $\xi$. As expected, for $\xi = 0$, the retrieval of SWH was very accurate. For non-zero $\xi$ an inevitable error is committed since our algorithm considers $\xi$ to be zero. In figure 5 we present the retrieved *vs* actual

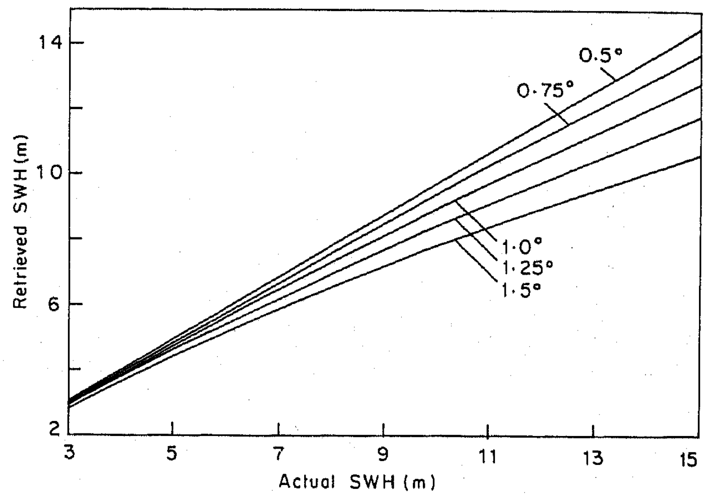


**Figure 5.** Retrieved *vs* actual SWH for five different off-nadir angles from 0·5° to 1·5° at intervals of 0·25°.

**Table 1.** Variation of retrieved SWH with off-nadir angle for Gaussian ocean.

| Actual SWH (m) | Retrieved SWH $\xi$(m) | | | | |
|---|---|---|---|---|---|
| | $=0{\cdot}5°$ | $=0{\cdot}75°$ | $=1{\cdot}0°$ | $=1{\cdot}25°$ | $=1{\cdot}5°$ |
| 3·0 | 3·0 | 3·0 | 3·0 | 2·9 | 2·8 |
| 6·0 | 5·9 | 5·8 | 5·6 | 5·4 | 5·2 |
| 9·0 | 8·8 | 8·5 | 8·1 | 7·7 | 7·2 |
| 12·0 | 11·6 | 11·1 | 10·5 | 9·8 | 9·0 |

SWH for different off-nadir angles and in table 1 we present some specific examples. It can be seen that for $\xi \leqslant 0{\cdot}75°$ the error in estimation is reasonably small, especially for low values of SWH. Even for high SWH, the error is of the order of 10% and is within the range of accuracy required. For higher off-nadir angles and for higher SWH values there is a systematic underestimation, but it is possible to correct for this by means of precomputed look-up tables (similar to table 1) or correction curve for the given value of $\xi$ (such as the ones in figure 5).

In order to further test our retrieval algorithm we have studied the case of noisy waveforms which is the case in practice. For this we have simulated noisy waveforms in the manner described below. First, we took the number of individual return pulses to be averaged to produce the mean return waveform to be 100. This approximately corresponds to 0·1 s averaging time employed in Seasat and Geosat altimeters. Next, we superimposed the Gaussian random noise of zero mean on the mean power return calculated previously by the analytical method. The standard deviation (SD) of the noise superimposed on the mean return waveform $W(t)$ at time $t$ is given by (Rodriguez and Chapman 1989):

$$\sigma_N(t) = N^{-\frac{1}{2}} W(t), \tag{12}$$

where $N$ is the number of individual pulses (= 100 in our study). The Gaussian random noise was generated by using IMSL Gaussian pseudorandom number generator routine. The SDs encountered were in the range of 4 to 6.

We simulated 10000 noisy realizations in this manner and retrieval was done for each mean return waveform. The SWHs employed were 2, 4, 6, 8 and 10 m. As in the noiseless case we carried out retrievals using various off-nadir angles (ONA). In table 2 we present the statistics of the retrieved SWH for the cases of three different ONA. In each case the statistics was computed from 100 samples.

For the case of zero ONA there are small positive biases in the retrieval. Since this case is theoretically most accurate we interpret that the bias arises due to the approximate way of numerically calculating the required slope at the origin. The bias decreases with increasing SWH but the SD increases leading to increased error, on the average, in the estimation. The scenario is similar to the noiseless case. However, even here, the standard accuracy requirement (10% of SWH, or, 0·5 m, whichever is higher) seems to be met generally. As regards the case of non-zero ONA the negative biases observed for higher SWH are due to the systematic underestimation observed in the noiseless case. However, the SDs are comparable to the case of zero ONA. As

**Table 2.** Statistics of retrieved SWH for three different off-nadir angles (all values are in m).

| Actual SWH (m) | Off-nadir angle (deg) 0·0 | | 0·5 | | 0·75 | |
|---|---|---|---|---|---|---|
| | Bias | SD | Bias | SD | Bias | SD |
| 2 | 0·13 | 0·04 | 0·10 | 0·04 | 0·07 | 0·04 |
| 4 | 0·05 | 0·13 | 0·02 | 0·14 | −0·03 | 0·13 |
| 6 | 0·06 | 0·33 | −0·03 | 0·27 | −0·20 | 0·24 |
| 8 | 0·06 | 0·44 | −0·16 | 0·48 | −0·28 | 0·37 |
| 10 | 0·02 | 0·59 | −0·29 | 0·62 | −0·55 | 0·72 |

Bias = mean retrieved SWH − actual SWH; SD = standard deviation of retrieved SWH

in practice the ONA of a satellite altimeter is very rarely more than 0·75° (usually less than 0·5°) we feel that the retrieval can still meet the accuracy requirement by correcting for the effect of ONA by means of precomputed look-up tables as mentioned in the study of noiseless case.

## 5. Conclusions

The analytical convolution form for the satellite altimeter mean return waveform employed by us could be useful either for system design studies for future space-borne altimeter missions or for ocean parameter retrieval. We have investigated the sensitivity of the return echo with respect to oceanic parameters and off-nadir angle. We have also demonstrated a simple iterative scheme for retrieving significant wave height using the modelled waveform including ideal and noisy cases typical of Seasat/Geosat altimeters. The method can be used for ERS-1 satellite altimeter data analysis and also for planning future Indian space missions with altimeters.

## Acknowledgements

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## Appendix

For the sake of completeness, we present here the expressions for the three functions used in the convolution equation (1) for the return echo. The average flat surface impulse response is

$$P_{FS}(t) = A \exp(-\delta t) I_0(t^{1/2}\beta) U(t), \tag{A1}$$

where

$$\delta = \frac{\ln 4}{\sin^2(\theta_W/2)}(c/h)\cos(2\xi) \tag{A2}$$

and

$$\beta = \frac{\ln 4}{\sin^2(\theta_W/2)}(c/h)^{1/2}\sin(2\,\xi). \tag{A3}$$

In (A1), $U(t)$ is a unit step function, $I_0$ the modified Bessel function, $h$ the satellite altitude and $\theta_W$ is the antenna beamwidth.

The radar observed ocean surface elevation distribution is the skewed Gaussian

$$q_s(t) = [(2\pi)^{-1/2}/\sigma_s][1 + (\lambda_s/6)H_3(t/\sigma_s)]\exp[-0{\cdot}5(t/\sigma_s)^2], \tag{A4}$$

where $\lambda_s$ is the skewness and $H_3$ is a Hermite polynomial.

The radar system point target response is the Gaussian

$$s_r(t) = [(2\pi)^{-1/2}/\sigma_r]\exp[-0{\cdot}5(t/\sigma_r)^2]. \tag{A5}$$

Convoluting $q_s(t)$ with $s_r(t)$ one obtains

$$\begin{aligned} B(t) &= q_s(t) * s_r(t) \\ &= [(2\pi)^{-1/2}/\sigma][1 + (\lambda/6)H_3(t/\sigma)]\exp[-0{\cdot}5(t/\sigma)^2], \end{aligned} \tag{A6}$$

where the composite risetime $\sigma$ is related to $\sigma_s$ and $\sigma_r$ by equation (7) in § 4 and

$$\lambda = \lambda_s(\sigma_s/\sigma)^3 \tag{A7}$$

contains the skewness information.

The return echo $W(t)$ is given by the final convolution

$$\begin{aligned} W(t) &= P_{\mathrm{FS}}(t) * B(t) \\ &= \int_{-\infty}^{\infty} P_{\mathrm{FS}}(z)B(t-z)\,\mathrm{d}z. \end{aligned} \tag{A8}$$

Expressions (A1), (A6) and a series expansion for $I_0$ lead to term-by-term integration in (A8) with the result that $W(t)$ is given by equation (2) in § 2.

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