SOME THREE DIMENSIONAL ELASTO-DYNAMIC SOLUTIONS OF LAYERED SHELLS

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ABSTRACT

Three dimensional solutions for natural frequencies and mode shapes of layered composite shells obtained by using the finite layer method, are presented in this paper. Higher order theories for laminated shells are discussed.

INTRODUCTION

Usage of layered composite fibre reinforced shells in modern engineering applications is increasing day by day. Design considerations of such structural elements are very different from similar metallic components, and so it is necessary that the theoretical models for the analysis of such components must have the capability to include second order effects such as shear deformation and rotory inertia, and to predict the frequency spectrum and the interlaminar stresses etc. Thus a new challange has emerged in modelling laminates, from various points of view. In this paper, we discuss the modelling of laminated cylindrical shells from the point of view of its natural vibrational behaviour and the development of higher order shear deformation theory.

Three-dimensional elasto-dynamic formulation is obviously, the most appropriate model, for the study of laminated shells. Unfortunately, it is not possible to utilize this very often, because of the difficulties associated with numerical work and computational limitations. On the other hand, it is much simpler and cheaper to use two-dimensional models if appropriate models are available. The classical laminated shell theory is generally believed to be inadequate due to non-inclusion of shear deformation and rotory inertia effects, and so the development of higher order models are being pursued. Three-dimensional elasto-dynamic solutions are useful in assessing the performance of such two-dimensional models. In this paper, we present the natural frequencies and mode shapes of typical isotropic and 3 layered composite shells, using the finite layer method.

ANALYSIS OF LAMINATED CYLINDRICAL SHELLS BY THE FINITE LAYER METHOD

Figure 1 shows a thick laminated cylindrical shell with the coordinate system employed. The material is assumed to be elastic, the deformations are considered to be small and three-dimensional theory of elasto-dynamics is used for the analysis. The boundary conditions at x = 0 and x = L are taken to be

$$\sigma_{xx} = 0; \; V = 0; \; W = 0 \qquad (1)$$

which corresponds to a form of simply supported end condition. At $z = \pm h/2$ the shell is stress free i.e.,

$$\sigma_{zz} = 0; \; \sigma_{zx} = 0; \; \sigma_{z} = 0 . \qquad (2)$$

Satisfying Eq. (1), the displacement field in the shell, when it is performing natural oscillations with a circular frequency, rad/sec, may be written as,

$$u = U(z) \operatorname{Cos} \frac{m\pi x}{L} \operatorname{Cos} n$$

$$v = V(z) \operatorname{Sin} \frac{m\pi x}{L} \operatorname{Sin} n$$

$$w = W(z) \operatorname{Sin} \frac{m\pi x}{L} \operatorname{Cos} n \qquad (3)$$

Fig. 1 shows a finite layer sub-division of the shell. The displacement field is chosen in terms of nodal (surface in the present case) variables $\{q\}$.

$$\{q\}^T = \{ U_1, V_1, W_1, U_2, V_2, W_2 \}$$

as,

$$\{\bar{u}\} = \begin{Bmatrix} u \\ v \\ w \end{Bmatrix} = [f] \; [Aq] \; \{q\} \qquad (4)$$

where

$$[f] = \begin{bmatrix} C.c & 0 & 0 \\ 0 & S.s & 0 \\ 0 & 0 & S.s \end{bmatrix}$$

with $C = \text{Cos}\,\dfrac{m\pi x}{L}$, $S = \text{Sin}\,\dfrac{m\pi x}{L}$

$c = \text{Cos}\, n\theta$, $s = \text{Sin}\, n\theta$

$$[Aq] = \begin{bmatrix} F_1 & 0 & 0 & F_2 & 0 & 0 \\ 0 & F_1 & 0 & 0 & F_2 & 0 \\ 0 & 0 & F_1 & 0 & 0 & F_2 \end{bmatrix}$$

with $F_1 = 1 - \xi$ and $F_2 = \xi$ (4a)

The element stiffness matrix takes the form

$$k = \iiint [Bq]^T [S^{-i}] [Bq] R \, d\theta \; dx \; dz$$

where,

$$[Bq] = [\Gamma] \; [f] \; [Aq]$$

with

$$[\Gamma] = \begin{bmatrix} \dfrac{\partial}{\partial z} & 0 & 0 \\ 0 & \dfrac{1}{R}\dfrac{\partial}{\partial \theta} & \dfrac{1}{R} \\ 0 & 0 & \dfrac{\partial}{\partial z} \\ \dfrac{1}{R}\dfrac{\partial}{\partial \theta} & \dfrac{\partial}{\partial x} & 0 \\ \dfrac{\partial}{\partial z} & 0 & \dfrac{\partial}{\partial x} \\ 0 & \left(\dfrac{\partial}{\partial z} - \dfrac{1}{R}\right) & \dfrac{1}{R}\dfrac{\partial}{\partial \theta} \end{bmatrix}$$

and $$[\bar{S}^{i}] = [T]^{T} [S^{i}] [T]$$

where [T] is a matrix which depend upon the fiber angle and $[S^i]$ is compliance matrix of the ith ply with reference to the material axes., which is taken as,

$$S_{11} = \frac{1}{E_L}, \quad S_{22} = S_{33} = \frac{1}{E_T}$$

$$S_{12} = S_{21} = S_{31} = S_{13} = S_{23} = S_{32} = \nu_{LT}/E_L$$

$$S_{44} = S_{55} = \frac{1}{G_{LT}}; \quad S_{66} = \frac{2(1+\nu_{TT})}{E_T}$$

All other components of the [S] matrix are zero and the ply indicator (i) is omitted for simplicity. L refers to the longitudinal direction of the fiber and T to the transverse direction of the fiber.

Five elastic constants E_L, E_T, G_{LT}, ν_{LT} and G_{LT} required to define the matrix [S] are taken as,

$$E_L = \sum_{i=1,2}^{N} V_n E_n; \quad \nu_{LT} = \nu_{TT} = \sum_{n=1,2}^{N} V_n \nu_n$$

$$E_T = \frac{1}{\sum_{n=1.2}^{N} (V_n/E_n)}; \quad G_{LT} = \frac{1}{\sum_{n=1,2}^{N} (V_n/G_n)}$$

where N is the number of constituent materials in the ply. V_n are volume fractions E_n, G_n and ν_n are the Young's and shear moduli and Poisson's ratio of the nth constituent of the ply material.

Similarly, the mass matrix is evaluated as,

$$[m] = \iiint [Aq]^T [f] \rho^i [f] [Aq] R \, d\theta \, dz \, dx$$

where ρ^i is the mass density of the ith ply, given by

$$\rho^i = \sum_{n=1,2}^{N} \rho_n V_n$$

The rest of the procedure is standard.

Table 1 gives frequency parameter of an isotropic shell by considering the thickness direction of the shell as a single finite layer element and these results are in close agreement with Ref. (1). Table 3 gives the natural frequencies of three composite shells and Table 2 contains the material properties used. The results given are obtained by considering the tnree plies as three finite layer elements. A typical set of mode shapes are shown in Fig. 2.

HIGHER ORDER MODELS FOR LAMINATED SHELLS

The shell theory attempts to provide a two-dimensional formulation to an essentially three-dimensional phenomenon. Following the recent developments in the modelling laminated plates, three modelling approaches, namely the displacement based (2), local-global (3) and iterative modelling possibilities (4) may be contemplated. The last two are not yet examined for their application to shells. The first one, is by far the simplest, readily amenable for finite element adaptation and have received considerable attention. In this approach the displacements are expressed in a series form in terms of thickness-wise coordinates. Retaining a finite number of terms and using the energy and/or equilibrium principles the necessary governing equations are generated. The most crucial step is the choice of the displacement field. Basic principles of this approach seems to have been stated as early as 1890 Ref (2) and over the years a number of modelling possibilities specialised from expressions of the type,

$$\bar{U}_i\,(x,y,z) = \sum_{i=1,} z^i\, U_i\,(x,y)\ .$$

have been examined. This expression is improved in Refs. (5,6) such that the displacement field satisfies zero shear stress conditions at the two surface of the shell. However no model seems to be available, at this time which satisfies the normal stress conditions also at the surface of the shell.

In Ref (5), natural frequencies and mode shapes estimated by higher order models are compared with three-dimensional elastodynamics solutions. Results indicated that as far as the frequency spectrum is concerned, the zeroth order approximation which corresponds to the classical laminated shell theory, is good enough for all practical purposes. Comparison of the mode shapes indicated more significant differences.

TABLE - 1

LOWEST FREQUENCY PARAMETER ψ FOR ISOTROPIC SHELLS
($m = 1$, $n = 1$, $\nu = 0.3$, $\psi = \omega/(\frac{m\pi}{L}\sqrt{G/\rho})$, $\beta = m\pi R_0/L$

	r = 0.5		r = 0.98	
β	Present study	Ref (1)	Present study	Ref (1)
0.1	0.090	0.090	0.111	0.117
0.2	0.175	0.175	0.214	0.208
0.3	0.254	0.250	0.304	0.292
0.4	0.324	0.318	0.378	0.370
0.5	0.387	0.383	0.439	0.431
0.6	0.440	0.438	0.487	0.480
0.7	0.483	0.483	0.525	0.519
0.8	0.525	0.519	0.555	0.552
0.9	0.558	0.551	0.578	0.578
1.0	0.587	0.580	0.594	0.607

TABLE - 2

PROPERTIES OF MATERIALS USED

Materials	Subscript used	Mass Density 10^{-4}	Youngs modulus 10^{6} PSi	Poisson's ratio
Epoxy	e	1.047	0.5	0.35
Glass	g	2.418	10.6	0.22
Steel	s	7.400	30.0	0.30
Boron	b	2.247	60.0	0.20

TABLE - 3

Natural Frequencies of Composite Shells (CPS)

R = 50", R/h = 20, L/R = 4, m = 1 , 3 Layers.

Number of circumferential waves	Isotropic steel shell		Composite Shells		
		V_e	0.4 +	0.4 +	0.4 ++
		V_g	0	0.6	0.6
		V_b	0.6	0	0
0	312.2		127.26	120.6	212.0
1	172.7		87.24	79.6	117.6
2	81.6		56.51	46.6	54.1
3	88.2		49.80	40.6	52.2
4	149.0		65.1	58.2	83.8
5	234.7		95.3	88.7	129.5
6	339.8		135.3	127.6	185.5
7	465.0		183.3	173.7	250.8
8	603.7		238.5	226.4	324.8
9	761.2		300.5	285.6	407.1
10	934.9		369.1	350.8	497.0
11	1123.9		444.0	422.0	594.0
12	1327.7		524.8	498.8	697.8
13	1545.5		611.3	581.0	807.6
14	1776.7		703.2	668.3	923.1
15	2020.4		800.3	760.5	1043.8

+ Fibre Orientation 0/0/0   ++ Fibre Orientation +45/0-45

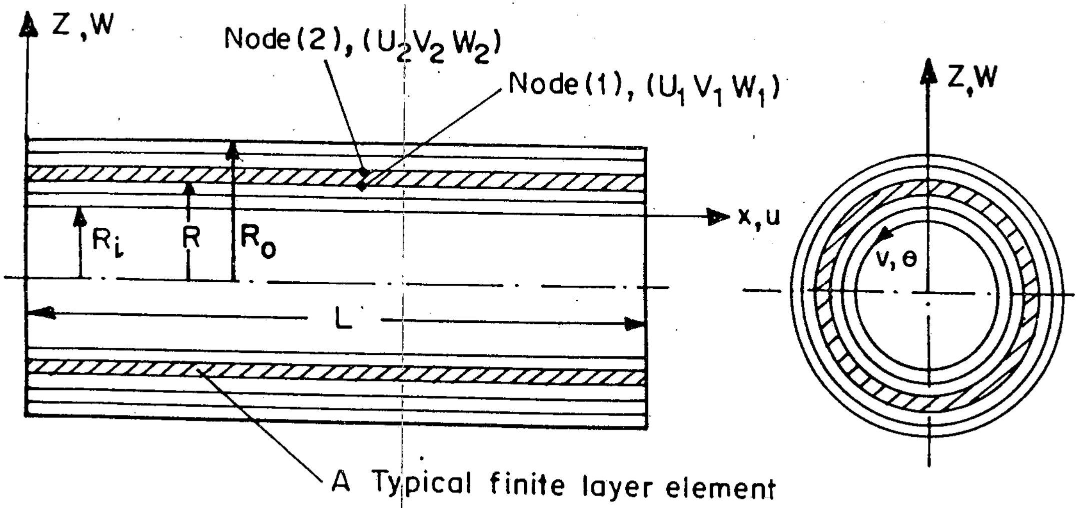

Fig. 1. A Composite cylindrical shell with finite layer element idealisation.

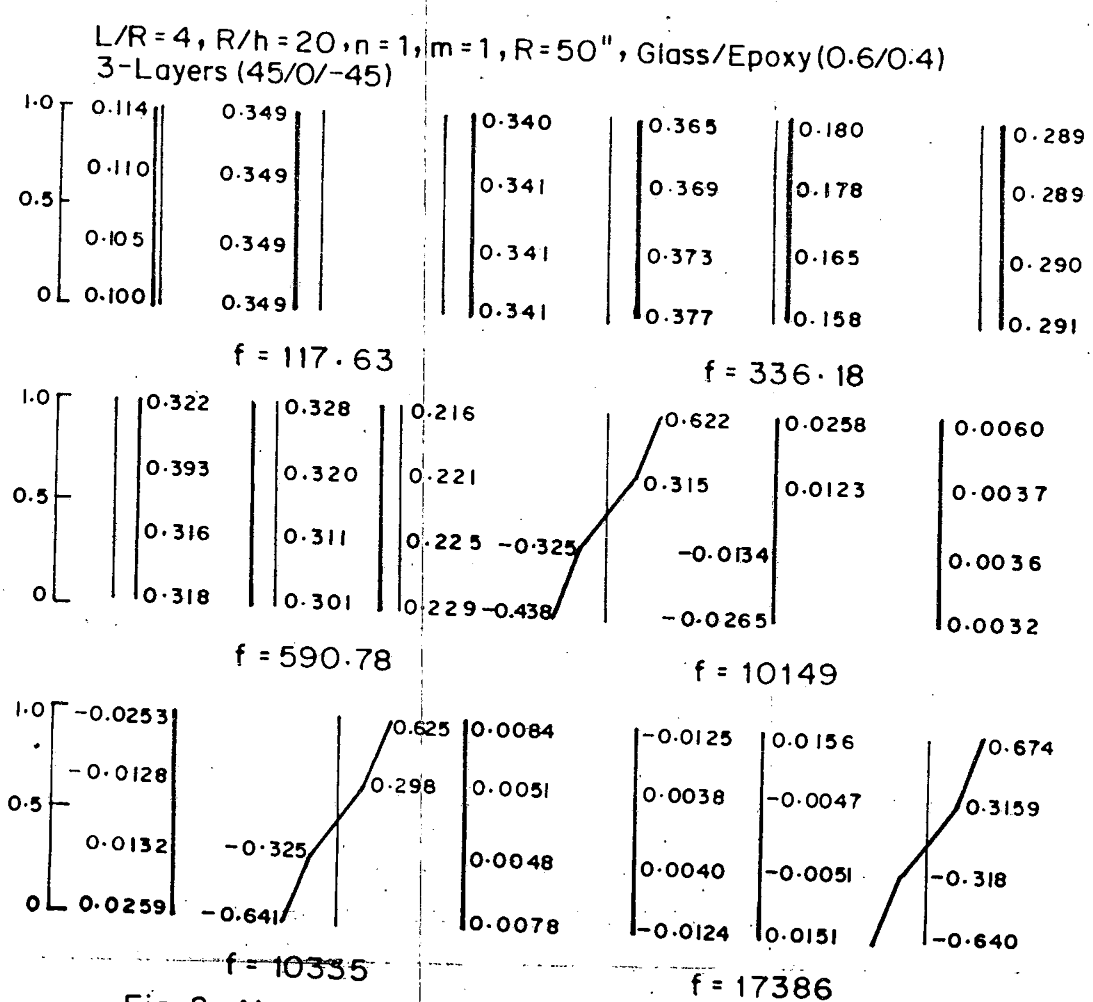

Fig. 2. Mode shapes of composite cylinder

CONCLUSIONS

In this paper, some three-dimensional elasto-dynamic solutions for simply-supported laminated shells are presented. Two-dimensional modelling possibilities are briefly discussed. It is recognised that a large variety of modelling possibilities exist and in this context, the example of simply-supported shell is ideally suited for assessing the new models, in view of its amenability for three-dimensional solution.

There are several aspects and criteria, with reference to which there is a need to assess the usefulness of higher order theories and establish an appropriate model for each application. Some of these important aspects include frequency spectrum, mode shapes, interlaminar stresses and strains.

As far as the frequency spectrum is concerned, the classical laminated plate theory may be considered adequate for practical purposes. Further investigations to assess various levels of higher order models, with regard to mode shapes and dynamic stress estimation are useful.

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