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Residual elastic field in two welded half spaces due to non uniform slip along a long strike slip fault

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Abstract

The problem of the static deformation of two homogeneous, isotropic and perfectly elastic half spaces in welded contact caused by non uniform slip along a vertical strike slip fault of infinite length and finite width is studied. Four slip profiles are considered : elliptic $b = b_0\ (1-h^2/L^2)^{1/2}$, parabolic $b = b_0\ (1-h^2/L^2)$, linear $b = b_0\ (1-h/L)$ and cubic $b = b_0\ (1-h^2/L^2)^{3/2}$, where b is the slip at distance h from the interface, b_0 is the interface slip and L is the fault width. The deformation corresponding to the four non uniform slip profiles is compared with the deformation due to uniform slip, assuming the source potency $\int_0^L b(h)\, dh$ to be the same. The parity in source potency is achieved by varying the fault width L, keeping the interface slip b_0 constant. Contour maps showing the displacement and stress fields around a long vertical strike slip fault are presented. It is found that the effect of non uniformity in slip in the near field is noteworthy. The far field is not affected significantly by the non uniformity in slip, i.e. the far field cannot see the details of the slip on the fault.

(**Keywords** : non uniform slip/static deformation/strike slip fault/welded half spaces)

Introduction

Several investigators have studied the static deformation of two welded half spaces caused by a long strike slip fault [see, e.g., Sharma *et al.*[1], Rani and Singh[2] and Singh and Rani[3]]. However, these studies assumed uniform slip on the fault. The assumption of uniform slip makes the edges of the fault plane singular where the displacement is indeterminate and the stress is infinite. For this reason, uniform slip models are not suitable in the near-field. There are a number of interesting phenomena that occur near the edge of the fault zone. These include the vertical movements associated with strike slip faulting and the formation of secondary faults (Chinnery and Petrak[4]). In order to study these phenomena, it is necessary to consider models of earthquake faulting with non uniform slip on the fault. The assumption of non uniform slip on the fault results in interesting theoretical models which might find useful applications in earthquake fault modelling. The purpose of the present paper is to study the effect of non uniform slip on the elastic field caused by a long strike slip fault in a half space in welded contact with another half space. The interface may represent, for example, the lithosphere/ asthenosphere boundary.

The problem of a fault with non-uniform slip in a half space has received some attention in geophysical literature. Chinnery and Petrak[4] used numerical intergration to compute the elastic field due to a vertical strike slip fault with slip that varies exponentially over the face of the fault. Freund and Barnett[5] gave a numerical solution for surface deformation due to 2-D dip slip faulting with variable slip on the fault plane. Mahrer and Nur[6] studied the deformation of an inhomogeneous half space, the shear modulus of which increases monotonically with depth due to strike slip faulting. They examined two general classes of faults : those that broke the surface smoothly reducing the slip to zero at some depth and those which were completely buried with smooth closure at both ends and evaluated the deformation numerically. Yang and Toksöz[7] used a finite element scheme to study a trapezoidal type of variable slip on a strike slip fault. Wang and Wu[8] obtained closed form analytical expressions for the displacements and stresses for the same model. Singh *et al.*[9] obtained closed form analytical expressions for the displacements in a half space caused by long vertical strike slip and dip slip faults with non uniform slip. In this paper, we have obtained closed form analytical expressions for the displacements and stresses at any point of two homogeneous, isotropic and elastic half spaces welded along a plane interface due to a long vertical strike slip fault with variable slip on the fault plane. Four slip profiles are considered : elliptic, parabolic, linear and cubic. It is assumed that the slip b decreases from a value b_0 at the interface to zero at the depth L. The value of the slip b_0 is the same for all the profiles, but the depth L is chosen in such a manner that the source potency (Ben-Menahem and Singh[10]) is the same for all the profiles. Analytical expressions for the displacement and shear stresses are used to compare the elastic field for non uniform slip with that for uniform slip. The effect of the rigidity contrast is examined. Contour maps showing the displacement and stress fields around a long vertical strike slip fault are also presented.

Theory

Consider two homogeneous, isotropic and perfectly elastic half-spaces that are welded along the plane $z = 0$. The upper half space $z < 0$ is called medium I and the lower half space ($z > 0$) is called medium II with rigidities μ_1 and μ_2 respectively. A vertical strike slip fault of infinite length and finite depth (width) occupies the region $-\infty < x < \infty, y = 0, 0 \le z \le L$ (Fig. 1). Let the slip on the fault be denoted by

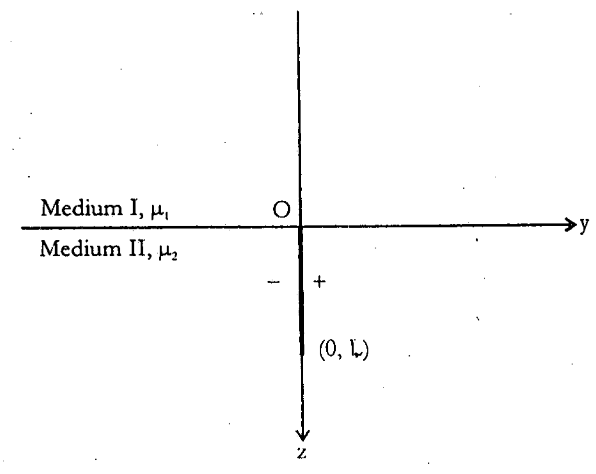

Fig. 1 – Two half-spaces in welded contact with a long, interface breaking, vertical and strike slip fault of width L in the lower half space. + and – indicate the displacements in the positive x direction and negative x direction, respectively. μ_1 is the rigidity of the upper half space (medium I) and μ_2 is the rigidity of the lower half space (medium II).

b which is non uniform in general. We are considering a 2-D approximation in which b is independent of x. In the following, the superscript (1) denotes displacement and stresses in medium I and the supersript (2) denotes those in medium II. Following Rani and Singh[2], the displacement parallel to the fault due to the slip b on the fault can be expressed in the form.

$$u_1^{(i)} = \int_0^L b(h)\, G_1^{(i)}(y, z; h)\, dh \quad (i = 1, 2). \tag{1}$$

The Green's functions appearing in equation (1) are given by

$$G_1^{1} = \frac{y}{\pi(1+\beta) R^2}, \tag{2}$$

$$G_1^{(2)} = \frac{y}{2\pi}\left[\frac{1}{R^2} + \left(\frac{1-\beta}{1+\beta}\right)\frac{1}{S^2}\right], \tag{3}$$

where

$$R^2 = y^2 + (z-h)^2,$$

$$S^2 = y^2 + (z+h)^2, \ \beta = \frac{\mu_1}{\mu_2}. \tag{4}$$

The expressions of the displacement for various slip profiles have been obtained from equation (1) by integrating analytically. The non zero shear stresses p_{12} and p_{13} are then obtained by using Hooke's law :

$$p_{12}^{(i)} = \mu_i \frac{\partial}{\partial y} u_1^{(i)}, \ p_{13}^{(i)} = \mu_i \frac{\partial}{\partial z} u_1^{(i)} \quad (i = 1, 2). \tag{5}$$

Uniform Slip

The case of uniform slip $b(h) = b_0$ over a strike slip fault has been discussed by Rani and Singh[2]. We use their results for numerical computations.

Elliptic

Let the slip on the fault vary according to the law

$$b(h) = b_0\, (1 - h^2L^2)^{1/2}, \quad (0 \le h \le L) \tag{6}$$

where b_0 is the interface slip and L is the fault depth. In this model, the slip decreases monotonically with depth from the interface value b_0 to zero at the buried edge of the fault. Inserting the expression for $b(h)$ in equation (1) and integrating, closed form expressions for the displacement are obtained. The corresponding stresses then follow from equation (5). The results valid at the interface ($z = 0$) are

$$u_i^{(1)} = u_1^{(2)} = \frac{b_0}{2(1+\beta)}\left[-Y \pm (1+Y^2)^{1/2}\right], \quad (7)$$

$$p_{13}^{(1)} = p_{13}^{(2)} = \frac{\mu_2 b_0 \beta}{\pi L(1+\beta)}$$

$$\times\left[\frac{1}{Y} - \frac{Y}{2(1+Y^2)^{1/2}}\ \ln\frac{(1+Y^2)^{1/2}-1}{(1+Y^2)^{1/2}+1}\right], \quad (8)$$

where $Y = \frac{y}{L}$. The upper sign in equation (7) is for $Y > 0$ and the lower sign for $Y < 0$.

Parabolic

Let the slip on the fault be given by

$$b(h) = b_0\left(1 - \frac{h^2}{L^2}\right), \quad (0 \le h \le L). \quad (9)$$

In this model, the slip decreases monotonically with depth from the interface value b_0 to zero at the buried edge of the fault. The expressions for the displacements and stresses obtained from equation (1) and (5) are

$$u_i^{(1)} = \frac{b_0}{\pi(1+\beta)}\left[(1+Y^2-Z^2)\left\{\tan^{-1}\left(\frac{1-Z}{Y}\right)\right.\right.$$

$$\left.\left.+\tan^{-1}\left(\frac{Z}{Y}\right)\right\} - Y - 2YZ\ \ln\left(\frac{A}{A_0}\right)\right], \quad (10)$$

$$u_i^{(2)} = \frac{b_0}{2\pi}\left[(1+Y^2-Z^2)\left\{\tan^{-1}\left(\frac{1-Z}{Y}\right)\right.\right.$$

$$\left.+\tan^{-1}\left(\frac{Z}{Y}\right)\right\} - Y - 2YZ\ln\left(\frac{A}{A_0}\right) + \frac{1-\beta}{1+\beta}$$

$$\times\left\{(1+Y^2-Z^2)\left[\tan^{-1}\left(\frac{1+Z}{Y}\right) - \tan^{-1}\left(\frac{Z}{Y}\right)\right]\right.$$

$$\left.\left.-Y + 2YZ\ \ln\left(\frac{B}{A_0}\right)\right\}\right], \quad (11)$$

$$p_{12}^{(1)} = \frac{\mu_2 b_0 \beta}{\pi L(1+\beta)}\left[2Y\left\{\tan^{-1}\left(\frac{1-Z}{Y}\right)\right.\right.$$

$$\left.\left.+\tan^{-1}\left(\frac{Z}{Y}\right)\right\} - \frac{Z}{A_0^2} - 2Z\ \ln\left(\frac{A}{A_0}\right) - 2\right], \quad (12)$$

$$p_{13}^{(1)} = \frac{2\mu_2 b_0 \beta}{\pi L(1+\beta)}\left[\frac{Y}{2A_0^2} - Y\ \ln\left(\frac{A}{A_0}\right) - Z\right.$$

$$\left.\times\left\{\tan^{-1}\left(\frac{1-Z}{Y}\right) + \tan^{-1}\left(\frac{Z}{Y}\right)\right\}\right], \quad (13)$$

$$p_{12}^{(2)} = \frac{\mu_2 b_0}{2\pi L}\left[2Y\left\{\tan^{-1}\left(\frac{1-Z}{Y}\right) + \tan^{-1}\left(\frac{Z}{Y}\right)\right\}\right.$$

$$-2Z\ \ln\left(\frac{A}{A_0}\right) - \frac{Z}{A_0^2} - 2 + \frac{1-\beta}{1+\beta}\left\{2Y\left[\tan^{-1}\left(\frac{1+Z}{Y}\right)\right.\right.$$

$$\left.\left.\left.-\tan^{-1}\left(\frac{Z}{Y}\right)\right] + 2Z\ \ln\left(\frac{B}{A_0}\right) + \frac{Z}{A_0^2} - 2\right\}\right], \quad (14)$$

$$p_{13}^{(2)} = \frac{\mu_2 b_0}{\pi L}\left[\frac{Y}{2A_0^2} - Y\ \ln\left(\frac{A}{A_0}\right) - Z\left\{\tan^{-1}\left(\frac{1-Z}{Y}\right)\right.\right.$$

$$\left.+\ \tan^{-1}\left(\frac{Z}{Y}\right)\right\} + \frac{1-\beta}{1+\beta}\left\{-\frac{Y}{2A_0^2} + Y\ \ln\left(\frac{B}{A_0}\right)\right.$$

$$\left.\left.-Z\tan^{-1}\left(\frac{1+Z}{Y}\right) + Z\tan^{-1}\left(\frac{Z}{Y}\right)\right\}\right], \quad (15)$$

where

$$Z = z/L, \qquad A_0^2 = Y^2 + Z^2$$

$$A^2 = Y^2 + (Z-1)^2,\ B^2 = Y^2 + (Z+1)^2. \quad (16)$$

Linear

For the slip profile

$$b(h)=b_0\left(1-h/L\right),\qquad (0\le h\le L).\quad (17)$$

the slip decreases linearly from the interface value b_0 to zero at a distance of L from the interface. For this profile, we obtain

$$u_i^{(1)}=\frac{b_0}{\pi(1+\beta)}\left[(1-Z)\left\{\tan^{-1}\left(\frac{1-Z}{Y}\right)+\tan^{-1}\left(\frac{Z}{Y}\right)\right\}-Y\ln\left(\frac{A}{A_0}\right)\right],\quad (18)$$

$$u_1^{(2)}=\frac{b_0}{2\pi}\left[(1-Z)\left\{\tan^{-1}\left(\frac{1-Z}{Y}\right)+\tan^{-1}\left(\frac{Z}{Y}\right)\right\}-Y\ln\left(\frac{A}{A_0}\right)+\frac{1-\beta}{1+\beta}\left\{(1+Z)\left[\tan^{-1}\left(\frac{1+Z}{Y}\right)-\tan^{-1}\left(\frac{Z}{Y}\right)\right]-Y\ln\left(\frac{B}{A_0}\right)\right\}\right],\quad (19)$$

$$p_{12}^{(1)}=-\frac{\mu_2 b_0\beta}{\pi L(1+\beta)}\left[\frac{Z}{A_0^2}+\ln\left(\frac{A}{A_0}\right)\right],\quad (20)$$

$$p_{13}^{(1)}=\frac{\mu_2 b_0\beta}{\pi L(1+\beta)}\left[\frac{Y}{A_0^2}-\tan^{-1}\left(\frac{1-Z}{Y}\right)-\tan^{-1}\left(\frac{Z}{Y}\right)\right],\quad (21)$$

$$p_{12}^{(2)}=-\frac{\mu_2 b_0}{2\pi L}\left[\frac{Z}{A_0^2}+\ln\left(\frac{A}{A_0}\right)+\frac{1-\beta}{1+\beta}\left\{-\frac{Z}{A_0^2}+\ln\left(\frac{B}{A_0}\right)\right\}\right],\quad (22)$$

$$p_{13}^{(2)}=\frac{\mu_2 b_0}{2\pi L}\left[\frac{Y}{A_0^2}-\tan^{-1}\left(\frac{1-Z}{Y}\right)-\tan^{-1}\left(\frac{Z}{Y}\right)+\frac{1-\beta}{1+\beta}\left\{\tan^{-1}\left(\frac{1+Z}{Y}\right)-\tan^{-1}\left(\frac{Z}{Y}\right)-\left(\frac{Y}{A_0^2}\right)\right\}\right].\quad (23)$$

Cubic

Let the slip on the fault vary according to the law

$$b(h)=b_0\left(1-h^2/L^2\right)^{3/2},\qquad (0\le h\le L).\quad (24)$$

This model describes slip which reaches the interface and smoothly reduces to zero with depth satisfying the smooth closure conditions (Mahrer and Nur[6]) : $b = 0$, $db/dh = 0$ at $h = L$. The expressions for the displacements and stresses valid at the interface ($z = 0$) are

$$u_1^{(1)}=u_1^{(2)}\frac{b_0}{2(1+\beta)}\left[-Y\left(Y^2+\frac{3}{2}\right)\pm\left(1+Y^2\right)^{3/2}\right],\quad (25)$$

$$p_{13}^{(1)}=p_{13}^{(2)}=\frac{\mu_2 b_0\beta}{\pi L(1+\beta)}\left[\frac{1+Y^2}{Y}+2Y-\frac{3}{2}\times Y\left(1+Y^2\right)^{1/2}\ln\frac{\left(1+Y^2\right)^{1/2}+1}{\left(1+Y^2\right)^{1/2}-1}\right].\quad (26)$$

The upper sign in equation (25) is for $Y > 0$ and the lower sign is for $Y < 0$.

Equation (7) to (26) yield the elastic field in two welded half spaces due to non uniform slip on a vertical strike slip fault. The corresponding results for a uniform half space given by Singh *et al.*[9] are obtained as a particular case of the present results on putting $\beta = \mu_1/\mu_2 = 0$.

Numerical Results and Discussion

There is a large volume of literature on the analysis of the stresses and strains in the vicinity of cracks. One of the successful crack models assumes the two surfaces of the crack to be in contact the stresses acting across the crack to be continuous and the relative displacement (slip) of the two sides specified. For mathematical simplification, the slip is usually assumed to be constant. In this paper we have attempted to study the effect of non uniform slip over the face of the crack.

We wish to compare the deformation due to non uniform along a long vertical strike slip fault in two welded half spaces with the corresponding deformation due to uniform slip. For all the slip profiles considered, the slip decreases from a value b_0 at the interface to zero at depth L. If the slip b_0 and the fault depth L are assumed to be the same for all the cases, then source potency $\int_0^L b(h)dh$ per unit length of the fault is different for different profiles. Source potency is the fault slip integrated over the fault face. For uniform slips, it is simply the slip multiplied by the fault area. Comparison of deformation of sources of different potency is not justified. Before making any comparison, a parity in source potency must be assured. A parity in source potency for different slip profiles can be achieved by adjusting either the interface slip or the fault depth. In our numerical computations, we have assumed that the interface slip b_0 is the same for all the slip profiles, but the fault depth L is so adjusted that parity in source potency is achieved. This yields the relationship (Singh *et al.*[9])

$$L_1 = \frac{\pi}{4}L_2 = \frac{2}{3}L_3 = \frac{1}{2}L_4 = \frac{3\pi}{16}L_5 = L \text{ (say)}, \quad (27)$$

where L_1 is the fault depth for the uniform slip model and L_2, L_3, L_4 and L_5 are, respectively, the fault depths for the elliptic, parabolic, linear and cubic profiles considered above. We measure the displacements in untis of the slip b_0 the distances in units of the fault depth $L_1 = L$ for the uniform slip model and the stresses in units of $\mu_2 b_0/L_1 = \mu_2 b_0/L$. The dimensionless quantities Y and Z for non uniform slip cases are to be suitably modified in accordance with the relation (27). For example in equation (10) to (16) Y should be replaced by $(2/3)Y$ and Z by $(2/3)Z$.

Variation of the dimensionless parallel displacement u_1/b_0 at the interface ($z = 0$) with the distance from the fault y for three values of the rigidity contrast $\beta = \mu_1/\mu_2 = 0.5$, 1, 2 is shown in Fig. 2 (a, b, c). The case $\beta = 0.5$ is an example in which the source medium is harder than the other medium; when $\beta = 2$ the source medium is softer than the other medium. When $\beta = 1$, $\mu_1 = \mu_2$ and the results for a source in a

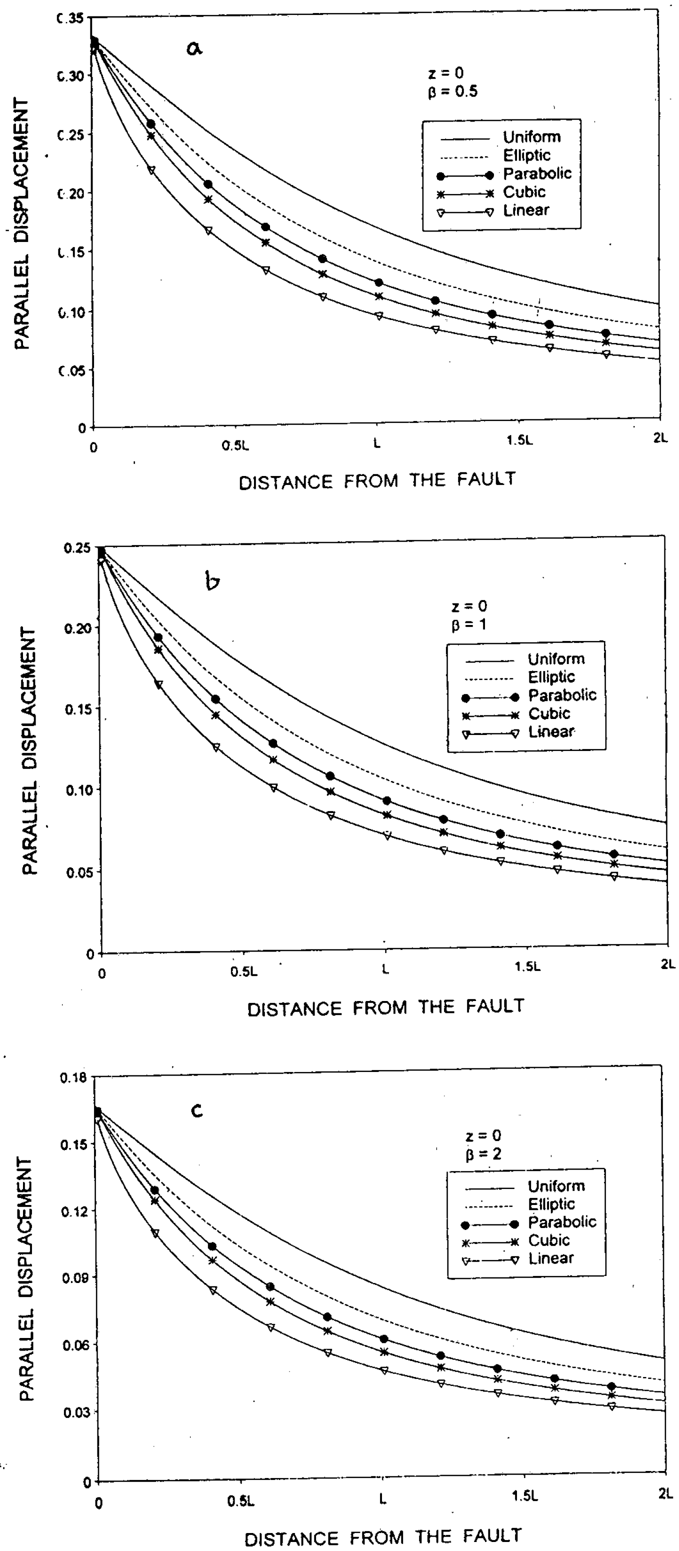

Fig. 2 – Variation of the dimensionless parallel displacement u_1/b_0 at the interface ($z = 0$) with the distance y from a vertical strike slip fault for different slip profiles for (a) $\beta = \mu_1/\mu_2 = 0.5$; (b) $\beta = 1$ and (c) $\beta = 2$.

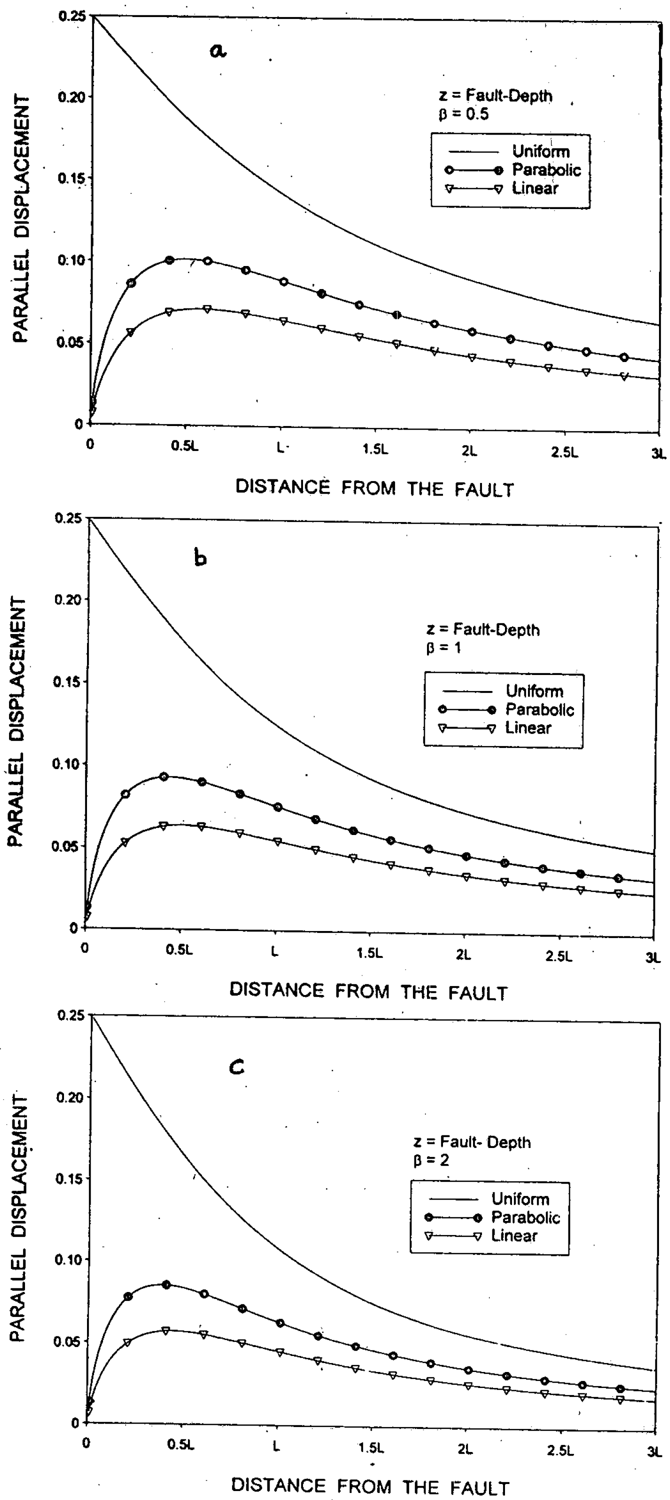

Fig. 3 – Variation of the parallel displacement at fault depth with the distance from the fault for different slip profiles for (a) $\beta = 0.5$; (b) $\beta = 1$ and (c) $\beta = 2$. Note that the fault depth for different slip profiles is different.

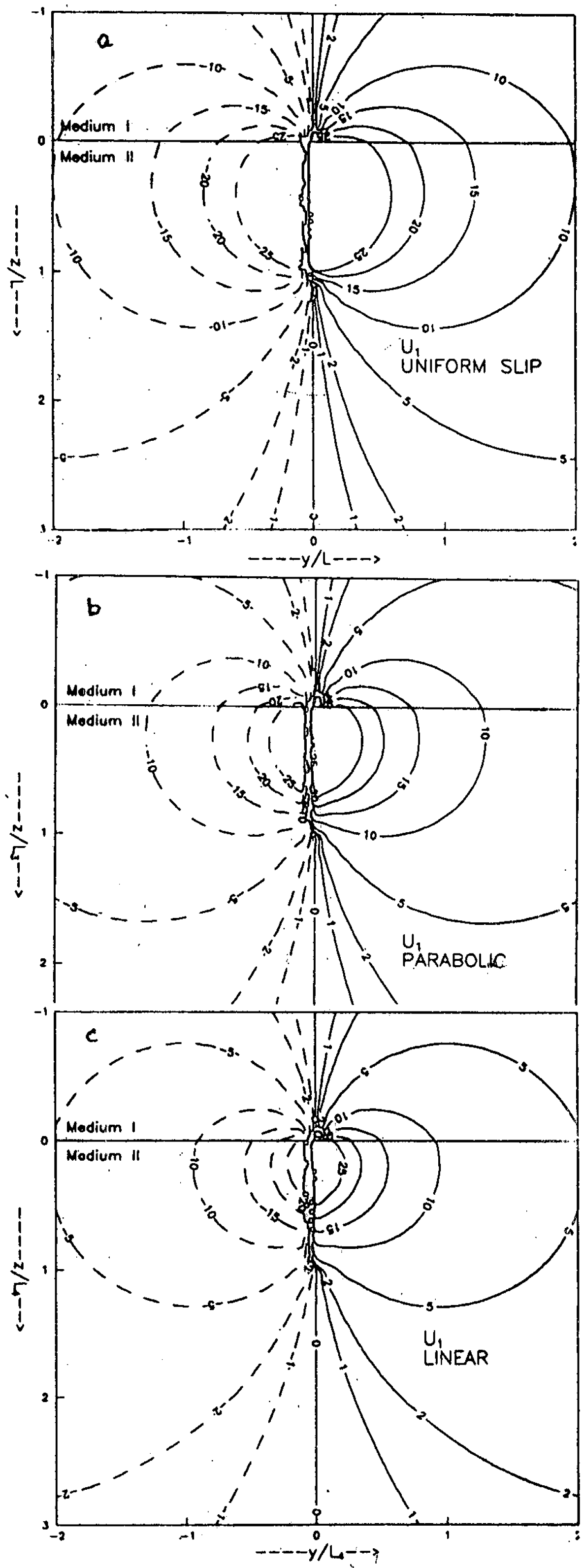

Fig. 4 – Contour maps for the dimensionless parallel displacement for $\beta = 0.5$. The displacement is measured in units of $b_0 \times 10^{-2}$ where b_0 is the interface slip. The distance is measured in units of fault depth for the slip profile under consideration. (a) uniform slip; (b) parabolic and (c) linear.

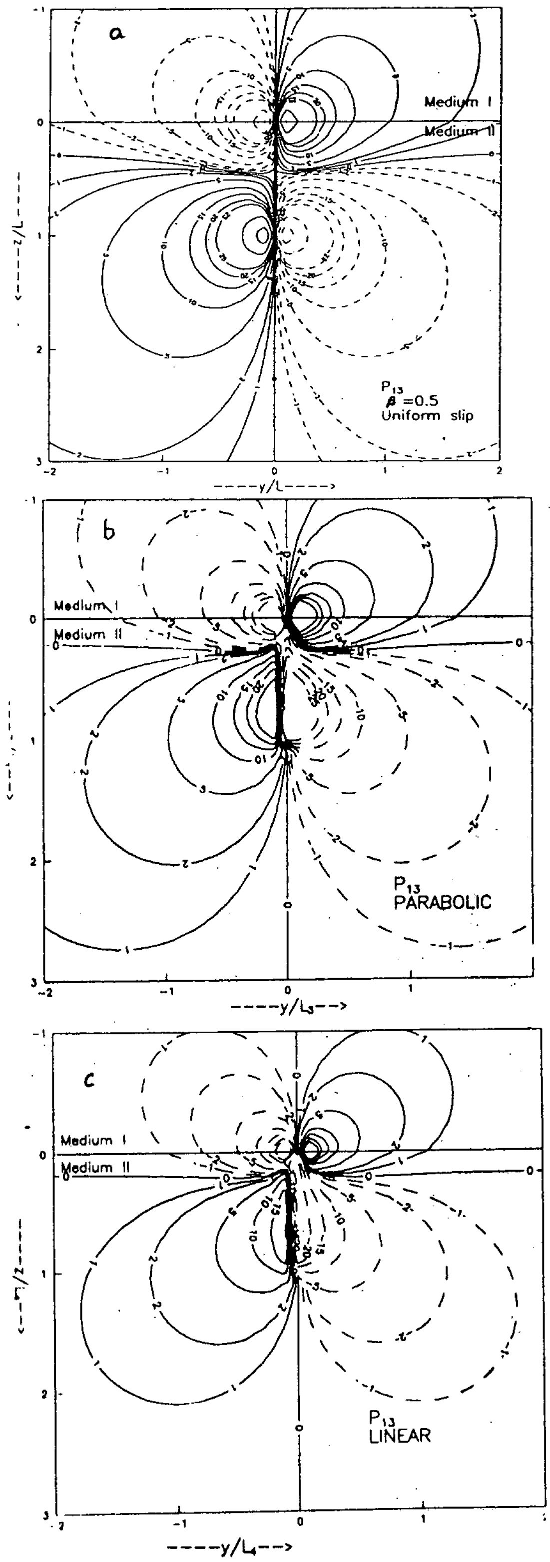

Fig. 5 – Contour maps for the shear stress p_{13} for $\beta = 0.5$. The stress is measured in units of $(\mu_2 b_0/L) \times 10^{-2}$, where L is the fault depth for uniform slip. The distance is measured in units of fault depth for the slip profile under consideration. (a) uniform slip; (b) parabolic and (c) linear.

uniform unbounded medium are obtained. We notice that the non uniformity in slip has only marginal effect on the interface displacement. Moreover, the interface displacement is greater when the source is in the harder medium. Fig. 3 (a, b, c) exhibits the variation of the parallel displacement at fault depth with the distance from the fault. We notice that on the fault tip ($y = 0$), the displacement for parabolic and linear profiles vanishes. Moreover, the near field displacements are significantly affected by the non uniformity of slip.

Contour maps for the dimensionless parallel displacement are given in Fig. 4 (a, b, c) and for the dimensionless shear stress in Fig. 5 (a, b, c) for uniform slip and for parabolic and linear profiles. These maps reveal that the deformation near the fault for different slip profiles is significantly different both qualitatively and quantitatively.

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References

1. Sharma, R.K., Garg, N.R. & Singh, K. (1991) *Proc. Indian Acad. Sci. (Earth Planet. Sci.)* **100** : 379-388.
2. Rani, S. & Singh, S.J. (1993) *Proc. Indian Natn. Sci. Acad.* **59A** : 455-464.
3. Singh, S.J. & Rani, S. (1996) *Proc. Natn. Acad. Sci. (India)* **66A** : 187-215.
4. Chinnery, M.A. & Petrak, J.A. (1968) *Tectonophysics* **5** : 513-529.
5. Freund, L.B. & Barnett, D.M. (1976) *Bull. Seismol. Soc. Am.* **66** : 667-675.
6. Mahrer, K.D. & Nur, A. (1979) *J. Geophys. Res.* **84** : 2296-2302.
7. Yang, M. & Toksöz, M.N. (1981) *J. Geophys. Res.* **86** : 2889-2901.
8. Wang, R. & Wu, H.L. (1983) *Pure Appl. Geophys.* **121** : 601-609.
9. Singh, S.J., Punia, M. & Rani, S. (1994) *Geophys. J. Int.* **118** : 411-427.
10. Ben-Menahem, A. & Singh, S.J. (1981) *Seismic Waves & Sources*. Springer-Verlag, New York.