Proc. Indian natn. Sci. Acad., **63**, A, No. 3, 1997, pp. 225-240.

DEFORMATION OF A LAYERED HALF-SPACE DUE TO A VERY LONG DIP-SLIP FAULT

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(Received 14 October 1996; Accepted 19 December 1996)

The problem of the static deformation of an elastic layer of uniform thickness overlying an elastic half-space caused by a very long dip-slip fault in the layer is solved analytically. Integral expressions for the surface displacements are obtained for a vertical dip-slip fault and a 45° dip-slip fault. The displacements for a dip-slip fault of arbitrary dip can be expressed in terms of the displacements for a vertical dip-slip fault and a 45° dip-slip fault. The integrals involved are evaluated approximately by replacing the integrand by a finite sum of exponential terms. Detailed numerical results showing the variation of the horizontal and vertical displacements with epicentral distance for various source locations in the layer are presented graphically.

Key Words: Dip-Slip Fault; Layered Half-Space; Static Deformation

Introduction

The static deformation of various Earth models caused by two-dimensional sources has been studied by many investigators. Maruyama[1] calculated the Green's functions for two-dimensional elastic dislocations in infinite and semi-infinite Poissonian media. The elastic residual field of a very long strike-slip fault in a layered half-space has been calculated by Rybicki[2], Chinnery and Jovanovich[3] and Singh and Rani[4], amongst others. Freund and Barnett[5] gave a two-dimensional analysis of surface deformation due to dip-slip faulting in a uniform half-space, using the theory of analytic functions of a complex variable.

Singh and Garg[6] obtained integral expressions for the Airy stress function in an unbounded medium due to various two-dimensional sources. Beginning with these results, Rani *et al.*[7] derived integral expressions for the Airy stress function, displacements and stresses in a homogeneous, isotropic, perfectly elastic half-space. The integrals were then evaluated analytically, obtaining closed-form expressions for the Airy stress function, displacements and stresses at any point of the half-space. Rani and Singh[8] began with the closed-form expressions for the Airy stress function for a dip-slip line source of arbitrary dip buried in a uniform half-space given by Rani *et al.*[7] to derive the elastic residual field due to a very long dip-slip fault of finite width. Singh *et al.*[9] obtained closed-form analytical expressions for the displacements and stresses at any point of either of two homogeneous, isotropic, perfectly elastic half-spaces in welded contact caused by various two-dimensional sources.

Two-dimensional dip-slip dislocation models have been used extensively to model the crustal deformation associated with thrust faulting at subduction zones. In the analytical approach, one has to solve the problem of a very long dip-slip fault in a layer (representing lithosphere) overlying a half-space (representing asthenosphere). To this end, Nur and Mavko[10] invoked the exact solution of Mura[11] for an edge dislocation parallel to the boundary of two elastic half-spaces in welded contact. They took into account the effect of the free surface approximately by considering Mura's solution for two equal and opposite edge dislocations with the free surface midway between them. By considering a simpler problem of a very long dip-slip fault in a uniform half-space, Singh and Punia[12] showed that the approximate method used by Nur and Mavko[10] does not yield satisfactory results. Singh and Punia[12] observed that for small dip angles and beyond a certain epicentral distance, while the exact solution predicts subsidence, the approximate solution predicts uplift. Moreover, the approximate solution yields non-zero values for the surface tractions at the free boundary. Thatcher and Rundle[13] have also pointed out that their results, which begin with the exact elastic solution, differ substantially from the approximate results of Nur and Mavko[10]. In fact, while the approximate method used by Nur and Mavko[10] may work very well in the antiplane strain problem of a very long strike-slip fault, it gives unsatisfactory results in the plane strain problem of a very long dip-slip fault.

In this paper, the problem of a very long dip-slip fault in a layer overlying a uniform half-space is solved analytically. The numerical solution is then obtained by using Sneddon's method of approximation (Sneddon[14]; Ben-Menahem and Gillon[15]).

Theory

We consider a two-dimensional approximation in which the displacement components u_i in the x_i-direction ($i = 1, 2, 3$) are independent of the Cartesian coordinate x_1 so that $\frac{\partial}{\partial x_1} \equiv 0$. Under this assumption the plane strain problem ($u_1 = 0$) and the antiplane strain problem ($u_2 = u_3 = 0$) are decoupled and, therefore, can be solved separately. We shall consider the plane strain problem only and use the notation $x = x_1$, $y = x_2$, $z = x_3$.

We consider a model consisting of a homogeneous, isotropic, elastic layer of uniform thickness H overlying a homogeneous, isotropic, elastic half-space. We place the origin of the Cartesian coordinate system (x,y,z) at the free surface with the z-axis vertically downwards. Let λ_1, μ_1 and λ_2, μ_2 be the Lamé constants for the layer and the half-space, respectively.

Let there be a line source parallel to the x-axis passing through the point (0,0,h) of the layer. As shown by Singh and Garg[6], the Airy stress function U_0 for a line source parallel to the x-axis passing through the point (0,0,h) in an infinite medium can be expressed in the form

$$U_0 = \int_0^\infty \left[(L_0 + M_0 k|z-h|)\sin ky + (P_0 + Q_0 k|z-h|)\cos ky\right] k^{-1} e^{-k|z-h|}\, dk \quad ...(1)$$

where the source coefficients L_0, M_0, P_0 and Q_0 are independent of k. Singh and Garg[6] have obtained these source coefficients for various sources.

For a line source parallel to the x-axis acting at the point (0,0,h) of the layer (h<H), the expressions for the Airy stress function for the layer and the half-space are of the form

$$U^{(1)} = U_0 + \int_0^\infty \left[(L_1 + M_1 kz)\sin ky + (P_1 + Q_1 kz)\cos ky\right] k^{-1} e^{-kz}\, dk$$

$$+ \int_0^\infty \left[(L_2 + M_2 kz)\sin ky + (P_2 + Q_2 kz)\cos ky\right] k^{-1} e^{kz}\, dk \quad ...(2)$$

$$U^{(2)} = \int_0^\infty \left[(L_3 + M_3 kz)\sin ky + (P_3 + Q_3 kz)\cos ky\right] k^{-1} e^{-kz}\, dk \quad ...(3)$$

where U_0 is given in equation(1) and the unknowns L_1, M_1, P_1, Q_1; L_2, M_2, P_2, Q_2 and L_3, M_3, P_3, Q_3 are to be determined from the boundary conditions.

The stresses and the displacements in terms of the Airy stress function are given by (Sokolnikoff[16], Section 71)

$$p_{22}^{(i)} = \frac{\partial^2 U^{(i)}}{\partial z^2},\; p_{23}^{(i)} = -\frac{\partial^2 U^{(i)}}{\partial y \partial z},\; p_{33}^{(i)} = \frac{\partial^2 U^{(i)}}{\partial y^2}$$

$$2\mu_1 u_2^{(i)} = -\frac{\partial U^{(i)}}{\partial y} + \frac{1}{2\alpha_i}\int \nabla^2 U^{(i)}\, dy$$

$$2\mu_i u_3^{(i)} = -\frac{\partial U^{(i)}}{\partial z} + \frac{1}{2\alpha_i}\int \nabla^2 U^{(i)}\, dz \quad ...(4)$$

(no summation over i; i = 1 for the layer and i = 2 for the half-space) where

$$\alpha_1 = \frac{\lambda_i + \mu_i}{\lambda_i + 2\mu_i} = \frac{1}{2(1-\sigma_i)}$$

$$\nabla^2 U^{(i)} = p_{22}^{(i)} + p_{33}^{(i)} \quad ...(5)$$

σ being the Poisson's ratio.

We assume that the surface of the layer (z = 0) is traction-free and the layer and the half-space are in welded contact along the plane z = H yielding the boundary conditions

$$
\begin{aligned}
&p_{23}^{(1)} = p_{33}^{(1)} = 0 && \text{at } z = 0 \\
&p_{23}^{(1)} = p_{23}^{(2)},\ p_{33}^{(1)} = p_{33}^{(2)} && \text{at } z = H \\
&u_2^{(1)} = u_2^{(2)},\ u_3^{(1)} = u_3^{(2)} && \text{at } z = H
\end{aligned}
\qquad \text{...(6)}
$$

Let L^-, M^-, P^-, Q^- be the values of L_0, M_0, P_0, Q_0, respectively, valid for z < h and L^+, M^+, P^+, Q^+ be the values of L_0, M_0, P_0, Q_0, respectively, for z > h. Inserting the expressions for the stresses and the displacements into the boundary conditions (6), we obtain two sets of equations for the determination of the twelve unknowns, namely, L_1, M_1, P_1, Q_1, ..., Q_3. These two sets are :

$$
J \begin{bmatrix} L_1 \\ L_2 \\ L_3 \\ M_1 \\ M_2 \\ M_3 \end{bmatrix} = \begin{bmatrix} -(L^- + M^- kh)\, e^{-kh} \\ (L^- - M^- + M^- kh)\, e^{-kh} \\ -(L^+ - M^+ + M^+ k(H-h))\, e^{-k(H-h)} \\ -(L^+ + M^+ k(H-h))\, e^{-k(H-h)} \\ (L^+ + M^+ k(H-h) - M^+/\alpha_1)\, e^{-k(H-h)} \\ -(L^+ - M^+ + M^+ k(H-h) + M^+/\alpha_1)\, e^{-k(H-h)} \end{bmatrix} \qquad \text{...(7)}
$$

and

$$
J \begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ Q_1 \\ Q_2 \\ Q_3 \end{bmatrix} = \begin{bmatrix} -(P^- + Q^- kh)\, e^{-kh} \\ (P^- - Q^- + Q^- kh)\, e^{-kh} \\ -(P^+ - Q^+ + Q^+ k(H-h))\, e^{-k(H-h)} \\ -(P^+ + Q^+ k(H-h))\, e^{-k(H-h)} \\ (P^+ + Q^+ k(H-h) - Q^+/\alpha_1)\, e^{-k(H-h)} \\ -(P^+ - Q^+ + Q^+ k(H-h) + Q^+/\alpha_1)\, e^{-k(H-h)} \end{bmatrix} \qquad \text{...(8)}
$$

where

$$\beta = \mu_1/\mu_2 \qquad \text{...(9)}$$

and J denotes the 6 x 6 matrix

$$
\begin{bmatrix}
1 & 1 & 0 & 0 & 0 & 0 \\
1 & -1 & 0 & -1 & -1 & 0 \\
e^{-kH} & -e^{kH} & -e^{-kH} & (kH-1)\,e^{-kH} & -(1+kH)\,e^{kH} & (1-kH)\,e^{-kH} \\
e^{-kH} & e^{kH} & -e^{-kH} & kHe^{-kH} & kHe^{kH} & -kHe^{-kH} \\
-e^{-kH} & -e^{kH} & \beta e^{-kH} & \left(\dfrac{1}{\alpha_1}-kH\right)e^{-kH} & -\left(\dfrac{1}{\alpha_1}+kH\right)e^{kH} & \beta\left(kH-\dfrac{1}{\alpha_2}\right)e^{-kH} \\
e^{-kH} & -e^{kH} & -\beta e^{-kH} & \left(\dfrac{1}{\alpha_1}-1+kH\right)e^{-kH} & -\left(1+kH-\dfrac{1}{\alpha_1}\right)e^{kH} & -\beta\left(\dfrac{1}{\alpha_2}-1+kH\right)e^{-kH}
\end{bmatrix}
$$

Solving the matrix equations (7) and (8) by Cramer's rule, we obtain

$$
\begin{aligned}
L_1 = \frac{1}{4v^2\,\delta^2\,\delta_1\Delta}\Bigg[\Bigg[&\left\{\delta^2 Z_1\,(1+2kH)\,(L^+ - M^+ kh) + \left(2\,\delta^2 Z_1 k^2 H^2 - \frac{\delta^2 Z_1}{2} + \frac{Z_4}{2}\right)M^+\right\}e^{-k(2H-h)} \\
&+\left\{\left(2\delta^2 Z_1\,k^2\,H^2 - \frac{\delta^2 Z_1}{2} + \frac{Z_4}{2}\right)M^- + \delta^2 Z_1\,(1-2kH)\,(L^- + M^-\,k)\right\}e^{-k(2H+h)} \\
&+\delta\, Z_3\,(L^- + M^- kh)\,e^{-kh} + \delta\, Z_2\,(L^+ - M^+ kh)\,e^{-k(4H-h)}\Bigg] \qquad \text{...(10)}
\end{aligned}
$$

$$
\begin{aligned}
M_1 = \frac{1}{4v^2\,\delta^2\,\delta_1\,\Delta}\Big[&\delta Z_3\,(2L^- - M^- + 2M^- kh)\,e^{-kh} + \delta Z_2 M^+ e^{-k(4H-h)} + \Big\{2\delta^2 Z_1\,(L^- + M^- kh) \\
&-\delta^2 Z_1\,(1+2kH)\,M^-\Big\}\,e^{-k(2H+h)} + \Big\{(Z_4 + \delta^2\,Z_1\,(4k^2\,H^2 - 2kH))\,M^+ \\
&+4\delta^2 Z_1 kH\,(L^+ - M^+ kh)\Big\}e^{-k(2H-h)}\Big] \qquad \text{...(11)}
\end{aligned}
$$

$$
\begin{aligned}
P_1 = \frac{1}{4v^2\delta^2\,\delta_1\,\Delta}\Bigg[\Bigg[&\left\{\delta^2 Z_1\,(1+2kH)\,(P^+ - Q^+ kh) + \left(2\delta^2 Z_1 k^2 H^2 - \frac{\delta^2 Z_1}{2} + \frac{Z_4}{2}\right)Q^+\right\}e^{-k(2H-h)} \\
&+\left\{\left(2\delta^2\,Z_1 k^2 H^2 - \frac{\delta^2 Z_1}{2} + \frac{Z_4}{2}\right)Q^- + \delta^2\,Z_1\,(1-2kH)\,(P^- + Q^- kh)\right\}e^{-k(2H+h)} \\
&+\delta Z_3\,(P^- + Q^- kh)\,e^{-kh} + \delta Z_2\,(P^+ - Q^+ kh)\,e^{-k(4H-h)}\Bigg] \qquad \text{...(12)}
\end{aligned}
$$

$$Q_1 = \frac{1}{4\nu^2 \delta^2 \delta_1 \Delta}\Big[\delta Z_3 (2P^- - Q^- + 2Q^- kh)\, e^{-kh} + \delta Z_2\, Q^+ e^{-k(4H-h)} + \big\{2\delta^2 Z_1 (P^- + Q^- kh)$$

$$-\delta^2 Z_1 (1+2kH)\, Q^-\big\}\, e^{-k(2H+h)} + \big\{(Z_4 + \delta^2 Z_1 (4k^2H^2 - 2kH))\, Q^+ + 4\delta^2 Z_1 kH (P^+ - Q^+ kh)\big\}\, e^{-k(2H-h)}\Big] \quad \ldots(13)$$

$$L_2 = \frac{1}{4\nu^2\delta^2\delta_1\Delta}\left[-\left\{\delta^2 Z_1 (1+2kH)(L^+ - M^+ kh) + \left(2\delta^2 Z_1 k^2H^2 - \frac{\delta^2 Z_1}{2} + \frac{Z_4}{2}\right) M^+\right\} e^{-k(2H-h)}\right.$$

$$+\left\{-\left(2\delta^2 Z_1 k^2 H^2 - \frac{\delta^2 Z_1}{2} + \frac{Z_4}{2}\right) M^- + (\delta^2 Z_1 (1+2kH) + 4\delta^2 k^2 H^2 Z_1 - \delta^2 Z_1 + Z_4)\right.$$

$$\left.\left.(L^- + M^- kh)\right\} e^{-k(2H+h)} + \delta Z_2 (L^- + M^- kh)\, e^{-k(4H+h)} - \delta Z_2 (L^+ - M^+ kh)\, e^{-k(4H-h)}\right] \quad \ldots(14)$$

$$M_2 = \frac{1}{4\nu^2\delta^2\delta_1\Delta}\Big[\delta^2 Z_1 \big\{-4kH(L^- + M^- kh) + (2kH-1) M^-\big\} e^{-k(2H+h)} + \delta^2 Z_1 \big\{2(L^+ - M^+ kh)$$

$$+(2kH-1)\, M^+\big\}\, e^{-k(2H-h)} - \delta Z_2 M^- e^{-k(4H+h)} + \delta Z_2 \big\{2(L^+ - M^+ kh) - M^+\big\} e^{-k(4H-h)}\Big] \quad \ldots(15)$$

$$P_2 = \frac{1}{4\nu^2\delta^2\delta_1\Delta}\left[-\left\{\delta^2 Z_1 (1+2kH)(P^+ - Q^+ kh) + (2\delta^2 Z_1 k^2 H^2 - \frac{\delta^2 Z_1}{2} + \frac{Z_4}{2})\, Q^+\right\} e^{-k(2H-h)}\right.$$

$$+\left\{-\left(2\delta^2 Z_1 k^2 H^2 - \frac{\delta^2 Z_1}{2} + \frac{Z_4}{2}\right) Q^- + (\delta^2 Z_1 (1+2kH) + 4\delta^2 k^2 H^2 Z_1 - \delta^2 Z_1 + Z_4)\right.$$

$$\left.\left.\times (P^- + Q^- kh)\right\} e^{-k(2H+h)} + \delta Z_2 (P^- + Q^- kh)\, e^{-k(4H+h)} - \delta Z_2 (P^+ - Q^+ kh)\, e^{-k(4H-h)}\right] \quad \ldots(16)$$

$$Q_2 = \frac{1}{4\nu^2\delta^2\delta_1\Delta}\Big[\delta^2 Z_1 \big\{-4kH (P^- + Q^- kh) + (2kH-1)\, Q^-\big\} e^{-k(2H+h)} + \delta^2 Z_1 \big\{2(P^+$$

$$- Q^+ kh) + (2kH-1)\, Q^+\big\}\, e^{-k(2H-h)} - \delta Z_2 Q^- e^{-k(4H+h)} + \delta Z_2 \big\{2(P^+ - Q^+ kh) - Q^+\big\}\, e^{-k(4H-h)}\Big] \ldots(17)$$

$$L_3 = \frac{1}{\Delta}\left[\left[\left\{\left(\frac{2}{\alpha_1 \nu\delta}(\nu+\delta) - \frac{1}{2\nu^2\delta^2\delta_1}(Z_4 - \delta^2 Z_1) kH\right)(L^- + M^- kh) - \frac{1-2kH}{8\nu^2\delta^2\delta_1}(Z_4 - \delta^2 Z_1) M^-\right\} e^{-kh}\right.\right.$$

$$+\left\{\frac{2kH-1}{8\nu^2\delta^2\delta_1}(Z_4 - \delta^2 Z_1) M^+ - \frac{2(\nu\delta_1 + 1)}{\alpha_1 \nu\delta_1}(L^+ - M^+ kh)\right\} e^{kh} + \left\{\frac{2}{\alpha_1 \nu}(\nu-1)(1-2kH)(L^- + M^- kh) + \right.$$

$$+\left.\left(\frac{1}{8\nu^2\delta^2\delta_1}(Z_4 - \delta^2 Z_1) + \frac{4(\nu-1)\, k^2H^2}{\alpha_1 \nu}\right) M^-\right\} e^{-k(2H+h)}$$

$$+\left\{\left(\frac{2(\delta-\nu\delta_1)}{\alpha_1\nu\delta\delta_1}+\frac{4}{\alpha_1\nu}(\nu-1)(1-2kH)kH\right)(L^+-M^+kh)\right.$$

$$\left.+\left(\frac{1}{8\nu^2\delta^2\delta_1}(Z_4-\delta^2Z_1)+\frac{4(\nu-1)k^2H^2}{\alpha_1\nu}\right)M^+\right\}e^{-k(2H-h)}\Bigg] \quad ...(18)$$

$$M_3=\frac{1}{\Delta}\Big[\left\{-2(\beta-1)(L^-+M^-kh)+(\beta-1)(1+2kH)M^-\right\}e^{-k(2H+h)}-\frac{(\nu+\delta)}{\nu\delta}M^+e^{kh}$$

$$+\left\{-4kH(L^+-M^+kh)+(1+2kH)M^+\right\}(\beta-1)e^{-k(2H-h)}+\frac{(\nu+\delta)}{\nu\delta}(2(L^-+M^-kh)-M^-)e^{-kh}\Big] \quad ..(19)$$

$$P_3=\frac{1}{\Delta}\Bigg[\left\{\left(\frac{2}{\alpha_1\nu\delta}(\nu+\delta)-\frac{1}{2\nu^2\delta^2\delta_1}(Z_4-\delta^2Z_1)kH\right)(P^-+Q^-kh)-\frac{1-2kH}{8\nu^2\delta^2\delta_1}(Z_4-\delta^2Z_1)Q^-\right\}e^{-kh}$$

$$+\left\{\frac{2kH-1}{8\nu^2\delta^2\delta_1}(Z_4-\delta^2Z_1)Q^+-\frac{2(\nu\delta_1+1)}{\alpha_1\nu\delta_1}(P^+-Q^+kh)\right\}e^{kh}+\left\{\frac{2}{\alpha_1\nu}(\nu-1)(1-2kH)(P^-+Q^-kh)\right.$$

$$\left.+\left(\frac{1}{8\nu^2\delta^2\delta_1}(Z_4-\delta^2Z_1)+\frac{4(\nu-1)k^2H^2}{\alpha_1\nu}\right)Q^-\right\}e^{-(2H+h)}+\left\{\left(\frac{2(\delta-\nu\delta_1)}{\alpha_1\nu\delta\delta_1}+\frac{4}{\alpha_1\nu}(\nu-1)(1-2kH)kH\right)\right.$$

$$\left.\times(P^+-Q^+kh)+\left(\frac{1}{8\nu^2\delta^2\delta_1}(Z_4-\delta^2Z_1+\frac{4(\nu-1)k^2H^2}{\alpha_1\nu}\right)Q^+\right\}e^{-k(2H-h)}\Bigg] \quad ...(20)$$

$$Q_3=\frac{1}{\Delta}\Big[\left\{-2(\beta-1)(P^-+Q^-kh)+(\beta-1)(1+2kH)Q^-\right\}e^{-k(2H+h)}-\frac{(\nu+\delta)}{\nu\delta}Q^+e^{kh}$$

$$+\left\{-4kH(P^+-Q^+kh)+(1+2kH)Q^+\right\}(\beta-1)e^{-k(2H-h)}+\frac{(\nu+\delta)}{\nu\delta}\left\{(2(P^-+Q^-kh)-Q^-)\right\}e^{-kh}\Big] \quad ...(21)$$

where

$$\Delta=|J|=-\frac{1}{4\nu^2\delta^2\delta_1}\left[\delta Z_3+(Z_4+\delta^2Z_1+4\delta^2k^2H^2Z_1)e^{-2kH}+\delta Z_2e^{-4kH}\right] \quad ...(22)$$

$$\frac{1}{\delta}=\frac{2}{\alpha_1}-1=3-4\sigma_1,\ \frac{1}{\delta_1}=\frac{2}{\alpha_2}-1=3-4\sigma_2$$

$$\nu=\mu_2/\mu_1=1/\beta$$
$$Z_1=4(\nu-1)(\nu\delta_1+1)$$
$$Z_2=4(\nu-1)(\nu\delta_1-\delta)$$
$$Z_3=4(\nu+\delta)(\nu\delta_1+1)$$
$$Z_4=4(\nu+\delta)(\nu\delta_1-\delta) \quad ...(23)$$

The determinant Δ of equation (22) coincides with the corresponding determinant given by Ben-Menahem and Singh[17] (equation 11-49) except for a multiplying constant. The quantities Z_1, Z_2, Z_3, Z_4 defined in equation (23) are also from Ben-Menahem and Singh[17] (equation 11-52).

Equations (1) - (5) yield the expressions for the stresses and the displacements. On putting $z = 0$ and inserting the values of the constants L_i, M_i, etc. given above, we obtain the following expressions for the surface displacements

$$
\begin{aligned}
2\mu_1 u_2^{(1)} = \int_0^\infty \Bigg[\Bigg\{ & \frac{1}{4\alpha_1 \nu^2 \delta^2 \delta_1 \Delta} \Big[\left\{ 2\delta Z_3 (L^- + M^- kh) - \delta Z_3 M^- \right\} e^{-kh} - 2\delta Z_2 \left\{ (L^+ - M^+ kh) \right. \\
& \left. - M^+ \right\} e^{-k(4H-h)} + \delta Z_2 M^- e^{-k(4H+h)} + \delta^2 Z_1 \left\{ 2(1+2kH)(L^- + M^- kh) - 4kHM^- \right\} e^{-k(2H+h)} \\
& + \left\{ 2\delta^2 Z_1 (2kH-1)(L^+ - M^+ kh) + (Z_4 + \delta^2 Z_1 (4k^2H^2 - 4kH + 1)) M^+ \right\} e^{-k(2H-h)} \Big] \\
& + \frac{M^-}{\alpha_1} e^{-kh} \Bigg\} \cos ky - \Bigg\{ \frac{1}{4\alpha_1 \nu^2 \delta^2 \delta_1 \Delta} \Big[\left\{ 2\delta Z_3 (P^- + Q^- kh) - \delta Z_3 Q^- \right\} e^{-kh} \\
& - 2\delta Z_2 \left\{ P^+ - Q^+ kh - Q^+ \right\} e^{-k(4H-h)} + \delta Z_2 Q^- e^{-k(4H+h)} + \delta^2 Z_1 \left\{ 2(1+2kH)(P^- + Q^- kh) \right. \\
& \left. - 4kHQ^- \right\} e^{-k(2H+h)} + \left\{ 2\delta^2 Z_1 (2kH-1)(P^+ - Q^+ kh) + (Z_4 + \delta^2 Z_1 (4k^2 H^2 \right. \\
& \left. - 4kH + 1)) Q^+ \right\} e^{-k(2H-h)} \Big] + \frac{Q^-}{\alpha_1} e^{-kh} \Bigg\} \sin ky \Bigg] dk
\end{aligned} \qquad \ldots(24)
$$

$$
\begin{aligned}
2\mu_1 u_3^{(1)} = \int_0^\infty \Bigg[\Bigg\{ & \frac{1}{4\alpha_1 \nu^2 \delta^2 \delta_1 \Delta} \Big[\left\{ 2\delta Z_3 (L^- + M^- kh) - \delta Z_3 M^- \right\} e^{-kh} + 2\delta Z_2 (L^+ \\
& - M^+ kh) e^{-k(4H-h)} - \delta Z_2 M^- e^{-k(4H+h)} + \delta^2 Z_1 \left\{ 2(1-2kH)(L^- + M^- kh) \right. \\
& \left. - 2M^- \right\} e^{-k(2H+h)} + \left\{ 2\delta^2 Z_1 (2kH+1)(L^+ - M^+ kh) + (Z_4 + \delta^2 Z_1 (4k^2 H^2 \right. \\
& \left. - 1)) M^+ \right\} e^{-k(2H-h)} \Big] - \frac{M^-}{\alpha_1} e^{-kh} \Bigg\} \sin ky + \Bigg\{ \frac{1}{4\alpha_1 \nu^2 \delta^2 \delta_1 \Delta} \\
& \times \Big[\left\{ 2\delta Z_3 (P^- + Q^- kh) - \delta Z_3 Q^- \right\} e^{-kh} + 2\delta Z_2 (P^+ - Q^+ kh) e^{-k(4H-h)} - \delta Z_2 Q^- e^{-k(4H+h)} \\
& + \delta^2 Z_1 \left\{ 2(1-2kH)(P^- + Q^- kh) - 2Q^- \right\} e^{-k(2H+h)} + \\
& + \left\{ 2\delta^2 Z_1 (2kH+1)(P^+ - Q^+ kh) + (Z_4 + \delta^2 Z_1 (4k^2 H^2 - 1)) Q^+ \right\} e^{-k(2H-h)} \Big] \\
& - \frac{Q^-}{\alpha_1} e^{-kh} \Bigg\} \cos ky \Bigg] dk
\end{aligned} \qquad \ldots(25)
$$

Dip-Slip Fault

The source coefficients for a vertical dip-slip line source (Fig. 1a) in the layer are given by (Singh and Garg[6], Rani *et al.*[7])

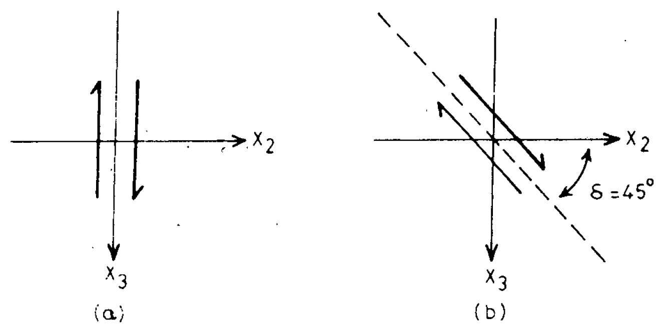

Fig 1 Representation of a (a) vertical dip-slip line source; (b) 45° dip-slip line source. X_1-axis is parallel to the line source

$$L^- = L^+ = P^- = P^+ = Q^- = Q^+ = 0$$

$$M^- = -(1/\pi)\alpha_1\mu_1\, bds,\quad M^+ = (1/\pi)\alpha_1\mu_1 bds \qquad \text{...(26)}$$

where b is the slip and ds is the width of line source. On putting these values of the source coefficients in equations(24) and (25), the expressions for the surface displacements for a vertical dip-slip line source in the layer are found to be

$$u_2^{(1)} = \frac{bds}{2\pi}\int_0^\infty \Bigg[\frac{1}{4\nu^2\delta^2\delta_1\Delta}\Big\{\delta Z_3(1-2kh)\,e^{-kh} + 2\delta Z_2(1+kh)\,e^{-k(4H-h)} - \delta Z_2\,e^{-k(4H+h)}$$
$$+2\delta^2 Z_1(2kH - kh - 2k^2hH)\,e^{-k(2H+h)} + \big\{Z_4 + \delta^2 Z_1(4k^2H^2$$
$$-4kH + 1 - 2kh(2kH-1))\big\}\,e^{-k(2H-h)}\Big\} - e^{-kh}\Bigg]\cos ky\, dk \qquad \text{...(27)}$$

$$u_3^{(1)} = \frac{bds}{2\pi}\int_0^\infty \Bigg[\frac{1}{4\nu^2\delta^2\delta_1\Delta}\Big\{\delta Z_3(1-2kh)\,e^{-kh} - 2\delta Z_2\, khe^{-k(4H-h)} + \delta Z_2\,e^{-k(4H+h)}$$
$$+2\delta^2 Z_1(1 - kh + 2k^2hH)\,e^{-k(2H+h)} + \big\{Z_4 + \delta^2 Z_1(4k^2H^2 - 2kH - 1$$
$$-4k^2hH)\big\}\,e^{-k(2H-h)} + e^{-kh}\Bigg]\sin ky\, dk \qquad \text{...(28)}$$

Taking the limit H_∞, equations (27) and (28) yield the following expressions for the surface displacements for a vertical dip-slip line source in a uniform half-space

$$u_2 = \frac{bds}{\pi} \int_0^\infty e^{-kh} (kh-1) \cos ky \, dk$$

$$= \frac{bds}{\pi} \left[-\frac{2hy^2}{(y^2+h^2)^2} \right]$$

$$u_3 = \frac{bds}{\pi} \int_0^\infty e^{-kh} kh \sin ky \, dk$$

$$= \frac{bds}{\pi} \left[\frac{2yh^2}{(y^2+h^2)^2} \right].$$

The above expressions coincide with the corresponding results of Maruyama[1].

The source coefficients for a dip-slip line source dipping at 45º (Fig. lb) situated in the layer are given by

$$L^- = L^+ = M^- = M^+ = P^- = P^+ = 0$$

$$Q^- = Q^+ = (1/\pi)\, \alpha_1 \, \mu_1 \, bds \qquad \text{...(29)}$$

From equations (24), (25) and (29) the corresponding expressions for the displacement components are

$$u_2^{(1)} = \frac{bds}{2\pi} \int_0^\infty \left[\frac{1}{4v^2\delta^2\delta_1\Delta} \left\{ \delta Z_3 (2kh-1)\, e^{-kh} + 2\delta Z_2 (1+kh)\, e^{-k(4H-h)} + \delta Z_2 \, e^{-k(4H+h)} \right. \right.$$

$$+ 2\delta^2 Z_1 (kh - 2kH + 2k^2hH)\, e^{-k(2H+h)} + \left\{ Z_4 + \delta^2 Z_1 \right.$$

$$\left. \left. \times (4k^2H^2 - 4kH + 1 - 2kh(2kH-1)) \right\} e^{-k(2H-h)} \right\} + e^{-kh} \Bigg] \sin ky \, dk \qquad \text{...(30)}$$

$$u_3^{(1)} = -\frac{bds}{2\pi} \int_0^\infty \left[\frac{1}{4v^2\delta^2\delta_1\Delta} \left\{ \delta Z_3 (2kh-1)\, e^{-kh} - 2\delta Z_2 \, khe^{-k(4H-h)} \right. \right.$$

$$-\delta Z_2 \, e^{-k(4H+h)} + 2\delta^2 Z_1 (kh - 2k^2hH - 1)\, e^{-k(2H+h)}$$

$$\left. + \left\{ Z_4 + \delta^2 Z_1 (4k^2H^2 - 1 - 2kh - 4k^2hH \right\} e^{-k(2H-h)} \right\} - e^{-kh} \Bigg] \cos ky \, dk \qquad \text{...(31)}$$

Taking the limit $H \to \infty$, equations (30) and (31) yield the following expressions for the surface displacements for a 45° dip-slip line source in a uniform half-space

$$u_2 = \frac{bds}{\pi} \int_0^\infty e^{-kh} (1 - kh) \sin ky \, dk$$

$$= \frac{bds}{\pi} \left[\frac{y(y^2 - h^2)}{(y^2 + h^2)^2} \right]$$

$$u_3 = \frac{bds}{\pi} \int_0^\infty e^{-kh} kh \cos ky \, dk$$

$$= \frac{bds}{\pi} \left[\frac{h(h^2 - y^2)}{(y^2 + h^2)^2} \right].$$

The displacement components due to dip-slip on an inclined plane can be expressed in the form

$$u = -\cos 2\delta \; u(VDS) + \sin 2\delta \, u\,(45^\circ\, DS) \qquad ...(32)$$

where δ is the dip angle and u(VDS) is given by equation (27) or (28) and u(45°DS) is given by equation (30) or (31), depending upon the component under reference.

Numerical Procedure

The integrals appearing in equations (27), (28), (30) and (31) are of the form

$$\int_0^\infty \frac{G}{\Delta} e^{-kp} k^q \begin{pmatrix} \cos & ky \\ \sin & ky \end{pmatrix} dk \qquad ...(33)$$

where

$$q = 0, 1, 2\,; G = -\frac{\delta Z_3}{4v^2\delta^2\delta_1}\,; p = h, 2H \pm h, 4H \pm h \qquad ...(34)$$

The occurrence of the factor $\frac{1}{\Delta}$ in the integrand makes integration by analytical methods difficult. However, Sneddon[14] suggested a method of approximate evaluation by which this difficulty is overcome. In this method, the factor Δ^{-1} is replaced by a finite sum of exponential terms in such a way that the error is made as small as desired. Once the integral is expressed in this way, an exact quadrature can be made. From (22) and (34), we have

$$\frac{G}{\Delta} = \frac{1}{1 + (A + Bk^2H^2)\, e^{-2kH} + De^{-4kH}} \qquad ...(35)$$

where

$$A = \frac{Z_4 + \delta^2 Z_1}{\delta Z_3}, B = \frac{4\delta Z_1}{Z_3}, D = \frac{Z_2}{Z_3} \qquad ...(36)$$

By binomial expansion, we have

$$\frac{G}{\Delta} = 1-(A+Bk^2H^2)e^{-2kH} + (A^2-D + 2ABk^2H^2 + B^2k^4H^4)e^{-4kH} + \ldots \quad \ldots (37)$$

As in Ben-Menahem and Gillon[15], we make the approximation

$$\frac{G}{\Delta} \simeq 1 - (A + Bk^2H^2)e^{-2kH} + (C + \alpha' k^n H^n)e^{-\beta' kH} \quad \ldots (38)$$

over the entire range of integration, where

$$C = \frac{A^2 + D(A-1)}{1+A+D}, \; n = 1, 2, 3, 4, \ldots.. \quad \ldots (39)$$

and α', β' (>2) are to be chosen in such a way so as to ensure a best fit in the least-square sense. The value of C is obtained by equating (35) and (38) for H = 0. Clearly α', β' and n have to be re-evaluated for each set of values of the parameters H, δ, δ_1 and ν. Using the approximation (38), the integral (33) can be expressed as a linear combination of standard integrals of the form (Gradshteyn and Ryzhik[18])

$$C_m(x,y) = \int_0^\infty k^m e^{-kx} \cos ky \, dk = (-1)^m \frac{\partial^m}{\partial x^m}\left(\frac{x}{x^2+y^2}\right) \quad \ldots (40)$$

$$S_m(x,y) = \int_0^\infty k^m e^{-kx} \sin ky \, dk = (-1)^m \frac{\partial^m}{\partial x^m}\left(\frac{y}{x^2+y^2}\right)$$

$$(m = 0, 1, 2, 3, \ldots.) \quad \ldots (41)$$

Ben-Menahem and Gillon[15] used the least-square procedure to obtain a suitable approximation of G/Δ for different values of n in equation (38) and found that for realistic Earth models, n = 2 yields satisfactory results. Therefore, we use the approximation

$$\frac{G}{\Delta} = 1-(A+Bk^2H^2)e^{-2kH} + (C + \alpha' k^2H^2)e^{-\beta' kH} \quad \ldots (42)$$

Inserting the expression (42) for G/Δ in equations (27), (28), (30) and (31), it is found that the surface displacements can be expressed as a linear combination of $C_m(x,y)$ and $S_m(x,y)$ with m = 0, 1, 2, 3, 4.

For numerical computations, the values for the parameters ν, σ_1, σ_2, α', β' are taken from Ben-Menahem and Gillon[15]. These values for three crustal models are given in Table I for ready reference.

Table I

$\nu = \frac{\mu_2}{\mu_1}$	σ_1	σ_2	α'	β'
1. 76 (oceanic)	0.27	0.27	0.438716	3.31986
2. 22 (continental)	0.27	0.27	0.703604	3.22888
5. 00	0.23	0.30	1.175744	2.92960

Figs. 2–4 show the variation of the horizontal and vertical displacements at the surface with the horizontal distance from the fault for the rigidity contrast $\frac{\mu_2}{\mu_1} = 1.76$ (oceanic crust model) for three values of the fault depth, viz., h = 0.1H, 0.5H and 0.9H, respectively. Figs 5–7 show the variation of the displacements with the distance from

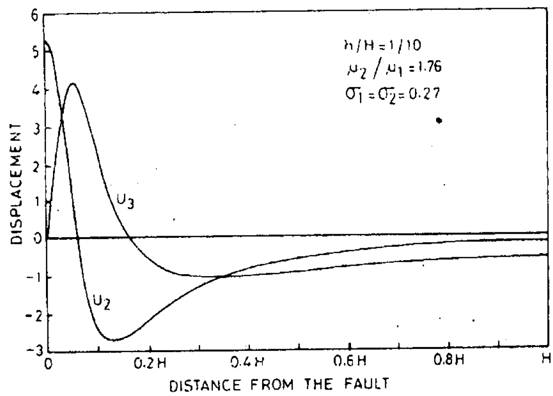

Fig 2 Variation of the dimensionless horizontal (U_2) and vertical (U_3) surface displacements with the distance from a vertical dip-slip line source situated at a depth h = 0.1H in an oceanic Earth model (μ_2/μ_1 = 1.76) consisting of a layer of uniform thickness H overlying a half-space. The displacements are measured in units of bds/πH, where *b* is the slip on the fault and *ds* is the fault width.

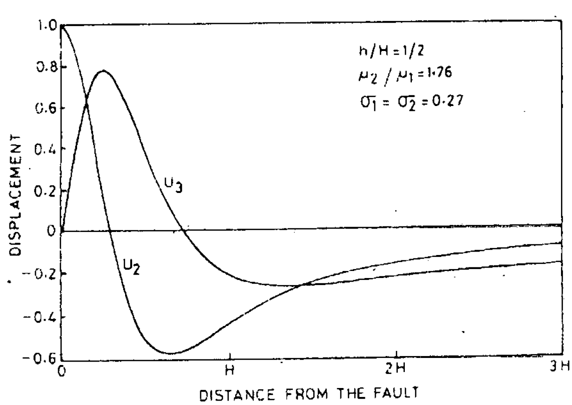

Fig 3 Same as in Fig 2 for the source depth h = 0.5H

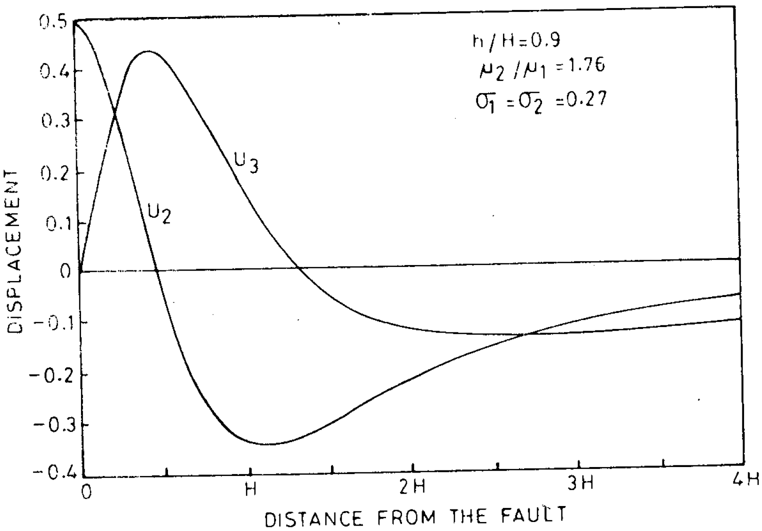

Fig 4 Same as in Fig 2 for the source depth h = 0.9H

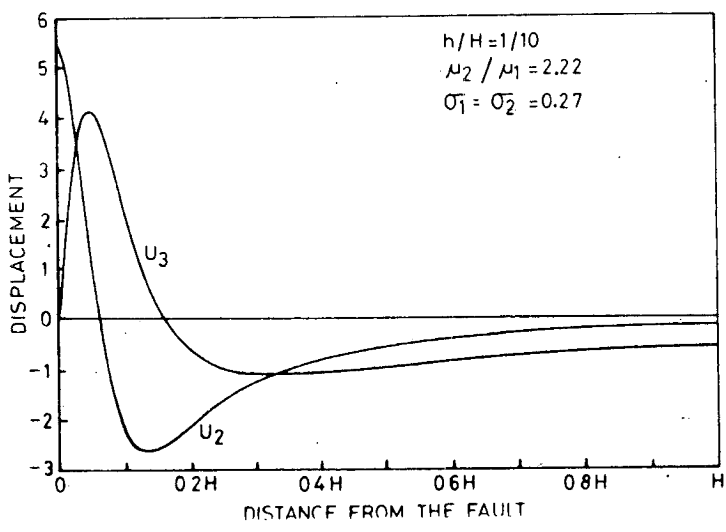

Fig 5 Variation of the dimensionless surface displacements when the line source is situated at a depth h = 0.1H in a continental Earth model (μ_2/μ_1 = 2.22)

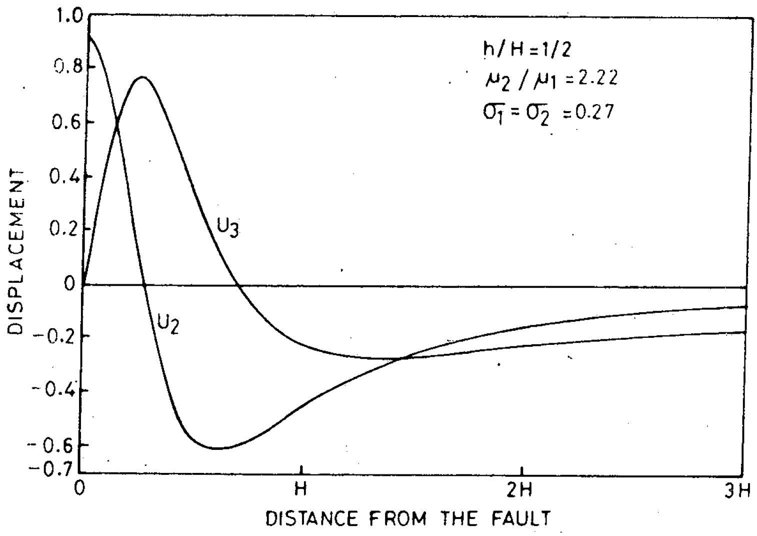

Fig 6 Same as in Fig. 5 for the source depth h = 0.5H

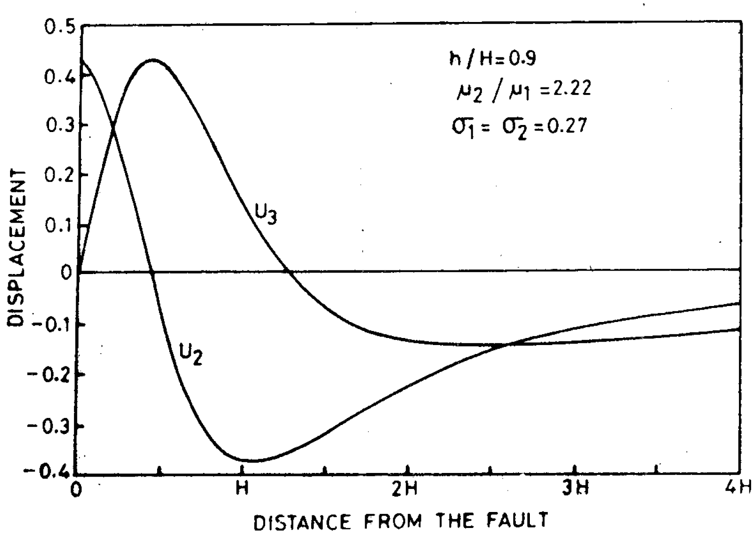

Fig 7 Same as in Fig 5 for the source depth h = 0.9H

the fault for the rigidity contrast $\frac{\mu_2}{\mu_1} = 2.22$ (continental crust model). In all the cases, the vertical displacement u_3 is zero at $y = 0$. Both vertical and horizontal displacements tend to zero as y tends to infinity. Source depth has a significant effect on the magnitude of the displacements. The maximum and the minimum values of the displacements decrease as the source depth increases. The epicentral distance at which the horizontal or the vertical displacement changes sign increases as the source depth increases. For realistic Earth models, the maximum and the minimum values of the displacements are almost inversely proportional to the source depth and the epicentral distance at which the horizontal or the vertical displacement changes sign is almost directly proportional to the source depth.

Acknowledgement

Financial support from the Council of Scientific and Industrial Research (CSIR), New Delhi through a Research Scheme sanctioned to SJS is gratefully acknowledged.

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