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2-D modelling of crustal deformation associated with strike-slip and dip-slip faulting in the Earth

SARVA JIT SINGH and SUNITA RANI*

Department of Mathematics, Maharshi Dayanand University, Rohtak-124001, India.

* *Department of Mathematics, Kurukshetra University Postgraduate Regional Centre, Hisar-125001, India.*

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Abstract

In this article, we review the step-by-step progress made in the recent past in the direction of forward modelling of the crustal deformation associated with strike-slip and dip-slip faulting in the Earth. We confine ourselves mainly to those studies which strive to find analytical closed-form solutions of the corresponding two-dimensional problems. Three Earth models are considered : a uniform half-space, two half-spaces in welded contact and a layered half-space. The effect of nununiform slip on the fault is also discussed.

(**Keywords** : co-seismic deformation/crustal deformation/dip-slip fault/displacement dislocation/half-space/long fault/nonuniform slip/strike-slip fault)

Introduction

The elasticity theory of dislocations developed by Steketee[1,2], Maruyama[3,4] and others, has proved to be a very useful tool in the analysis of processes taking place in tectonically active areas. Dislocations are commonly used as models of earthquake foci and are also applied in the interpretation of observed aseismic deformations. As a mathematical model of faulting, Steketee[1,2] assumed a displacement dislocation surface, i.e. a surface across which the displacement vector is discontinuous. Extensive reviews of the applications of the elasticity theory of dislocations to earthquake faulting have been given by Savage[5], Mavko[6], and Rybicki[7]. These reviews include thorough discussions of 2-D models of faulting. Although a 2-D model is an oversimplification of the physical system, these models are very useful in gaining insight into the relationsip among various fault parameters. Moreover, there are faults, the most obvious being the San Andreas fault in California, which are sufficiently long and shallow that the 2-D approximation may be used.

The crustal deformation cycle can be divided into four time phases relative to the earthquake : interseismic, preseismic, coseismic and postseismic. Coseismic deformation fields have been observed for strike-slip and subduction zone thrust earthquakes. This deformation is produced by the strain relief upon dynamic faulting. The duration of this process is relatively short, of the order of a few minutes at the most. The medium in which faulting occurs can be considered as perfectly elastic for this time scale. The coseismic phase is well explained by dislocation models of faulting in the Earth.

In order to model crustal deformations associated with faulting at a transform plate boundary, the problem of a long inclined strike-slip fault in a layer overlying a uniform half-space is considered (see, e.g. Rybicki[8], Nur and Mavko[9], Garg and Singh[10], Singh and Rani[11,12]). Closed-form expressions for the static displacements, strains and stresses are obtained when the two media are elastic. The correspondence principle[13-16] is then used to obtain the viscoelastic quasi-static field when the layer is elastic and the half-space is Maxwell viscoelastic. In this model the elastic layer represents the lithosphere and the Maxwell half-space represents the asthenosphere. The coseismic field is given by the static response and the postseismic field is given by the quasi-static response minus the static response.

In order to model crustal deformations associated with thrust faulting at a subduction zone, the problem of a long inclined dip-slip fault in a uniform half-space is considered (see, e.g.Freund and Barnett[17], Rani *et al.*[18], Rani and Singh[19], Singh and Rani[20]). Closed-form expressions for the static displacements, strains and stresses are first obtained. The quasi- static viscoelastic field is then obtained from the static elastic field with the help of the correspondence principle. It is customary to assume Maxwell rheology for the half-space for which the initial viscoelastic response coincides with the elastic response. The coseismic deformation is modelled with the static elastic field. To model the postseismic deformation field, the static reponse is subtracted from the quasi-static response. It is found that surface displacements are independent of the elastic moduli. Therefore, at the surface, the quasi-static displacements are identical with the static displacements. It is thus apparent that a simple half-space model is not adequate to explain time-dependent postseismic surface displacements. To calculate the deformation field due to a thrust fault in the lithosphere-asthenosphere composite, one must solve the problem of a long inclined dip-slip fault in an elastic layer overlying a viscoelastic half-space. A beginning has been made by Punia[21] by solving analytically the problem of a dip-slip line source in an elastic layer overlying an elastic half-space. The integrals occuring in the elastic solution have been evaluated approximately by Sneddon's method (see, Sneddon[22], Ben-Menahem and Gillon[23]). For multilayered half-space, one can use the Thomson-Haskell matrix method[24–26] to compute the deformation field.

Uniform Half-Space

Okada[27] obtained a complete set of analytical expressions for the deformation due to three-dimensional shear and tensile faults in a half-space. We present in a uniform notation a complete set of closed-form analytical expressions for the subsurface displacements and strains due to two-dimensional, inclined, strike-slip and dip-slip faults in a uniform half-space. The expressions for the displacements have appeared in the literature in various forms. Most of the results for the strains are taken from Singh and Rani[28]. The stresses can be obtained by a direct application of the generalized Hooke's law.

The problem of the static deformation of a uniform half-space by a long strike-slip fault has been discussed by several investigators. Maruyama[4] calculated the Green's functions for two-dimensional elastic dislocations in a Poissonian half-space. Freund and Barnett[17] gave a two - dimensional analysis of surface deformation due to dip -slip faulting in a half-space, using the theory of analytic functions of a complex variable. Rani *et al.*[18] obtained closed-form expressions for the displacements and stresses at any point of a uniform half-space caused by various two-dimensional sources. Rani and Singh[19] derived the expressions for the displacements and stresses at any point of a uniform half-space due to a dip-slip fault of finite width and infinite length.

Let the Cartesian coordinates be denoted by (x_1, x_2, x_3) with the x_3-axis vertically downwards. Consider a two-dimensional approximation in which the displacement componets u_1, u_2 and u_3 are independent of x_1 so that $\partial/\partial x_1 \equiv 0$. Under this assumption, the plane strain problem ($u_1 = 0$) and the antiplane strain problem ($u_2 = u_3 = 0$) are independent of each other. The problem of a long strike-slip fault striking in the x_1-direction located in a uniform half-space occupying the region $x_3 \geq 0$ is an antiplane strain problem. In contrast, the problem of a long dip-slip fault striking in the x_1-direction located in a uniform half-space occupying the region $x_3 \geq 0$ is a plane strain problem. We assume that the boundary $x_3 = 0$ of the half-space is traction-free and that the line source, which is parallel to the x_1-axis, passes through the point $(0, y_2, y_3)$. We use the notation

$$R^2 = (x_2 - y_2)^2 + (x_3 - y_3)^2 \tag{1}$$

$$= (x_2 - s\cos\delta)^2 + (x_3 - s\sin\delta)^2, \tag{2}$$

$$S^2 = (x_2 - y_2)^2 + (x_3 + y_3)^2 \tag{3}$$

$$= (x_2 - s\cos\delta)^2 + (x_3 + s\sin\delta)^2, \tag{4}$$

where (s, δ) are the polar coordinates corresponding to the Cartesian coordinates (y_2, y_3), so that (Fig. 1)

$$y_2 = s \cos \delta, \; y_3 = s \sin \delta. \tag{5}$$

We also put

$$\alpha = \frac{\lambda + \mu}{\lambda + 2\mu} = \frac{1}{2(1 - \sigma)}, \tag{6}$$

where λ, μ are the Lamé constants and σ is the Poisson's ratio.

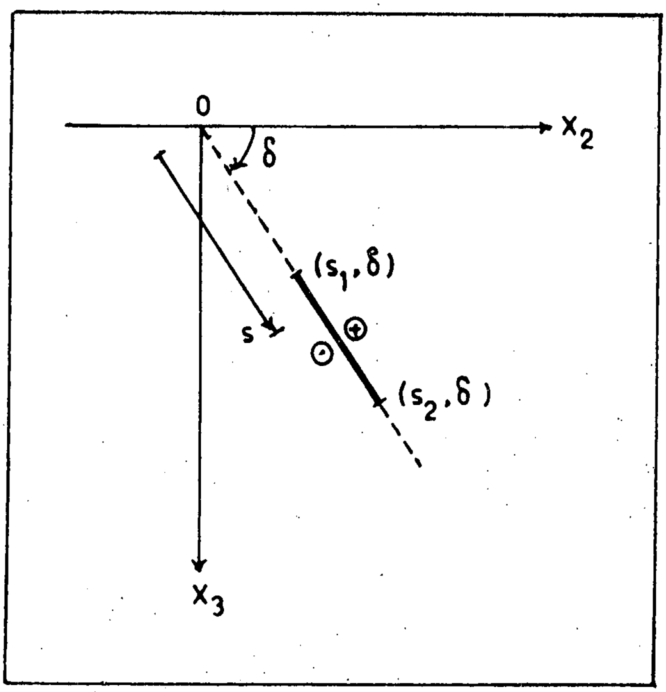

Fig. 1 – Geometry of a long strike-slip fault. The displacement discontinuity on the fault is parallel to the x_1-axis. The sign ⊙ indicates displacement in the direction of the x_1-axis, the sign ⊗ in the opposite direction. The Cartesian coordinates of a point on the fault are (y_2, y_3) and its polar coordinates (s, δ), where δ is the dip angle and $s_1 \leq s \leq s_2$.

Strike-Slip Fault in a Uniform Half-Space

(i) *Line source*

For a line source of the strike-slip type parallel to the x_1 -axis and passing through the point $(0, y_2, y_3)$ of a uniform half-space $x_3 \geq 0$, the deformation field is given by

$$u_1 = \frac{bds}{2\pi}\left[\cos\delta\left(\frac{x_3 - y_3}{R^2} - \frac{x_3 + y_3}{S^2}\right) - \sin\delta\,(x_2 - y_2)\left(\frac{1}{R^2} + \frac{1}{S^2}\right)\right] \tag{7}$$

$$\frac{\partial u_1}{\partial x_2} = \frac{bds}{2\pi}\left\{2(x_2 - y_2)\cos\delta\left(\frac{y_3 - x_3}{R^4} + \frac{x_3 + y_3}{S^4}\right)\right.$$

$$\left. - \sin\delta\left[-\frac{1}{R^2} + 2\frac{(x_3 - y_3)^2}{R^4} - \frac{1}{S^2} + 2\frac{(x_3 + y_3)^2}{S^4}\right]\right\} \tag{8}$$

$$\frac{\partial u_1}{\partial x_3} = \frac{bds}{2\pi}\left\{\cos\delta\left[\frac{1}{R^2} - 2\frac{(x_3 - y_3)^2}{R^4} - \frac{1}{S^2} + 2\frac{(x_3 + y_3)^2}{S^4}\right]\right.$$

$$\left. + 2\sin\delta\,(x_2 - y_2)\left(\frac{x_3 - y_3}{R^4} + \frac{x_3 + y_3}{S^4}\right)\right\} \tag{9}$$

where b is the displacement discontinuity (slip) in the x_1-direction, ds is the width of the line dislocation and δ is the dip angle (Fig. 1).

(ii) *Finite fault*

For a strike-slip fault of finite width $L = s_2 - s_1$ and infinite length (Fig. 1) located in a uniform half-space $x_3 \geq 0$ parallel to the x_1-axis, the deformation field is given by

$$u_1 = \frac{b}{2\pi}\left[\tan^{-1}\left(\frac{s - x_2\cos\delta - x_3\sin\delta}{x_3\cos\delta - x_2\sin\delta}\right)\right.$$

$$- \tan^{-1} \left(\frac{s - x_2 \cos\delta + x_3 \sin\delta}{x_3 \cos\delta + x_2 \sin\delta} \right) \Bigg] \Bigg|_{s_1}^{s_2} \tag{10}$$

$$\frac{\partial u_1}{\partial x_2} = \frac{b}{2\pi} \left[\frac{s \sin\delta - x_3}{R^2} + \frac{x_3 + s \sin\delta}{S^2} \right] \Bigg|_{s_1}^{s_2} \tag{11}$$

$$\frac{\partial u_1}{\partial x_3} = \frac{b}{2\pi} \left[(x_2 - s\cos\delta) \left(\frac{1}{R^2} - \frac{1}{S^2} \right) \right] \Bigg|_{s_1}^{s_2} \tag{12}$$

where

$$f(s) \Bigg|_{s_1}^{s_2} = f(s_2) - f(s_1) . \tag{13}$$

On putting $\delta = 90^\circ$ in eqn. (10) - (12) we get the results for a long vertical strike-slip fault in a half-space (Chinnery[29]).

Dip-Slip Fault in a Uniform Half-space

(i) *Line source*

For a line source of the dip-slip type, the deformation field is given by

$$u_2 = \frac{\alpha b ds}{2\pi} \Bigg\{ \cos 2\delta \Bigg[\left(\frac{1+\alpha}{\alpha} \right) \frac{x_3 - y_3}{R^2} - 2 \frac{(x_3 - y_3)^3}{R^4} - \frac{x_3 - y_3}{S^2} - \frac{x_3 + 3y_3}{\alpha S^2}$$

$$- 2 (x_3 + y_3) \left\{ (x_3 + y_3)(y_3 - x_3 - 2y_3/\alpha) - 6 x_3 y_3 \right\} \frac{1}{S^4}$$

$$- 16 x_3 y_3 \frac{(x_3 + y_3)^3}{S^6} \Bigg] + (x_2 - y_2) \sin 2\delta \Bigg[- \frac{1}{\alpha R^2} + 2 \frac{(x_3 - y_3)^2}{R^4} - \frac{1}{\alpha S^2}$$

$$+ 2 \left\{(x_3 + y_3)(x_3 - y_3 + 2y_3/\alpha) + 2x_3 y_3\right\} \frac{1}{S^4} - 16 x_3 y_3 \frac{(x_3 + y_3)^2}{S^6} \Bigg]\Bigg]\Bigg\} \quad (14)$$

$$u_3 = \frac{\alpha b ds}{2\pi} \Bigg\{ (x_2 - y_2) \cos 2\delta \Bigg[(1/\alpha - 1) \frac{1}{R^2} + 2 \frac{(x_3 - y_3)^2}{R^4} - \frac{(1 - \alpha)}{\alpha S^2}$$

$$- 2 \left\{(x_3 + y_3)(x_3 + y_3 - 2y_3/\alpha) + 2x_3 y_3\right\} \frac{1}{S^4} + 16 x_3 y_3 \frac{(x_3 + y_3)^2}{S^6} \Bigg]$$

$$+ \sin 2\delta \Bigg[\left(\frac{1 - 2\alpha}{\alpha}\right) \frac{x_3 - y_3}{R^2} + 2 \frac{(x_3 - y_3)^3}{R^4} - 2 \frac{x_3 + y_3}{S^2} + \frac{x_3 + 3y_3}{\alpha S^2}$$

$$+ 2(x_3 + y_3) \left\{(x_3 + y_3)(x_3 + y_3 - 2y_3/\alpha) + 6 x_3 y_3\right\} \frac{1}{S^4}$$

$$- 16 x_3 y_3 \frac{(x_3 + y_3)^3}{S^6} \Bigg]\Bigg\} \quad (15)$$

$$\frac{\partial u_2}{\partial x_2} = \frac{\alpha b ds}{2\pi} \Bigg\{ 2(x_2 - y_2) \cos 2\delta \Bigg[- \frac{(1 + \alpha)(x_3 - y_3)}{\alpha R^4} + 4 \frac{(x_3 - y_3)^3}{R^6}$$

$$+ \left\{x_3 - y_3 + (x_3 + 3y_3)/\alpha\right\} \frac{1}{S^4} + 4(x_3 + y_3)(y_3^2 - x_3^2 - 6 x_3 y_3) \frac{1}{S^6}$$

$$- 8y_3 \frac{(x_3 + y_3)^2}{\alpha S^6} + 48 x_3 y_3 \frac{(x_3 + y_3)^3}{S^8} \Bigg] + \sin 2\delta \Bigg[\frac{1}{\alpha R^2}$$

$$- 2(3 + 1/\alpha) \frac{(x_3 - y_3)^2}{R^4} + 8 \frac{(x_3 - y_3)^4}{R^6} + \frac{1}{\alpha S^2}$$

$$- 6\,(x_3^2 - y_3^2 + 2x_3\,y_3)\,\frac{1}{S^4} - 2\,(x_3 + y_3)\,\frac{x_3 + 7y_3}{\alpha\,S^4}$$

$$+ 8\,(x_3 + y_3)^2\,\left\{(x_3 + y_3)\,(x_3 - y_3 + 2y_3/\alpha) + 12x_3\,y_3\right\}\frac{1}{S^6}$$

$$\left.\left. - 96\,x_3\,y_3\,\frac{(x_3 + y_3)^4}{S^8}\right]\right\} \tag{16}$$

$$\frac{\partial u_3}{\partial x_3} = \frac{\alpha bds}{2\pi}\left\{2\,(x_2 - y_2)\,\cos 2\,\delta\left[(3 - 1/\alpha)\,\frac{x_3 - y_3}{R^4} - 4\,\frac{(x_3 - y_3)^3}{R^6} - \frac{3x_3 + 5y_3}{S^4}\right.\right.$$

$$+ \frac{x_3 + 3y_3}{\alpha\,S^4} + 4\,(x_3 + y_3)\,(x_3^2 + 3\,y_3^2 + 10\,x_3\,y_3)\,\frac{1}{S^6}$$

$$\left. - 8y_3\,\frac{(x_3 + y_3)^2}{\alpha\,S^6} - 48\,x_3\,y_3\,(x_3 + y_3)^3\,\frac{1}{S^8}\right] + \sin 2\,\delta\left[(1/\alpha - 2)\,\frac{1}{R^2}\right.$$

$$+ 2\,(5 - 1/\alpha)\,\frac{(x_3 - y_3)^2}{R^4} - 8\,\frac{(x_3 - y_3)^4}{R^6} + (1/\alpha - 2)\,\frac{1}{S^2}$$

$$+ 2\,(5x_3^2 + 11\,y_3^2 + 22\,x_3\,y_3)\,\frac{1}{S^4} - 2\,(x_3 + y_3)\,(x_3 + 7y_3)\,\frac{1}{\alpha\,S^4}$$

$$- 8\,(x_3 + y_3)^2\,(x_3^2 + 3\,y_3^2 + 16\,x_3\,y_3)\,\frac{1}{S^6} + 16\,y_3\,(x_3 + y_3)^3\,\frac{1}{\alpha\,S^6}$$

$$\left.\left. + 96\,x_3\,y_3\,\frac{(x_3 + y_3)^4}{S^8}\right]\right\} \tag{17}$$

$$\frac{\partial u_2}{\partial x_3} = \frac{\alpha\,bds}{2\pi}\left\{\cos 2\,\delta\left[(1 + 1/\alpha)\,\frac{1}{R^2} - 2\,(4 + 1/\alpha)\,\frac{(x_3 - y_3)^2}{R^4} + 8\,\frac{(x_3 - y_3)^4}{R^6}\right.\right.$$

$$- (1 + 1/\alpha)\ \frac{1}{S^2} + \left\{8\,(x_3 + y_3)^2 + 12\,x_3\,y_3\right\}\frac{1}{S^4} + 2\,(x_3 + y_3)\,(x_3 + 7y_3)\ \frac{1}{\alpha\,S^4}$$

$$- 8\ (x_3 + y_3)^2\ (x_3^2 + y_3^2 + 14\,x_3\,y_3)\ \frac{1}{S^6} - 16\,y_3\ \frac{(x_3 + y_3)^3}{\alpha\,S^6}$$

$$+ 96\,x_3\,y_3\,\frac{(x_3 + y_3)^4}{S^8}\Bigg] + 2\,(x_2 - y_2)\,\sin 2\,\delta\left[(2 + 1/\alpha)\ \frac{x_3 - y_3}{R^4} - 4\,\frac{(x_3 - y_3)^3}{R^6}\right.$$

$$+ \frac{x_3 + 3y_3}{\alpha\,S^4} + 2\ \frac{x_3 + y_3}{S^4} - 4\,(x_3 + y_3)\ (x_3^2 + y_3^2 + 8\ x_3\,y_3)\ \frac{1}{S^6}$$

$$\left. - 8\ y_3\ \frac{(x_3 + y_3)^2}{\alpha\,S^6} + 48\ x_3\,y_3\ (x_3 + y_3)^3\ \frac{1}{S^8}\right]\Bigg\} \tag{18}$$

$$\frac{\partial u_3}{\partial x_2} = \frac{\alpha b d s}{2\,\pi}\left\{\cos 2\,\delta\left[\ (1 - 1/\alpha)\ \frac{1}{R^2} + 2\,(1/\alpha - 4)\ \frac{(x_3 - y_3)^2}{R^4} + 8\,\frac{(x_3 - y_3)^4}{R^6}\right.\right.$$

$$- (1 - 1/\alpha)\,\frac{1}{S^2} + \left\{8\,(x_3 + y_3)^2 + 12x_3\,y_3\right\}\frac{1}{S^4} - 2\,(x_3 + y_3)\,(x_3 + 7y_3)\,\frac{1}{\alpha\,S^4}$$

$$- 8\ (x_3 + y_3)^2\ (x_3^2 + y_3^2 + 14\,x_3\,y_3)\ \frac{1}{S^6} + 16\,y_3\ (x_3 + y_3)^3\ \frac{1}{\alpha\,S^6}$$

$$+ 96\ x_3\,y_3\ (x_3 + y_3)^4\ \frac{1}{S^8}\Bigg] + 2\,(x_2 - y_2)\ \sin 2\,\delta\left[(2 - 1/\alpha)\ \frac{x_3 - y_3}{R^4}\right.$$

$$- 4\,\frac{(x_3 - y_3)^3}{R^6} + 2\,\frac{x_3 + y_3}{S^4} - \frac{x_3 + 3y_3}{\alpha\,S^4} - 4\,(x_3 + y_3)\,(x_3^2 + y_3^2 + 8x_3\,y_3)\,\frac{1}{S^6}$$

$$\left.\left. + 8y_3\ \frac{(x_3 + y_3)^2}{\alpha\,S^6} + 48x_3\,y_3\ (x_3 + y_3)^3\ \frac{1}{S^8}\right]\right\} \tag{19}$$

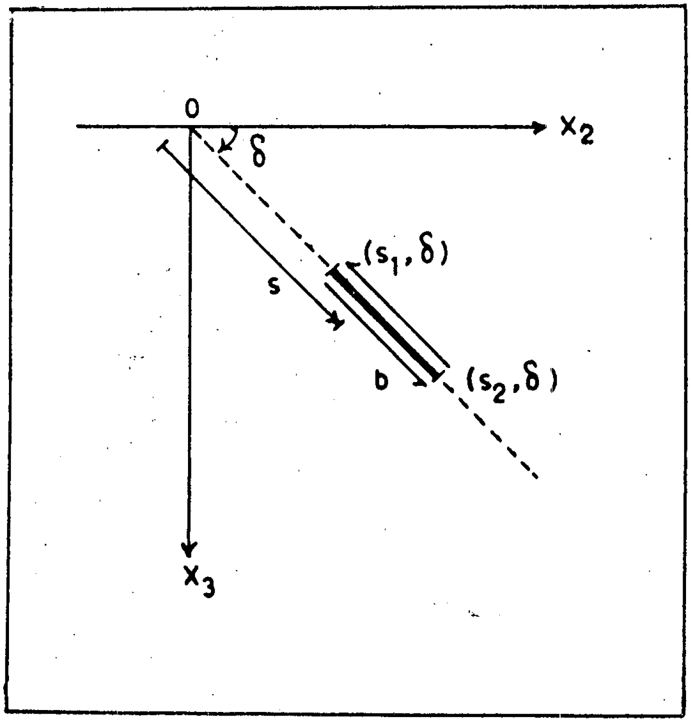

Fig. 2 – Geometry of a long dip-slip fault. The displacement discontinuity b is perpendicular to the x_1-axis, parallel to the fault plane.

where b is the displacement discontinuity (slip) prependicular to the x_1-axis, parallel to the fault plane, ds is the width of the line dislocation and δ is the dip angle (Fig. 2).

(ii) *Finite fault*

For a dip-slip fault of finite width $L = s_2 - s_1$ and infinite length (Fig. 2), the deformation field is given by

$$u_2 = \frac{\alpha b}{2\pi} \left\{ (1/\alpha - 1) \sin\delta \ln(R/S) + \frac{\cos\delta}{\alpha} \left[\tan^{-1}\left(\frac{x_2 - s\cos\delta}{x_3 + s\sin\delta}\right) \right.\right.$$

$$\left. - \tan^{-1}\left(\frac{x_2 - s\cos\delta}{x_3 - s\sin\delta}\right)\right] - (x_2 \sin\delta - x_3 \cos\delta)\,(x_2 - s\cos\delta)\left(\frac{1}{R^2} - \frac{1}{S^2}\right)$$

$$+ 2 \sin\delta \left[\frac{1}{\alpha}(x_3 \sin\delta - x_2 \cos\delta + s) - x_3 \sin\delta\right]\frac{s}{S^2}$$

$$+ 4\, x_3 \sin\delta\,(x_2 \sin\delta + x_3 \cos\delta)\,(x_2 - s\cos\delta)\,\frac{s}{S^4}\Bigg\}\Bigg|_{s_1}^{s_2} \tag{20}$$

$$u_3 = \frac{\alpha b}{2\pi}\Bigg\{(1 - 1/\alpha)\cos\delta\,\ln(R/S) + \frac{\sin\delta}{\alpha}\left[\tan^{-1}\left(\frac{x_3 - s\sin\delta}{x_2 - s\cos\delta}\right)\right.$$

$$\left. - \tan^{-1}\left(\frac{x_3 + s\sin\delta}{x_2 - s\cos\delta}\right)\right] - (x_2 \sin\delta - x_3 \cos\delta)\left[\frac{x_3 - s\sin\delta}{R^2}\right.$$

$$\left. - \frac{x_3 + s\sin\delta}{S^2}\right] + 2\sin\delta\left[\frac{x_2 \sin\delta + x_3 \cos\delta}{\alpha} - (x_2 \sin\delta + 2x_3 \cos\delta)\right]\frac{s}{S^2}$$

$$+ 4\,x_3 \sin\delta\,(x_2 \sin\delta + x_3 \cos\delta)\,(x_3 + s\sin\delta)\,\frac{s}{S^4}\Bigg\}\Bigg|_{s_1}^{s_2} \tag{21}$$

$$\frac{\partial u_2}{\partial x_2} = \frac{\alpha b}{2\pi}\Bigg\{\left[\frac{x_2 \sin\delta - x_3 \cos\delta}{\alpha} - (x_2 \sin\delta + x_3 \cos\delta - s\sin 2\delta)\right]$$

$$\times\left(\frac{1}{R^2} - \frac{1}{S^2}\right) + \frac{2 s \sin 2\delta}{\alpha S^2} - 2\,(x_2 \sin\delta - x_3 \cos\delta)\left[\frac{(x_3 - s\sin\delta)^2}{R^4}\right.$$

$$\left. - \frac{(x_3 + s\sin\delta)^2}{S^4}\right] - 4\sin\delta\,(x_2 \sin\delta + x_3 \cos\delta)\left[x_3 + \frac{x_3 + s\sin\delta}{\alpha}\right]\frac{s}{S^4}$$

$$- \frac{4\,x_3 \sin 2\delta\,(x_3 + s\sin\delta)\,s}{S^4}$$

$$+ 16\,x_3 \sin\delta\,(x_2 \sin\delta + x_3 \cos\delta)\,(x_3 + s\sin\delta)^2\,\frac{s}{S^6}\Bigg\}\Bigg|_{s_1}^{s_2} \tag{22}$$

$$\frac{\partial u_3}{\partial x_3} = \frac{\alpha b}{2\pi} \left\{ \left[(3x_3 \cos\delta - x_2 \sin\delta) + \frac{x_2 \sin\delta - x_3 \cos\delta}{\alpha} \right] \left(\frac{1}{R^2} - \frac{1}{S^2} \right) \right.$$

$$- s \sin 2\delta \left(\frac{1}{R^2} + \frac{3}{S^2} - \frac{2}{\alpha S^2} \right) + 2 (x_2 \sin\delta - x_3 \cos\delta) \left[\frac{(x_3 - s \sin\delta)^2}{R^4} \right.$$

$$\left. - \frac{(x_3 + s \sin\delta)^2}{S^4} \right] + 4 \sin\delta \, (x_2 \sin\delta + x_3 \cos\delta) \left[x_3 - \frac{x_3 + s \sin\delta}{\alpha} \right] \frac{s}{S^4}$$

$$+ 8 \sin\delta \, (x_2 \sin\delta + 2x_3 \cos\delta) (x_3 + s \sin\delta) \frac{s}{S^4}$$

$$\left. - 16 x_3 \sin\delta \, (x_2 \sin\delta + x_3 \cos\delta) (x_3 + s \sin\delta)^2 \frac{s}{S^6} \right\} \Bigg|_{s_1}^{s_2} \tag{23}$$

$$\frac{\partial u_2}{\partial x_3} = \frac{\alpha b}{2\pi} \left\{ (1 + 1/\alpha) \; (x_2 \cos\delta + x_3 \sin\delta - s) \left(\frac{1}{R^2} - \frac{1}{S^2} \right) \right.$$

$$- 2 \, (x_2 \cos\delta + x_3 \sin\delta - s) \left[\frac{(x_3 - s \sin\delta)^2}{R^4} - \frac{(x_3 + s \sin\delta)^2}{S^4} \right]$$

$$+ 4 \sin\delta \, (x_2 \cos\delta - x_3 \sin\delta - s) \left[x_3 + \frac{x_3 + s \sin\delta}{\alpha} \right] \frac{s}{S^4}$$

$$- 8 x_3 \sin^2\delta \, (x_3 + s \sin\delta) \frac{s}{S^4}$$

$$\left. - 16 x_3 \sin\delta \, (x_2 \cos\delta - x_3 \sin\delta - s) (x_3 + s \sin\delta)^2 \frac{s}{S^6} \right\} \Bigg|_{s_1}^{s_2} \tag{24}$$

$$\frac{\partial u_3}{\partial x_2} = \frac{\alpha b}{2\pi} \left\{ (1 - 1/\alpha) \; (x_2 \cos\delta + x_3 \sin\delta - s) \left(\frac{1}{R^2} - \frac{1}{S^2} \right) \right.$$

$$- 2 (x_2 \cos \delta + x_3 \sin \delta - s) \left[\frac{(x_3 - s \sin \delta)^2}{R^4} - \frac{(x_3 + s \sin \delta)^2}{S^4} \right]$$

$$+ 4 \sin \delta \, (x_2 \cos \delta - x_3 \sin \delta - s) \left[x_3 - \frac{x_3 + s \sin \delta}{\alpha} \right] \frac{s}{S^4}$$

$$- 8 \, x_3 \sin^2 \delta \, (x_3 + s \sin \delta) \frac{s}{S^4}$$

$$- 16 \, x_3 \sin \delta \, (x_2 \cos \delta - x_3 \sin \delta - s) \, (x_3 + s \sin \delta)^2 \left. \frac{s}{S^6} \right\} \Bigg|_{s_1}^{s_2} \tag{25}$$

On putting $x_3 = 0$ in eqn. (20) and (21), we get the surface displacements (Freund and Barnett[17]). There is a printing error in eqn. (10) and (11) of Freund and Barnett[17]. The second term on the right hand side of these equations should be multiplied by $1/\pi$ (see also Savage[5]). The deformation field for a strike-slip fault is independent of the elastic constants. For a dip-slip fault, the elastic constants occur in combinations of the form Q_2 -A, where

$$Q_2 = \frac{2\mu}{3(\lambda + 2\mu)} = \frac{2\mu}{3k + 4\mu} \quad (k = \text{bulk moldulus}) \tag{26}$$

and A can assume the values 0, 1/3, 4/9, 2/3, 1 and 4/3. From the correspondence principle, the quasi-static field can be obtained on replacing $Q_2 - A$ by (Singh and Singh[16])

$$\left[\frac{2\mu}{3k + 4\mu} \exp\left(- \frac{3k}{3k + 4\mu} \times \frac{t}{t_2} \right) - A \right] H(t) \, ,$$

where t_2 is the relaxation time and $H(t)$ is the unit step function. The above result assumes the medium to be elastic in dilatation and Maxwell viscoelastic in distortion and the time-variation of the source to be the unit step function. For the Poisson's case ($\lambda = \mu$), $Q_2 - A$ is to be replaced by

$$\left[\frac{2}{9} \exp\left(- \frac{5t}{9t_2} \right) - A \right] H(t)$$

Nonuniform Slip on a Long Fault in a Half-space

The problem of the static deformation of a uniform half-space caused by long strike-slip and dip-slip faults has been discussed by several investigators. However, most of these studies assume uniform slip on the fault. The assumption of uniform slip makes the edges of the fault plane singular where the displacement is indeterminate and the stress is infinite. For this reason, uniform slip models cannot be used in the near field. There are a number of interesting phenomena which occur near the edge of the fault zone; e.g. vertical movements associated with strike-slip faulting.In order to study these phenomena, it is necessary to consider models of earthquake faulting with nonuniform slip on the fault.

Chinnery and Petrak[30] used numerical integration to compute the elastic field due to a vertical strike-slip fault with slip that varies over the face of the fault. The exponential variation in slip was chosen so as to remove the stress singualrity that occurs at the edge of the fault plane in some earlier models. Freund and Barnett[17] gave a two-dimensional analysis of surface deformation due to dip-slip faulting. They also resorted to numerical integration for computing the elastic field for variable slip on the fault plane. Mahrer and Nur[31] studied the deformation of an inhomogenous half-space, the shear modulus of which increases monotonically with depth, due to strike-slip fauling. They examined two general classes of faults : those which broke the surface, smoothly reducing the slip to zero at some depth; and those faults which were completely buried, with smooth closure at both upper and lower ends, and evaluated the deformation numerically. Yang and Toksoz[32] used a finite element scheme to study a trapezoidal type of variable slip on a strike-slip fault. Wang and Wu[33] obtained closed-form analytical expressions for the displacements and stresses for the same model. Singh *et al.*[34] obtained closed-form analytical expressions for the displacements caused by nonuniform slip on long, vertical, strike-slip and dip-slip faults in a uniform half-space. They considered four slip profiles : elliptic, parabolic, linear and cubic. They assumed that the slip b decreases from a value b_0 at the surface to zero at the depth L. The value of the surface-slip b_0 is common to all the porfiles but the depth L for a particular porfile is chosen in such a manner that the source potency is the same for all the profiles. By performing detailed numerical computations, Singh *et. al*[34] demonstrated that, at the surface, the fall - off of the displacement with the distance from the fault is not much affected by the details of the slip at depth. However, this is not true for subsurface deformations. At depth, the deformation near the fault depends upon the slip porfile. There is qualitative as well as quantitative change in the subsurface deformation near the fault for any significant change in the details of the slip profile.

Consider a homogeneous, isotropic, perfectly elastic half-space ocupying the region $x_3 \geq 0$. A vertical stike-slip fault of infinite length and finite depth (width) occupies the region $-\infty < x_1 < \infty$, $x_2 = 0$, $0 \leq x_3 \leq L$ (Fig. 3). Let the slip (dislocation) on the fault be denoted by b which is nonuniform in general.We are considering a two-dimensional

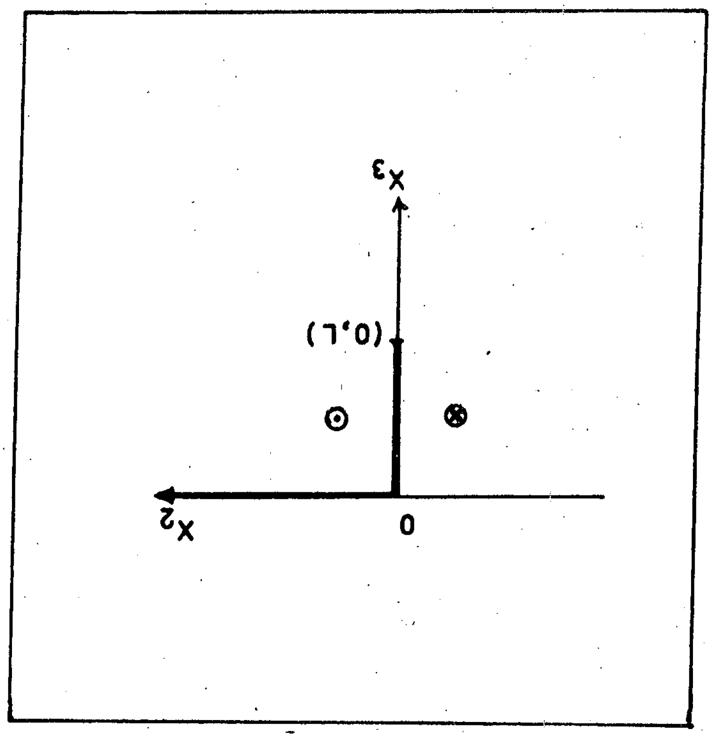

Fig. 3 – Geometry of a surface-breaking, vertical, long strike-slip fault in a uniform half-space $x_3 \geq 0$.

approximation in which $\bar{b}$ is independent of x_1. Following Maruyama[4], the displacement at any point of the half-space due to the slip b on the fault can be expressed in the form

$$u_1 = \int_0^L b(h)\ G_1\ (x_2\,,\ x_3\,,\ h)\ dh\,, \tag{27}$$

where

$$G_1 = \frac{x_2}{2\,\pi}\left(\frac{1}{R^2} + \frac{1}{S^2}\right), \tag{28}$$

$$R^2 = x_2^2 + (x_3 - h)^2 \ , \quad S^2 = x_2^2 + (x_3 + h)^2 \ , \tag{29}$$

$$\alpha = \frac{1}{2(1-\sigma)} \ , \tag{30}$$

σ being the Poisson's ratio.

The expressions for the displacements for various slip profiles can be obtained from eqn. (27) by integrating analytically. Following Singh *et. al.*[34], we have the following results.

(i) *Elliptic*

Let the slip on the fault very according to the law

$$b(h) = b_0 \left(1 - \frac{h^2}{L^2}\right)^{1/2} \tag{31}$$

$$(0 \le h \le L)$$

where b_0 is the surface-slip and L is the fault-depth. Inserting the expression for $b(h)$ in eqn. (27) and integrating, closed-form expression for the displacement can be obtained. The expression valid at the surface ($x_3 = 0$) is

$$u_1 = \frac{b_0}{2} \left[- Y \pm (Y^2 + 1)^{1/2}\right] \tag{32}$$

$$(Y \gtrless 0)$$

where

$$Y = x_2/L .$$

(ii) *Parabolic*

Let the slip on the fault be given by

$$b(h) = b_0 (1 - h^2/L^2) \tag{34}$$

$$(0 \le h \le L)$$

The expression for the displacement obtained from eqn. (27) and (34) is

$$u_1 = \frac{b_0}{2\pi}\left\{- 2Y - 2YZ \ln (A/B)\right.$$

$$\left. + (1 + Y^2 - Z^2)\left[\tan^{-1}\left(\frac{1-Z}{Y}\right) + \tan^{-1}\left(\frac{1+Z}{Y}\right)\right]\right\}, \tag{35}$$

where

$$Z = x_3/L\ ,$$

$$A^2 = Y^2 + (Z - 1)^2\ ,\ B = Y^2 + (Z + 1)^2 \tag{36}$$

(iii) *Linear*

For the linear slip porfile

$$b(h) = b_0\ (1 - h/L)\ , \tag{37}$$

$$(0 \leq h \leq L)$$

we obtain

$$u_1 = \frac{b_0}{2\pi}\left[(1 - Z)\tan^{-1}\left(\frac{1-Z}{Y}\right) + (1 + Z)\tan^{-1}\left(\frac{1+Z}{Y}\right)\right.$$

$$\left. - Y \ln (AB) + 2\ Y \ln A_0 - 2\ Z \tan^{-1}(Z/Y)\right], \tag{38}$$

where

$$A_0^2 = Y^2 + Z^2$$

(iv) *Cubic*

Let the slip on the fault vary according to the law

$$b(h) = b_0\ (1 - h^2/L^2)^{3/2} \tag{39}$$

$$(0 \leq h \leq L)$$

The expression for the displacement obtained from eqn. (27) and (39) valid at the surface ($x_3 = 0$) is

$$u_1 = \frac{b_0}{2}\left[- Y\,(Y^2 + 3/2) \pm (1 + Y^2)^{3/2}\right] \tag{40}$$

$$(Y \gtrless 0)$$

Singh *et al.*[34] have also obtained closed-form analytical expressions for the displacements caused by nonuniform slip on a long, vertical dip-slip fault in a uniform half-space for the four slip profiles described above. For all the slip profiles considered, the slip decreased from a value b_0 at the surface to zero at the depth L. If the surface-slip b_0 and the fault-depth L are assumed to be the same for all the cases, then the source potency $\int_0^L b(h)\,dh$ per unit length of the fault is different for different porfiles. One is not justified in comparing deformation of sources of different potency. Before making any comparison, a parity in source potency must be assured. A parity in source potency for different slip porfiles can be achieved by adjusting either the surface-slip or the fault-depth. If the surface-slip b_0 is the same for all the slip profiles, but the fault-depth L is so adjusted that parity in source potency is achieved, then we find

$$L_1 = \frac{\pi}{4} L_2 = \frac{2}{3} L_3 = \frac{1}{2} L_4 = \frac{3\pi}{16} L_5 , \tag{41}$$

where L_1 is the fault-depth for the uniform slip model and L_2, L_3 L_4 and L_5 are, respectively, the fault-depths for the elliptic, parabolic, linear and cubic porfiles.

Two Half-Spaces in Welded Contact

To investigate the effect of a structural discontinuity on the elastic field, it is instructive to find analytical solution of the problem of a long fault in a model consisting of two homogeneous half-space in welded contact. Sharma *et al.*[35] obtained closed-form analytical expressions for the displacements and stresses at any point of either of the two homogeneous, isotropic, perfectly elastic half-spaces in welded contact due to a horizontal or a vertical long stike-slip fault. Beginning with the expressions for a strike-slip line source given by Sharma *et al.*[35], Rani and Singh[36] obtained the displacement field for a long strike-slip fault of arbitrary dip placed in a half-space in welded contact with another half-space by integration over the width of the fault. The inclinaton of the fault introduces asymmetry of various degrees in the displacement field depending upon the dip angle.

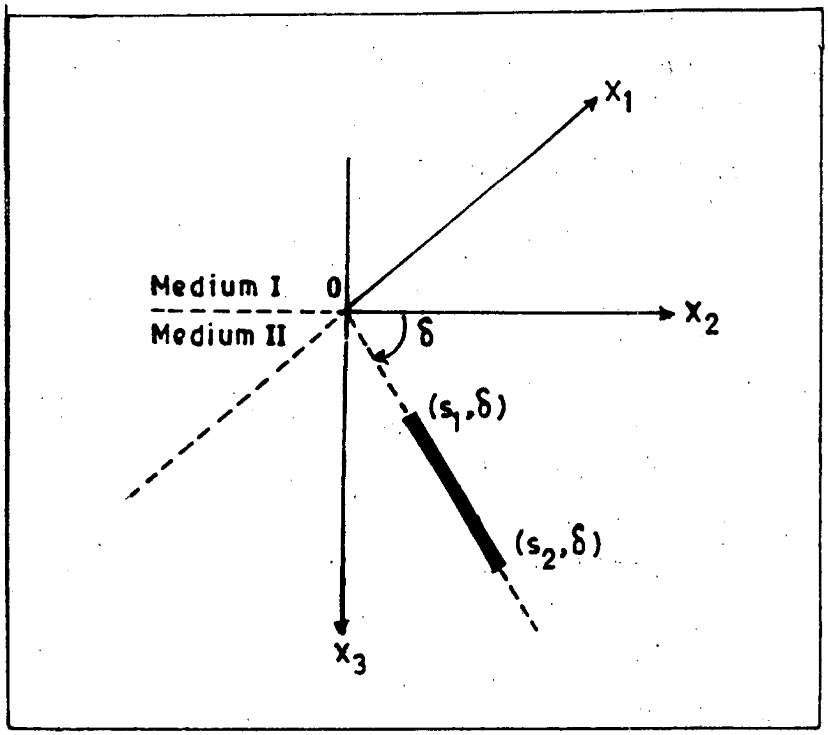

Fig. 4 – Two half-spaces in welded contact with a long strike - slip fault in the lower half-space.

Singh *et al.*[37] obtained the elastic field at any point of either of the two homogeneous, isotropic, perfectly elastic half-spaces in welded contact due to various two-dimensional sources. Beginning with the expressions for a dip-slip line source given by Singh *et al.*[37] and integrating over the width of the fault, Rani and Singh[38] obtained the field due to a long dip-slip fault of arbitrary dip placed in a half-space in welded contact with another half-space.

Let the Cartesian coordinates be denoted by (x_1, x_2, x_3) with x_3-axis vertically downwards. Consider two homogeneous, isotropic, perfectly elastic half-spaces that are welded along the plane $x_3 = 0$. The upper half-space $(x_3 < 0)$ is called medium I and the lower half-space $(x_3 > 0)$ is called medium II with rigidities μ_1 and μ_2, respectively (Fig. 4). In the following the superscript (1) denotes quantities related to medium I and the superscript (2) denotes those related to medium II.

Using the results of Sharma *et. al.*[35], the expressions for the displacements in the two half-spaces due to an inclined strike-slip line dislocation parallel to the x_1-axis and passing through the point (y_2, y_3) in the lower half space (medium II) are found to be[36]

$$u_1^{(1)} = \frac{bds}{\pi\ (1+\beta)\ R^2} \left[(x_3 - y_3) \cos\delta - (x_2 - y_2) \sin\delta \right], \tag{42}$$

$$u_1^{(2)} = \frac{bds}{2\pi} \left[\frac{(x_3 - y_3) \cos\delta - (x_2 - y_2) \sin\delta}{R^2} \right.$$

$$\left. - \frac{(1-\beta)}{(1+\beta)\ S^2} \left\{ (x_3 + y_3) \cos\delta + (x_2 - y_2) \sin\delta \right\} \right], \tag{43}$$

where

$$\begin{aligned} b &= \text{displacement discontinuity (slip)} \\ ds &= \text{width of the line dislocation} \\ (x_2, x_3) &= \text{receiver location} \\ (y_2, y_3) &= \text{source location} \\ R^2 &= (x_2 - y_2)^2 + (x_3 - y_3)^2, \\ S^2 &= (x_2 - y_2)^2 + (x_3 + y_3)^2, \\ \beta &= \mu_1 / \mu_2. \end{aligned} \tag{44}$$

We put (Fig. 4)

$$y_2 = s \cos\delta, \qquad y_3 = s \sin\delta \tag{45}$$

Inserting the values of y_2 and y_3 into eqn. (42) and (43) and integrating over s between the limits (s_1, s_2), Rani and Singh[36] obtainbed the following expressions for the displacements for a long strike-slip fault of width $L = s_2 - s_1$:

$$u_1^{(1)} = \frac{b}{\pi\ (1+\beta)} \left[\tan^{-1} \left(\frac{s - x_2 \cos\delta - x_3 \sin\delta}{x_3 \cos\delta - x_2 \sin\delta} \right) \right] \Bigg|_{s_1}^{s_2}, \tag{46}$$

$$u_1^{(2)} = \frac{b}{2\pi} \left[\tan^{-1} \left(\frac{s - x_2 \cos\delta - x_3 \sin\delta}{x_3 \cos\delta - x_2 \sin\delta} \right) \right.$$

$$\left. - \frac{1-\beta}{1+\beta} \tan^{-1} \left(\frac{s - x_2 \cos\delta + x_3 \sin\delta}{x_3 \cos\delta + x_2 \sin\delta} \right) \right] \Bigg|_{s_1}^{s_2} \tag{47}$$

The corresponding expressions for the stresses are

$$p_{12}^{(1)} = \frac{\mu_1 b}{\pi\,(1+\beta)} \left[\frac{s \sin\delta - x_3}{R^2} \right] \Bigg|_{s_1}^{s_2}, \tag{48}$$

$$p_{13}^{(1)} = \frac{\mu_1 b}{\pi\,(1+\beta)} \left[\frac{x_2 - s \cos\delta}{R^2} \right] \Bigg|_{s_1}^{s_2}, \tag{49}$$

$$p_{12}^{(2)} = \frac{\mu_2 b}{2\pi} \left[\frac{s \sin\delta - x_3}{R^2} + \frac{1-\beta}{1+\beta} \left(\frac{x_3 + s \sin\delta}{S^2} \right) \right] \Bigg|_{s_1}^{s_2}, \tag{50}$$

$$p_{13}^{(2)} = \frac{\mu_2 b}{2\pi} \left[(x_2 - s \cos\delta) \left\{ \frac{1}{R^2} - \frac{(1-\beta)}{(1+\beta)\,S^2} \right\} \right] \Bigg|_{s_1}^{s_2}, \tag{51}$$

where now

$$R^2 = (x_2 - s \cos\delta)^2 + (x_3 - s \sin\delta)^2, \tag{52}$$

$$S^2 = (x_2 - s \cos\delta)^2 + (x_3 + s \sin\delta)^2. \tag{53}$$

Eqn. (46) - (51) give the residual elastic field at any point of the two half-spaces due to a long strike-slip fault of finite width and arbitrary dip. The corresponding results for a dip-slip fault have been given by Rani and Singh[38].

Layered Half-Space

A model for the crustal deformation field should consist of a layer (representing lithosphere) overlying a viscoelastic half-space (representing asthenosphere). Rybicki[8] found a closed-form analytical solution for the problem of a long vertical strike- slip fault in a two-layer model of the Earth. Rybicki's solution has been used by Nur and Mavko[9] and Cohen[39], amongst others, to explain the postsiesmic surface deformation. While Nur and Mavko[8] assumed the lithosphere-asthenosphere composite as an elastic layer overlying a standard linear viscolastic half-space, Cohen[39] assumed a standard linear viscoelastic layer overlying a Maxwell viscoelastic half-space. Bonafede *et al.*[40] modelled a microplate as an elastic plate with two long strike-slip boundaries lying over a Maxwell viscoelastic asthenosphere.

Singh and Rani[12] generalized Rybicki's solution to a fault of arbitrary dip. The correspondence principle of linear viscoelasticity was used to obtain the quasi-static field when the layer is elastic and half-space Maxwell viscoelastic. The static field was used to model the coseismic deformation following a strike-slip earthquake at a transform plate boundary and the quasi-static field minus the static field was used to model the postseismic deformation. They performed detailed numerical computations and showed that the field caused by a surface-breaking fault is characteristically different from the field caused by a fault at depth. The main advantage of considering the 2-D problem of a long strike-slip fault in an elastic layer over a viscoelastic half-space instead of the corresponding 3-D problem is that one is in a position to obtain closed-form analytical solution of the problem. In the 3-D case, one is forced either to use approximate Green's functions or to resort to numerical integration.

We consider an Earth model consisting of a homogeneous, isotropic, elastic layer of thickness H lying over a homogeneous, isotropic, elastic half-space (Fig. 5). We place the origin of a Cartesian coordinate system (x_1, x_2, x_3) at the free surface and the x_3-axis is drawn into the medium. A long inclined strike-slip fault, with strike along the x_1-axis, is

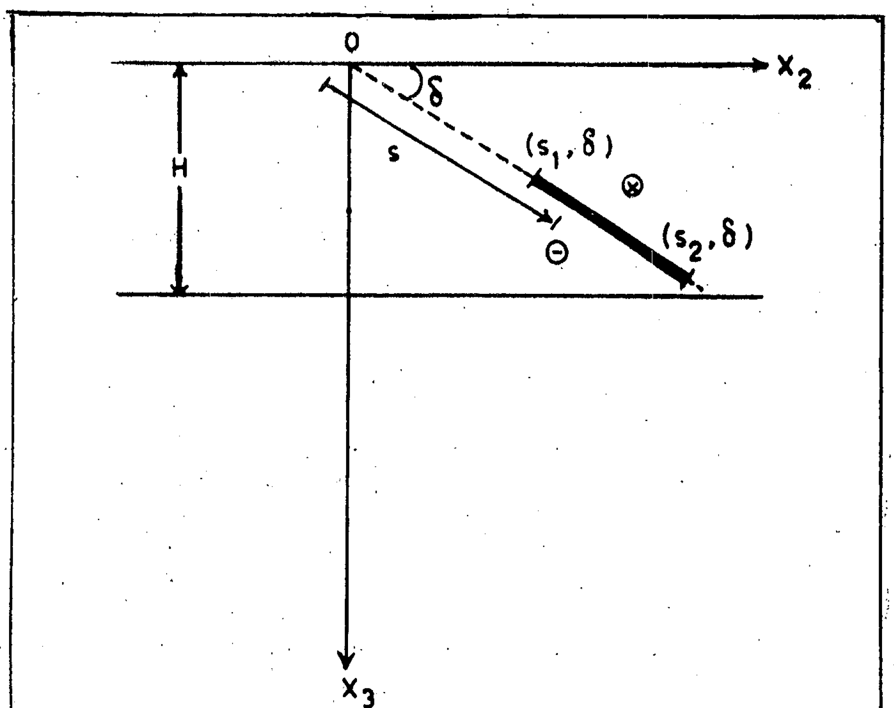

Fig. 5 – Geometry of a long strike-slip fault situated in a layer of uniform thickness H lying over a half-space.

situated in the layer and (y_1, y_2, y_3) is any point on the fault $(0 \le y_3 \le H)$. Let μ_1 and μ_2 be the rigidities of the layer and of the half-space, respectively. The superscript (1) denotes quantities related to the layer and the superscript (2) denotes those related to the half-space. Singh and Rani[12] have shown that the displacement field due to an inclined strike-slip line dislocation passing through the point $(0, y_2, y_3)$ is given by

$$u^{(1)} = \frac{b\,ds}{2\pi}\left\{\cos\delta\left[\frac{x_3 - y_3}{R^2} - \frac{x_3 + y_3}{S^2} + \sum_{n=1}^{\infty} r^n \left\{\frac{2nH - x_3 - y_3}{T^2} + \frac{2nH + x_3 - y_3}{V^2} - \frac{2nH - x_3 + y_3}{U^2} - \frac{2nH + x_3 + y_3}{W^2}\right\}\right] - \sin\delta\,(x_2 - y_2)\left[\frac{1}{R^2} + \frac{1}{S^2} + \sum_{n=1}^{\infty} r^n \left(\frac{1}{T^2} + \frac{1}{V^2} + \frac{1}{U^2} + \frac{1}{W^2}\right)\right]\right\} \tag{54}$$

$$u^{(2)} = \frac{b\,ds}{2\pi}(1+r)\left\{\cos\delta\left[\frac{x_3 - y_3}{R^2} - \frac{x_3 + y_3}{S^2} + \sum_{n=1}^{\infty} r^n \left\{\frac{2nH + x_3 - y_3}{V^2} - \frac{2nH + x_3 + y_3}{W^2}\right\}\right] - \sin\delta\,(x_2 - y_2)\left[\frac{1}{R^2} + \frac{1}{S^2} + \sum_{n=1}^{\infty} r^n \left(\frac{1}{V^2} + \frac{1}{W^2}\right)\right]\right\} \tag{55}$$

where

$$\begin{aligned}
r &= (\mu_1 - \mu_2)/(\mu_1 + \mu_2)\ , \\
R^2 &= (x_2 - y_2)^2 + (x_3 - y_3)^2\ , \\
S^2 &= (x_2 - y_2)^2 + (x_3 + y_3)^2\ , \\
T^2 &= (x_2 - y_2)^2 + (2nH - x_3 - y_3)^2\ , \\
U^2 &= (x_2 - y_2)^2 + (2nH - x_3 + y_3)^2\ , \\
V^2 &= (x_2 - y_2)^2 + (2nH + x_3 - y_3)^2\ , \\
W^2 &= (x_2 - y_2)^2 + (2nH + x_3 + y_3)^2\ ,
\end{aligned} \tag{56}$$

b is the slip and ds is the width of the line dislocation.

Changing to polar coordinates (Fig. 5)

$$y_2 = s \cos\delta\,, \quad y_3 = s \sin\delta$$

in eqn. (54) and (55) and integrating over s between the limits (s_1, s_2), Singh and Rani[12] obtained the following expressions for the displacements due to a long inclined strike-slip fault of finite width $s_2 - s_1$:

$$u^{(1)} = \frac{1}{2\pi}\left\{ M_0 \left[\tan^{-1}\left(\frac{s - x_2\cos\delta - x_3\sin\delta}{x_3\cos\delta - x_2\sin\delta}\right) - \tan^{-1}\left(\frac{s - x_2\cos\delta + x_3\sin\delta}{x_3\cos\delta + x_2\sin\delta}\right)\right]\right.$$

$$+ \sum_{n=1}^{\infty} M_n \left[\tan^{-1}\left(\frac{s - x_2\cos\delta + x_3\sin\delta - 2nH\sin\delta}{2nH\cos\delta - x_3\cos\delta - x_2\sin\delta}\right)\right.$$

$$+ \tan^{-1}\left(\frac{s - x_2\cos\delta - x_3\sin\delta - 2nH\sin\delta}{2nH\cos\delta + x_3\cos\delta - x_2\sin\delta}\right)$$

$$- \tan^{-1}\left(\frac{s - x_2\cos\delta - x_3\sin\delta + 2nH\sin\delta}{2nH\cos\delta - x_3\cos\delta + x_2\sin\delta}\right)$$

$$\left.\left. - \tan^{-1}\left(\frac{s - x_2\cos\delta + x_3\sin\delta + 2nH\sin\delta}{2nH\cos\delta + x_3\cos\delta + x_2\sin\delta}\right)\right]\right\}\Bigg|_{s_1}^{s_2}, \tag{57}$$

$$u^{(2)} = \frac{1}{\pi}\left\{ N_0 \left[\tan^{-1}\left(\frac{s - x_2\cos\delta - x_3\sin\delta}{x_3\cos\delta - x_2\sin\delta}\right)\right.\right.$$

$$- \tan^{-1}\left(\frac{s - x_2\cos\delta + x_3\sin\delta}{x_3\cos\delta + x_2\sin\delta}\right)$$

$$+ \sum_{n=1}^{\infty} N_n \left[\tan^{-1} \left(\frac{s - x_2 \cos\delta - x_3 \sin\delta - 2nH \sin\delta}{2nH \cos\delta + x_3 \cos\delta - x_2 \sin\delta} \right) \right.$$

$$\left. \left. - \tan^{-1} \left(\frac{s - x_2 \cos\delta + x_3 \sin\delta + 2nH \sin\delta}{2nH \cos\delta + x_3 \cos\delta + x_2 \sin\delta} \right) \right] \right\} \Bigg|_{s_1}^{s_2} , \tag{58}$$

where

$$f(s) \Bigg|_{s_1}^{s_2} = f(s_2) - f(s_1) ,$$

$$M_n = b \left(\frac{\mu_1 - \mu_2}{\mu_1 + \mu_2} \right)^n , \tag{59}$$

$$N_n = b\,\mu_1 \frac{(\mu_1 - \mu_2)^n}{(\mu_1 + \mu_2)^{n+1}} .$$

The corresponding expressions for the stresses are

$$p_{12}^{(1)} = \frac{\mu_1}{2\pi} \left\{ M_0 \left(- \frac{x_3 - s \sin\delta}{R^2} + \frac{x_3 + s \sin\delta}{S^2} \right) \right.$$

$$+ \sum_{n=1}^{\infty} M_n \left[\frac{x_3 + s \sin\delta - 2nH}{T^2} - \frac{x_3 - s \sin\delta + 2nH}{V^2} \right.$$

$$\left. \left. - \frac{x_3 - s \sin\delta - 2nH}{U^2} + \frac{x_3 + s \sin\delta + 2nH}{W^2} \right] \right\} \Bigg|_{s_1}^{s_2} , \tag{60}$$

$$p_{13}^{(1)} = \frac{\mu_1}{2\pi} (x_2 - s \cos\delta) \left[M_0 \left(\frac{1}{R^2} - \frac{1}{S^2} \right) \right.$$

$$+ \sum_{n=1}^{\infty} M_n \left(-\frac{1}{T^2} + \frac{1}{V^2} + \frac{1}{U^2} - \frac{1}{W^2} \right) \Bigg] \Bigg|_{s_1}^{s_2} , \tag{61}$$

$$p_{12}^{(2)} = \frac{1}{\pi} \Bigg\{ P_0 \left(-\frac{x_3 - s \sin \delta}{R^2} + \frac{x_3 + s \sin \delta}{S^2} \right)$$

$$+ \sum_{n=1}^{\infty} P_n \left[-\frac{x_3 - s \sin \delta + 2nH}{V^2} + \frac{x_3 + s \sin \delta + 2nH}{W^2} \right] \Bigg\} \Bigg|_{s_1}^{s_2} , \tag{62}$$

$$p_{13}^{(2)} = \frac{1}{\pi} (x_2 - s \cos \delta) \left[P_0 \left(\frac{1}{R^2} - \frac{1}{S^2} \right) + \sum_{n=1}^{\infty} P_n \left(\frac{1}{V^2} - \frac{1}{W^2} \right) \right] \Bigg|_{s_1}^{s_2} , \tag{63}$$

where

$$P_n = b \mu_1 \mu_2 \frac{(\mu_1 - \mu_2)^n}{(\mu_1 + \mu_2)^{n+1}} \tag{64}$$

In order to calculate the elastic deformation due to a thrust fault in the lithosphere - asthenosphere composite, Nur and Mavko[9] used the exact solution of Mura[41] for an edge dislocation parallel to the boundary of two elastic half-spaces in welded contact. They took into account the effect of the free surface approximately by cosidering Mura's solution for two equal and opposite edge dislocations with the free surface midway between them. By considering a simpler problem of a long dip-slip fault in a uniform half-space, Singh and Punia[42] have shown that the approximate method used by Nur and Mavko does not yield satisfactory results. For small dip angles and beyound a certain epicentral distance, while the exact solution predicts subsidence, the approximate solution predicts uplift. Moreover, the approximate solution yields non-zero values for the surface tractions at the free surface.

Recently, Punia[21] has found analytical solution of the porblem of a long dip-slip fault in a layer overlying a uniform half-space. He used the expressions for the Airy stress function in an unbounded medium given by Singh and Garg[43] to obtain the integral expressions for the displacements at the surface by applying suitable boundary conditions. As expected, these expressions are rather lengthy. Punia[21] used Sneddon's method of approximations (Sneddon[22], Ben-Menahem and Gillon[23]) to perform numerical computations.

Conclusions

The purpose of this article has been to describe closed-form analytical solutions of the porblems relating to the permanent deformation of plane layered Earth models caused by long displacement dislocations. These analytical solutions are useful in modelling the crustal deformation associated with strike-slip and dip-slip faulting in the Earth.

Closed-form analytical solutions are always superior to numerical solutions which are prone to computational errors, beside taking more time on the computer. We have seen that closed - form solutions are available for long strike-slip and dip-slip faults in a uniform half-space and in a two phase medium consisting of two uniform half-spaces in welded contact.Such a solution is also available for a long strike-slip fault in a layered half-space. However, in the case of a long dip-slip fault in a layered half-space,the analytical solution is available only in the form of an infinite integral. This integral can be modified by using Sneddon's method of approximating the integrand in such a manner that analytical integration is possible. For multilayered Earth models, one can take recourse to the Thomson- Haskell matrix method, modified by Singh[24] to compute permanent residual deformations in 3-D configurations and by Singh and Garg[26] in 2-D configurations. This method has been successfully applied by several investigators[44-58] in the recent past for modelling the crustal deformation of the Earth.

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References

1. Steketee, J.A. (1958) *Can. J. Phys.* **36** : 192.
2. Steketee, J. A. (1958) *Can. J. Phys.* **36** : 1168.
3. Maruyama, T. (1964) *Bull. Earthq. Res. Inst.* **42** : 289.
4. Maruyama, T. (1966) *Bull. Earthq. Res. Ist.* **44** : 811.
5. Savage, J.C. (1980) *Dislocations in Seismology, in Dislocations in Solids, Vol 3 ; Moving Dislocations,* ed. Nabarro, F.R.N., North-Holland, Amsterdam, pp. 251-339.
6. Mavko, G. M. (1981) *Ann. Rev. Earth. Planet Sci.* **9** : 81.
7. Rybicki, K. (1986) *Dislocations and Their Geophysical Applications, in Continuum Theories in Solid Earth Physics*, ed. Tiesseyre, R., Elsevier, Amsterdam, pp. 18 - 186.
8. Rybicki, K. (1971) *Bull.Seism. Soc. Am.* **61** : 79.
9. Nur, A. & Movko, G.M. (1974) *Science* **183** : 204.
10. Garg, N.R. & Singh, S.J. (1988) *Proc. Indian Acad.Sci. EarthPlanet. Sci.* **97** : 11.

11. Singh, S.J. & Rani, S. (1994) *Current Science* **66** : 219
12. Singh, S.J. & Rani, S. (1994) *J. Phys. Earth* **42** : 197.
13. Lee, E.H. (1955) *Q. Appl. Math.* **13** : 183.
14. Fung, Y.C. (1965) *Foundations of Solid Mechanics,* Prentice- Hall, Englewood Cliffs, New Jersey.
15. Singh, S. J. & Rosenman, M. (1974) *Phys. Earth Planet. Int.* **8** : 87.
16. Singh, K. & Singh, S.J. (1990) *Bull. Seism. Soc. Am.* **60** : 488.
17. Freund, L.B. & Barnett, D.M. (1976) *Bull. Seism. Soc. Am.* **66** : 667.
18. Rani, S., Singh, S.J. & Garg, N.R. (1991) *Phys. Earth Planet. Int.* **65** : 276.
19. Rani, S. & Singh, S.J. (1992) *Geophys. J. Int.* **109** : 469.
20. Singh, S. J. & Rani, S. (1993) *J. Phys. Earth* **41** : 87.
21. Punia, M. (1995) *Ph. D. Thesis,* Maharshi Dayanand University, Rohtak.
22. Sneddon, I.N. (1951) *Fourier Transforms,* McGraw-Hill, New York.
23. Ben-Menahem, A. & Gillon, A. (1970) *Bull. Seism. Soc. Am.* **60** : 193.
24. Singh, S.J. (1970) *J. Geophys. Res.* **75** : 3257.
25. Jovanovich, D.B., Husseini, M.I. & Chinnery, M.A. (1974) *Geophys. J. Roy.Astr. Soc.* **39** : 205.
26. Singh, S.J. & Garg, N.R., (1985) *Phys. Earth Planet. Int.* **40** : 135.
27. Okada, Y. (1992) *Bull. Seism. Soc. Am.* **82** : 1018.
28. Singh, S.J. & Rani, S. (1995) submitted for publication.
29. Chinnery, M. A. (1963) *Bull Seism. Soc. Am.* **53** : 921.
30. Chinnery, M.A. & Petrak, J.A. (1968) *Tectonophysics* **5** : 513
31. Mahrer, K. D. & Nur, A. (1979) *J. Geophys. Res.* **84** : 2296.
32. Yang, M. & Toksoz, M.N. (1981) *J. Geophys. Res.* **86** : 2889.
33. Wang, R. & Wu, H.L. (1983) *Pure Appl. Geophys.* **121** : 601.
34. Singh, S.J., Punia, M. & Rani, S. (1994) *Geophys. J. Int.* **118** : 411.
35. Sharma, R.K., Garg, N.R. & Singh, K. (1991) *Proc. Indian Acad.Sci. (Earth Planet. Sci.)* **100** : 379.
36. Rani, S. & Singh, S.J. (1993) *Proc. Indian. Natn. Sci. Acad.* **59A** : 455.
37. Singh, S.J., Rani, S. & Garg, N.R. (1992) *Phys. Earth Planet. Int.* **70** : 90.
38. Rani, S. & Singh, S.J. (1992) *Proc. Indian Acad. Sci. (Earth Planet. Sci.)* **101** : 269.
39. Cohen, S.C. (1979) *Geophys.Res. Lett.* **6** : 129.
40. Bonafede, M., Boschi, E. & Dragoni, M. (1984) *Geophys. J. Roy. Astr. Soc.* **76** : 515.
41. Mura. T. (1968) *The Continuum Theory of Dislocations, in Advances in Materials Research,* **Vol. 3** (H. Herman ed.).Wiley, New York, pp. 1-108.
42. Singh, S.J. & Punia, M. (1994) *J. Phys. Earth* **42** : 89.
43. Singh, S.J. & Garg, N.R. (1986) *Acta Geophys. Pol.* **34** : 1.
44. Sato, R. (1971) *J. Phys. Earth* **19** : 31.

45. Sato, R. (1972) *J. Phys. Earth* **20** : 357.
46. Sato, R. & Matsu'ura, M. (1973) *J. Phys. Earth* **21** : 227.
47. Turcotte, D.L. & Spence, D.A. (1974) *J. Geophys. Res.* **79** : 4407.
48. Matsu'ura, M. & Sato, R. (1975) *J. Phys. Earth* **23** : 1.
49. Rundle, J. B. (1980) *J. Geophys. Res.* **85** : 5355.
50. Rundle, J.B. (1981) *J. Phys. Earth* **29** : 173.
51. Leary, P.C. (1985) *J. Geophys. Res.* **90** : 1901.
52. Leary, P.C. (1985) *Geophys. Res. Lett.* **12** : 29.
53. Pan, E. (1989) *Phys. Earth Planet. Int.* **58** : 103.
54. Roth, F. (1990) *Geophys. J. Int.* **103** : 147.
55. Pan, E. (1991) *Acta Mech.* **87** : 105.
56. Ma, X. Q. & Kusznir, N.J. (1992) *Geophys, J. Int.* **111** : 542.
57. Roth. F. (1993) *J. Geophys. Res.* **98** : 4543.
58. Ma, X.Q. & Kusznir, N.J. (1994) *Geophys. J. Int.* **118** : 201.