Fractal analysis of sea level variations in coastal regions of India

N. K. Indira, R. N. Singh*† and K. S. Yajnik

CSIR Centre for Mathematical Modelling and Computer Simulation, National Aerospace Laboratories, Belur Campus, Bangalore 560 037, India
*National Geophysical Research Institute, Hyderabad 500 007, India
†Present address: CSIR Centre for Mathematical Modelling and Computer Simulation, National Aerospace Laboratories, Belur Campus, Bangalore 560 037, India

Sea level variations reflect the dynamics of the nonlinear atmosphere–ocean–cryosphere–lithosphere system. The nature of the nonlinearity can be deciphered from the analysis of the sea level records. Fractal analysis of the data recorded at the Indian tide gauge stations during the past several decades has been done using the Iterated Function Systems (IFS) technique. The calculated fractal dimensions vary between 1.2 and 1.3. This is slightly more than the values of the dimension of the sea level record over the duration of thousands of years[1]. It is inferred that the long term behaviour of sea level changes can be modelled by a nonlinear dynamical system having a small number of variables.

THE changes in the global sea level are due to a variety of endogenic and exogenic causes and have gained current interest because of their relevance to greenhouse effect. Greenhouse effect causes global warming and this results in rise in sea level. Over a long span of time (say decades/centuries) the following additional factors affect the sea level: melting of large polar ice sheets; thermal expansion of ocean water and lithospheric subsidence, etc. The major impact of sea level rise is on the socio-economic conditions of the coastal regions as a large part of the world's population and food production activities are situated along the coastal zone. It is, therefore, essential that the nature of sea level changes is analysed and modelled in order to assess its implications to coastal resources and environment.

As the sea level is a response of a nonlinear system consisting of ocean, atmosphere, cryosphere and lithosphere, it would be desirable to apply nonlinear time series analysis techniques, such as the fractal analysis to this data, which yields the fractal dimension of the time series. The importance of the fractal dimension lies in its association with an attractor and the measure of attractor provides the information about the structure, complexity and nonuniformity of the asymptotics of the associated dynamical system. They also provide approximations to the number of degrees of freedom necessary to effectively model the system asymptotics. A non-integer value for the dimension of the fractal curve representing sea level variations at a station will indicate that the underlying (or causative) process is non-linear.

Hsui *et al.*[1] had conducted a fractal analysis of long term sea level changes and obtained fractal dimensions. Longer period data show values closer to one except the Missourian sea level data (296 to 292 m.y. before present) showing a large value of 1.4. They use this result to imply that sea level shows a persistent behaviour in long term. Larger value for the Missourian data is ascribed due to both climatic and tectonic causes or different climatic responses than at present. In this paper fractal analysis of the annual sea level data over the Indian coastal stations is performed to see the existence of any such behaviour. The tide gauge data of India using stochastic modelling techniques have been analysed[2]. Further, Indian tide gauge records have also been analysed to identify possible trends in sea level variations[3]. Shetye *et al.*[4] have examined tide gauge records in order to study the vulnerability of the Indian coast to a possible sea level rise.

The fractal dimension can be obtained by several methods. The power spectrum analysis is the frequently used method provided the data length is large. However, it is known that the fractal dimension obtained by spectral method is over-estimated when values of the dimensions are small[5]. Since the fractal dimension for long period sea level changes[1] is small and the data length in the present study is between 30 and 60 years, IFS technique[6] is being used here to obtain the fractal dimension of sea level variations at Indian coastal stations. IFS technique has been applied to analyse the marine bathymetric data[7].

Method of analysis and results

Tide gauge data at the Indian coastal stations have been collected by the Survey of India. The annual data for five coastal stations in India Bombay, Cochin, Madras, Vizag (Vishakhapatnam) and Sagar Island (Calcutta) are used for the analysis.

The details on the implementation of the IFS technique, for the construction of the fractal interpolation algorithm, are given by Barnseley[6]. In this technique, a synthetic curve of known fractal dimension is generated and is fitted to the observed sea level curve. For the generation of synthetic curve, a few points are chosen at first from the observed data points $\{(x_i, z_i) \in \mathbb{R}^2,$

$i = 0, 1, \ldots, N\}$, where x_i is on the time axis and z_i is the corresponding sea level and $x_0 < x_1 < \ldots < x_N$. A continuous interpolation function $f: [x_0, x_N] \rightarrow \mathbb{R}$ is constructed such that $f(x_i) = z_i$ for $i = 0, 1, \ldots, N$. An IFS is defined in the form $\{\mathbb{R}^2; w_n, n = 1, \ldots, N\}$, where N is a positive integer greater than one and w_n is such that the maps are affine transformation given as

$$w_n\begin{pmatrix} x \\ z \end{pmatrix} = \begin{pmatrix} a_n & 0 \\ c_n & d_n \end{pmatrix}\begin{pmatrix} x \\ z \end{pmatrix} + \begin{pmatrix} e_n \\ f_n \end{pmatrix}. \quad (1)$$

Here a_n, c_n, e_n, and f_n are constant real numbers and d_n ($0 \le d_n \le 1$) is the vertical scaling factor in the transformation w_n. These constants are determined by using the following constraints by the data on the transformation

$$w_n\begin{pmatrix} x_0 \\ z_0 \end{pmatrix} = \begin{pmatrix} x_{n-1} \\ z_{n-1} \end{pmatrix} \text{ and } w_n\begin{pmatrix} x_N \\ z_N \end{pmatrix} = \begin{pmatrix} x_n \\ z_n \end{pmatrix} \quad (2)$$

for $n = 1, \ldots, N$. Points in between the chosen points are generated by using this transformation and by applying iteratively the interpolation algorithm which calls for the random number generator. Larger number of iteration leads to better quality of the synthetic curve. The fractal dimension D of the synthetic data is obtained by solving

$$\sum |d_n| a_n^{D-1} = 1, \quad (3)$$

where the numbers a_n are

$$a_n = (x_n - x_{n-1})/(x_N - x_0) \quad (4)$$

obtained from the data. The solution of D in equation (3) is valid if the interpolation points do not lie on a straight line and $\sum |d_n| > 1$, otherwise the fractal dimension of the synthetic data is one.

Using the IFS technique, the fractal dimensions of sea level variations in the coastal regions of India have been obtained. As the value of the fractal dimension is sensitive to the choice of the scaling factors than to the choice of data points, the selection of the scaling factor has been made carefully for each data set. For each data set the algorithm iterates 2000 times to generate interpolated points. Table 1 gives the number of points chosen for interpolation along with the number of scaling factors and also the range of the scaling factor (d_n) selected for the interpolation.

Table 1. Parameters used in IFS technique

Station	Year	Number of chosen points	Number of scaling factors	Range of d_n
Bombay	1921–1988	33	32	(–0.21, 0.24)
Cochin	1956–1988	15	14	(–0.26, 0.24)
Madras	1952–1988	19	18	(–0.21, 0.18)
Vizag	1953–1988	18	17	(–0.32, 0.12)
Sagar	1948–1988	20	19	(–0.15, 0.21)

Table 2. Fractal dimensions

Station	Year	Fractal dimension
Bombay	1921–1988	1.28
Cochin	1956–1988	1.21
Madras	1952–1988	1.23
Vizag	1953–1988	1.24
Sagar	1948–1988	1.21

Figures 1 to 5 show fractal curves fitted to the observed sea level data for the five coastal stations in India. The observed data points are shown by both squares and triangles. The points shown by squares are the points chosen for the application of IFS. The fractal interpolation algorithm is applied to generate points in between the chosen points. The series of filled circles forming a curve passing through the observed data points, are the points generated through IFS technique between the chosen points shown by squares. Table 2 gives the dimension of the fractal curve for each of the five coastal stations studied here with their respective data length. It can be noticed that the fractal dimensions of sea level curves are non-integer and lie between 1.2 and 1.3 for the five coastal stations taken up here for analysis. We have also seen that no matter which points are chosen for fractal interpolation, the fractal dimension of the curves remains the same. Thus, the dimension of the sea level variations can be said with confidence to be a non-integer, thereby demonstrating the non-linearity in the sea level variations.

Discussions

Fractal dimensions have been used to investigate the persistence behaviour in the past. For instance, the Gaussian noise process showing no persistence behaviour has fractal dimension D equivalent to 1.5. The processes with values of D different from 1.5 show the features of fractional Brownian motion which leads to persistence and antipersistence behaviour[8]. For $D < 1.5$ the process is said to have persistence behaviour, i.e. if there has been an increase/decrease in the past for some period then it is expected to continue for a similar period in future. Antipersistence is the case when $D > 1.5$, i.e. the process tends to show a decrease in value following previous increase and increase in the value following previous decrease. Hsui *et al.*[1] have argued that the fractal dimension approaching the value 1 shows a long term persistence behaviour. Thus, the sea level data studied here show persistent behaviour as D values lie between 1.2 and 1.3.

The fractal dimensions lying between 1.2 and 1.3 obtained in this paper for sea level changes over a short time interval of around 40 years are slightly more than the fractal dimension of sea level curves over the past

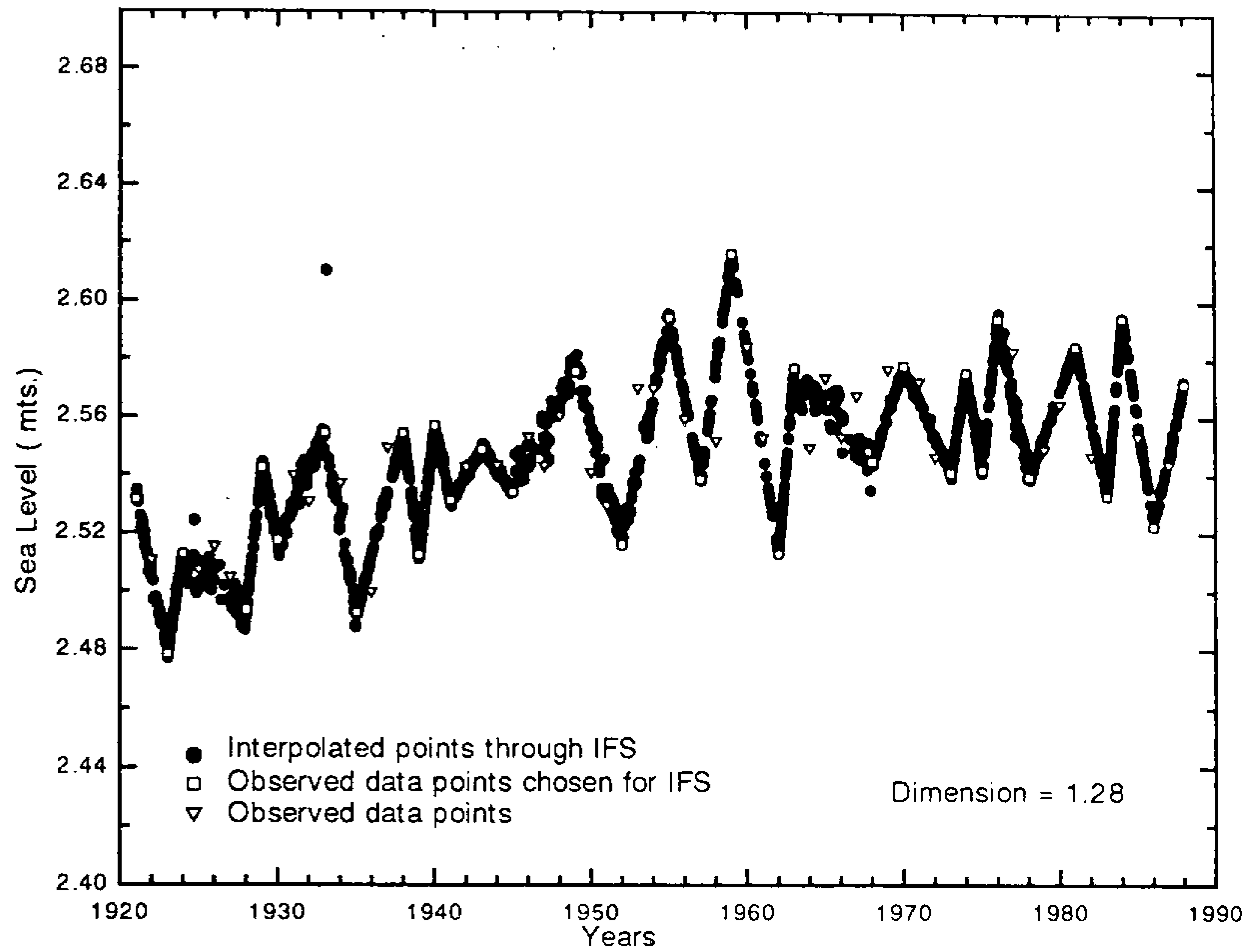

Figure 1. Sea level in Bombay (1921–1988).

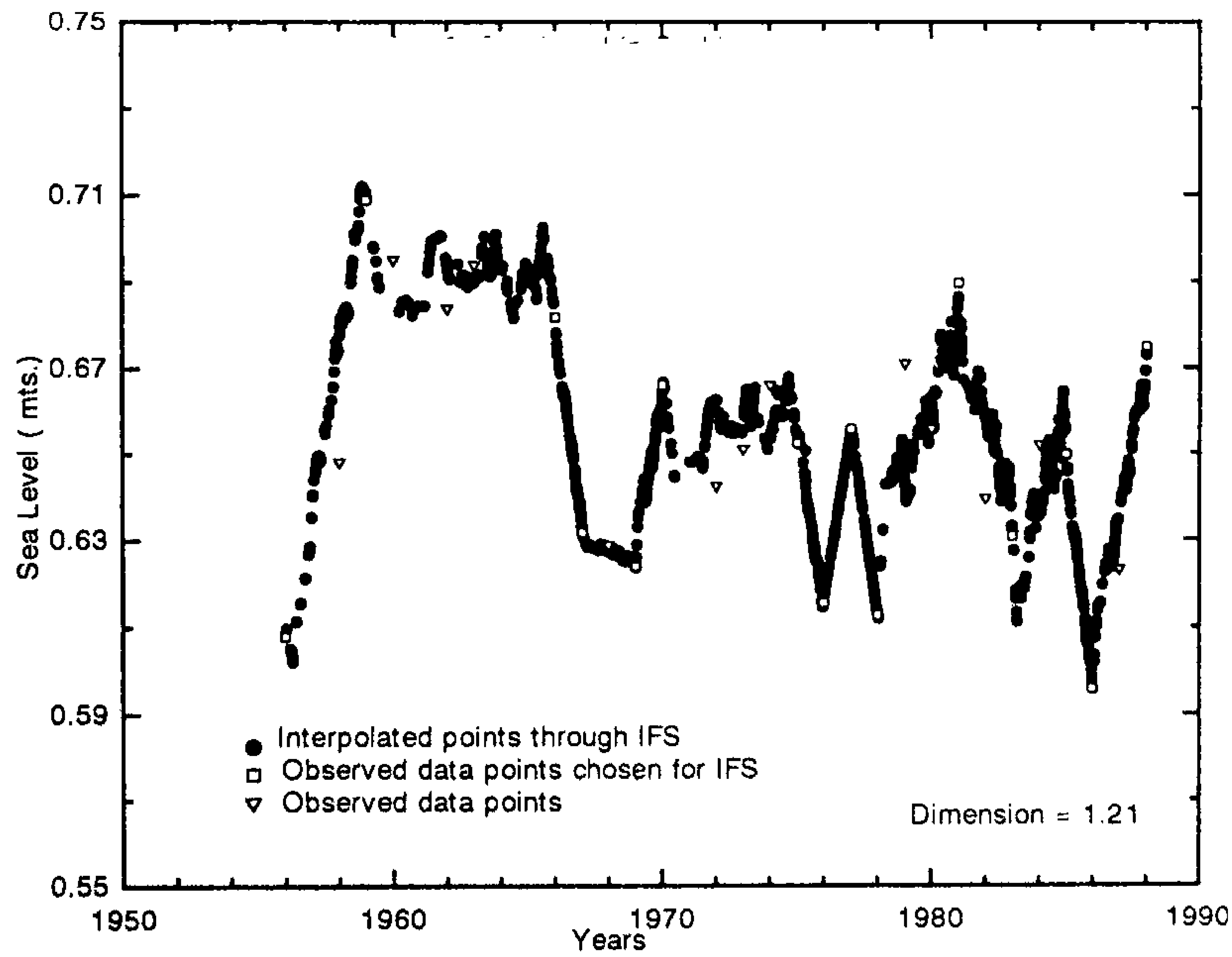

Figure 2. Sea level in Cochin (1956–1988).

thousands of years as obtained by Hsui *et al.* This difference can be caused by a variety of internal/external geophysical processes. It is interesting to note that the dimensions obtained here are similar to the fractal

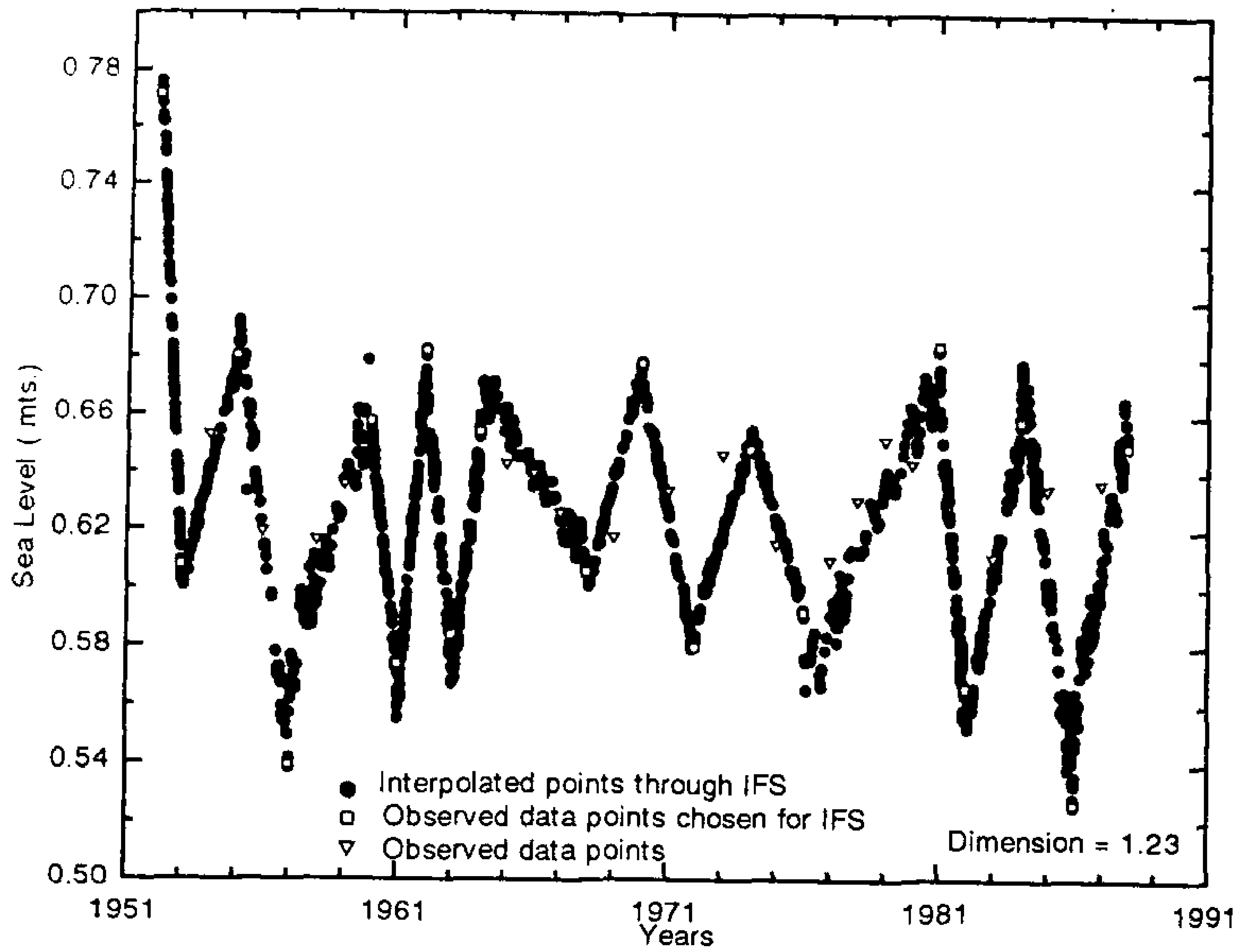

Figure 3. Sea level in Madras (1952–1988).

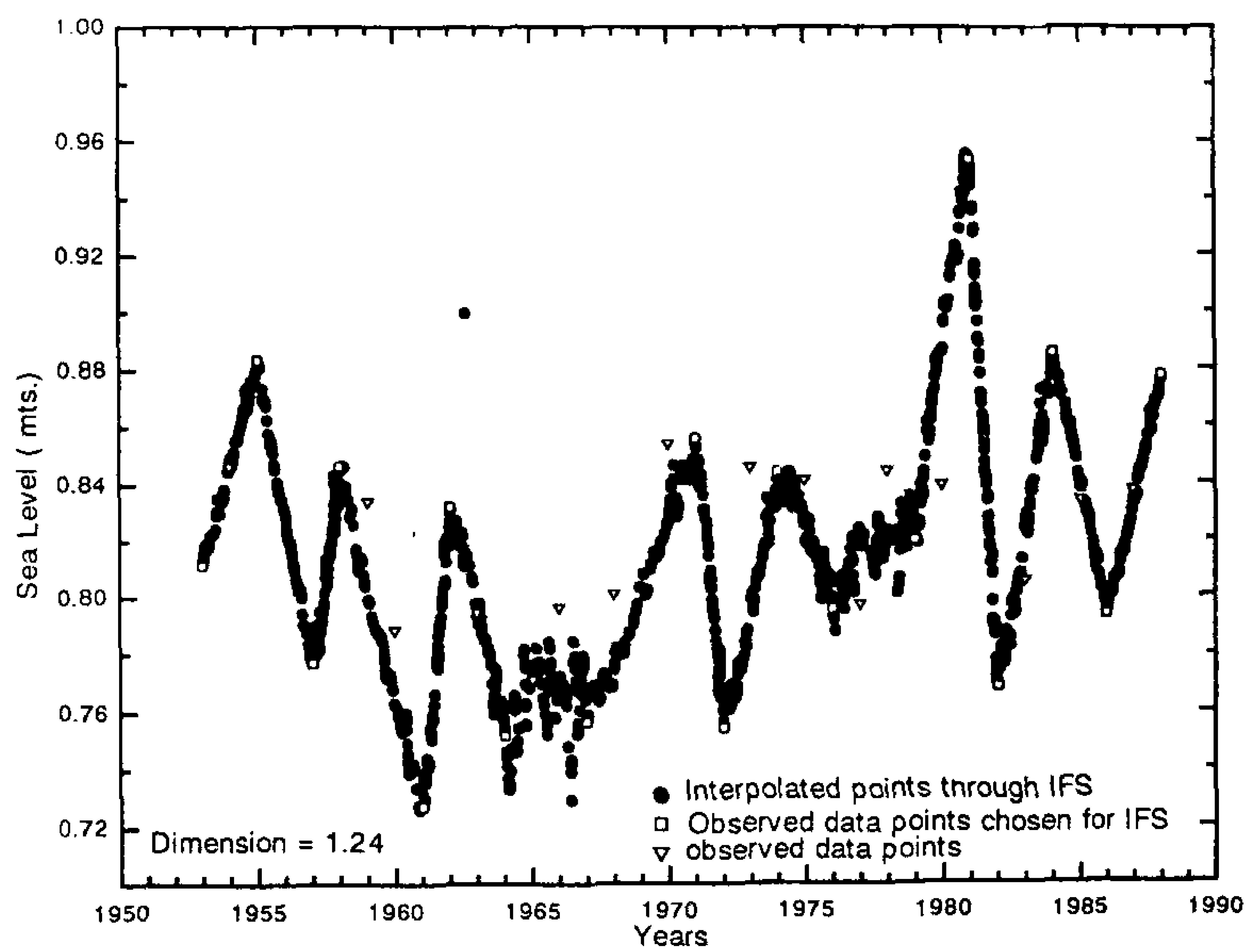

Figure 4. Sea level in Vizag (1953–1988).

dimension of 1.22 and 1.26 of the climatic series[1]. This also establishes a close relationship between sea level changes analysed here and the climatic variations.

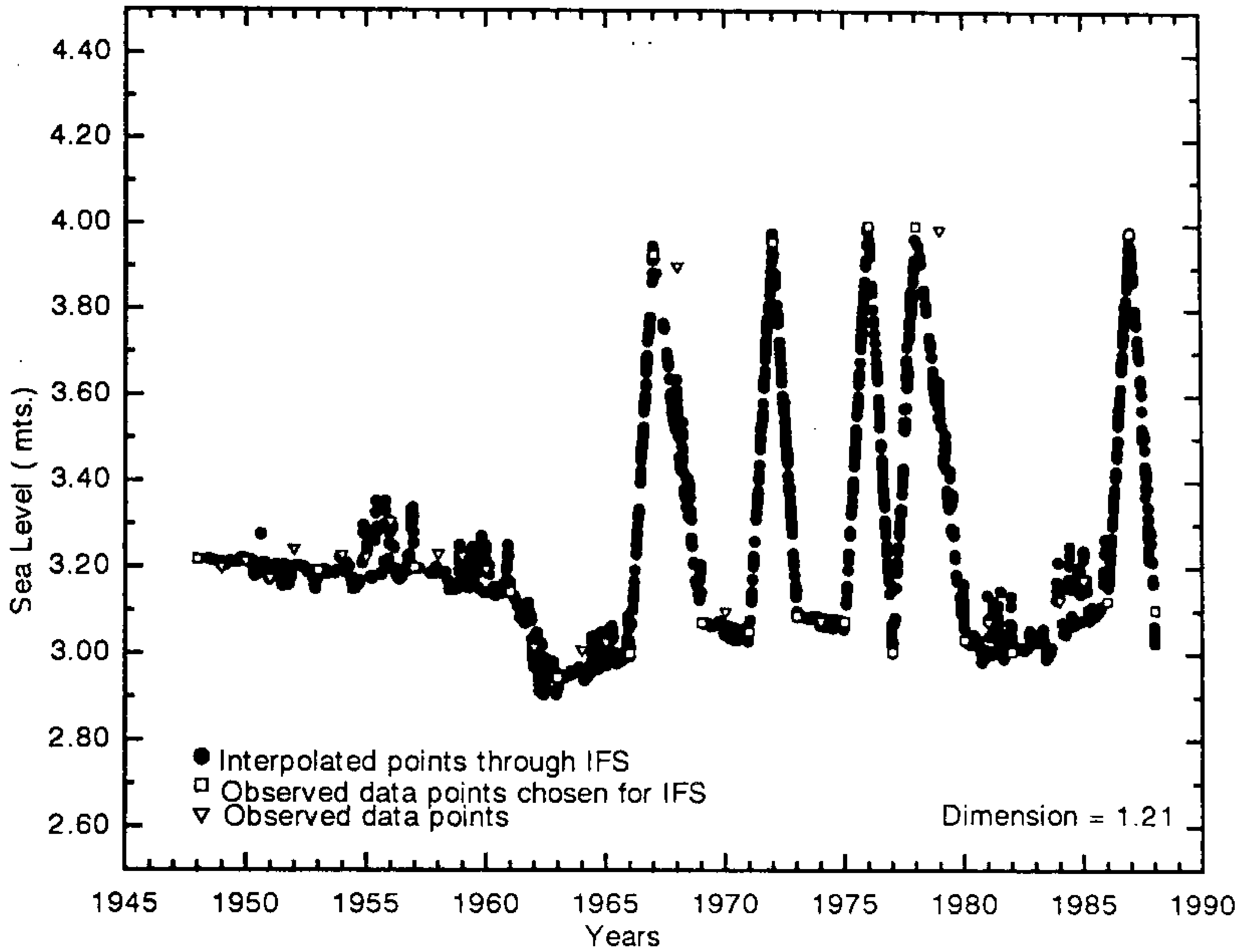

Figure 5. Sea level in Sagar (1948–1988).

It has been suggested[9] for the case of climate fluctuations that the fractal dimensions help to determine the number of variables in the nonlinear dynamical system responsible for characterizing long term behaviour. Thus from low dimensions obtained in the present case, it can also be surmised that the long term behaviour of sea level fluctuation can be determined by a nonlinear dynamical system which has small number of variables[10].

Conclusions

The fractal dimensions of annual sea level data of about 40 years duration using the IFS technique are obtained as non-integer values lying between 1.2 and 1.3. Thus, sea level variations show a nonlinear nature and should be represented by a nonlinear dynamical process. The low values of dimensions suggest the small number of variables necessary to represent the system. The dimensions also indicate the long term persistence behaviour.

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