BU TH-94/04
hep-ph/9407210

Structure of proton and nuclei *

Rohini M. Godbole
Dept. of Physics, Univ. of Bombay, Kalina Campus,
Santa Cruz(E), Bombay 400 098, India.

Abstract

We discuss in this set of lectures the structure of proton/neutron as revealed through a study of form-factors. This is followed by a discussion of the structure functions of proton/nuclear targets as measured in the deep inelastic scattering (DIS) of leptons off these targets. We discuss the parton model in DIS as well as outline the usage of parton model in processes other than the DIS. We then go on to dicsuss the EMC effect: the nuclear dependence of the structure functions. After a discussion of different models of the EMC effect we end by pointing out the possibility of distinguishing between these different models by studying the correlation between the A–dependence of different hard processes and the EMC effect.

*Lectures given at the S.E.R.C. school in Nuclear Physics, Goa University, Feb. 1993

Introduction

Structure of hadrons, the strongly interacting particles, has been the subject of theoretical and experimental investigations over the period of past four decades and has played a crucial role in the birth of the subject of particle physics. Traditionally, information about the constituents of a system has come from two sources: one is the study of static properties such as mass, magnetic moments, spin, parity, etc., and the other is scattering experiments. The famous Rutherford scattering experiment is the prototype of the latter. In the case of hadrons, the clues to quark structure came in the form of the quark model put forward by Gell-Mann [1], but the final confirmation came from the Deep Inelastic Scattering (DIS) experiments [2] where the observed phenomenon of 'scaling' was explained [3] in terms of 'partons'(pointlike constituents of proton) [4]. The discovery of 'asymptotic freedom' [5] of 'Quantum Chromodynamics' (QCD) [6]: the gauge theory of quarks and gluons, coupled with the operator product expansion gave an explanation, in the context of perturbative QCD(pQCD), why the parton model works so well[7]. By now QCD is the accepted theory of strong interactions. In the set of these six lectures an effort will be made to review the phenomenological understanding of DIS and how its study led to the parton model, along with some aspects of parton structure of nuclei. We begin with an introduction as to how one probes the structure of a nucleus via electromagnetic interactions and explain the concept of the electric and magnetic form factors of the proton/neutron. Following this, we will discuss the DIS kinematics and introduce the idea of two structure functions in terms of which the electromagnetic DIS cross-section can be parametrised. This will be followed by a discussion of scaling of the structure functions and parton model. The scaling violations predicted by QCD will be alluded to very briefly. In the case of the DIS processes mediated by the electroweak gauge bosons, $W^\pm/Z^0$, three independent structure functions are required to parametrise the DIS cross-section. We introduce these and briefly discuss the important role played by neutrino induced DIS processes in providing conclusive evidence for the parton model. The momentum distribution of partons measured in both the electromagnetic and the weak DIS processes is found to be affected by the nuclear environment. This is the so called EMC effect [8, 9]. This will be discussed in the last lecture. Various models [9, 10] have been put forward as a theoretical explanation of this effect. Some of the models will be summarized along with a discussion of possible experimental tests to distinguish among these models [10].

1 Electromagnetic Structure of Hadrons

Historically, hadrons were classified as particles with large masses ($m \simeq 1$ GeV) which participate in strong interactions. The wide variety of these hadrons (p, n, Λ, $\Sigma^0, \Sigma^\pm, \Xi^\pm$, $\pi^\pm, \pi^0, K^\pm, K^0, \bar{K}^0, \cdots$) already indicated that these are not elementary. Further, for an elementary, spin $\frac{1}{2}$ particle, Dirac theory predicts gyromagnetic ratio g to be 2. Even the deviation of g from 2 can be computed in perturbation theory.

For example, for an e^-, theoretical computations [11] predict

$$\left(\frac{g-2}{2}\right)_e^{th} = \frac{\alpha}{2\pi} - 0.328478966\left(\frac{\alpha}{\pi}\right)^2 + 1.1765(13)\left(\frac{\alpha}{\pi}\right)^3 - .8(2.5)\left(\frac{\alpha}{\pi}\right)^4$$
$$= 0.001159652\underline{460}(192) \tag{1.1}$$

where α is the fine structure constant given by $e^2/4\pi$.[†] The experimentally measured value [12] is

$$\left(\frac{g-2}{2}\right)_e^{expt.} = 0.0011596521\underline{93}(10) \tag{1.2}$$

In eqs. (1.1) and (1.2) the numbers in the bracket indicate the theoretical and experimental uncertainty respectively. The excellent agreement between the theoretical prediction and the experimentally measured value indicates that the electron is indeed elementary. However, for a proton and neutron, the measured values [12] of magnetic moments are

$$\mu_p = 2.79\ \mu_N = 2.79\frac{|e|}{2M_p} \quad ,$$
$$\mu_n = -1.91\ \mu_N = -1.91\frac{|e|}{2M_p} \quad . \tag{1.3}$$

These correspond to values of 1.79 μ_N and -1.91 μ_N for anomalous magnetic moments of the proton and neutron respectively. This clearly indicates that they are not elementary particles *i.e.* they have spatial extension. They have a structure. But this is, strictly speaking, an energy dependent statement. At low energies, the scattering experiments are not able to 'resolve' this structure. After all, Rutherford concluded from his scattering experiments [13] with α particles that all the positive charge in an atom is concentrated in a *point* nucleus and the electrons occupy the rest of the atomic volume. But what this conclusion really meant was that the nuclear radius, r_N, is much smaller than atomic sizes ($\sim \mathcal{O}$ (few $\mathring{A} \approx 10^{-10}\ m$)) . One had to perform the scattering experiments [14] with higher energy electrons ($\sim 100 - 500$ MeV) to reveal the structure of nuclei at a distance scale $\sim$ few fm $= 10^{-15}\ m$.

1.1 Effect of structure of the scattering centers on scattering amplitudes

To understand this let us consider a general scattering process

$$e^- + A \to e^- + A\ . \tag{1.4}$$

The process is represented digramatically in fig. 1. Let P_1 and P_2[‡] denote the four momentum of the target nucleus and the incoming electron respectively and P_3, P_4

[†] I will use units $\hbar = c = 1$ throughout these lectures.

[‡] I use Pauli metric such that $P_i^2 = -m_i^2$. Details of notation are given in Appendix A.

be the four momenta of the e^- and nucleus in the final state, respectively. Let

$$P_1 \equiv (\vec{0}, iM_A), \quad P_2 = (\vec{p}_0, iE_0)\ ,$$
$$P_3 \equiv (\vec{p}, iE), \qquad P_4 = (\vec{p}\,', iW)\ . \tag{1.5}$$

As shown in the figure, the electromagnetic scattering takes place via the exchange of a photon. Application of four-momentum conservation at each vertex implies that the four momentum of the photon is given by

$$q = P_2 - P_3 = P_4 - P_1\ . \tag{1.6}$$

In the notation of eq. (1.5) we have,

$$q \equiv (\vec{q}, iq_0) = (\vec{p}^{\,0} - \vec{p},\ i(E_0 - E))$$
$$= (\vec{p}\,',\ i(W - M_A))\ . \tag{1.7}$$

Using eqs. (1.5) and (1.6), we get for the invariant mass of the photon,

$$q^2 = (P_2 - P_3)^2 = -2m_e^2 - 2|\vec{p}_2|\ |\vec{p}_3| \cos\theta + 2E_2E_3\ . \tag{1.8}$$

Neglecting the electron mass m_e (which also implies $|\vec{p}_2| = E_0 = |\vec{p}_0| = p_0$ and $|\vec{p}_3| = E = p$) we get,

$$q^2 = 2pp_0(1 - \cos\theta) = 4pp_0 \sin^2\frac{\theta}{2}\ . \tag{1.9}$$

Thus for a scattering process one always has $q^2 > 0$. This means that the exchanged photon is not a real photon but a virtual one.

It can be shown, on very general grounds, that the net effect of the presence of a structure in the scattering center on the scattering amplitude, calculated from a diagram such as shown in fig. 1, is to multiply it by a form factor $F(q^2)$. This form factor is given by,

$$F(q^2) = \int e^{iq\cdot y}\ f(y)\ d^4y \tag{1.10}$$

where $f(y)$ is the distribution function describing the target. In the limit of recoilless, elastic scattering ($W \approx M_A$) eq. (1.6) gives $\vec{q} \equiv (\vec{q}, i0)$. In this case the four-fourier transform reduces to the spatial Fourier transform.

1.2 Relation of spatial charge distribution to form factor

Let us consider scattering of a spinless electron from a spinless charge distribution and derive an expression for the form factor in this case. Consider an e^- of energy E_0 and momentum $\vec{p}_0$ incident on a target nucleus of charge $Z|e|$ at rest. The four momenta of various particles are then given by eq. (1.5). For a point nucleus, the classical Rutherford scattering formula for the differential cross-section[§] is given by

$$\left.\frac{d\sigma}{d\Omega}\right|_{\text{Rutherford}} = \frac{m_e^2 Z^2 (e^2/4\pi)^2}{4p_0^4 \sin^4\frac{\theta}{2}} = \frac{Z^2\alpha^2 m_e^2}{4p_0^4 \sin^4\frac{\theta}{2}}\ , \tag{1.11}$$

[§]Note that this formula can be derived classically as well as in non-relativistic quantum mechanics using Born approximation for the scattering of an e^- in the screened, Coulomb field of a nucleus.

where Ω is the solid angle and $d\Omega = \sin\theta d\theta d\phi$, θ being the scattering angle. For a spinless, relativistic e^- scattered by the screened, Coulomb potential of a point nucleus, this expression is,

$$\frac{d\sigma}{d\Omega} = \frac{Z^2\alpha^2}{4E_0^2\sin^4\frac{\theta}{2}} \quad . \tag{1.12}$$

If we assume the initial electron to be incident along the z axis, the kinematics is as depicted in fig. 2. Now using eqs. (1.5) and (1.6), we see that

$$\vec{p}_0 = \vec{p} + \vec{p}\,' \quad , \tag{1.13a}$$

$$M_A + E_0 = E + W \quad . \tag{1.13b}$$

If E_i and E_f are used to indicate total initial and final state energies, we have

$$E_i = M_A + E_0 = M_A + p_0 = E_f = W + E = W + p \quad . \tag{1.14}$$

Let us suppose that the nuclear charge $Z|e|$ is distributed with a distribution function $\rho(R)$, normalised to unity by,

$$\int \rho(\vec{R}) d^3R = 1 \quad . \tag{1.15}$$

Hence the total electric charge in a volume element d^3R is given by $Z|e|\rho(\vec{R})d^3R$. The scattering cross-section is

$$d\sigma = \frac{\mathcal{W}}{v} \quad , \tag{1.16}$$

where $\mathcal{W}$ is the transition probability and v is the velocity of the incident e^- w.r.t. the center (which for relativistic electrons is 1 in our units).¶ The transition probability $\mathcal{W}$ is calculated using Fermi's golden rule

$$\mathcal{W} = 2\pi\rho_f|\mathcal{M}_{if}|^2 \quad , \tag{1.17}$$

where ρ_f is the density of states in the final state and $\mathcal{M}_{if}$ is the scattering amplitude, computed in Born approximation for the screened, Coulomb nuclear potential. For the two particle final states, the density of final states ρ_f is given by (with $\hbar = 1$),

$$\rho_f = \frac{dN}{dE_f} = p^2 d\Omega \frac{dp}{dE_f} \tag{1.18}$$

and $\mathcal{M}_{if}$ is given in the Born approximation as

$$\mathcal{M}_{if} = \int \psi_f^\star(\vec{r}) V(\vec{r}) \psi_i(\vec{r}) d^3r \quad . \tag{1.19}$$

dp/dE_f in eq. (1.18) can be computed using eq. (1.13a) as follows. Eq. (1.13a) gives

$$\begin{aligned} E_f = p + W &= \sqrt{|\vec{p}\,'|^2 + m_A^2} + p \\ &= \sqrt{M_A^2 + p_0^2 + p^2 - 2pp_0\cos\theta} + p \quad . \end{aligned}$$

¶Note here that I am normalising with one particle per unit volume.

This in turn gives,

$$\frac{dp}{dE_f} = \frac{W}{E_f - p_0 \cos\theta} \quad . \tag{1.20}$$

The energy and three momentum conservation of eqs. (1.13a) , (1.13b) gives

$$E_f = p_0 + M_A = p + W = p + \sqrt{p^2 + p_0^2 - 2pp_0 \cos\theta + m_A^2} \quad . \tag{1.21}$$

This gives us,

$$\frac{p_0}{p} M_A = E_f - p_0 \cos\theta \quad . \tag{1.22}$$

This can also be rewritten as ,

$$\frac{p}{p_0} = \frac{1}{1 + 2p_0/M_A \sin^2\frac{\theta}{2}} \quad . \tag{1.23}$$

Combining eqs. (1.20) and (1.22) we have

$$\frac{dp}{dE_f} = \frac{W}{p_0}\frac{p}{M_A} \quad . \tag{1.24}$$

To calculate $\mathcal{M}_{if}$ in eq. (1.19) we need $V(\vec{r})$. If we take into account the screening of the nuclear charge due to atomic electrons, the potential $V(\vec{r})$ felt by the electron at $\vec{r}$, due to the nuclear charge distribution shown in fig. 3 will be,

$$V(\vec{r}) = -\frac{Ze^2}{4\pi} \int \frac{\rho(\vec{R}) d^3R}{|\vec{r} - \vec{R}|} e^{-|\vec{r}-\vec{R}|/a} \quad , \tag{1.25}$$

where a is a damping factor $\sim \mathcal{O}$ (atomic radius) which arises from the screening of the nuclear charge by the atomic electrons and is called the screening radius. Clearly $a \gg R$. If one uses the plane wave approximation for the wave functions of the incident and scattered electrons, with momenta $\vec{p}_0$ and $\vec{p}$ respectively, we have

$$\psi_f^\star(\vec{r}) = e^{-i\vec{p}\cdot\vec{r}}, \quad \psi_i(\vec{r}) = e^{i\vec{p}_0\cdot\vec{r}} \qquad (\hbar = c = 1) \quad . \tag{1.26}$$

This then gives for $\mathcal{M}_{if}$,

$$\begin{aligned}
\mathcal{M}_{if} &= \frac{-Ze^2}{4\pi} \int e^{i\vec{q}\cdot\vec{r}} d^3r \int \frac{\rho(\vec{R}) e^{-|\vec{r}-\vec{R}|/a}}{|\vec{r} - \vec{R}|} d^3R \\
&= \frac{-Ze^2}{4\pi} \int e^{i\vec{q}\cdot\vec{R}} \rho(\vec{R}) d^3R \int \frac{e^{i\vec{q}(\vec{r}-\vec{R}) - |\vec{r}-\vec{R}|/a}}{|\vec{r} - \vec{R}|} d^3R \quad .
\end{aligned}$$

Choosing the z axis along $\vec{q}$ and $\vec{s} = \vec{r} - \vec{R}$, we get

$$\mathcal{M}_{if} = \frac{-Ze^2}{4\pi} \int e^{i\vec{q}\cdot\vec{R}} \rho(\vec{R}) d^3R \int_0^\infty 2\,\pi\ s ds \int_{-1}^{1} d\cos\alpha\ e^{iqs\cos\alpha}\ e^{-s/a} \quad . \tag{1.27}$$

The first integral is a function only of $Q^2 = |\vec{q}|^2$. Then we have

$$\mathcal{M}_{if} = -F(Q^2)\frac{Ze^2}{4\pi}\frac{4\pi}{Q^2+1/a^2} \quad , \tag{1.28}$$

where

$$F(Q^2) = \int e^{i\vec{q}\cdot\vec{R}}\rho(R)d^3R \quad . \tag{1.29}$$

$F(Q^2)$ is called the form factor. Note here that the screening factor $e^{-s/a}$ in the integrand is essential to make this integral convergent. $Q^2 = |\vec{q}|^2 = p_0^2+p^2-2pp_0\cos\theta$ is $\sim \mathcal{O}(\mathrm{MeV}^2)$ for the energies under consideration whereas $a \sim \mathcal{O}(10^{-8}\mathrm{cm})$; *i.e.* $1/a^2 \simeq 4(\mathrm{KeV}^2)$. [‖] Hence for $E_0 \gtrsim O$ (MeV) , $1/a^2 \ll Q^2$. Therefore the expression for $\mathcal{M}_{if}$ can be approximated as

$$\mathcal{M}_{if} \simeq \frac{-Ze^2}{q^2} \simeq \frac{-Ze^2}{Q^2} \quad . \tag{1.30}$$

Substituting eqs. (1.30), (1.20) and (1.18) in (1.17), we get

$$d\sigma = 2\pi\frac{p^2d\Omega}{(2\pi)^3}\frac{W}{E_f - p_0\cos\theta}\left(\frac{Ze^2}{4\pi}\right)^2\frac{(4\pi)^2}{Q^4}|F(Q^2)|^2 \quad . \tag{1.31}$$

Using eq. (1.23) we get finally,

$$\frac{d\sigma}{d\Omega} = 4p^2\frac{W}{M_A}\frac{p}{p_0}\left(\frac{Ze^2}{4\pi}\right)^2\frac{|F(Q^2)|^2}{Q^4} \quad . \tag{1.32}$$

Since the nuclear mass $M_A \sim \mathcal{O}$ (GeV) $\sim \mathcal{O}$(1000 MeV), for electron beam energies of $\sim \mathcal{O}$ (MeV) , $W \simeq M_A$. Hence eq. (1.13b) tells us $p \approx p_0$ and hence $Q^2 \simeq 4p_0^2\sin^2\theta/2 = 4E_0^2\sin^2\theta/2$. Using all this the expression for the differential cross-section becomes

$$\frac{d\sigma}{d\Omega} = \frac{Z^2\alpha^2}{4E_0^2\sin^4\frac{\theta}{2}}\,|F(Q^2)|^2 \quad . \tag{1.33}$$

Comparing eqs. (1.33) and (1.12) we see that the net effect of the distribution of the nuclear charge over a spatial volume according to a distribution function $\rho(\vec{R})$ is to multiply the cross-section by the square of the form factor $|F(Q^2)|^2$. $F(Q^2)$ is nothing but the three dimensional Fourier transform of the charge distribution $\rho(\vec{R})$. If $|F(Q^2)| = 1$ for all Q^2, then eq. (1.33) reduces to eq. (1.12) which is the Rutherford scattering cross-section formula for the scattering of a relativistic electron from a spinless point nucleus; *i.e.*,

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{charge distn.}} = |F(Q^2)|^2\left(\frac{d\sigma}{d\Omega}\right)_{\text{point nucleus}} \quad , \tag{1.34}$$

[‖] Use $\hbar c = 197$ MeV fm to arrive at this result.

Table 1: The form factors and the Root Mean Square (r.m.s.) radii $\langle R^2\rangle^{1/2}$ for different charge distributions.

$\rho(R)$	$F(Q^2)$	$\langle R^2\rangle^{1/2}$	
$\delta^3(\vec{R})$	1	0.0	
$\frac{e^{-mR}m^2}{4\pi R}$	$\frac{1}{(1+\frac{Q^2}{m^2})}$	$\frac{\sqrt{6}}{m}$	Monopole
$\frac{e^{-mR}m^3}{8\pi}$	$\frac{1}{(1+\frac{Q^2}{m^2})^2}$	$\frac{\sqrt{12}}{m}$	Dipole

where the form factor is given by eq. (1.29). For a spherically symmetric charge distribution, choosing the z axis along $\vec{q}$, the equation for $F(Q^2)$ becomes

$$F(Q^2) = 4\pi \int_0^\infty \rho(R) \frac{\sin QR}{QR} R^2 dR \quad . \tag{1.35}$$

The charge distribution $\rho(R)$ is given in terms of $F(Q^2)$ by the inverse Fourier transform,

$$\rho(R) = \frac{1}{2\pi^2} \int F(Q^2) \frac{\sin QR}{QR} Q^2 dQ \quad . \tag{1.36}$$

At values of $Q = |\vec{q}|$ such that $QR \ll 1$ over all the region where $\rho(R)$ is appreciable, one can expand $\sin QR/QR$ in the integrand in eq. (1.35) and we get

$$F(Q^2) = 4\pi \int_0^\infty R^2 \rho(R) dR - \frac{Q^2}{6} \int_0^\infty 4\pi R^2 \rho(R) R^2 dR + O(Q^4 R^4) \quad . \tag{1.37}$$

Hence for a charge distribution normalised as given by eq. (1.15) we have,

$$F(Q^2) = 1 - \frac{Q^2\langle R^2\rangle}{6} \quad . \tag{1.38}$$

Thus a measurement of $F(Q^2)$ at small Q^2 such that $Q^2\langle R^2\rangle \ll 1$ gives us $\langle R^2\rangle$ for different charge distributions. Table 1 summarises the form factors $F(Q^2)$ and $\langle R^2\rangle$ for different charge distributions where these can be calculated analytically. As can be seen, all of them satisfy eq. (1.38).

Recall now from eq. (1.7) that the square of the four-momentum transfer q^2 is given by

$$q^2 = |\vec{p}\,'|^2 - (W - M_A)^2 = -2M_A^2 + 2WM_A \quad . \tag{1.39}$$

This gives us

$$\frac{q^2}{2M_A^2} + 1 = \frac{W}{M_A}. \tag{1.40}$$

Further for $W \simeq M_A$ (*i.e.* $q^2 \ll 2m_A^2$; small nuclear recoil), we also have $q^2 \simeq Q^2$. This means that in this approximation we can replace Q^2 by q^2 in all the earlier equations.

If we repeat the analysis above for a point e^- incident on a point spinless charge $Z|e|$ but keeping the effect of e^- spin we get (see problem 4, Appendix B)

$$\left(\frac{d\sigma}{d\Omega}\right)_{\substack{\text{point } e^- \\ \text{point, spinless nucleus}}} = \frac{Z^2\alpha^2\cos^2\frac{\theta}{2}}{4E_0^2\sin^4\frac{\theta}{2}\left[1+\frac{2p_0}{M_A}\sin^2\frac{\theta}{2}\right]} \quad . \tag{1.41}$$

Neglecting the recoil, for relativistic e^--nucleus scattering this becomes,

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott.}} = \frac{Z^2\alpha^2\cos^2\frac{\theta}{2}}{4E_0^2\sin^4\frac{\theta}{2}} = \frac{4Z^2\alpha^2\cos^2\frac{\theta}{2}\ E^2}{q^4} \quad . \tag{1.42}$$

The factor of $\cos^2\frac{\theta}{2}$ in the numerator in eq. (1.42) indicates the impossibility of 180° scattering for longitudinally polarised, spin $\frac{1}{2}$ electrons (see problem 3, Appendix B for a further discussion of this point). Once again, repeating the exercise for a nuclear charge distribution, we get

$$\left(\frac{d\sigma}{d\Omega}\right)_{\substack{\text{spin } \frac{1}{2}e^- \\ \text{spinless, charge distn.}}} = \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} |F(q^2)|^2 \quad , \tag{1.43}$$

where $F(q^2)$ is the form factor given by eq. (1.29). Hence for a spinless nucleus the form factor can be measured by the ratio:

$$|F(q^2)|^2 = \frac{(d\sigma/d\Omega)^{eA\to eA}}{(d\sigma/d\Omega)_{\text{Mott}}} \quad . \tag{1.44}$$

The charge distribution of the nucleus is then given by the inverse Fourier transform of $F(q^2)$. Hofstadter studied [14] the e^--nucleus scattering for $E_{e^-} \lesssim 600$ MeV. Hence the electrons were relativistic. Since $M_A \simeq 1000$ A MeV, the slow recoil assumption is also justified in this case. Hence, the nuclear form factors could be determined in this case using eq. (1.44). Thus a deviation of the angular dependence of the cross-section from the one expected for a point target, measures the form-factor. Therefore to get information about a possible structure of the scattering center, we must know the theoretical predictions for the results expected for a point target. These, along with the experimental information on the form-factor, for the case of proton/neutron will be discussed in the next lecture. It must be noted here that the interpretation of the form-factor as a three dimensional Fourier transform of the charge distribution is strictly true only in the case of recoilless scattering. At higher energies, when this assumption is not justified, the form-factor $F(q^2)$ can be looked upon as the ratio of the scattering amplitudes from an extended and point target.

2 Form Factors of Proton and Neutron

2.1 $e^- p \to e^- p$ for pointlike "Dirac" proton

The discussion in the last section and eq. (1.3) in particular indicates that the proton and neutron are not pointlike and elementary. It also makes it clear that one can obtain information about the charge distribution in a proton (neutron) by studying the electromagnetic scattering process

$$e^- + p\ (n) \to e^- + p\ (n) \quad . \tag{2.1}$$

Eq. (1.44) indicates that to be able to do this it is essential to know the expected cross-section $d\sigma/d\Omega$ for a point proton. In the discussion below we will try to indicate how such a calculation is done.

At the energies which we are considering the proton and neutron also need to be treated relativistically. For a Dirac (*i.e.* point) $e^-(p)$, the electromagnetic current is given by

$$\begin{aligned} J_\mu^{e(p)} &= i q_{e(p)} \overline{\psi} \gamma_\mu \psi \\ &= i \frac{q_{e(p)}}{2m_e(M_p)} \left(\frac{\partial \overline{\psi}}{\partial x_\mu} \psi - \overline{\psi} \frac{\partial \psi}{\partial x_\mu} \right) - \frac{q_{e(p)}}{2m_e(M_p)} \frac{\partial}{\partial x_\nu} (\overline{\psi} \sigma_{\nu\mu} \psi) \quad , \end{aligned} \tag{2.2}$$

where γ_μ are the usual Dirac matrices and $\sigma_{\mu\nu} = \frac{1}{2i}[\gamma_\mu, \gamma_\nu]$; $q_{e(p)}$ is the charge of the $e^-(p)$ and $m_e(M_p)$ is the mass of the $e^-(p)$. The fermion index (e^- or p) on the spinor is suppressed. It can be shown easily that this expression for the electromagnetic current means $g_e = 2$, $\mu_e = 1$ B.m., $g_p = 1$ and $\mu_p = 1$ n.m. To calculate $d\sigma/d\Omega$ one needs the matrix element for the scattering process $e^- p \to e^- p$. This is given by

$$\mathcal{M} \sim \frac{1}{q^2} J_\mu^e J_\mu^p \quad , \tag{2.3}$$

where q^2 is the square of the four momentum transfer in this process. The Feyman diagram for the scattering process and the four momentum assignments of various particles involved in it are shown in fig. 4. The kinematics of the process is the same as given by eqs. (1.5) - (1.9), (1.13a, 1.13b) and (1.22) - (1.24), after replacing M_A by M_p. Again the square of invariant mass of exchanged photon $q^2 \neq 0$, and hence it is virtual.

To calculate $d\sigma/d\Omega$ now one has to use the rules of quantum field theory. The cross-section for a process $A + B \to C + D$ is given by

$$d\sigma = \frac{1}{2E_A} \frac{1}{2E_B} \frac{(2\pi)^4 \delta^4(P_A + P_B - P_C - P_D)}{(2\pi)^6} \frac{d^3 p_C}{2E_C} \frac{d^3 p_D}{2E_D} \overline{|\mathcal{M}|^2} \tag{2.4}$$

where P_A, P_B, P_C and P_D are the four momenta of the four particles involved and $\overline{|\mathcal{M}|^2}$ is the square of the matrix element averaged over initial state spins and summed over the final state spins. The matrix element $\mathcal{M}$ itself for the scattering process is given by eq. (2.3). Using $J_\mu^{e(p)}$ given by eq. (2.2) and the trace rules given in Appendix A , we get for a point proton (see problem 4, Appendix B) ,

$$\frac{d\sigma}{d\Omega}(e^-p \to e^-p)\Big|_{\text{Point e}^-/\text{p}} = \frac{\alpha^2 \cos^2\frac{\theta}{2}}{4E_0^2 \sin^4\frac{\theta}{2}} \frac{1}{1+\frac{2p_0}{M_p}\sin^2\frac{\theta}{2}} \left[1 + \frac{q^2}{2M_p^2}\tan^2\frac{\theta}{2}\right]$$

$$= \left(\frac{d\sigma}{d\Omega}\right)_{\substack{\text{Mott}\\(Z=1)}} \frac{p}{p_0}\left[1+\frac{q^2}{2M_p^2}\tan^2\frac{\theta}{2}\right] \quad . \tag{2.5}$$

The first term is just the Mott electrostatic scattering cross-section of eq. (1.42) with the nuclear charge $Z = 1$, the factor of p/p_0 is the recoil factor which goes to unity for recoil-less scattering (recall eq. (1.13b)) and the last factor is due to the magnetic moment of the proton and electron which reduces to unity if proton were to be replaced by a spinless point charge. Thus for a scalar, point proton we can write (see problem 4, Appendix B),

$$\frac{d\sigma}{d\Omega}(e^-p \to e^-p)\Big|_{\text{Point e}^-/\text{p}} = \frac{\alpha^2 \cos^2\frac{\theta}{2}}{4E_0^2 \sin^4\frac{\theta}{2}} \frac{1}{1+\frac{2p_0}{M_p}\sin^2\frac{\theta}{2}} \left[1 + \frac{q^2}{2M_p^2}\tan^2\frac{\theta}{2}\right]$$

$$= \left(\frac{d\sigma}{d\Omega}\right)_{\substack{\text{Mott}\\(Z=1)}} \frac{p}{p_0} \quad . \tag{2.6}$$

Note also that we are no longer restricted to $W \simeq M_p$ and hence $q^2 \neq |\vec{q}|^2 = Q^2$.

2.2 Effect of anomalous magnetic moment of proton

The above discussion assumes that the proton is a Dirac particle with $g_p = 2$. In reality, even if we were to neglect the possibility that the proton is an extended object, the anomalous magnetic moment of the proton cannot be neglected, *i.e.*, eq. (2.2) cannot give the electromagnetic current for the physical proton even if were to be point like. The anomalous magnetic moment gives an additional contribution to the electromagnetic current of a proton and it can be written as**

$$\text{“}J_\mu^p\text{”} = \frac{iq_p}{2M_p}\left[\frac{\partial\overline{\psi}_p}{\partial x_\mu}\psi_p - \overline{\psi}_p\frac{\partial\psi_p}{\partial x_\mu}\right] - \frac{q_p}{2M_p}(1+\kappa_p)\frac{\partial}{\partial x_\nu}\overline{\psi}_p\sigma_{\nu\mu}\psi_p \quad . \tag{2.7}$$

Note here that the effects of a possible spatial structure of the proton are not taken into account apart from the nonzero value of the anomalous magnetic moment. The interaction Hamiltonian $\mathcal{H}^{int}$ is given by $\sim$ “J_μ^p”A_μ and hence the scattering amplitude will be determined by this. Recall also $\psi(x) \sim u(P)\exp(iP\cdot x)$. Since P_1 and P_4 are the four momenta of the initial and final state respectively, the expression for the current of eq. (2.7), in momentum space, becomes

$$\text{“}J_\mu^p\text{”} = J_\mu^{Dirac} + \frac{iq_p}{2M_p}\bar{u}_p(P_4)\sigma_{\nu\mu}u_p(P_1)q_\nu \quad , \tag{2.8}$$

**see for example Advanced Quantum Mechanics, J.J. Sakurai, (Addison Wesley, Reading, Mass., USA) where J_μ^{Dirac} represents the current of eq. (2.2), in momentum space, given by

$$\begin{aligned} J_\mu^{Dirac} &= iq_p \bar{u}_p(P_4)\gamma_\mu u_p(P_1) \\ &= \frac{q_p}{2M_p}(P_{4\mu}+P_{1\mu})\bar{u}_p(P_4)u_p(P_1) + \frac{iq_p}{2M_p} q_\nu \bar{u}_p(P_4)\sigma_{\nu\mu}u_p(P_1). \end{aligned} \tag{2.9}$$

This is called the Gordon decomposition of the electromagnetic current of a spin 1/2 particle.

2.3 Effect of the spatial extension of proton

The discussion upto now has neglected a possible spatial extension for the proton. However, the large value of κ_p given by eq. (1.3) implies that the proton is not a spin 1/2, point particle. Our discussions on nuclear form factor indicate that the proton structure will be reflected in a multiplicative form factor for the scattering amplitude. Following this, we can try to write down the most general expression for the electromagnetic current of a spin 1/2 object by constructing a four vector using the various bilinear covariants made up of $\overline{\psi}_p(P_4)$ and $\psi_p(P_1)$ along with the independent four momenta available in the scattering process of eq. (2.1). The requirement that parity be conserved, *i.e.*, the electromagnetic current be a vector and not a pseudovector, rules out all the bilinears containing γ_5. The possible 4-vectors that can be constructed are then:

1. $\bar{u}_p(P_4)\gamma_\mu u_p(P_1)$
2. $q_\nu \bar{u}_p(P_4)\sigma_{\nu\mu}u_p(P_1)$
3. $(P_{1\nu}+P_{4\nu})\bar{u}_p(P_4)\sigma_{\nu\mu}u_p(P_1)$
4. [a] $(P_{1\mu}+P_{4\mu})\bar{u}_p(P_4)u_p(P_1)$
   [b] $(P_{4\mu}-P_{1\mu})\bar{u}_p(P_4)u_p(P_1)$

Out of these, (3) can be shown proportional to (4.b) by using Dirac equation, $\not{P} u_p(P) = iM_p u_p(P)$ as well as the usual γ-matrix algebra. It is also clear from eq. (2.9) that (1), (2) and (4.a) are linearly dependent. Hence (1), (2) and (4.b) exhaust the set of linearly independent fourvectors that can be constructed out of the spinors $\bar{u}_p(P_4)$, $u_p(P_1)$ and the fourmomenta P_1, P_4 of the incoming and outgoing proton respectively. Hence the most general expression for the electromagnetic current of a spin 1/2 particle, which has a structure, can be written as,

$$\begin{aligned} J_\mu^{p,\mathrm{ext}} = iq_p \Bigg[& \mathcal{F}_1^p(q^2)\; \bar{u}_p(P_4)\gamma_\mu u_p(P_1) + \mathcal{F}_2^p(q^2)\; \frac{\kappa_p}{2M_p} q_\nu \bar{u}_p(P_4)\sigma_{\nu\mu}u_p(P_1) \\ & + \mathcal{F}_3^p(q^2)\; \bar{u}_p(P_4) q_\mu u_p(P_1) \Bigg] \;, \end{aligned} \tag{2.10}$$

where due to Lorentz invariance, $\mathcal{F}^p_1$, $\mathcal{F}^p_2$ and $\mathcal{F}^p_3$ are three arbitrary functions of the scalars that can be constructed out of the available linearly independent fourvectors. Since the incoming and outgoing proton are on mass-shell, this means that $\mathcal{F}^p_i$ $(i = 1, 2, 3)$ can be functions of q^2 alone. Eq. (2.10) means that the structure of the proton can be parametrised by three arbitrary functions.

We know, however, that the electromagnetic current is a conserved quantity, *i.e.*, $\partial_\mu J^{p,\mathrm{ext}}_\mu = 0$. This translates in momentum space into

$$q_\mu J^{p,\mathrm{ext}}_\mu = 0 \quad \text{where} \quad q_\mu = P_{4\mu} - P_{1\mu}. \tag{2.11}$$

Applying this condition to (2.10) we get,

$$\begin{aligned} 0 = q_\mu J^{p,\mathrm{ext}}_\mu = iq_p \Bigg[& \mathcal{F}^p_1(q^2)\bar{u}_p(P_4)\gamma_\mu q_\mu u_p(P_1) + \mathcal{F}^p_2(q^2)\frac{\kappa_p}{2M_p} q_\mu q_\nu \bar{u}_p(P_4)\sigma_{\nu\mu} u_p(P_1) \\ & + \mathcal{F}^p_3(q^2)\bar{u}_p(P_4) q^2 u_p(P_1) \Bigg] \ . \end{aligned} \tag{2.12}$$

The first term in the square bracket vanishes identically as can be seen by using Dirac equation for proton. The second term is identically zero since $q_\mu q_\nu$ is a symmetric tensor in (μ, ν) and it is contracted with $\sigma_{\mu\nu}$ which is antisymmetric under $(\mu \leftrightarrow \nu)$. Hence eq. (2.12) becomes

$$\mathcal{F}^p_3(q^2) q^2 \bar{u}_p(P_4) u_p(P_1) = 0 \ . \tag{2.13}$$

This then tells us that $\mathcal{F}^p_3(q^2) \equiv 0$. Hence the most general expression for the electromagnetic current of a spin 1/2 proton with structure, which is consistent with Lorentz and gauge invariance is given by

$$\begin{aligned} J^{p,\mathrm{ext}}_\mu &= iq_p \left[\mathcal{F}^p_1(q^2)\bar{u}_p(P_4)\gamma_\mu u_p(P_1) + \mathcal{F}^p_2(q^2)\frac{\kappa_p}{2M_p} q_\nu \bar{u}_p(P_4)\sigma_{\mu\nu} u_p(P_1) \right] \\ &\equiv \bar{u}_p(P_4)\Gamma_\mu u_p(P_1) \ . \end{aligned} \tag{2.14}$$

Thus we see that the most general expression for the electromagnetic current of an extended, spin 1/2 object is specified completely in terms of just two arbitrary functions $\mathcal{F}_1(q^2)$ and $\mathcal{F}_2(q^2)$, *i.e.*, the effect of the spatial extension of the scattering centre on the scattering amplitude is simply parametrised in terms of these two functions. Comparing eq. (2.14) with eq. (2.8) we see that we recover the case for a proton with an anomalous magnetic moment κ_p but without a structure if $\mathcal{F}^p_2(q^2)$ and $\mathcal{F}^p_1(q^2)$ both are unity. Our discussions on nuclear form factors tell us that for $q^2 \ll 1/\langle R^2 \rangle_p$, any form factor associated with the proton will be close to unity (recall eq. (1.38)). In particular eq. (2.8) gives us,

$$\mathcal{F}^p_1(0) = 1, \quad \mathcal{F}^p_2(0) = 1 \ . \tag{2.15}$$

The two functions $\mathcal{F}_i^p(q^2)$ are called the two form factors of the proton and are associated with two linearly independent fourvectors in terms of which the electromagnetic current can be decomposed. For a neutron which has no electric charge and only an anomalous magnetic moment we will have,

$$\mathcal{F}_1^n(0) = 0; \quad \mathcal{F}_2^n(0) = 1, \tag{2.16}$$

and κ_p in eq. (2.14) will be replaced by κ_n.

2.4 Electric and Magnetic Form Factors

Having established the most general expression for the electromagnetic current of an extended object with spin 1/2, it is now necessary to calculate $d\sigma/d\Omega$ for the scattering process of eq. (2.1) in terms of these form factors. We can calculate $d\sigma/d\Omega$ by using eq. (2.4) for $d\sigma$, with $\mathcal{M}$ given by eq. (2.3), where J_μ^p is given by eq. (2.14) and J_μ^e by eq. 2.9 with a replacement of $q_p \to q_e$, $M_p \to m_e$, $P_4 \to P_3$ and $P_1 \to P_2$. We then get (see problems 4, Appendix B)

$$\begin{aligned} \frac{d\sigma}{d\Omega}(e^- p \to e^- p)\bigg|_{p,\mathrm{ext}} &= \left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{Mott}} \left(\frac{p}{p_0}\right) \left\{ \left((\mathcal{F}_1^p)^2 + \frac{\kappa_p^2 q^2}{4M_p^2}(\mathcal{F}_2^p)^2 \right) \right. \\ &\left. + \frac{q^2}{2M_p^2}(\mathcal{F}_1^p + \kappa_p \mathcal{F}_2^p)^2 \tan^2\frac{\theta}{2} \right\}. \end{aligned} \tag{2.17}$$

In the limit $q^2 \ll 1/\langle R^2\rangle_p$ and with $\kappa_p = 0$ this reduces to eq. (2.5) as it should. Since the term $(q^2/2M_p^2)\tan^2\frac{\theta}{2}$ in eq. (2.5) is known to arise from the magnetic moment of the "Dirac" proton and electron, on comparing eqs. (2.5) and (2.17) we see that while the form factors $\mathcal{F}_1^p$ and $\mathcal{F}_2^p$ were the natural ones while considering the tensor decomposition of $J_\mu^{p,\mathrm{ext}}$, the combination $(\mathcal{F}_1^p + \kappa_p \mathcal{F}_2^p)$ has the more natural interpretation as the magnetic form factor. If we define the electric and magnetic form factors of the proton by

$$\begin{aligned} G_M^p(q^2) &\equiv \mathcal{F}_1^p(q^2) + \kappa_p \mathcal{F}_2^p(q^2), \\ G_E^p(q^2) &\equiv \mathcal{F}_1^p(q^2) - \kappa_p \frac{q^2}{4M_p^2}\mathcal{F}_2^p(q^2). \end{aligned} \tag{2.18}$$

the expression for the differential cross–section becomes,

$$\frac{d\sigma}{d\Omega}(e^- p \to e^- p) = \left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{Mott}} \frac{p}{p_0} \left[\frac{{G_E^p}^2(q^2) + \frac{q^2}{4M_p^2}{G_M^p}^2(q^2)}{1 + q^2/4M_p^2} + \frac{q^2}{2M_p^2}{G_M^p}^2(q^2)\tan^2\frac{\theta}{2} \right]. \tag{2.19}$$

The physical significance of $G_M^p(q^2)$ and $G_M^n(q^2)$ can be understood in terms of electromagnetic scattering of protons with definite helicity in the Breit frame. The latter choice means that the scattering involves no change in energy but only in the sign of the three momentum. To see this let us analyse the expression for $J_\mu^{p,\rm ext}$ of (2.14), but for proton states of definite helicity (spin projection along direction of motion). Using eq. (2.9) we can rewrite this as

$$\begin{aligned} J^{\,p,\rm ext}_{\,\mu,\lambda,\lambda'} &= \bar u(P_4,\lambda')\Gamma_\mu u(P_1,\lambda) \\ &= iq_p \bar u(P_4,\lambda')\left[\gamma_\mu G_M^p(q^2) + \frac{i(P_{4\mu}+P_{1\mu})}{2M_p}\frac{(G_M^p(q^2)-G_E^p(q^2))}{(1+q^2/4M_p^2)}\right]u(P_4,\lambda). \quad (2.20)\end{aligned}$$

The Dirac spinor for a particle with helicity λ when its momentum is along the z axis $\vec p_1 = \hat z|\vec p_1|$ is given by

$$u(\vec p_1,\lambda) = N\begin{pmatrix}\chi_\lambda \\ \dfrac{|\vec p_1|\sigma_3}{E+M_p}\chi_\lambda\end{pmatrix}, \qquad (2.21)$$

where

$$N = \sqrt{\frac{E+M_p}{2M_p}}, \quad \chi_{+1/2} = \begin{pmatrix}1\\0\end{pmatrix}, \text{ and } \chi_{-1/2} = \begin{pmatrix}0\\1\end{pmatrix}. \qquad (2.22)$$

$\chi_{+1/2}$ and $\chi_{-1/2}$ correspond to states with helicity +1 and -1 respectively. The Dirac spinor for a particle with helicity λ, but moving along the negative z direction will be obtained by considering the transformation of the above spinor under rotation through 180^o about y axis. This transformation is given by $\exp(-i\pi\sigma_y)$, *i.e.*, $-i\sigma_y$. (The negative sign comes from the fact that we are rotating the physical state and not the co-ordinates.) Hence we get,

$$u(-\vec p_1,\lambda) = N\begin{pmatrix}-i\sigma_y\chi_\lambda \\ \dfrac{|\vec p_1|\sigma_x}{E+M_p}\chi_\lambda\end{pmatrix}. \qquad (2.23)$$

In the Breit frame $\vec p_4 = -\vec p_1$ and therefore we can use the above expressions for the spinors for a fixed helicity. We then get,

$$\begin{aligned} J^{p,\rm ext}_{\mu,+1/2,+1/2} &= G_M^p(q^2)\frac{|\vec p_1|}{2M_p}q_p(1,i,0,0), \\ J^{p,\rm ext}_{\mu,+1/2,-1/2} &= q_p G_E^p(q^2)(\vec 0,-i). \quad (2.24)\end{aligned}$$

The first amplitude in eq. (2.24) corresponds to no helicity flip (since the three momenta of the scattered and incident electron are opposite to each other, this means spin flip) scattering and this is proportional to the magnetic form factor, whereas the second amplitude represents the contribution with helicity flip (hence no spin flip) which corresponds to the electric form factor. Also note that the spin-flip amplitude due to the magnetic scattering vanishes in the non-relativistic (NR) limit. This observation also explains clearly the terminology used, viz., the nomenclature of electric/magnetic form factors. Thus electrostatic scattering cannot flip the spin of the electron whereas magnetic scattering does in this kinematical configuration (see discussion in problems).

The discussion for $\frac{d\sigma}{d\Omega}(e^- p \to e^- p)$ will be applicable to the case of neutrons equally well. An expression similar to eq. (2.19) can be written for $\frac{d\sigma}{d\Omega}(e^- n \to e^- n)$ as well. Only the boundary conditions on $G_{M,E}(q^2 = 0)$ will change appropriately. Eqs. (2.15), (2.16) and (2.17) imply,

$$\begin{aligned} G_E^p(0) = 1, \quad & G_M^p(0) = 1 + \kappa_p \equiv \mu_p \text{ (in n.m.)} , \\ G_E^n(0) = 0, \quad & G_M^n(0) = \kappa_n \equiv \mu_n \text{ (in n.m.)} . \end{aligned} \tag{2.25}$$

Experimental information on the form factors $G_{M,E}^{p(n)}(q^2)$ was obtained by studying $\frac{d\sigma}{d\Omega}(ed \to ed)$ and $\frac{d\sigma}{d\Omega}(ep \to ep) \equiv \left(\frac{d\sigma}{d\Omega}\right)_{ep}$. The differential cross–section $\frac{d\sigma}{d\Omega}(en \to en)$ is then obtained from the first two measurements applying correction factor for nuclear physics effects. It is clearly not possible to do experiments with neutron targets as the neutron is not stable. Since we are discussing elastic scattering, eq. (1.23) ensures that there is only one independent variable characterising the final state. This can be chosen to be either $\cos\theta$ or $q^2 = (4p_0^2 \sin^2\theta/2) \Big/ \left(1 + \dfrac{2p_0}{M}\sin^2\dfrac{\theta}{2}\right)$. Hence eq. (2.19) implies that

$$\frac{\left(\frac{d\sigma}{d\Omega}\right)_{ep}}{\left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}}} = \frac{p}{p_0}\left(A^p(q^2) + B^p(q^2)\tan^2\frac{\theta}{2}\right) , \tag{2.26}$$

where

$$\begin{aligned} A^p(q^2) &= \frac{1}{(1 + q^2/4M_p^2)} \left(G_E^{p\,2}(q^2) + \frac{q^2}{4M_p^2} G_M^{p\,2}(q^2)\right) \\ &= \left((\mathcal{F}_1^p)^2 + \frac{\kappa_p^2 q^2}{4M_p^2}(\mathcal{F}_2^p)^2\right); \\ B^p(q^2) &= \frac{q^2}{2M_p^2}\, G_M^{p\,2}(q^2) \\ &= \frac{q^2}{2M_p^2}(\mathcal{F}_1^p + \kappa_p \mathcal{F}_2^p)^2. \end{aligned} \tag{2.27}$$

The above formula is called Rosenbluth formula. This also shows clearly how one can determine the two functions $G_E^p(q^2)$ and $G_M^p(q^2)$ from a measurement of $\left(\frac{d\sigma}{d\Omega}\right)_{ep}$. From the kinematic considerations of eq. (1.23), one sees that it is possible to obtain data at a fixed q^2 and fixed angle θ, by changing the incident electron beam energy.

The form factors for both the proton and neutron have been measured now over a wide range of q^2 ($q^2 \lesssim 30$ GeV^2 for the proton and $q^2 \lesssim 5$ GeV^2 for the neutron). The experimentally measured form-factors for proton and neutron seem to obey a scaling law in the following sense:

$$G_E^p(q^2) = \frac{G_M^p(q^2)}{|\mu_p|} = \frac{G_M^n(q^2)}{|\mu_n|} \equiv G(q^2) = \frac{1}{(1 + q^2/M_V^2)^2} , \tag{2.28}$$

with $M_V^2 = (0.84)^2$ GeV^2. This is the dipole form factor of Table 1. $G_E^n(q^2)$ is identically zero as is to be expected. If we again compare (2.19) and (2.5), we see that to leading order in q^2,

$$\frac{\left(\frac{d\sigma}{d\Omega}\right)_{ep}}{\left(\frac{d\sigma}{d\Omega}\right)_{\text{ep,point}}} \approx G^2(q^2) \simeq \frac{1}{(1+q^2/M_V^2)^4} \quad . \tag{2.29}$$

This means that the elastic cross-section falls off very sharply with increasing values of q^2 for scattering off an extended object. This behaviour then tells us that the probability of elastic scattering falls off very sharply with increasing q^2 (*i.e.*, increasing energy of the incident e^- beam). Some of the SLAC data on $G_M^p(q^2)$ taken from [15] are shown in fig. 5.

The experimentally observed q^2-dependence of the form factors , eq. (2.29), along with eq. (1.38) implies that the r.m.s. radius of the electric charge/magnetic moment distribution in a proton or neutron is

$$\langle R^2 \rangle^{1/2} = \frac{\sqrt{12}}{0.84 \text{ GeV}} \simeq 0.81 \text{ fm} \quad . \tag{2.30}$$

The dipole form factor of eq. (2.29) can also be interpreted (see Table 1) as an exponential charge/magnetic moment distribution given by

$$\rho(R) = \frac{M_V^3}{8\pi} \exp(-M_V R) \quad . \tag{2.31}$$

While it is true that the electric and magnetic form factors provide a neat way to parametrise the effects of spatial extension of the target on a scattering process, attempts to calculate these form factors from first principles, in some model, did not add to our knowledge of strong interaction dynamics or that of the structure of proton, beyond the information provided by eq. (2.31). Real progress in the information on proton/neutron structure as well as strong interaction dynamics was made by increasing the energy of the electron beams incident on the proton/neutron target and studying the scattering process, which will be discussed in the next section.

2.5 Summary of the dependence of the cross–sections for elastic scatterring on the nature of the target

In this section we saw how the differential cross–section for elastic scattering depends on the nature of the scatterer and how this can be parametrised in terms of one (two) form factors for a scalar (spin 1/2) target. This information can be summarised in a compact form as in Table 2.

As we can see, in the limit of the point proton, the form factors reduce to 1 and the Rosenbluth formula reduces to eq. (2.5) as mentioned in the line 4 of the table. This does not incorporate the anomalous magnetic moment either. The term proportional to $\tan^2\theta/2$ in both these equations is due to the magnetic scattering

Table 2: Dependence of the elastic differential cross–section on the nature of the target and projectile (Y: effect included, N : Not included).

Formula	Projectile e		Target $Z\|e\|$			
	Spin	Energy	Spin	Anom. μ κ	Size	Recoil (Target mass)
Rutherford eq. (1.11)	N	Non Rel. (NR)	N	N	N	N
Mott eq. (1.42)	Y	Rel. $E/m \to \infty$	N	N	N	N
eq. (2.6) spinless point p	Y	”	N	N	N	Y
eq. (2.5) point spin 1/2 p	Y	”	Y	N	N	Y
Rosenbluth eq. (2.17)	Y	”	Y	Y	Y	Y

and is present when the spin 1/2 nature of *both* the target and the projectile is taken into account. As we have seen in our discussions at lower energies, the electrostatic scattering dominates and hence the second term can be dropped. If we drop this term from eq. (2.5) we recover the expression for the cross–section for the scattering of a spin 1/2 electron from a spinless, pointlike particle. For a spinles non–pointlike particle this would be multiplied by an appropriate form factor which at low energies of the projectile will reduce to one. If the target has no spin and has infinite mass (as compared with the projectile energy) then the target recoil in the scattering process can be neglected and $E \simeq E_0$ for the projectile. In this limit this formula reduces to the Mott scattering cross–section of eq. (1.42). At lower projectile energies, the effects of the spin 1/2 nature of the projectile also become negligible and the relativistic Mott scattering cross–section reduces to the Rutherford scattering cross–section. Again if the target is not a pointlike charge but a charge distribution instead, then this formula gets multiplied by a form factor as we have seen before. Thus we see that as we go to lower and lower projectile energies, various factors which represent effects of the projectile spin (e.g. the factor $\cos^2(\theta/2)$ in the Mott scattering cross–section of the second line in the table) or the target spin, size and mass (the term with $\tan^2\theta/2$ factor, the form factor or the recoil factor respectively) go either to one or zero, giving us in the end the simple Rutherford scattering cross–section in the complete non–relativistic (NR) limit.

3 Deep Inelastic Electron-Nucleon Scattering

As we saw in the last section, study of elastic scattering of an electron off a proton target gives information about the spatial charge/magnetic moment distribution for a proton. However, eq. (2.29) and fig. 5 tell us that the elastic cross-section falls through four orders of magnitude as q^2 changes from $2 \to 25$ GeV^2. These larger values of q^2 are reached using higher energy electron beams. With increasing q^2, quasi–elastic scattering with excitation of baryon resonances becomes possible (recall the case of nuclei in nuclear reactions), and at still higher energies (and hence at higher q^2 values), the scattering is dominated by the inelastic process: the so called Deep Inelastic Scattering (DIS).

3.1 Kinematics of inelastic scattering

The kinematics of the inelastic $e^- - p$ scattering is shown in fig. 6, which is almost the same as fig. 4, except that the hadronic final state is no longer a proton. The reaction now is

$$e^- + p \to e^- + X \quad . \tag{3.1}$$

X stands for the hadronic final state. The four momenta of the various particles involved in scattering are as shown in fig. 6. The invariant mass of the hadronic final state is now,

$$M_X^2 = -P_4^2 = -|\vec{p}\,'|^2 + W^2 \quad . \tag{3.2}$$

Energy momentum conservation of eq. (1.6) gives us

$$q^2 = (P_2 - P_3)^2 = (P_4 - P_1)^2 = |\vec{p}\,'|^2 - (W - M_p)^2 = -M_X^2 + M_p^2 + 2M_p\nu \quad , \tag{3.3}$$

where $\nu = E_0 - E$ is the energy transfer from the electron to proton in the laboratory frame. Using eq. (1.9) for q^2 we then get,

$$q^2 = 4EE_0 \sin^2\frac{\theta}{2} = -M_X^2 + M_p^2 + 2M_p(E_0 - E) \quad . \tag{3.4}$$

As we can see, if $M_X^2 = M_p^2$, *i.e.*, the scattering is elastic, then eq. (3.4) reduces to

$$q^2 = 2M_p\nu = 2M_p(E_0 - E) \quad . \tag{3.5}$$

Using the expression for q^2 given in eq. (3.4), we can see that eq. (3.5) leads to eq. (1.23) with $M_A \to M_p$. Eq. (3.5) means that q^2 and ν are not independent variables and the reaction is characterised by only one independent variable.

For quasi–inelastic scattering, *i.e.*, excitation of a baryon resonance (say $N^{\star +}$ in the reaction $e^- p \to e^- N^{\star +} \to e^- p \pi^0$), $M_X = M^\star$. Eq. (3.3) then becomes,

$$q^2 = 4EE_0 \sin^2\frac{\theta}{2} = -M^{\star 2} + M_p^2 + 2M_p\nu \quad . \tag{3.6}$$

Again there is only one independent variable except that the relation between q^2 and ν is different from that for elastic scattering (cf. eq. (3.3)).

If M_X^2 does not have a fixed value but changes continuously, then both q^2 and ν are independent variables. Clearly the proton no longer remains intact. This region is called the continuum region. This completely inelastic scattering is characterised by two independent variables. From eq. (1.23) and the definition of ν it is clear that both of these are completely specified once the energy of the scattered electron E and its angle θ are measured; *i.e., the kinematics of the event is independent of the precise details of the hadronic final state* X. It then makes sense to think of a measurement of the cross-section where one sums over all possible hadronic final states X. Such a measurement is called an inclusive measurement.

The kinematically allowed region in the $q^2 - \nu$ plane for the elastic, quasi-elastic and inclusive inelastic scattering is shown in fig. 7. For elastic scattering the allowed region is the straight line given by eq. (3.5). For quasi-elastic scattering, again the allowed region is a straight line but now with an intercept $(M^{\star 2} - M_p^2)/2M_p$, as given by eq. (3.6). The discussions of last section, particularly eqs. (2.19) and (2.29), make it quite clear that the probability of elastic scattering goes down with increasing q^2 as more inelastic channels open up. When both q^2 and ν are large then the cross-section is dominated by continuum excitation. The allowed region in the $q^2 - \nu$ plane in this case is the entire region to the right of the straight line given by eq. (3.5).

The kinematics can be described in terms of any of the pairs of variables: (E, θ) or (q^2, ν). Equivalently, one can also define two dimensionless variables

$$x = \frac{q^2}{2M_p\nu} = \frac{-q^2}{2P_1 \cdot q} \quad ; \tag{3.7a}$$

$$y = \frac{\nu}{E_0} = \frac{E_0 - E}{E_0} \quad . \tag{3.7b}$$

For elastic scattering q^2 and ν are related to each other via eq. (3.5). Hence the variables x and y become,

$$\begin{aligned} x &= 1 \quad ; \\ y &= \frac{2E_0/M_p \, \sin^2\frac{\theta}{2}}{1 + \frac{2E_0}{M_p}\sin^2\frac{\theta}{2}} \quad . \end{aligned} \tag{3.8}$$

For inclusive, inelastic scattering both q^2 and ν (or equivalently x and y) can vary independently. We notice from eqs. (3.4), (3.7) that

$$\begin{aligned} q^2 = 2M_p\nu x = 2M_pE_0xy &= -M_X^2 + M_p^2 + 2M_p\nu \\ &= 4EE_0\sin^2\theta/2 \quad . \end{aligned} \tag{3.9}$$

Since $M_X > M_p$, the equations above trivially yield the kinematically allowed region for variables (x, y) as

$$0 < x < 1, \quad 0 < y < \frac{2E_0}{2E_0 + xM_p} \quad . \tag{3.10}$$

Using eq. (3.9) we can also derive the allowed region in the (q^2, ν) plane and it is given by

$$0 < q^2 < \frac{4M_p E_0^2}{2E_0 + M_p} \approx 2M_p E_0 \quad , \quad \frac{q^2}{2M_p} < \nu < E_0 - \frac{q^2}{4E_0} \quad . \qquad (3.11)$$

The early DIS experiments at SLAC [2] used electron beams with energy $E_0 = 20$ GeV which corresponds to $q^2 \leq 40$ GeV^2. The current experiments at Fermilab use ν beams with energy 500 GeV and hence can reach q^2 values upto 1000 GeV^2, whereas the $e^- - p$ collider HERA at DESY operating with an electron beam of 30 GeV and proton beam of 800 GeV is capable of measuring DIS cross-sections upto $q^2 \leq 10^5$ GeV^2.

3.2 Inelastic cross-section and structure functions

We saw in the first lecture that for elastic scattering of eq. (2.1), the most general expression for the current $J_\mu^{p,\text{ext}}$ and hence that for the cross-section could be written in terms of two arbitrary functions $\mathcal{F}_1^p(q^2)$ and $\mathcal{F}_2^p(q^2)$. In case of the inclusive, inelastic measurement described above, again the cross-section can be parametrised in terms of two arbitrary functions $W_1(q^2,\nu)$ and $W_2(q^2,\nu)$ which have to be determined experimentally. First let us see how the number of these arbitrary functions, which parametrise the effect of the structure of a proton on the DIS process, can be restricted to two using gauge invariance, Lorentz invariance, parity invariance and time reversal invariance.

Let us start by recalling that in case of elastic scattering we started off with $\mathcal{M} \sim (J_\mu^e J_\mu^p)/q^2$, where J_μ^e was given by eq. (2.9) and J_μ^p for a point Dirac proton was given by eq. (2.9) whereas it was given by eq. (2.14) for a proton with structure. In the present case, we need to sum over all possible hadronic states. Since now there is no spinor available to describe the hadronic state, unlike the earlier case of elastic scattering, it is not possible to repeat the steps which led us to eq. (2.14). Recall, however, the calculation leading to eq. (2.5), which gave the cross-section for an electron scattering off a point Dirac proton. Then we had,

$$|\mathcal{M}^2| \sim e^4 \; L_{\mu\nu} H'_{\mu\nu} \quad \text{where} \quad L_{\mu\nu} = \frac{1}{2e^2} \sum_{S,S'} J_\mu^e J_\mu^{e\dagger} \, , \quad H'_{\mu\nu} = \frac{1}{2e^2} \sum_{S_p,S'_p} J_\mu^p J_\mu^{\dagger p} . \quad (3.12)$$

$S, S'(S_p, S'_p)$ denote the spins of the initial and final state electron (proton). In the case of a non-point-like proton all we had to do was to replace J_μ^p of eq. (2.9) by $J_\mu^{p,\text{ext}}$ of eq. (2.14). Comparing figs. 4 and 6, we see that the lepton end in both the elastic and inelastic case is the same. Hence we can once again write $|\mathcal{M}|^2$ in a form analogous to eq. (3.12). However, in this case we know nothing about the tensor involving hadronic variables.

To get more insight into the structure of the hadronic tensor for the case of inclusive scattering, it might be instructive to look at a few steps that led us to eq. (2.5). The differential cross-section for the elastic process was written as:

$$d\sigma^{e\ell} = \frac{1}{2M_p} \, \frac{1}{2E_0} \frac{d^3p}{(2\pi)^3} \, \frac{1}{2E} \frac{d^3p'}{(2\pi)^3} \frac{1}{2W} \, \frac{e^4}{q^4} \times$$

$$\left[\frac{1}{2}\sum_{S,S'}\bar{u}_e(\vec{p},S')\gamma_\mu u_e(\vec{p}_0,S)\bar{u}_e(\vec{p}_0,S)\gamma_\nu u_e(\vec{p},S')\right]\times$$
$$\left[\frac{1}{2}\sum_{S_p,S'_p}\bar{u}_p(\vec{p}_1,S_p)\gamma_\mu u_p(\vec{p}\,',S'_p)\bar{u}_p(\vec{p}\,',S'_p)\gamma_\nu u_p(\vec{p}_1,S_p)\right]\times$$
$$(2\pi)^4\delta^4(P_1+q-P_4)\quad . \tag{3.13}$$

The first factor in the square bracket is $L_{\mu\nu}$ and the second is $H'_{\mu\nu}$ of eq. (3.12). The above can be rewritten as,

$$d\sigma^{e\ell} = \frac{1}{2M_p}\frac{1}{2E_0}\frac{d^3p}{(2\pi)^3\,2E}\;\frac{d^3p'}{(2\pi)^3 2W}\frac{e^4}{q^4}\;L_{\mu\nu}\times$$
$$\frac{1}{2}\sum_{S_p,S'_p}\langle P_1,S_p|\tilde{J}^\dagger_\mu|P_4,S'_p\rangle\langle P_4,S'_p|\tilde{J}_\nu|P_1,S_p\rangle\times$$
$$(2\pi)^4\delta^4(P_1+q-P_4)$$

with

$$\langle\;P_4,S'_p|\tilde{J}_\mu|P_1,S_p\rangle = \bar{u}_p(\vec{p}',S'_p)\gamma_\mu u_p(\vec{p}_1,S_p)\quad . \tag{3.14}$$

Here $\tilde{J}_\mu$ represents the electromagnetic current operator from which certain constants have been removed. The above equation can be used to write the expression for the double differential cross-section for elastic scattering as,

$$\frac{d^2\sigma^{e\ell}}{d\Omega dE} = \frac{\alpha^2}{q^4}\;\frac{E}{E_0}\;L_{\mu\nu}H^{\mu\nu}\quad , \tag{3.15}$$

where

$$H_{\mu\nu} = \frac{1}{4\pi M_p}\frac{1}{2}\sum_{S_p,S'_p}\int\frac{d^3p'}{(2\pi)^3}\;(2\pi)^4\;\delta^4(P_1+q-P_4)$$
$$\langle P_1,S_p|\tilde{J}^\dagger_\mu|P_4,S'_p\rangle\langle S'_pP_4|\tilde{J}_\nu|P_1,S_p\rangle\quad . \tag{3.16}$$

For the case of inclusive scattering we can generalise the expression for $H_{\mu\nu}$ given by eq. (3.16) as:

$$H_{\mu\nu} = \frac{1}{2}\sum_{S_p}\sum_{X}\int\frac{1}{4\pi}\frac{1}{M_p}\prod_{i=1}^{n}\frac{d^3\ell_i}{2E_i(2\pi)^3}\;(2\pi)^4\delta^4\left(P_1+q-\sum_{i=1}^{n}\ell_i\right)$$
$$\langle P_1,S_p|\tilde{J}^\dagger_\mu|X\rangle\langle X|\tilde{J}_\nu|P_1,S_p\rangle\quad , \tag{3.17}$$

where $|X\rangle$ denotes a state containing n particles with four momenta $\ell_1,\ell_2,\cdots,\ell_n$ and we sum over all such states. The differential cross-section $d\sigma^{ine\ell}$ for this case of inclusive scattering is then given by (using eqs. (3.13) to (3.16))

$$d\sigma^{ine\ell} = \frac{1}{2M_p}\;\frac{1}{2E_0}\;\frac{d^3p}{(2\pi)^3}\frac{1}{2E}\;\frac{e^4}{q^4}\;4\pi M_p\;L_{\mu\nu}H_{\mu\nu}\quad . \tag{3.18}$$

All we know about $H_{\mu\nu}$ given by eq. (3.17) are its symmetry properties. Gauge invariance requires $\partial_\mu \tilde{J}_\mu = 0$. In momentum space, this translates into

$$q_\mu H_{\mu\nu} = H_{\mu\nu} q_\nu = 0 \quad . \tag{3.19}$$

We begin by writing $H_{\mu\nu}$ as the most general tensor consistent with gauge invariance and parity invariance that can be constructed out of the linearly independent four-momenta available at the hadronic end; q_μ and $P_{1\mu}$. Also note that $L_{\mu\nu}$ is symmetric in $\mu \leftrightarrow \nu$. Hence only the symmetric part of $H_{\mu\nu}$ will be relevant. The most general form for $H_{\mu\nu}$ can therefore be written as

$$\begin{aligned} H_{\mu\nu} = W_1(q^2,\nu)\delta_{\mu\nu} + W_2(q^2,\nu)\frac{P_{1\mu}P_{1\nu}}{M_p^2} \\ +W_4(q^2,\nu)\frac{q_\mu q_\nu}{M_p^2} + W_5(q^2,\nu)\left(\frac{P_{1\mu}q_\nu + q_\mu P_{1\nu}}{M_p^2}\right) \quad , \end{aligned} \tag{3.20}$$

where $W_i(q^2,\nu)$ $(i = 1,2,4,5)$ are arbitrary functions of q^2 and ν. The requirement of gauge invariance of eq. (3.19) gives

$$\begin{aligned} q_\nu\left[W_1(q^2,\nu) + \frac{q^2}{M_p^2}W_4(q^2,\nu) + \frac{P_1\cdot q}{M_p^2}W_5(q^2,\nu)\right] \\ + P_{1\nu}\left[W_5(q^2,\nu)\frac{q^2}{M_p^2} + \frac{P_1\cdot q}{M_p^2}W_2(q^2,\nu)\right] = 0 \end{aligned}$$

Since q_ν and $P_{1\nu}$ are two linearly independent fourvectors and the terms in the square brackets multiplying them are Lorentz scalars, it follows that each of the brackets must be identically zero. Hence we get,

$$\begin{aligned} W_5(q^2,\nu) &= -\frac{P_1\cdot q}{q^2}W_2(q^2,\nu) \quad , \\ W_1(q^2,\nu) + \frac{q^2}{M_p^2}&W_4(q^2,\nu) + \frac{P_1\cdot q}{M_p^2}W_5(q^2,\nu) = 0 \quad . \end{aligned} \tag{3.21}$$

Using this, the expression for $H_{\mu\nu}$ becomes,

$$H_{\mu\nu} = W_1(q^2,\nu)\left[\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right] + \frac{W_2(q^2,\nu)}{M_p^2}\left[P_{1\mu} - \frac{P_1\cdot q}{q^2}q_\mu\right]\left[P_{1\nu} - \frac{P_1\cdot q}{q^2}q_\nu\right] \quad . \tag{3.22}$$

Thus we see that in effect the most general expression for the hadronic tensor involves only two arbitrary functions. $L_{\mu\nu}$ of eq. (3.18) is the same as that for the elastic scattering and is given by

$$L_{\mu\nu} = -\frac{1}{2}Tr\left[\not{P}\gamma_\mu \not{P}_0\gamma_\nu\right] = -2\left[P\cdot P_0\ \delta_{\mu\nu} - P_\mu P_{0\nu} - P_\nu P_{0\nu}\right] \quad . \tag{3.23}$$

To calculate $d\sigma^{\rm inel}$ of eq. (3.18) we need to know $L_{\mu\nu}H_{\mu\nu}$. Using eqs. (3.22) and (3.23), we get

$$L_{\mu\nu}H_{\mu\nu} = -4P\cdot P_0\ W_1(q^2,\nu) + \frac{W_2(q^2,\nu)}{M_p^2}\left(2M_p^2 P\cdot P_0 + 4P\cdot P_1 P_1\cdot P_0\right) \quad . \tag{3.24}$$

With our choice of normalisation for $W_i(q^2,\nu)$, (cf. eqs. (3.18) and (3.20), we have

$$\frac{d^2\sigma^{\rm inel,em}}{d\Omega dE} = \frac{\alpha^2}{q^4}\,\frac{E}{E_0}\; L_{\mu\nu}H_{\mu\nu}\quad . \tag{3.25}$$

The superscript 'em' denotes here the nature of interaction involved in the scattering process, viz. electromagnetic interaction. In the laboratory $P_1\cdot P_0 = -M_pE_0$, $P\cdot P_1 = -M_p(E_0-\nu)$, and $P\cdot P_0 = -q^2/2 = -2EE_0\sin^2\theta/2$. Using these relations and eq. (3.24), we get

$$\frac{d^2\sigma^{\rm inel,em}}{d\Omega dE} = \frac{4\alpha^2}{q^4}E^2\cos^2\frac{\theta}{2}\left[W_2(q^2,\nu)+2W_1(q^2,\nu)\tan^2\frac{\theta}{2}\right]\quad . \tag{3.26}$$

Thus the cross-section for the inelastic scattering process $e^-p\to e^-X$ is parametrised in terms of two functions $W_1(q^2,\nu)$ and $W_2(q^2,\nu)$. Since these functions contain all the information about the proton structure as revealed to an electromagnetic probe, these are called the structure functions. The differential cross-section of eq. (3.26) can be equivalently written in terms of the pair of variables (q^2,ν) or the pair of dimensionless variables (x,y) introduced earlier. We can show that

$$\begin{aligned}\frac{d^2\sigma}{dxdy} = 2ME_0x\frac{d^2\sigma}{dxdq^2} = 2ME_0^2y\frac{d^2\sigma}{dq^2d\nu} &= 2M_p\pi\frac{E_0}{E}\;y\frac{d^2\sigma}{d\Omega dE}\\ &= M_p\frac{E_0}{E}\;y\frac{d^2\sigma}{d\cos\theta dE}\quad .\end{aligned} \tag{3.27}$$

Before we go on to discuss the subject of experimental measurements of functions $W_1(q^2,\nu)$, $W_2(q^2,\nu)$ and the information they yield about the proton structure, it is instructive to see to what the general functions W_1, W_2 reduce to for the special cases of elastic scattering of an electron off a pointlike "Dirac" proton (eq. (2.5)) and a proton with structure (eq. (2.19)). The cross-section of eq. (2.5) can be rewritten as a double differential cross-section by using the identity,

$$\frac{d\sigma^{e\ell}}{d\Omega} = \int dE\;\delta\left(E-\frac{E_0}{1+\frac{2E_0}{M_p}\sin^2\frac{\theta}{2}}\right)\frac{d\sigma^{e\ell}}{d\Omega}\quad . \tag{3.28}$$

The above equation follows from realising that for elastic scattering the energy of the scattered electron is fixed via eq. (1.23), once the angle is fixed. Using the definitions of ν and q^2, as well as properties of δ-function, we can rewrite the above expression for $d\sigma^{e\ell}/d\Omega$ as

$$\frac{d\sigma^{e\ell}}{d\Omega} = \int dE\;\delta\left(-\nu+\frac{q^2}{2M_p}\right)\left(1+\frac{2E_0}{M_p}\sin^2\frac{\theta}{2}\right)\frac{d\sigma^{e\ell}}{d\Omega}\quad . \tag{3.29}$$

From this equation it is obvious that the double differential cross-section for the elastic case is

$$\frac{d^2\sigma^{e\ell}}{d\Omega dE} = \;\delta\left(-\nu+\frac{q^2}{2M_p}\right)\left(1+\frac{2E_0}{M_p}\sin^2\frac{\theta}{2}\right)\frac{d\sigma^{e\ell}}{d\Omega}\quad . \tag{3.30}$$

Formally, eq. (3.30) can also be derived by using,

$$\int \frac{d^3p'}{2W} \frac{\delta^4(P_1+q-P_4)}{(2\pi)^3} = \frac{1}{2M_p}\delta\left(\nu - \frac{q^2}{2M_p}\right) \quad . \tag{3.31}$$

This is merely a restatement of the relation between E, E_0 and $\sin^2\theta/2$ given by eq. (1.23).

Using eqs. (3.30) and (2.5) we can therefore write,

$$\left.\frac{d^2\sigma^{ep\to ep}}{d\Omega dE}\right|_{\text{“Dirac''}} = \delta\left(\nu - \frac{q^2}{2M_p}\right)\left[1 + \frac{q^2}{2M_p^2}\tan^2\frac{\theta}{2}\right]\left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} \quad . \tag{3.32}$$

Using the expression for $(d\sigma/d\Omega)_{\text{Mott}}$ we therefore get,

$$\left.\frac{d^2\sigma^{ep\to ep}}{d\Omega dE}\right|_{\text{“Dirac''}} = \delta\left(\nu - \frac{q^2}{2M_p}\right)\frac{4\alpha^2E^2}{q^4}\cos^2\frac{\theta}{2}\left[1 + \frac{q^2}{2M_p^2}\tan^2\frac{\theta}{2}\right] . \tag{3.33}$$

Similarly, we can see from eq. (2.6) that for the imaginary case of a spinless, pointlike proton we will get,

$$\left.\frac{d^2\sigma^{ep\to ep}}{d\Omega dE}\right|_{\text{spinless point proton}} = \delta\left(\nu - \frac{q^2}{2M_p}\right)\frac{4\alpha^2E^2}{q^4}\cos^2\frac{\theta}{2} \quad . \tag{3.34}$$

Using eq. (2.19) we get similarly,

$$\left.\frac{d^2\sigma}{d\Omega dE}\right|_{\text{ext.}} = \delta\left(\nu - \frac{q^2}{2M_p}\right)\frac{4\alpha^2}{q^4}E^2\cos^2\frac{\theta}{2}\left[A^p(q^2) + B^p(q^2)\tan^2\frac{\theta}{2}\right] \quad , \tag{3.35}$$

where $A^p(q^2)$ and $B^p(q^2)$ are given by eq. (2.27). Comparing eqs. (3.33) – (3.35) with eq. (3.26) we get,

$$\begin{aligned}
W_2^{\text{point},p}(q^2,\nu) &= \delta\left(\frac{q^2}{2M_p} - \nu\right) = \frac{1}{\nu}\delta(1-x) \quad ; \\
W_1^{\text{point},p}(q^2,\nu) &= \frac{q^2}{4M_p^2}\delta\left(\frac{q^2}{2M_p} - \nu\right) = \frac{1}{2M_p}\frac{q^2}{2M_p\nu}\delta(1-x) \quad ,
\end{aligned} \tag{3.36}$$

while

$$\begin{aligned}
W_2^{\text{scalar},p} &= \frac{1}{\nu}\,\delta(1-x) \quad ; \\
W_1^{\text{scalar},p} &= 0 \quad ,
\end{aligned} \tag{3.37}$$

and

$$\begin{aligned}
W_2^{e\ell}(q^2,\nu) &= \frac{1}{\nu}A^p(q^2,\nu)\delta(1-x) \quad ; \\
W_1^{e\ell}(q^2,\nu) &= \frac{B^p(q^2,\nu)}{2\nu}\delta(1-x) = \frac{1}{2M_p}\frac{q^2}{2M_p\nu}{G_M^p}^2(q^2)\,\delta(1-x) \quad .
\end{aligned} \tag{3.38}$$

Table 3: The structure functions $M_p W_1(x, Q^2)$ and $\nu W_2(x, Q^2)$ for elastic scattering from different type of scatters.

Scatterer	$M_p W_1(x, Q^2)$	$\nu W_2(x, Q^2)$
pointlike spin 1/2 proton	$\frac{q^2}{4M_p\nu}\ \delta(1-x)$	$\delta(1-x)$
pointlike scalar proton	0	$\delta(1-x)$
spin 1/2 proton with structure	$\frac{q^2}{4M_p\nu}(G_M^p)^2\ \delta(1-x)$	$\frac{\left((G_E^p)^2+\frac{q^2}{4M_p^2}(G_M^p)^2\right)}{\left(1+\frac{q^2}{4M_p^2}\right)}\ \delta(1-x)$

The functions $G_E^p(q^2)$ and $G_M^p(q^2)$ have the very steep q^2 dependence given by eq. (2.29). This means that the structure functions $W_{1,2}^{e\ell}(q^2, \nu)$ fall off very steeply with increasing q^2. More interesting is the observation that the structure functions $W_{1,2}^{\rm point}$ for a point scatterer do not depend upon the variables q^2 and ν separately, but are functions only of the combination $x = q^2/2M_p\nu$. This is of course obvious. In the case of a point scatterer, the structure functions should depend only on dimensionless variables as there is no intrinsic length scale associated with the scatterer. Thus if the scatterer has a finite size then the structure functions for the elastic scattering $W_1^{e\ell}$, $W_2^{e\ell}$, fall off as a power of q^2 whereas for a pointlike scatterer they depend on q^2 only through the combination $q^2/2M_p\nu$. Table 3 summarises the behaviour of the structure function for different types of scatterers.

3.3 Scaling of structure functions and partons

The discussions of the earlier section tell us that the q^2 and ν dependence of the structure functions $W_{1,2}(q^2, \nu)$ depends on the nature of scatterer. Hence it is worthwhile asking, what do the experimental measurements of $W_{2,1}^{{\rm inel},p}(q^2, \nu)$ look like? With increasing q^2, the inelastic scattering begins to dominate the elastic process for $q^2 \gtrsim 2$ – 3 GeV^2 (recall eq. (2.29)). For quasi-inelastic scattering, the resonance excitation corresponds to $M_X = M^\star$ in eq. (3.4). Hence ν is fixed once q^2 is. This means that $d^2\sigma/dq^2 d\nu$ will be a δ-function in ν for a fixed q^2. Experimental measurements of $d^2\sigma/dq^2 d\nu$ do indeed show peaks in ν at a fixed q^2. Fig. 8 taken from [16] shows this. With increasing q^2, the proton-resonance transition form factors also show a power law fall off with q^2, just like $G_M^p(q^2)$. At still higher values of q^2, continuum production takes over. The interesting observation of *Bjorken scaling* was the fact that for this inelastic scattering the structure functions $W_{1,2}^{\rm inel}(q^2,\nu)$ do not fall off with increasing q^2 but they are found to become independent of q^2 (for a fixed value of $q^2/2M_p\nu$) as both q^2 and ν become large, *i.e.*, they are functions of $x = q^2/2M_p\nu$ alone. This is illustrated for some data on $\nu W_2^{\rm inel}(q^2,\nu)$ in fig. 9 taken from the first of Ref. [2]. Thus the experimental observation is,

$$\begin{aligned}
\nu W_2^{\rm inel}(q^2,\nu); & \xrightarrow[\nu\to\infty,\, q^2\to\infty]{\text{scaling}} F_2^{ep}(x) \;\; ; \\
M_p W_1^{\rm inel}(q^2,\nu) & \xrightarrow[\nu\to\infty,\, q^2\to\infty]{\text{scaling}} F_1^{ep}(x) \;\; .
\end{aligned} \tag{3.39}$$

This phenomenon of 'scaling' of structure functions was interpreted by Bjorken [3] as an indication of the existence of pointlike scatterers inside the proton. This interpretation can be understood by recalling eq. (3.36). Thus the observed scaling of the DIS cross-sections indicates that inelastic electron-proton scattering can be understood in terms of *incoherent*, elastic scattering of electron off the individual, pointlike constituents of the proton termed 'partons' [4]. The charged partons which take part in the electromagnetic scattering are termed quarks. Whether these quarks are to be identified with the quarks whose existence is inferred from spectroscopic studies is best discussed later on in the context of the parton model. Since there is no scale associated with these pointlike objects, eq. (3.36) indicates that the individual elastic cross-sections must scale. We will show later, in a detailed discussion of the parton model, that the variable x can be interpreted as the fraction of proton momentum that the parton carries. Hence the intuitive picture is as shown in fig. 10.

In the scaling limit of eq. (3.39), using eq. (3.27), eq. (3.26) becomes

$$\frac{d^2\sigma^{{\rm inel},em}}{dx\; dy} = \frac{4\pi\alpha^2}{Sx^2y^2}\left[xy^2F_1^{ep}(x) + \left(1 - y - \frac{M_p}{2E_0}xy\right)F_2^{ep}(x)\right] \;\; .$$

S is the square of the total c.m. energy of the scattering process $\simeq 2M_pE_0$. The scaling limit corresponds to high energies of the incident electron, hence particle masses can be neglected and we get,

$$\frac{d^2\sigma^{{\rm inel},em}}{dx\; dy} = \frac{4\pi\alpha^2}{Sx^2y^2}\left[xy^2F_1^{ep}(x) + (1-y)\,F_2^{ep}(x)\right] \;\; . \tag{3.40}$$

Here $F_1^{ep}(x)$ and $F_2^{ep}(x)$ are the electromagnetic structure functions of the proton.

It is worth noting here that according to eq. (3.37), $W_1^{\rm scalar} = 0$. Hence if the pointlike constituents inside the proton are scalars, the term proportional to $\tan^2(\theta/2)$ in eq. (3.26) (or equivalently the term proportional to xy^2 in eq. (3.40)) will be absent. Note also that all the discussions will be completely unchanged if one were to use μ^- beams instead of e^- beams. The structure functions are characterised by the target and the type of interactions used as a probe, in the present case a proton. So we have

$$\begin{aligned}
F_1^{ep}(x) &= F_1^{\mu p}(x) \;\; ; \\
F_2^{ep}(x) &= F_2^{\mu p}(x) \;\; .
\end{aligned}$$

3.4 Neutrino Deep Inelastic Scattering

So far, in our discussions the scattering process that we considered was e^-–p scattering. This probes the structure of proton via electromagnetic interactions and, as shown before, is characterised by two independent, arbitrary functions $W_i(q^2, \nu)$ ($i = 1, 2$) which are to be determined experimentally. One can also probe the structure of the proton via weak interactions in the charged current reaction

$$\nu_\ell + p \to \ell^- + X \tag{3.41}$$

and the neutral current reaction,

$$\nu_\ell + p \to \nu_\ell + X \quad . \tag{3.42}$$

Here we have started the discussion directly with the inclusive, inelastic process. The analogues of the form factors G_M^p, G_E^p and G_M^n for the elastic processes involving neutrinos exist. Symmetries of strong interactions relate the electromagnetic form–factors as measured in the e^-–p or μ^-–p scattering and the weak form–factors as measured in $\nu_\ell\ p$ scattering. The charged current weak form–factors have been measured and indeed played a very important role in confirming these features of strong interaction symmetries. Here, however, we concentrate only on the DIS process shown in fig. 11.

Analogous to the earlier discussions (cf. eqs. (3.13) – (3.19)) for the charged current reaction of eq. (3.41), we can write

$$d\sigma^{\text{inel},\nu} = \frac{1}{4M_pE_0}\frac{d^3p}{(2\pi)^2 2E}\left(\frac{g}{\sqrt{2}}\right)^2\left(\frac{1}{q^2 - M_W^2}\right)^2 4\pi M_p L'_{\mu\nu} H''_{\mu\nu}, \tag{3.43}$$

where $L'_{\mu\nu}$ is the leptonic tensor evaluated using the weak current $J_\mu^{\text{weak}}(\ell, \nu_\ell)$ instead of the electromagnetic current used in eq. (3.12), $H''_{\mu\nu}$ is the hadronic tensor (again analogue of eq. (3.17) but replacing the electromagnetic current by the weak current), g is the weak coupling of ℓ, ν_ℓ to the weak gauge boson W and M_W is the mass of the W boson. The matrix element of the weak current is given by [17]

$$\langle P_3, S_\ell | \hat{J}_\mu^{\text{weak}} | P_2, S_{nu_\ell}\rangle \equiv J_\mu^{\text{weak}}(\ell, \nu_\ell) \equiv \frac{ig}{\sqrt{2}}\bar{u}_\ell(S_\ell, P_3)\gamma_\mu(1+\gamma_5)u_{\nu_\ell}(S_{\nu_\ell}, P_2) \quad . \tag{3.44}$$

Hence

$$\begin{aligned} L'_{\mu\nu} &= \frac{1}{2}\sum_{S,S'} J_\mu^{\text{weak}} J_\nu^{\dagger\ \text{weak}} \\ &= \frac{1}{2}\,\frac{g^2}{2}\,\sum_{S,S'}\left[\bar{u}_\ell(S', P_3)\gamma_\mu(1+\gamma_5)u_{\nu_\ell}(S, P_2)\right] \times \\ &\qquad\qquad \left[\bar{u}_{\nu_\ell}(S, P_2)\gamma_4(1+\gamma_5)\gamma_\nu\gamma_4 u_\ell(S', P_3)\right] \\ &= -\frac{1}{2}g^2\ \mathrm{Tr}\left[\gamma_\mu(1+\gamma_5)(-i\not{P}_2)\gamma_\nu(1+\gamma_5)(-i\not{P}_3)\right] \end{aligned}$$

$$= \frac{g^2}{2} \mathrm{Tr}\,[\not{P}_3 \gamma_\mu \not{P}_2 \gamma_\nu (1+\gamma_5)]$$

$$= \frac{g^2}{2}\, 4\,[P_{3\mu} P_{2\nu} + P_{3\nu} P_{2\mu} - \delta_{\mu\nu} P_2 \cdot P_3 - \epsilon_{\mu\nu\alpha'\beta'} P_{3\alpha'} P_{2\beta'}] \quad .$$

Here, as before, masses of the leptons are neglected. We note now that $L'_{\mu\nu}$ is no longer symmetric under an exchange $\mu \leftrightarrow \nu$. Hence the most general expression for $H''_{\mu\nu}$ must involve antisymmetric tensors too. A tensor decomposition of $H''_{\mu\nu}$ now involves six linearly independent tensors that one can construct out of the four momenta P_1 and q, and hence six arbitrary functions of q^2 and ν. This is to be contrasted with the electromagnetic case where one needed only two arbitrary functions due to the its symmetric nature and the requirement of gauge invariance. The most general expression for $H''_{\mu\nu}$ in this case can be written as

$$\begin{aligned} H''_{\mu\nu} &= W'_1(q^2,\nu)\delta_{\mu\nu} + W'_2(q^2,\nu)\frac{P_{1\mu}P_{1\nu}}{M_p^2} \\ &+ W'_3(q^2,\nu)\frac{P_{1\alpha}q_\beta}{2M_p^2}\epsilon_{\mu\nu\alpha\beta} + W'_4(q^2,\nu)\frac{q_\mu q_\nu}{M_p^2} \\ &+ W'_5(q^2,\nu)\left(\frac{P_{1\mu}q_\nu + q_\mu P_{1\nu}}{M_p^2}\right) + W'_6(q^2,\nu)\left(\frac{P_{1\mu}q_\nu - q_\mu P_{1\nu}}{M_p^2}\right) \quad . \end{aligned} \tag{3.45}$$

Thus, in general, the DIS process $\nu_\ell + p \to \ell^- + X$ requires six structure functions. However, it should be noted at this point that the cross-section involves contraction of $H''_{\mu\nu}$ with $L'_{\mu\nu}$, and contributions from W'_4, W'_5 and W'_6 to the cross-section can be seen to be proportional to the lepton masses and can therefore be dropped. This is demonstrated for the term containing $W'_4(q^2,\nu)$. Consider

$$Y = W'_4(q^2,\nu)\frac{q_\mu q_\nu}{M_p^2}\, L'_{\mu\nu} = \frac{W'_4(q^2,\nu)}{M_p^2}\left[2(P_3\cdot q)(P_2\cdot q) - q^2(P_2\cdot P_3)\right] \quad .$$

Using $-P_3 \cdot q = P_2 \cdot q = q^2/2$; $P_2 \cdot P_3 = -q^2/2 - m_e^2$ we get,

$$Y = \frac{W'_4(q^2,\nu)}{M_p^2}\left[2\,\frac{q^2}{2}\,\left(-\frac{q^2}{2}\right) - q^2\left(-\frac{q^2}{2} - m_e^2\right)\right] = \frac{W'_4(q^2,\nu)}{M_p^2} m_e^2 q^2 \simeq 0(m_e^2)$$

Similarly terms proportional to W'_5 and W'_6 in $H''_{\mu\nu}L'_{\mu\nu}$ can be shown to be small.

The contraction of the first two symmetric terms in eq. (3.45) with the symmetric terms in $L'_{\mu\nu}$ gives results similar to the electromagnetic case. The contraction between the symmetric and antisymmetric terms will obviously yield zero. The contraction of the antisymmetric term in $H''_{\mu\nu}$ with the one in $L'_{\mu\nu}$ gives,

$$X = -4\,\frac{g^2}{2}\,\frac{W'_3(q^2,\nu)}{2M_p^2}\, P_{1\alpha} q_\beta \epsilon_{\mu\nu\alpha\beta}\epsilon_{\mu\nu\alpha'\beta'} P_{3\alpha'} P_{2\beta'} \quad .$$

Using, $\epsilon_{\mu\nu\alpha\beta}\epsilon_{\mu\nu\alpha'\beta'} = 2\,[\delta_{\alpha\alpha'}\delta_{\beta\beta'} - \delta_{\alpha\beta'}\delta_{\beta\alpha'}]$, we get

$$X = -4\,\frac{W'_3(q^2,\nu)}{2M_p^2}\,\frac{g^2}{2}\,2\,[P_1\cdot P_3 P_2\cdot q - P_1\cdot P_2 P_3\cdot q].$$

Using $P_1 \cdot P_3 = -M_p E$, $P_1 \cdot P_2 = -M_p E_0$ and the expressions for $P_2 \cdot q$, $P_3 \cdot q$ as well as q^2 quoted earlier, we get

$$X = 8EE_0 \sin^2\theta/2 \, \frac{g^2}{2} \, \frac{W_3'(q^2,\nu)}{M_p}(E+E_0) \ .$$

Neglecting the lepton masses, in the limit $q^2 \ll M_W^2$, the general expression for the inelastic, inclusive, differential cross-section for ν reactions becomes,

$$\frac{d^2\sigma^{\rm inel}}{d\Omega dE}(\nu_\ell\, p \to \ell^- X) = \frac{G_F^2}{2\pi^2}E^2 \left[2W_1'(q^2,\nu)\sin^2\frac{\theta}{2} + W_2'(q^2,\nu)\cos^2\frac{\theta}{2} \right.$$
$$\left. + W_3'(q^2,\nu)\frac{E_0+E}{M_p}\sin^2\frac{\theta}{2} \right] \ , \qquad (3.46)$$

where $G_F/\sqrt{2} = g^2/8M_W^2$. The maximum ν-beam energy that has been reached in current experiments is $E_0 = 500$ GeV. From eq. (3.11) this means that the maximum q^2 that can be reached in these experiments is ≈ 1000 GeV^2. Since $M_W \simeq \mathcal{O}$ (1000 GeV) , the approximation $q^2 \ll M_W^2$ is valid even at these highest attainable values of q^2. The charged current weak structure functions at higher values of q^2 are accessible only in the study of the reaction

$$e^- + p \to \nu + X$$

at the HERA collider where q^2 can reach upto 10^5 GeV^2.

Again, in the Bjorken limit of large q^2 and ν the structure functions are found to scale just as in eq. (3.39), *i.e.*, $M_p W_1'(q^2,\nu) \to F_1^{\nu \rm p}(x)$, $\nu W_2'(q^2,\nu) \to F_2^{\nu \rm p}(x)$ and $\nu W_3'(q^2,\nu) \to F_3^{\nu \rm p}(x)$, giving

$$\frac{d^2\sigma^{\rm inel}}{dxdy}(\nu_\ell\, p \to \ell^- X) = \frac{G_F^2}{2\pi}\, S \left[xy^2 F_1^{\nu p}(x) + F_2^{\nu p}(x)(1-y) \right.$$
$$\left. + F_3^{\nu p}(x) xy \left(1-\frac{y}{2}\right) \right] \ . \qquad (3.47)$$

S in the above equation stands for the square of the centre of mass energy given by $M_p^2 + 2M_pE_0 \simeq 2M_pE_0$. Note that the result of eq. (3.47) is obtained from eq. (3.46) in the limit of vanishing particle masses just as in the case of eq. (3.40). Here we have three structure functions as opposed to the electromagnetic case. The third structure function arises from the term containing $\epsilon_{\mu\nu\alpha\beta}$ in the tensor $L'_{\mu\nu}$ $(H''_{\mu\nu})$. This is the parity violating structure function. If instead of neutrino scattering we were to consider

$$\bar\nu_\ell + p \to \ell^+ + X$$

the corresponding term in $L'_{\mu\nu}$ will change sign. Hence in the cross-section the term containing F_3 will change sign. Hence,

$$F_3^{\bar\nu p} = -F_3^{\nu p}.$$

This can be physically understood by realising that $\bar{\nu}(\nu)$ is right (left) handed and the term containing F_3 essentially arises from the $V-A$ interference term in $L'_{\mu\nu} H''_{\mu\nu}$, which changes sign as we go from a right handed $\bar{\nu}$ to a left handed ν.[††]

At this stage of analysis, it is not at all clear that the functions $W'_1(q^2, \nu)$, $W'_2(q^2, \nu)$ and $W'_3(q^2, \nu)$ (or equivalently $F_i^{\nu p}(x)$, $i = 1, 3$) have anything to do with the electromagnetic structure functions $W_i(q^2, \nu)$, $i = 1, 2$ (or equivalently $F_i^{ep}(x)$ or $F_i^{\mu p}(x)$, $i = 1, 2$). One can make predictions for relations between these only in the framework of the parton model. As a matter of fact, experimental test of these relations was an important step in establishing the parton model firmly. This will become clearer as we go on to discuss DIS and the parton model in the next section.

3.5 Relationship between scaling of cross-section and point-like constituents

The phenomenon of scaling of the cross–sections reflecting the existence of pointlike scattering centers occurs at different energy scales twice as we go from e^-A to e^-p elastic scattering to e^-p deep inelastic scattering. This is illustrated by the data on nuclear scattering very nicely. Fig. 12 taken from ref. [18] shows this schematically whereas fig. 13 [14] shows some of the actual data. The "large"ness or "small"ness of a particular q^2 value has always to be understood with respect to the inverse size. Recall, eg., that for $q^2 \ll (0.71)$ GeV^2 the $W_{1,2}^{el}$ of eq. (3.38) will look just like their counterpart for elastic scattering from a pointlike scatterer of eq. (3.36). In general therefore, for elastic scattering we can write,

$$(M_p W_1^{e\ell})\ \nu W_2^{e\ell} \sim f_{(1)2}(x)\ g(q^2)\ \ .$$

For q^2 values such that $qR_{\text{target}} \ll 1$ (*i.e.*, $q^2 \ll \Lambda^2_{\text{target}}$, where Λ_{target} = inverse size of the target $\sim (1/R)$), $\nu W_2^{e\ell}$ $(M_p W_1^{e\ell})$ will scale, *i.e.*, will not show any extra q^2 dependence. At these values of q^2, $g(q^2) \approx 1$. In the case of elastic scattering, be it from a nucleon or a nucleus, the variables q^2 and ν always satisfy the relation

$$q^2 = 2M_{\text{target}}\ \nu\ \ .$$

This also indicates that the function $f_{1(2)}(x)$ must also have a factor $\delta\left(\frac{q^2}{2M_{\text{target}}\nu} - 1\right)$ or equivalently a factor $= \delta(x-1)$, *i.e.*, $f_{1(2)}(x)$ will thus have a peak at $x = q^2/(2M_{\text{target}}\ \nu) = 1$. As q^2 values increase, the function $g(q^2)$ starts differing from 1 and falls off with increasing q^2. For $q^2R^2 = 1$, *i.e.*, $q^2 \approx \Lambda^2_{\text{target}}$, form factors cause a measurable suppression of the cross-section. For $qR \gg 1$, the function $g(q^2)$ falls off very steeply indeed and the elastic peak at $x = 1$ disappears.

In fig. 12(a) the elastic peak at $x = q^2/2M_c\nu = 1$ at $q^2 = 0.01$ GeV^2 in $eC \to eC$ is shown. The quasi elastic excitation of the resonance $C^\star$ in $eC \to eC^\star$ appears as a peak at a lower x values (cf. eq. (3.3)). At this value of q^2, we have $q^2 \gg \Lambda_C^2$ and the Carbon nucleus appears like a point particle and elastic scattering dominates. As q^2 increases further, at $q^2 = (0.1)$ GeV^2, we have the situation

$$\Lambda^2_{\text{proton}} \gg q^2 \gg \Lambda_C^2\ \ .$$

[††]The terminology of left (right) handedness of the $\nu(\bar{\nu})$ corresponds to its helicity being -1(1).

In this regime the proton appears to be pointlike and the elastic peak for $eC \to eC$ disappears. The dominant process is no longer the elastic process $eC \to eC$, but the inelastic one which is now an incoherent sum of scattering off the N nucleons inside the Carbon nucleus. In principle the elastic scattering off a nucleon should show up as a peak at $x = q^2/2M_C\ \nu = 1/N$ (for the elastic ep scattering $q^2 = 2M_p\nu$; hence $x = q^2/2M_C\ \nu = 1/N$). The Fermi motion of the proton in the nucleus however changes the kinematics and smears the δ-function peak. This is shown in fig. 12(b). Now

$$\nu W_2^C \sim f_2^C(x)\ g^p(q^2)\ .$$

But $g^p(q^2) \approx 1$ since $q^2 \ll \Lambda^2_{\rm proton}$. Thus $f_2^C(x, Q^2)$ now scales. This scaling thus reveals the existence of N pointlike nucleons in the nucleus C. Fig. 13 shows data on $e\alpha$ scattering at two different q^2 values $q^2 \simeq 0.08$ GeV^2 and $q^2 \simeq 0.1$ GeV^2. This figure illustrates the same point, only the value of N here is 4.

If we now consider a proton at rest, at higher values of q^2 ($\simeq 0.5$ GeV ^{2}) and higher ν values, the elastic peak for $ep \to ep$ will occur at $x = \frac{1}{N}$ and the quasi-elastic excitations corresponding to ,say, Δ^{++} will show up as a peak at still lower values of x. This is the proton analogue of Fig. 12(a) and which was discussed already in section 2; except for the fact that the variable defined there had been in terms of proton mass ($x_p = (q^2/2M_p\nu)$). Now the cross-section (more precisely νW_2^p) again will consist of two factors: one an x dependent kinematic function and a q^2 dependent form factor reflecting the target size. This is the violation of scaling.

Now if q^2 is further increased to $q^2 \gg 0.71$ GeV^2 (Fig. 12(d) corresponds to $q^2 \simeq 5$ GeV^2), then the rapid fall-off of the form factor, $g^p(q^2)$, with increasing q^2 will cause the elastic peak to vanish as well as the inelastic Δ^{++} peak. Now the incoherent scattering from the pointlike partons (quarks) $eq \to eq$ will show up as a smeared peak in x distribution. The x value at which this peak appears should give information about the number of partons off which the electrons get scattered. The structure function scales again, revealing the existence of pointlike constituents inside the proton and this peak will be at $x \simeq 1/3N$ indicating the existence of three valence quarks.

At still higher q^2 values, for the proton target, ($q^2 \simeq 200 - 400$ GeV^2 for the ep , μp and νp experiments discussed earlier or $q^2 \lesssim 10^4 - 10^5$ GeV^2 at the ep collider, HERA) $q\bar{q}$ production begins and that increases the number of pointlike constituents in the proton effectively. This shifts the peak in the structure function to lower and lower x values. These scaling violations are better discussed in terms of QCD and have been studied extensively in current DIS experiments[19]. However, these will not be discussed here any further.

4 Parton Model

4.1 Formalism

As discussed in the last chapter, the DIS $e^-p \to e^-X$ cross-section scales at large q^2 and large ν (in the Bjorken limit). The observed scaling is evidence that the

DIS cross-section is given by an incoherent sum of scattering of the electron against individual partons inside the proton. This interpretation is basically the parton model. Implicit in this model are two assumptions:

- Interactions among the partons are negligible during the time of interaction between the electron and the parton. The higher the energy, shorter is the interval of time of this interaction. Before the proof of asymptotic freedom [5] of QCD, this assumption was justified only by the success of the parton model, but asymptotic freedom provides now justification for this assumption.
- The second assumption is that final state interactions can be neglected. If the struck parton receives a huge kick then it gets removed from the parton and the final state interactions are hence negligible (recall here fig. 10).

Fig. 14 shows a schematic description of DIS in the parton model. Let ξ be the momentum fraction of the proton carried by the struck parton and e_q denote its electric charge in units of the proton charge. Let $f_{i/p}(\xi)$ be the probability that the parton i carries momentum fraction ξ of the proton. Momentum conservation implies

$$\sum_i \int_0^1 d\xi \; f_{i/p}(\xi)\, \xi = 1 \quad . \tag{4.1}$$

Note here that the sum is over all types of partons, not just the charged ones which the incident electron (or equivalently the virtual photon) sees. In this model, one has neglected transverse momentum of the partons. This is justified by the experimental observation that apart from the struck parton which causes hadrons to emerge at large angles, the remaining particles in the final state emerge at small angles w.r.t. the beam direction.

Let us choose the z axis to be the direction of motion of the proton and hence of the parton. The magnitude of the three momentum is $|\vec{p}_1|$ and $\xi|\vec{p}_1|$ respectively for the proton and the parton. The four momenta are given by $P_1 = (\vec{0}, P_L, iE_1)$ and $P_q = (\vec{0}, \xi P_L, i\xi E_1)$, where $P_L = |\vec{p}_1|$. This gives $P_1^2 = -M_p^2$ and $P_q^2 = -\xi^2 M_p^2$. It appears as if the partons have a variable mass ξM_p. This clearly is not what we mean. This is just a reflection of the fact that the above kinematics is strictly correct only in the limit where all masses can be neglected. In this case the kinematics given above simply corresponds to a collinear emission of a massless particle from another massless particle.

The frame of reference in which the above kinematics is strictly valid is called the infinite momentum frame. In the infinite momentum frame, time dilation slows down the rate at which partons interact with one another and this time scale is now much bigger than the time taken by the current (*i.e.*, the electron or the virtual photon) to interact with the parton. Hence the impulse approximation (assumption (i) above) is justified in this frame.

After scattering the struck parton (quark) appears in the detectors as a stream of hadrons. The time scale of hadronisation ($\sim 10^{-23}$ sec.) is much bigger than the interaction time scale ($\lesssim 10^{-25}$ sec.) for $\nu \gtrsim \mathcal{O}(10$ GeV). This description of scattering as a two step process is the second basic tenet of parton model. It is clear from the above discussion that for both these assumptions to be justified and the picture of fig. 14 to be true, q^2, ν and W all need to be large.

Since the partons are pointlike objects, $m_q W_1^{\rm parton}(q^2,\nu)$ and $\nu W_2^{\rm parton}(q^2,\nu)$ are given by eqs. (3.36), replacing $M_p \to m_q = \xi M_p$. Hence we have,

$$m_q W_1^{\rm parton}(q^2,\nu) = e_q^2 \frac{q^2}{4m_q}\delta\left(\nu - \frac{q^2}{2m_q}\right) \quad ;$$

$$\nu W_2^{\rm parton}(q^2,\nu) = e_q^2 \delta\left(1 - \frac{q^2}{2m_q\nu}\right) \quad . \tag{4.2}$$

If we now define

$$\omega = \frac{2M_p\nu}{q^2} = \frac{-2P_1\cdot q}{q^2} = \frac{1}{x} \quad , \tag{4.3}$$

then we get

$$M_p W_1^{\rm parton}(q^2,\nu) \equiv e_q^2 \, \frac{q^2}{4m_q\nu\xi}\delta\left(1 - \frac{1}{\xi\omega}\right) e_q^2 = \frac{q^2}{4M_p\nu\xi^2}\delta\left(\frac{-x}{\xi} + 1\right) \quad .$$

The right hand side of this equation is clearly a function of x alone, and we can write (with an analogous discussion for $\nu W_2^{\rm parton}(q^2,\nu)$),

$$F_1^{\rm parton}(x) = M_p W_1^{\rm parton}(q^2,\nu) = \frac{e_q^2}{2}\frac{q^2}{2M_p\nu\xi^2}\delta\left(1 - \frac{x}{\xi}\right) \quad ;$$

$$F_2^{\rm parton}(x) = \nu W_2^{\rm parton}(q^2,\nu) = \delta\left(1 - \frac{x}{\xi}\right) e_q^2 \quad .$$

Since in the limit of large q^2, ν and W, the total cross-section for ep scattering (and hence $M_p W_1^{\rm parton}$, $\nu W_2^{\rm parton}$) are given by an incoherent addition over all the charged partons (quarks) we get in the scaling limit,

$$\begin{aligned}
F_2^{ep}(x) &= \sum_q \int_0^1 d\xi F_2^{\rm parton}(\xi)\; f_{q/p}(\xi) \\
&= \sum_q e_q^2 \int_0^1 d\xi \delta\left(1 - \frac{x}{\xi}\right) f_{q/p}(\xi) \\
&= \sum_q e_q^2 \; f_{q/p}(x)\; x \quad .
\end{aligned}$$

Similarly,

$$\begin{aligned}
F_1^{ep}(x) &= \frac{1}{2}\sum_q \int_0^1 d\xi e_q^2 \frac{q^2}{2M_p\nu\xi}\delta(\xi - x) f_{q/p}(\xi) \\
&= \frac{1}{2}\sum_q e_q^2 \; f_{q/p}(x) \quad .
\end{aligned}$$

It should be noted here that the summation is over particles as well as antiparticles.

Thus we find that the scaling variable x which we defined earlier can be identified with the momentum fraction ξ that the parton q carries. The structure functions appearing in eq. (3.39) (which are the scaling limits of the arbitrary functions $W_i(q^2, \nu)$, $i = 1, 2$ which appeared in the tensor decomposition of eq. (3.22)) are related to the probability of finding a parton of charge e_q (in units of proton charge) with momentum fraction x of the proton. There is yet another way of seeing the same thing. In the infinite momentum frame all masses are negligible. The final four momentum of the struck parton is given by $(\xi P_1 + q)$. Hence we have

$$(\xi P_1 + q)^2 \simeq 0 \ ,$$

which gives us,

$$\xi = \frac{-q^2}{2P_1 \cdot q} \ . \tag{4.4}$$

This is same as the variable x defined in eq. (3.7a). The above expressions for $F_i^{ep}(x)$, $i = 1, 2$ also imply

$$F_2^{ep}(x) = 2xF_1^{ep}(x) \ , \tag{4.5}$$

which is known as the Callan-Gross relation. Using eq. (4.5) we get from eq. (3.40)

$$\begin{aligned} \frac{d^2\sigma^{inel}}{dxdy}(e^- p \to e^- X) &= \frac{2\pi\alpha^2}{Sx^2y^2}\left[\sum_q e_q^2 x f_{q/p}(x)\right]\left[1 + (1-y)^2\right] \\ &= \frac{4\pi\alpha^2}{Sx^2y^2} F_2^{ep}(x)\left[\frac{1}{2} + \frac{(1-y)^2}{2}\right] \ . \end{aligned} \tag{4.6}$$

In the parton model picture which we have developed above, now the structure function $F_2^{ep}(x)$ is expressed in terms of the probability density functions $f_{q/p}(x)$. Since this probability should be independent of the probe that is used to extract it from the data, one expects that structure functions $F_i^{\nu p}(x)$ $(i = 1, 2, 3)$, measured in νp DIS, must also be some combinations of the same functions $f_{q/p}(x)$ that are extracted from the data on electromagnetic DIS processes, the specific form being decided by the nature of νq interactions. As a matter of fact, identifying the charged partons in the proton with the constituent quarks of $SU(3)$ flavour, one can make definite predictions for the weak structure functions of the parton as well as electromagnetic and weak structure functions of other targets such as neutron or nuclei and relate them to each other. An experimental verification of these relations in the ν DIS experiment [20] played a very important role in establishing the parton model on a firm footing.

An alternative definition of eq. (4.6) can be given as follows. Consider the cross-section $d\sigma/d\Omega$ for scattering of an e^- from a pointlike object of charge e_q (in units of proton charge). The corresponding expression is given by eq. (2.5). For the case of elastic scattering, we know that y is given by (recall eq. (3.8)) ,

$$y = \frac{\nu}{E_0} = \frac{q^2}{S} = \frac{2E_0/M_p \sin^2\frac{\theta}{2}}{1 + \frac{2E_0}{M_p}\sin^2\frac{\theta}{2}} \ .$$

Hence

$$\frac{d\sigma^{el}}{dy} = \frac{d\sigma^{el}}{d\Omega} \left(\frac{2\pi E_0 M_p}{E^2}\right) = \frac{S}{E^2}\frac{d\sigma^{el}}{d\Omega} \quad ,$$

where S is the square of the cm energy. In the present case we have to consider scattering of the e^- from a charged parton carrying a fraction x of the four-momentum P_1 of the proton. The square of the cm energy $\hat{S}$ of the elastic electron-parton scattering is given by,

$$\hat{S} = -(xP_1 + P_2)^2 = -2xP_1 \cdot P_2 = 2xM_p E_0 = xS \quad . \tag{4.7}$$

Using the expression for $d\sigma^{el}/d\Omega$ given by eq. (2.5) but now for c.m. energy $\sqrt{\hat{S}}$ as given by eq. (4.7), we have (again neglecting particle masses),

$$\begin{aligned}\frac{d\sigma^{el}}{dy}(e^- q \to e^- q) &= \frac{4\pi\alpha^2}{q^4}\frac{\hat{S}}{E^2} e_q^2\, E^2 \left(\frac{E}{E_0}\right)\left[\cos^2\frac{\theta}{2} + \frac{q^2}{2M_p^2}\sin^2\frac{\theta}{2}\right] \\ &\simeq \frac{4\pi\alpha^2 e_q^2}{q^4} S\, x \left[\frac{1+(1-y)^2}{2}\right] \quad . \end{aligned} \tag{4.8}$$

If $f_{q/p}(x)dx$ is the probability of finding a parton with momentum fraction of proton between x and $x + dx$, we have

$$\frac{d\sigma}{dy}(e^- p \to e^- p) = \frac{4\pi\alpha^2}{q^4}\left[\sum_q e_q^2 \int_0^1 f_{q/p}(x) x\, dx\right]\left[\frac{1+(1-y)^2}{2}\right] \quad .$$

Hence we get

$$\frac{d\sigma}{dx\, dy}(e^- p \to e^- p) = \frac{4\pi\alpha^2 S}{q^4}\left[\sum_q e_q^2 f_{q/p}(x) x\right]\left[\frac{1+(1-y)^2}{2}\right] \quad . \tag{4.9}$$

Note that above equation is the same as eq. (4.6) or eq. (3.40) where,

$$F_2^{ep}(x) = 2xF_1^{ep}(x) = \sum_q e_q^2\, x\, f_{q/p}(x) \quad . \tag{4.10}$$

This alternative derivation of eq. (4.6) indicates that, to derive the corresponding expressions for the double differential DIS cross-section for νp or $\bar{\nu} p$ processes, we need to know $d\sigma^{el}/dy(\nu_\ell\, q \to \ell\, q')$, $d\sigma^{el}/dy(\bar{\nu}_\ell\, q \to \bar{\ell}\, q')$ etc. Due to the parity violating nature of weak interactions, the angular distribution (and hence y distribution) of νq and $\nu \bar{q}$ scattering are different in nature. This is in contrast to the situation in the case of electromagnetic scattering. The differential distributions $d\sigma^{el}/dy$ for the elementary scattering process can be obtained after a simple calculation to be

$$\begin{aligned}\frac{d\sigma^{el}}{dy}(\nu_\ell\, q) &= \frac{d\sigma^{el}}{dy}(\bar{\nu}_\ell\, \bar{q}) = \frac{G_F^2\, S}{\pi} \quad ; \\ \frac{d\sigma^{el}}{dy}(\bar{\nu}_\ell\, q) &= \frac{d\sigma^{el}}{dy}(\nu_\ell\, \bar{q}) = \frac{G_F^2\, S}{\pi}(1-y)^2 \quad .\end{aligned}$$

The second equation above indicates the impossibility of $\nu_\ell\, \bar{q}$ ($\bar{\nu}_\ell\, q$) scattering in the backward direction ($\theta = \pi$ corresponds to $y = 1$). This can be easily understood from fig. 15. This shows that the backward scattering in this case will correspond to $|\Delta J_Z| = 2$ which is not possible. We can use the above expressions for $d\sigma^{el}/dy(\nu_\ell\, q)$ etc. in a manner similar to the one used in ariving at eq. (4.9). In this case we get for the charged current DIS cross-section,

$$\begin{aligned}\frac{d^2\sigma}{dxdy}(\nu_\ell\, p \to \ell^- n) &= \frac{4G_F^2\, S}{2\pi}\left[2x\sum_q f_{q/p}(x) + 2x\sum_q f_{\bar{q}/p}(x)(1-y)^2\right] \\ &= \frac{G_F^2\, S}{2\pi}\left[2x\sum_q \left(f_{q/p}(x) + f_{\bar{q}/p}(x)\right)\left(\frac{1+(1-y)^2}{2}\right)\right. \\ &\quad \left. +2x\sum_q \left(f_{q/p}(x) - f_{\bar{q}/p}(x)\right)\left(\frac{1-(1-y)^2}{2}\right)\right] \quad .\end{aligned}$$

Of course it is understood that the sum is to be taken over those quarks or antiquarks which can take part in allowed transitions. The first term in the bracket is parity conserving whereas the second one violates parity. This equation is the same as eq. (3.47) if we identify

$$F_2^{\nu p}(x) = 2xF_1^{\nu p}(x) = 2x\left[\sum_q f_{q/p}(x) + f_{\bar{q}/p}(x)\right] \quad ; \tag{4.11}$$

$$xF_3^{\nu p}(x) = 2x\left[\sum_q f_{q/p}(x) - f_{\bar{q}/p}(x)\right] \quad . \tag{4.12}$$

4.2 Structure functions for proton, neutron and isoscalar targets

In this section we will write the form for the electromagnetic and weak structure functions expected in the parton model, if one identifies the charged parton with the Gell-Mann-Zweig constituent quarks. In the $SU(3)_f$ picture, e.g., the proton contains two u-quarks with charge $\frac{2}{3}e_p$ and one d-quark with charge $-\frac{1}{3}e_p$. Hence one expects that F_2^{ep} is given by,

$$F_2^{ep} = \frac{4}{9}xu^p(x) + \frac{1}{9}xd^p(x) \quad . \tag{4.13}$$

Here $u^p(x) = f_{u/p}(x)$ and so on. Isospin invariance would imply,

$$u^n(x) = d^p(x);\ d^n(x) = u^p(x) \quad .$$

Hence we expect,

$$F_2^{en}(x) = \frac{4}{9}d^p(x) + \frac{1}{9}u^p(x) \quad . \tag{4.14}$$

However, this presupposes the picture that the proton (neutron) has 2(1) u-quarks and 1(2) d-quarks. However, all the conclusions, verified by experiments, about static properties of proton/neutron will remain unchanged, if in addition to these partons one had large number of $u\bar{u}$, $d\bar{d}$, $s\bar{s}$ pairs forming an $SU(3)$ singlet, which have been radiated from those "valence" (large momentum) partons. One would expect the proton/neutron to contain such radiated partons in the parton model picture. Since these arise from radiation, these will have carry less momentum than the parent quarks, they will have a softer momentum distribution than the "valence" quarks. These were called "wee" or "sea" partons by Feynman. Then we will have, assuming $SU(3)$ symmetry for the sea (*i.e.*, the sea quark density is the same for all three types of quarks u, d and s),

$$
\begin{aligned}
u^p(x) &= u_V^p(x) + u_S^p(x), \\
d^p(x) &= d_V^p(x) + d_S^p(x), \\
u_S^p(x) &= \bar{u}_S^p(x) = d_S^p(x) = \bar{d}_S^p(x) = S_S^p(x) = \bar{S}_S^p(x) = K(x) \quad . \qquad (4.15)
\end{aligned}
$$

In this case eq. (4.13) will modify to

$$
\begin{aligned}
F_2^{ep}(x) &= x\Bigg[\frac{4}{9}(u_V^p(x) + u_S^p(x)) + \frac{1}{9}(d_V^p(x) + d_S^p(x)) + \frac{4}{9}\bar{u}_S^p(x) \\
&\qquad + \frac{1}{9}\bar{d}_S^p(x) + \frac{2}{9}\bar{S}_S^p(x)\Bigg] \\
&= x\left[\frac{4}{9}u_V^p(x) + \frac{1}{9}d_V^p(x) + \frac{4}{3}K(x)\right] \quad . \qquad (4.16)
\end{aligned}
$$

Again isospin invariance gives us

$$
F_2^{en}(x) = x\left[\frac{4}{9}d_V^p(x) + \frac{1}{9}u_V^p(x) + \frac{4}{3}K(x)\right] \quad . \qquad (4.17)
$$

In writing eqs. (4.16) and (4.17) we have assumed, in addition to the flavour symmetry of the sea, also absence of heavier charm and bottom quarks in the sea. At higher energies, even these can be radiated. In that case the factor before $K(x)$ will change. Using eqs. (4.16) and (4.17) we get,

$$
\frac{F_2^{en}(x)}{F_2^{ep}(x)} = \frac{u_V^p(x) + 4d_V^p(x) + \frac{4}{3}K(x)}{4u_V^p(x) + d_V^p(x) + \frac{4}{3}K(x)} \quad . \qquad (4.18)
$$

Can we compute these structure functions $u_V^p(x)$, $d_V^p(x)$ and $K(x)$? The answer is no, not without a model. However, we can try to measure them experimentally. The experimentally measured DIS cross-sections for ep (μp) reactions will yield $F_2^{ep}(F_2^{\mu p})$. How does one extract the specific parton densities from it? Before one goes into this question, let us see how one can obtain some qualitative information about $u_V^p(x)$, $d_V^p(x)$ and $K(x)$. Consider the ratio of eq. (4.18). If $K(x)$ dominates over $u_V^p(x)$, $d_V^p(x)$, this ratio will be 1. If either $u_V^p(x)$ or $d_V^p(x)$ dominates over all the other densities then this ratio will go to 1/4 or 4 respectively. Hence, under the assumption of valence dominance we will get

$$\frac{1}{4} < \frac{F_2^{en}(x)}{F_2^{ep}(x)} < 4 \tag{4.19}$$

If, on the other hand, we have $K(x) = 0$ and $u_V^p(x) = 2d_V^p(x)$ (as can be expected in a pure constituent picture), this ratio will have the value 2/3.

With these qualitative predictions we turn to the question as to what the experiments say. As was the case with the form factors, $F_2^{en}(F_2^{\mu n})$ is measured by combining the data on ep, ed ($\mu p, \mu d$) scattering. Early data[21] tell us:

(a) $K(x)$ dominates at small x and the ratio is indeed close to 1 at small x.

(b) At large $x(x \to 1)$, $F_2^{ep}(x)/F_2^{en}(x) \to 4$. This means that $u_V^p(x)$ dominates over $d_V^p(x)$, as well as over $K(x)$ at large x.

(c) Observation (b) also tells us that the naive expectation of $u_V^p(x) = 2d_V^p(x)$ is not fullfilled.

Fig. 16 shows the recent high statistics data of the NMC collaboration on the ratio $F_2^{\mu n}/F_2^{\mu p}$ [22]. What are the other theoretical constraints on these densities? We know that the net number of $u(d)$ quarks in a proton is 2(1) and the net number of strange quarks in both is zero. Hence we have,

$$\begin{aligned}
&\int_0^1 \left(u^p(x) - \bar{u}^p(x)\right) dx = 2 = \int_0^1 u_V^p(x) dx \quad , \\
&\int_0^1 \left(d^{\,p}(x) - \bar{d}^p(x)\right) dx = 1 = \int_0^1 d_V^p(x) dx \quad , \\
&\int_0^1 \left(S^p(x) - \bar{S}^p(x)\right) dx = 0 \quad .
\end{aligned} \tag{4.20}$$

The first two constraints of eq. (4.20) can also be obtained by considering electromagnetic charge conservation, which implies, for the proton and neutron respectively,

$$\begin{aligned}
1 &= \int_0^1 \left[\frac{2}{3}\left(u^p(x) - \bar{u}^p(x)\right) - \frac{1}{3}\left(d^p(x) - \bar{d}^p(x)\right)\right] dx \quad , \\
0 &= \int_0^1 \left[\frac{2}{3}\left(d^p(x) - \bar{d}^p(x)\right) - \frac{1}{3}\left(u^p(x) - \bar{u}^p(x)\right)\right] dx \quad .
\end{aligned} \tag{4.21}$$

We have to use the fact that both the proton and neutron have zero strangeness, in order to get the third of eq. (4.20). It should be emphasized here that the actual determination of $u^p(x)$, $d^p(x)$ and $K(x)$ has to be done by combining data on electromagnetic structure functions as well as the weak structure functions for different targets; p, n and isoscalar nuclei and then fitting a form to the densities $u_V^p(x)$, $d_V^p(x)$ and $K(x)$, using the data, subject to the above sum rules. Some of these details will be discussed below.

4.3 Properties of partons as determined from DIS

4.3.1 Spin of partons

Let us start from the Callan-Gross relation of eq. (4.5). Note here that if the charged partons were scalars then, according to eq. (3.37), $F_1^{\rm parton}(x) = 0$. Hence the ratio $F_2^{ep}(x)/(2xF_1^{ep})(x) \to \infty$ whereas eq. (4.5) predicts the ratio to be 1 for spin 1/2 partons. It is customary to define a longitudinal structure function $F_L^{ep}(x)$ by

$$F_L^{ep}(x) = F_2^{ep}(x) - 2xF_1^{ep}(x) \quad . \tag{4.22}$$

Since $F_2^{ep}(x)$ and $F_1^{ep}(x)$ can be extracted from the measured DIS cross-section, an experimental verification of the above relation will imply that the pointlike constituents inside the proton revealed in DIS have spin 1/2. $F_L^{ep}(x)$ is termed the longitudinal structure function as it is proportional (in the large q^2, ν limit) to the virtual photoabsorption cross-section for longitudinal photons (*i.e.*, photons with helicity $\lambda = 0$). In fig. 17, we see the experimental data as the ratio

$$R = \frac{F_L^{ep}(x)}{2xF_1^{ep}(x)} \quad . \tag{4.23}$$

In the large q^2, ν limit this is the ratio of virtual photoabsorption cross-section for longitudinal and transverse photons ($\lambda = \pm 1$). As noticed before, the denominator is zero for scalar partons. This can be physically understood as the impossibility of absorption of transverse photons by a scalar target. The data of fig. 17 taken from [21] show clear evidence that the charged partons are spin 1/2 objects and not spin 0 objects. Deviation of this quantity from zero is yet another 'check' for QCD but again will not be discussed here further.

4.3.2 Momentum carried by charged partons

As can be seen from eqs. (4.16) and (4.17) the area under the $F_2^{ep}(F_2^{en})$ vs. x curve will measure the weighted sum of the momentum fractions carried by the charged partons in the proton (neutron). Hence

$$\begin{aligned} \textstyle\int_0^1 F_2^{ep}(x)dx &= \tfrac{4}{9}\epsilon_u + \tfrac{1}{9}\epsilon_d + \tfrac{1}{9}\epsilon_s \quad , \\ \textstyle\int_0^1 F_2^{en}(x)dx &= \tfrac{1}{9}\epsilon_u + \tfrac{4}{9}\epsilon_d + \tfrac{1}{9}\epsilon_s \quad , \end{aligned} \tag{4.24}$$

where ϵ_u, ϵ_d and ϵ_s are the fractions of the proton momentum carried by $(u+\bar{u})$, $(d+\bar{d})$ and $(s+\bar{s})$. If we define $\delta = \epsilon_s/(\epsilon_u+\epsilon_d)$, we can write

$$\int_0^1 \left[F_2^{ep}(x) + F_2^{en}(x)\right] dx = \frac{5}{9}(\epsilon_u+\epsilon_d) + \frac{2}{9}\delta(\epsilon_u+\epsilon_d) \quad ,$$

$$\frac{9(\delta+1)}{5+2\delta}\int_0^1 \left[F_2^{ep}(x) + F_2^{en}(x)\right] dx = \epsilon_u + \epsilon_s + \epsilon_d \quad . \tag{4.25}$$

It is clear from eq. (4.25) that a knowledge of δ is necessary to determine $(\epsilon_u+\epsilon_s+\epsilon_d)$. An extraction of quark densities from the DIS data using $ep, en, \nu p, \nu n$ experiments and eq. (4.28) shows that $\delta \leq 0.06$. Using the data on $(F_2^{ep}(x)+F_2^{en}(x))$, this gives,

$$\epsilon_u + \epsilon_d + \epsilon_s \simeq (0.54 - 0.56) \pm 0.04 \quad . \tag{4.26}$$

Eq. (4.26) above implies that some momentum of the proton ($\sim 50\%$) is carried by partons which are neutral to the probe, *i.e.*, partons which do not have electromagnetic or weak interactions. These are called gluons which hold the quark-partons together in a proton.

4.3.3 Charge assignment of different partons

To confirm the fractional charge assignment used in arriving at, e.g., eqs. (4.13), (4.14) one has to combine data on F_2^{ep}, F_2^{en} with the data on $F_2^{\nu p}, F_2^{\nu n}$. For this we should obtain expressions for $F_2^{\nu p}, F_2^{\nu n}$ analogous to eqs. (4.13), (4.14). To do this one has to note the following :

(i) A neutrino can scatter off only the charge $-\frac{1}{3}e_p$ quarks and charge $-\frac{2}{3}e_p$ anti-quarks, as it has to have a ℓ^- in the final state. Hence only $\nu_\ell\ d$, $\nu_\ell\ s$ and $\nu_\ell\ \bar{u}$ processes take place. (Recall, we have at present neglected the heavier, charm and bottom, quark content of the proton/neutron).

(ii) At low energies, $\nu_\ell\ s \to \ell^- c$ transition can be neglected and $\nu_\ell\ s \to \ell^- u$ transition is suppressed in the limit of the mixing angle in the s–d sector (Cabibbo angle) $\theta_c \simeq 0$. This is a good approximation as $\cos\theta_c = 0.98$. Under these approximations, the only transitions that contribute to the DIS cross-section for incident ν_ℓ are $\nu_\ell\ d\ (\bar{u}) \to \ell^- u\ (\bar{d})$ and $\bar{\nu}_\ell\ u\ (\bar{d}) \to \ell^+ d\ (\bar{u})$ for incident $\bar{\nu}_\ell$.

Using eq. (4.12) then we have, at low energies and with the above approximations,

$$\begin{aligned} F_2^{\nu p}(x) &= 2x\left[d^{\,p}(x) + \bar{u}^p(x)\right] \quad , \\ F_2^{\nu n}(x) &= 2x\left[u^p(x) + \bar{d}^p(x)\right] \quad . \end{aligned} \tag{4.27}$$

Using eqs. (4.16), (4.17) and (4.27) we get,

$$\frac{F_2^{ep}(x) + F_2^{en}(x)}{F_2^{\nu p}(x) + F_2^{\nu n}(x)} = \frac{5}{18}\frac{\left[u^p(x) + d^{\,p}(x) + \bar{u}^p(x) + \bar{d}^p(x) + S^p(x) + \bar{S}^p(x)\right]}{\left[u^p(x) + d^p(x) + \bar{u}^p(x) + \bar{d}^p(x)\right]} \quad . \tag{4.28}$$

Excess of this ratio above 5/18 is a measure of the momentum carried by the $s(\bar{s})$ quarks. Production of charmed particles in DIS in νp reactions provides a direct measurement of this strange sea content of the proton. The factor of 5/18 in the eq. (4.28) above is the average squared charge of the u, d quarks.

The electromagnetic and weak structure functions are usually measured not only for light targets such as proton/deuterium, but more often experiments are performed with heavier, nuclear targets so as to get large cross-sections. Normally one expects that for nuclear targets, the cross-section from different nuclei will add incoherently. For an isoscalar target therefore, the structure function per nucleus becomes,

$$\frac{1}{A}F_2^{eA} = \frac{1}{2}(F_2^{en} + F_2^{ep}) \quad .$$

With eqs. (4.16) and (4.17), we get

$$\frac{1}{A} F_2^{eA} = x \Bigg\{ \frac{5}{18} \left[u^p(x) + d^p(x) + \bar{u}^p(x) + \bar{d}^p(x) + S^p(x) + \bar{S}^p(x) \right]$$

$$- \frac{1}{3} \left[S^p(x) + \bar{S}^p(x) \right] \Bigg\} \ .$$

If we include also the charm sea contribution then we will have

$$\frac{1}{A} F_2^{eA} \simeq \frac{5}{18} \sum_q x q^p(x) \ . \tag{4.29}$$

Note again the factor of 5/18. It is clear from this discussion that the ratio $\frac{F_2^{eA}}{F_2^{\nu A}}$ is the same as the r.h.s. of eq. (4.28). A collection of some of the data taken from the third of Ref. [20] presented in fig. 18 shows experimental proof of this relation. This confirms the identification of the charged partons with the Gell-Mann-Zweig[1] quarks.

Note also that a measurement of $\frac{1}{A} \int_0^1 F_2^{eA} dx$ for an isoscalar target can directly give information about the total momentum carried by the quark-partons. The recent high Q^2 data on $\mu p/\mu d$ DIS scattering by the different collaborations like EMC, BCDMS, NMC and $\nu p/\nu Fe$ DIS data by the CCFR collaboration [19] confirm all the parton model features quite beautifully.

4.4 Interpretation of sea densities

The existence of gluons gives a very simple understanding of the sea-quark densities. At some very low q^2-scale, the proton can be looked upon as made up of only three valence quarks u, u and d, all carrying $(\frac{1}{3})^{\rm rd}$ of the proton momentum. The probability distributions $u_V^p(x)$ and $d_V^p(x)$ are just $\delta(1 - x/3)$ with appropriate normalisation. Emission of gluons, by bremsstrahlung, by these quarks causes the probability distributions to shift to lower x values. The emitted gluons give rise to $q\bar{q}$ pairs thus giving rise to sea quarks. Since the process of bremsstrahlung is naturally peaked at small values of momentum fractions, it generates $q\bar{q}$ sea. At higher and higher energies the number of $q\bar{q}$ pairs produced goes on increasing. Actually this process will also give rise to scaling violations as it causes $u^p(x)$, $d^p(x)$ to shift to lower x values with increasing q^2 and makes $F_2^{ep}(x)$ q^2 dependent. However, this clearly takes us out of the realm of the Quark-Parton-Model (QPM) and causes corrections to the simple QPM picture. These corrections (scaling violations) actually played an important role in establishing the nature of interactions among quarks and gluons. But this will not be discussed further here.

Another way of understanding more about the sea-quark densities is to try to construct a quantity which is independent of sea quark densities. Consider the following combination:

$$F_2^{ep}(x) - F_2^{en}(x) = \frac{x}{3} \left[u_V^p(x) - d_V^p(x) \right] . \tag{4.30}$$

This difference does not involve the sea quark densities at all *if one assumes isospin–symmetric sea densities.* The experimental data on $(F_2^{ep}(x) - F_2^{en}(x))$ [21] show a peak at $x = 1/3$. Fig. 19 shows (just to show the increased accuracy of the newer data) the much more recent and high statistics data taken from [22]. This clearly supports the picture of three constituents of mass $\frac{1}{3}M_p$ and also the interpretation of sea quark pairs as arising due to bremsstrahlung from valence quarks.

4.5 Sum rules on parton densities

Since there is, as yet, no theory which can compute parton densities in a proton, all the knowledge about parton densities is to be obtained from experiments, which can then be used for testing models of the proton structure functions. Within the framework of the quark–parton model, various relations, sum rules have been derived for F_2^{ep} and F_2^{en} or combinations thereof. Some of these sum rules have already been written down in eqs. (4.20) and (4.21). From eq. (4.30) we get

$$\int_0^1 \frac{(F_2^{ep}(x) - F_2^{en}(x))}{x} dx = \frac{1}{3} \int_0^1 (u_V^p(x) - d_V^p(x))\, dx = \frac{1}{3} \quad . \tag{4.31}$$

This is called the Gottfried sum rule. This sum rule is arrived at by assuming $\bar{u}_s^p(x) = \bar{d}_s^p(x)$. If that is not the case, this will be violated. Current data from μ DIS experiments [22] show that the sum rule is violated, and the above integral (obtained from extrapolation of the data for the region $x > 0.8$ and $x < 0.004$) is,

$$\int_0^1 \frac{F_2^{ep}(x) - F_2^{en}(x)}{x} dx = 0.258 \pm 0.017 \quad .$$

These data use measurements of the structure function at $q^2 = 4$ GeV^2. These indicate a departure from the expected value of 1/3 and a breaking of the isospin symmetry of the sea densities which has been assumed in arriving at eq. (4.31).

One can derive yet another sum rule using $F_2^{\nu p}$ and $F_2^{\nu n}$. We see from eq. (4.27) that ,

$$\int_0^1 \frac{(F_2^{\nu n} - F_2^{\nu p})dx}{x} = 2 \int_0^1 (u^p(x) - d^{\,p}(x))\, dx = 2 \quad . \tag{4.32}$$

This sum rule is called the Adler sum rule. Again it assumes isospin symmetry for the sea-quark densities. The sum rule is known to be satisfied. But the ν scattering data have intrinsically much larger errors compared to the $\mu p(\mu D)$ scattering data. As a result the neutrino data are not in a position to test the violation of isospin symmetry of sea densities implied by the μ DIS data from EMC.

Yet another sum rule that has been written down is for the parity violating structure function $F_3^{\nu p}$. One can write down expressions for $F_3^{\nu p}$ and $F_3^{\nu n}$ using eq. (4.12) and using the isospin invariance of the neutron/proton parton densities. Using eq. (4.12) we get,

$$\int_0^1 (F_3^{\nu p}(x) + F_3^{\nu n}(x)) dx = 3 \quad .$$

This is called the Gross–Llewellyn Smith sum rule. The earliest ν experiments [20] verified this sum rule. Currently it has been a focus of lot of discussions as the deviation of the r.h.s from three can provide important information about and test of pQCD. The currently measured value is $2.50 \pm 0.018(stat.) \pm 0.078(syst.)$.

4.6 Parton model in processes other than DIS

DIS processes provided the first measurement of parton densities. We saw in the previous sections that a combination of the different (weak and electromagnetic) structure functions with different targets (proton, neutron, isoscalar nuclei) can be used to establish different properties of the charged partons such as their spin, charge assignment etc. The individual parton densities can be extracted from the data on F_2 only under certain assumptions and one usually fits a form to these. The fits are constrained by various sum rules which are derived on general principles. However, none of the DIS processes can give information about the gluon densities. In the framework of perturbative QCD, outside the realm of quark-parton model, some information about gluon densities is obtained by studying the scaling violation (q^2 dependence) of the structure functions. But no direct information on the gluons is available from these processes. It is also important to note that since there are, as yet, no theoretical predictions for either quark/gluon densities, it is imperative to find processes other than DIS to extract combination of parton densities different from those measured in DIS, to supplement our knowledge. This, in turn, can help to get a more complete picture of parton densities.

A better knowledge of parton densities is not only essential to check our ideas about perturbative QCD but it is also necessary to be able to make correct predictions for the cross-sections of different physical processes expected at the high energy e–p or hadron-hadron colliders. High energy collisions of hadrons (say p–p, π–p or π–A) with each other can be described in parton model as an incoherent sum of interactions among partons. This means that our predictions as to what is likely to happen at high energies in collisions of these hadrons will depend on our knowledge of these parton densities.

As an illustration of the above, consider the production of a vector boson W^+ in $p\bar{p}$ collisions as shown in fig. 20. The production cross-section is given by

$$d\sigma(p\bar{p} \to W^+ x) = \int_{M_W^2/S}^{1} dx_1 \int_{M_W^2/Sx_1}^{1} dx_2 u^p(x_1) \bar{d}^{\bar{p}}(x_2)\, d\hat{\sigma}(u\bar{d} \to W^+)\Big|_{\hat{s}=Sx_1x_2} . \tag{4.33}$$

The important point here is that the densities u^p or $\bar{d}^{\bar{p}}$ in eq. (4.33) are exactly the same as those extracted from DIS. That the same probability functions are applicable in all 'hard' processes be it DIS or be it (say) W^+ production, is an assumption in the QPM. In the framework of perturbative QCD this has been proved quite rigorously for a large number of hard processes[7]. So any hard processes in hadron-hadron collision is computed as an incoherent sum over all the partons where the individual $2 \to n$ parton subprocess is convoluted with the parton distribution functions. An agreement of measured cross–section with the predictions made in the QPM provides consistency checks on our knowledge of these parton densities. Further, by singling out final states which are sensitive to a specific parton in the initial state, we can better our knowledge of parton densities. For example, a comparison of W^+ production discussed above with W^-, can be a good test of the isospin symmetry of the sea.

This simple QPM picture has been justified theoretically in perturbative QCD, has been used extensively and has strong experimental support. The high energy processes of interest to the Nuclear physics community are the heavy ion collisions. The plain QCD contributions to different final states in these collisions, arising from the partons in the nuclei, need to be computed correctly before one can assess the observability of different signals of Quark-Gluon-Plasma (QGP) formation such as J/ψ suppression, dilepton production or direct photon production [23]. For this one needs a good knowledge of parton densities inside the nuclei. This is the topic of the next lecture.

4.6.1 Jet Production

The most common hard process in hadronic collisions is called jet production. This arises basically from the scattering of two partons in the hadrons which produce light partons in the final state. As already said in eq. (4.33), in QPM the total cross-section at the hadronic level is obtained by convoluting the parton densities with the cross–section of this basic $2 \to 2$ subprocess. Since this scattering is between pointlike particles, it involves large momentum transfers and the final state partons are thus produced at large angles (*i.e.*, with large momenta transverse to the beam direction, (p_T)) as opposed to the remaining partons in the two hadrons which go along the beam direction. There are in all 8 hard scattering $2 \to 2$ subprocesses which will produce light partons in the final state. The q or g in the final state does not appear in the detector as a free gluon or quark due to the phenomenon of 'colour confinement'. They appear in the detector as a 'stream' or 'jet' of particles clustered around the original direction of the parton. As said earlier, the major feature of QPM is that these jets are predicted to have large p_T due to the hard scattering of the two partons. Observation of this large p_T jet production was one of early confirmation of QPM in processes other than the DIS.

Quantitative information about the parton densities can be obtained using jet production, however, only in the framework of perturbative QCD. The qualitative success of the QPM is formally justified only in the context of perturbative QCD (pQCD) any way. Use of jet-production as a probe of parton densities is a little complicated as both quarks and gluons in the initial state contribute and over most of the range of p_T neither quarks or gluons dominate. In the framework of pQCD one can show that [24] the large p_T jet production in hadronic collisions is given by,

$$\begin{aligned}\frac{d\sigma(H_1H_2 \to j_1j_2)}{dp_T dy_1 dy_2} &= F^{H_1}(x_1)F^{H_2}(x_2)f(p_T, y_1, y_2)\\ &= x_1x_2\left[\frac{4}{9}\sum_q \left(q^{H_1}(x_1) + \bar{q}^{H_1}(x_2)\right) + g^{H_1}(x_1)\right] \times\end{aligned}$$

$$\left[\frac{4}{9}\sum\left(q^{H_2}(x_2)+\bar{q}^{H_2}(x_2)\right)+g^{H_2}(x_2)\right]\times$$
$$f(p_T,y_1,y_2)\quad , \tag{4.34}$$

where $y_i = \frac{1}{2}\ell n\left(\frac{E_i+P_L^i}{E_i-P_L^i}\right)$ is the rapidity of the ith jet in terms of its energy E_i and longitudinal momentum P_L^i. The momentum fractions $x_1(x_2)$ of the hadrons $H_1(H_2)$ carried by the partons are related to rapidities y_1, y_2 and p_T of the jets and $f(p_T, y_1, y_2)$ is a function approximating the dominant part of all the subprocess cross-sections $d\hat{\sigma}/dp_T^2(p_1p_2 \to p_3p_4)$ where p_i stand for partons. Measurement of large p_T-jet, triple differential cross-section of eq. (4.34) by UA-1 collaboration in $p\bar{p}$ collisions [25] allowed extraction of $F^p(x)$ and its comparison with the parton densities extracted from DIS, has tested the above picture quite well.

4.6.2 Drell-Yan process

Another hard process which was studied in the early days of QPM and does not involve pQCD in the subprocess (apart from its role as a justification of QPM) is the production of a massive $\mu^+\mu^-$ pair with zero p_T, via $q\bar{q}$ fusion [24]. The hard subprocess, shown in fig. 21, is

$$q\bar{q} \to \gamma^\star \to \mu^+\mu^- \quad . \tag{4.35}$$

The process at the level of hadrons is

$$H_1H_2 \to \mu^+\mu^-X \quad . \tag{4.36}$$

Let us take the special case when $H_1 = p$, $H_2 = \bar{p}$. Then the total cm energy available for the subprocess (say) $q^p + \bar{q}^{\bar{p}} \to \mu^+\mu^-$ is given by,

$$\begin{aligned} m^2_{\mu^+\mu^-} &= \hat{s} = (P_qp + P_{\bar{q}}\bar{p})^2 = (x_1P^p + x_2P^{\bar{p}})^2 \\ &\simeq 2x_1x_2P^p\cdot P^{\bar{p}} \simeq x_1x_2(P^p+P^{\bar{p}})^2 \simeq x_1x_2S \quad , \end{aligned} \tag{4.37}$$

where S is the centre of mass energy of the $p\bar{p}$ system. The process shown in fig. 20 is an electroweak variant of the same. The $\mu^+\mu^-$ pair so produced has no net momentum transverse to the relative direction of motion of p and $\bar{p}$, as the transverse momenta of the initial state partons is negligible in the QPM.

The cross-section for the Drell-Yan (DY) process obviously reflects the quark and anti-quark content of both the beam and target. Since the cross-section for $\mu^+\mu^-$ pair production is proportional to e_q^2, the information available from its study is essentially the same as that obtained from a study of DIS. In the QPM some simple relations exist between DY cross-sections for a fixed target and different beams, e.g., the valence quarks in π^+ are u and $\bar{d}$, while those in π^- are $\bar{u}$ and d. Since the valence quarks in proton are u and d, the dominant contribution to DY pair production with $\pi^+(\pi^-)$ beam and proton target comes from $d^p\bar{d}^{\pi^+}(u^p\bar{u}^{\pi^-})$ annihilation. For an isoscalar target like Carbon nucleus, we know from our discussion in the earlier section, $d^C = u^C$. Hence we expect

$$\frac{\sigma(\pi^+ C)}{\sigma(\pi^- C)} \simeq \frac{1}{4}. \tag{4.38}$$

This very basic prediction of QPM, which tests the ideas of valence quarks, sea-quarks and the quark-content of $\pi^+/\pi^-/p$ etc. as given by the static quark-model, was tested quite adequately in the early experiments and thus played an important role in establishing the QPM.

Going outside the framework of plain QPM, in the context of perturbative QCD, one can also compute production of massive $\mu^+\mu^-$ pair, at large p_T, in hadron-hadron collisions. It takes place via the hard scattering process,

$$q\bar{q} \to \gamma^\star g \to g\mu^+\mu^-; \quad qg \to q\gamma^\star \to q\mu^+\mu^-. \tag{4.39}$$

As discussed for jet production, the final state light parton appears as a jet in each case. The $\gamma^\star$ (equivalently the $\mu^+\mu^-$ pair) produced in the hard $2 \to 2$ scattering subprocess carries a large p_T, which is balanced by the jet, even though the initial state partons have zero p_T. The process at the hadron level is

$$H_1 + H_2 \to \mu^+ + \mu^- + jet + X, \tag{4.40}$$

where the $\mu^+\mu^-$ pair is produced with non-zero p_T. It is clear from eq. (4.39) that the large p_T DY $\mu^+\mu^-$ pair can yield 'direct' information about the gluon content of the hadron.

4.6.3 Direct photon-production

A related process which can also probe the gluon content of a hadron is the 'direct' photon production [24] via the subprocess

$$q\bar{q} \to g\gamma, \quad qg \to q\gamma. \tag{4.41}$$

The only difference from eq. (4.39) is that the photon in the final state is a 'real' photon; *i.e.*, the invariant mass of the photon here is zero. The nomenclature 'direct' (or prompt) has to do with the fact that this photon is produced on the short time scale of the hard scattering process as opposed to the photons which are produced in the decay of the final state hadrons like π^0 and hence appear on a longer (hadronisation) time scale. Of course the latter are produced in much more plentiful numbers as the corresponding cross-sections are much higher. Therefore, a study of 'direct' or 'prompt' photons is experimentally quite challenging. But the effort is well worth it, as one can isolate certain kinematic regions of the final state where the cross-section is dominated by the gluon content of the colliding particles and hence the measurement offers an almost 'direct' probe of the gluon densities.

4.6.4 Heavy quark production

An even better probe of the gluon densities is provided by production of heavy quark [24] (charm, bottom etc.) in hadronic collisions via the subprocesses,

$$gg \to Q\bar{Q}; \quad q\bar{q} \to Q\bar{Q} \ , \tag{4.42}$$

where Q stands for c/b quark. This final state has a very distinctive signature and can be separated from the very big background of jet production (from the same initial states) with comparative ease. The corresponding process at the hadronic level is

$$H_1 H_2 \to Q\bar{Q}X \ . \tag{4.43}$$

This heavy quark production is dominated by gluon densities due to dynamical reasons that make the corresponding cross-sections larger. The situation can be further improved (from the point of view the determination of the gluon densities) by considering photoproduction of heavy quarks [24] in the process

$$\gamma h \to Q\bar{Q}X \ . \tag{4.44}$$

In this case the basic subprocess is

$$\gamma g \to Q\bar{Q}X \ . \tag{4.45}$$

This is just an analogue of the corresponding process for the hadronic production of heavy flavour, where a gluon is replaced by γ. Thus a study of photoproduction (production with incident photons) of heavy quarks can give pretty good information about the gluon content of the target.

A process with smaller cross-section but a more distinctive signature is the production of a bound $Q\bar{Q}$ pair (quarkonium) instead of a free $Q\bar{Q}$ pair. The processes which contribute to the hadroproduction (production in hadronic collisions) and photoproduction of a quarkonium[26] are the same as those in eqs. (4.42) and (4.45) respectively.

If we relax the restriction to the $2 \to 2$ subprocess which is inherent to the the QPM and allow $2 \to 3$ subprocesses (which is justified by pQCD) then we can describe large p_T, photo- and hadro-production of quarkonium in the above picture (*i.e.*, convolution of parton densities with a subprocess). Some of the subprocesses are,

$$q\bar{q} \to Q\bar{Q}g, \quad gg \to Q\bar{Q}g, \quad \gamma g \to Q\bar{Q}g. \tag{4.46}$$

A process which is closely related to the above is the associated production of a quarkonium with a photon in hadronic collisions, via the subprocess, e.g.,

$$gg \to \bar{Q}Q\gamma. \tag{4.47}$$

This was recently suggested as a probe of gluon densities. The cross–sections are quite a bit smaller than hadroproduction of a quarkonium but, due to the associated photon, is much cleaner for detection. It is better than the 'direct' or 'prompt' photon in association with jet (eq. (4.41)), again due to the ease of discrimination against background. It also has the advantage of being 'directly' proportional to gluon densities for certain spin-parities of quarkonia. In the next section we will discuss how these processes can be used to glean information about the gluon densities in the nucleus.

5 EMC effect

5.1 EMC effect : Experimental situation

As alluded to in the earlier discussion, because of the high energies involved in DIS, it was expected that the nuclear structure function F_2^{eA} should simply be an incoherent sum of the structure functions of the individual nucleons. This picture, based on the impulse approximation, depicted in fig. 22 was implicitly assumed in all our discussions of the parton model and various tests of the parton model using isoscalar nuclei involved using this approximation. The success of these tests, albeit qualitative, supports the assumption. However, experiments which set out to test this quantitatively met with a surprise [8]. This experiment compared $\frac{1}{A}F_2^{\mu A}$ with $F_2^{\mu p}$. The original experiment studied the EMC ratio ($\rho^{EMC}(x)$) defined by

$$\rho^{EMC}(x) = \frac{1}{A}\frac{F_2^{\mu A}(x)}{F_2^{\mu p}(x)} \quad , \tag{5.1}$$

as a function of the scaling variable x for A = Fe. The deviation of this ratio from unity is a measure of the failure of the impulse picture. The initial data gave $\rho^{EMC} > 1$ for $x < 0.3$ and a suppression for $0.3 < x < 0.8$. Since then a lot of DIS experiments verified this non-trivial nuclear dependence of F_2^{lA} ($\rho^{EMC}(x) \neq 1$) for a wide range of nuclear targets, with a variety of lepton types (e^- beams, μ beams, ν beams) and over a wide range of q^2 ($4 < q^2 < 200$ GeV^2)[27]. The newer data by NMC[9] probed the effect to very *low values of* x upto $x = 0.0035$. Fig. 23 shows a collection of some of these data [12, 9, 19]. The data show the following features :

1. There is no low $x (x < 0.2)$ enhancement of the nuclear structure functions which was present in the original EMC data.

2. There is a definite 'shadowing' effect, *i.e.*, $\rho^{EMC} < 1$ for $x < 0.05$ even at large values of q^2. The suppression of the nuclear structure function F_2^{lA} w.r.t. the nucleon structure function F_2^{lp} rises with x for $x < 0.2$. The 'shadowing' means that the DIS cross–section per nucleon is reduced as compared to that for a free nucleon, due to the presence of the other nucleons in the nucleus. This shadowing effect depends only weakly on q^2.

3. For $0.05 - 0.1 < x < 0.2$ $\rho^{EMC}(x)$ goes slightly above unity and then falls below 1 for $0.3 < x < (0.8 - 0.9)$. This depletion of $\frac{1}{A}F_2^{lA}$ w.r.t F_2^{lp} is $\sim 10 - 15\%$.

4. For values of $x > 0.8$ the EMC ratio goes above unity.

5. The effect does not show any q^2 dependence.

6. Nor does the effect show strong dependence on the mass number A of the target.

7. An experimental measurement of the ratio $R = F_L(x)/(2xF_1(x))$ does not show any appreciable nuclear dependence.

Due to the intuitive appeal of the impulse approximation, observation of this nontrivial nuclear dependence of F_2^{lA} was almost unexpected by theorists except for a suggestion[28] of a possible enhancement of the nuclear sea–densities. For the same reason, the observation of the EMC effect also gave rise to a large amount of theoretical activity and a large number of models for nuclear structure functions. Considering that we do not as yet have a credible model (let alone a theory) even of the nucleon structure function, it is clear that all the models proposed to explain the nuclear structure functions do so only by giving a recipe to calculate the nuclear structure function in terms of that of the nucleon. Different models differ in the theoretical ideas about the effect of the nuclear environment on the parton densities. All these models of course involve parameters some of which are estimated and some are usually fitted to reproduce the observed EMC effect. Since the DIS experiments probe only the quark–parton densities directly, it is not surprising that all the models agree on the form of the nuclear quark–parton densities. However, the nuclear gluon densities which are unconstrained by the DIS data are *predictions* of these different models and generally differ greatly from model to model. The different models differ radically in the physics phenomenon they invoke to explain the EMC effect. Hence, to arrive at the correct theoretical understanding of the EMC effect, it is essential to be able to distinguish between the various models. This can be done effectively if one can extract nuclear gluon densities and hence measure the gluonic EMC ratio

$$\rho_g = \frac{1}{A}\frac{g^A(x)}{g^p(x)} \quad . \tag{5.2}$$

There is yet another reason which makes such a determination imperative. This has to do with the signals of the Quark–Gluon–Plasma (QGP) mentioned in the earlier sections. To assess the observability of any 'hard' signal of QGP formation in Heavy ion collisions it is absolutely essential to understand the contributions to the 'hard' final state under question, coming from a combination of the nuclear dependence of the parton densities and perturabtive QCD. This of course requires a good knowledge of ρ_g defined above.

The discussions of the last section outlined various hard processes other than the DIS which can be used to glean information about the gluonic EMC ratio. Hence, a study of correlation between the nontrivial nuclear dependence of the structure functions (the EMC effect) and the A–dependence of the different hard processes such as large p_T jet–production, DY $\mu^+\mu^-$ pair production(including large p_T DY), electro- and photo-production of quarkonia as well as their production in hadronic collisions etc. can help shed light on the EMC effect. It is worthwhile to ask at this point whether there exists any evidence of a nontrivial nuclear dependence for the abovementioned hard processes, before we turn to a discussion of such correlations. Below we first discuss some of the models that have been suggested to explain the EMC effect, summarize the experimental evidence of nontrivial nuclear effects in hard processes other than the DIS and then examine the implications of these data for various models of the EMC effect.

5.2 EMC effect: Theoretical Models

As mentioned in the earlier sub–section, observation of the till then unexpected nuclear dependence of the structure functions gave rise to a large variety of models[9] for the EMC effect. Almost all the models address primarily the region $0.2 < x < 0.8$. The small x 'shadowing' region is interesting and has been recently the focus of theoretical discussions[29] but will not be discussed here. Broadly speaking the models can be divided into different classes:

5.2.1 Nuclear Physics based models

Models based on 'conventional' nuclear physics try to explain the depletion of F_2^A in the valence region as being due to the virtual pions present in the nucleus (as a result of the nuclear force). The pions can carry a momentum fraction up to M_π/M_p and hence will cause a depletion of quarks in the valence region and also a low x enhancement. This idea [30] almost always will give rise to a enhancement of the anti–quark content of the nuclear structure function as the pions contain more valence anti–quarks. The nuclear structure function is given by

$$F_2^A(x,q^2) = \int_x^1 dy f_N(y) F_2^N(x/y,q^2) + \int_x^1 dy f_\pi(y) F_2^\pi(x/y,q^2) \ , \qquad (5.3)$$

where f_N and f_π denote the nucleon and the pion distribution functions in the nucleus. Free parameters of this model are the average number of pions in the nucleus and the momentum fraction η carried by the pions. Of course the form of the quark–distributions in the free proton and pion, *i.e.*, $F_2^A \text{and} F_2^\pi$, are also an input to the model. The original idea was extended further by a large number of authors [9] including also the effect of $\Delta's$ in the nucleus.

Attempts[31] were made to calculate the two abovementioned parameters in nuclear physics framework by using the measured values of the nuclear separation energies. There was much excitement initially as the original calculations seemed to yield values of η and the average number of pions required by the best fit to the data on EMC effect. However, since then the issue has been revisited by a lot of nuclear physicists[9] and there is no clear consensus about the exact size of the nuclear binding contribution to the EMC effect; but it is fair to say that it can, at the most, account for only 10-15% of the EMC effect[32]. Since the model always predicts an enhancement of the anti–quark content and also in the low x region, the experimental information on the nuclear dependence of hard processes other than the DIS constrain these types of model rather strongly, as we will see later.

5.2.2 Cluster models

Several authors conjectured that the nontrivial nuclear dependence of the structure function can be understood in terms of nucleon clusters inside the nucleus. The idea is that clusters of nucleons deconfine in the nucleus and share quarks with each other. So the quarks belonging to these clusters now occupy the volume of the cluster rather than that of a single nucleon. Idea of such clusters was initially suggested to explain the deep inelastic data from ^3He in the q^2 range $1 < q^2 < 4\text{GeV}^2$. The probabilities of N–quark cluster formation can be computed theoretically. A feature of the cluster models is the existence of large momentum partons in the nucleus. For an N–quark cluster, the variable $x = q^2/2M_p\nu$ can take values up to $N/3$. Thus the fractional momentum x carried by the quark can be greater than unity in this case. The cluster models(which involve 6 or 9–quark clusters) invoke only a partial deconfinement of the quarks in the nucleus. Furthermore the quark distribution functions in the free–nucleon and in the N–quark clusters are both input functions. There exist also another class of models where the quarks from the nucleons are postulated to deconfine to the whole nucleus and these deconfined partons are assumed to form a gas. These 'gas' models then compute the parton distributions of the 'deconfined' quarks in terms of the parameters of the model. Below two representative models of this kind are discussed.

1) The Gas Model

The parton densities of a nucleus with atomic number A are defined in the Gas model[33] as a sum of two components.

$$f_{i/A}(x) = (1-\omega)\tilde{f}_{i/N}(x) + \frac{1}{A}\sum_{r=1}^{A}\omega^r(1-\omega)^{A-r} f_{i,r}^{\text{gas}}(x;\mu,T) \quad . \tag{5.4}$$

The first term, occurring with a weight $(1-\omega)$, is that for a free nucleon parton density after corrections for its Fermi motion inside the nucleus. The second component is written in terms of thermal distributions of momenta at a temperature T, leading to the following functions $f_{i,r}^{\text{gas}}(x;\mu,T)$:

$$f_{q,r}^{\text{gas}}(x;\mu,T) = \frac{2r_0^3T^3}{\pi}\left[\phi\ \ell n(1+ze^{-\phi}) + \frac{1}{2}\ell n^2(1+ze^{-\phi}) + Li_2\left(\frac{z}{z+e^{\phi}}\right)\right] \tag{5.5}$$

and

$$f_{g,r}^{\text{gas}}(x;\mu,T) = \frac{2r_0^3T^3}{\pi}\left[Li_2(e^{-\phi}) - \phi\ \ell n(1-e^{-\phi})\right] \quad . \tag{5.6}$$

Here $r_0 = 1.2$ fm, $\phi = Mx/2T$, $z = \exp(-\mu/T)$, where M is the nucleon mass and $Li_2(x)$ is the Euler dilogarithm function. Using the constraint on the total baryon number of the nucleus to eliminate the chemical potential μ, one has two model parameters, T and ω, for each nucleus.

Using the CDHS parametrisations[34],

$$F_2^{\nu p}(x) \ = \ 1.1(1+3.7x)(1-x)^{3.9},$$

$$
\begin{aligned}
x\sigma_p(x) &= 0.17(1-x)^{8.54} \quad , \\
xf_{g/p}(x) &= 2.62(1+3.5x)(1-x)^{5.9} \quad ,
\end{aligned}
\tag{5.7}
$$

for $F_2^p(x)$ (here $\sigma_p(x)$ is the total sea density) and the data on $\rho(x) = F_2^A(x)/AF_2^p(x)$ these parameters have been fixed[33] for many nuclei, including the ones used for the E772 experiment. The corresponding $\rho_g(x)$ is then predicted uniquely. They are summarized in Table 4.

It may be mentioned here that, using a different set of structure functions for proton instead of eq. (5.7) necessitates a time-consuming re-analysis of the EMC-data to obtain T and ω. For this reason we have not used more recent parametrisations for the proton structure function and we are also constrained to use *different* proton structure functions in different models.

2) The six-quark cluster model

The six-quark cluster model[35] is also a representative of the two-component models for EMC effect. In this model, it is assumed that when two nucleons get closer to each other than a certain critical radius they merge together to form a six-quark cluster. By assuming the probability to form higher clusters to be negligible, the remaining model inputs are the probability of forming such a cluster and the form of the parton distributions in the 3-quark and 6-quark clusters. The latter are chosen using the quark-counting rules and the constraints of i) normalisation of valence densities (an N-quark cluster has N valence quarks), and ii) conservation of momentum. It is further assumed that the average momentum carried by the sea partons is the same for the three- and six-quark clusters and that it is ~ 0.2 of that of the gluons. The forms of nuclear densities per nucleon in this model are,

$$
f_{i/A}(x) = (1-\epsilon) f_{i,3}(x) + \frac{\epsilon}{2}\, f_{i,6}\left(\frac{x}{2}\right) \quad , \tag{5.8}
$$

where ϵ is the probability to find a six quark cluster which increases with A [36] and the subscripts denote the cluster size. The values of ϵ used are given in Table 4. Specific choices[35] for the valence density $V(x) = f_{u_V}(x) + f_{d_V}(x)$, sea density $S(x)$ and the gluon $G(x)$ for an N quark cluster ($N = 3, 6$) from Ref.[35, 37] are given by,

$$
\begin{aligned}
xV_N(x) &= \frac{1}{B\,(1/2,\ 2N-2)}\ N x^{0.5} (1-x)^{2N-3} \\
xS_N(x) &= \frac{N-1}{2(4N-3)}\ (a_N+1)\ (1-x)^{a_N} \\
xf_{g,N}(x) \equiv xG_N(x) &= \frac{5(N-1)}{2(4N-3)}\ (c_N+1)\ (1-x)^{c_N} \quad ,
\end{aligned}
\tag{5.9}
$$

with $a_3 = 9,\ a_6 = 11,\ \ c_3 = 7$, and $c_6 = 10$[37]. Here B is the usual Euler Beta function and $S_N(x)$ represents the sum of sea quark densities over all flavours. With a further assumption of $\bar{f}_{\bar{s},N}(x) = \frac{1}{2}\ f_{\bar{u},N}(x) = \frac{1}{2}\ f_{\bar{d},N}(x)$, the $\bar{u}$ distribution for

Table 4: Model parameters for nuclei used in E772 experiment for gas model(T,ω) and six quark cluster model(ϵ).

A	T (MeV)	ω	ϵ
12	54	0.069	0.112
40	47	0.057	0.170
56	45	0.117	0.186
184	42	0.132	0.230

the N-quark cluster is given by $f_{\bar{u},N}(x) = \frac{1}{5}\, S_N(x)$. The six quark–cluster model described is an updated version (in their choice of the parameters ϵ and the input proton densities) of the original six quark–bag model[38].

5.2.3 Rescaling models

A large class of models[39, 40, 41] try to model the effect of the nuclear medium in terms of the different length scales associated with the nucleon and the nucleus. The precise fashion in which it is done varies from model to model. In the q^2 rescaling models the nuclear parton densities at a scale q^2 are obtained from the parton densities in a proton at the same q^2 by evolving them to a scale $\xi_A q^2$, *i.e.*, the nuclear parton density per nucleon $f_{i/A}(r, q^2)$ is given by

$$f_{i/A}(x, q^2) = f_{i/p}(x, \xi_A q^2) \quad . \tag{5.10}$$

Here the results for the rescaled nuclear densities as obtained in Refs. [40, 41] are shown where $\xi_A = A^{2/3}$ where the starting nucleon parton densities are taken[41] to be a parametrisation of the EMC Deuterium data at $q^2 = 20$ GeV^2. It should be added here that the rescaling models along with the nuclear physics based models always tend to enhance the nuclear structure function at small values of x.

There exist also hybrid models [42] which combine the ideas of both, the rescaling and the cluster models. In these types of models the nuclear parton densities(per nucleon) are given by,

$$f_{i/A}(x, q^2) = f_{i/p}(x/\alpha_A, \xi_A q^2) \quad . \tag{5.11}$$

The two parameters are introduced to model the change in the scale in the nuclear case as well as the the possibility of cluster formation. The two parameters are then fitted to reproduce the data on ρ_{EMC}. Of course the fitted values of the parameters depend on the choice of the parametrisation for the parton densities in the proton. The values we obtained[43] are

$$\alpha_A = 0.012, \;\; \xi_A = A^{0.4}.$$

5.3 Comparisons of different model predictions for the gluon density

In fig. 24 are shown the fits to the data obtained in, e.g., the rescaling and the hybrid models. This makes the point that the fits to the data on ρ_{EMC} in different models are all of the same quality and all have similar quark parton distributions. Fig. 5.4.2 shows the expectations for the gluonic EMC ratio ρ_g of eq. (5.2), for some of the models of the EMC effect discussed above. It should be noted here that the different fits to the data use different parametrisations for the proton densities and hence it is more meaningful to compare the predictions for ρ_g for the different models of the EMC effect rather than the absolute gluon densities. The figure shows clearly that the differences in the predictions of the various models are indeed sizable.

5.4 A dependence of the hard processes

5.4.1 Experimental situation

Even before the EMC effect was discovered [8], there existed a few experiments which reported an anomalous nuclear enhancement of cross–sections for large–p_T particle/jet production [44] and DY $\mu^+\mu^-$ pair production [45] with nuclear targets. The experiments parametrised the cross–section for nuclear targets with a beam B as,

$$\frac{1}{A}\sigma^{BA} = A^{\alpha-1}\sigma^{Bp} \quad . \tag{5.12}$$

Similarly the ratio of the differential cross–sections, e.g., $\dfrac{d\sigma}{dp_T}$ is parameterised in terms of $\alpha(p_T)$. Again for no nontrivial nuclear dependence we must have $\alpha = 1$. A deviation of α from unity signals an anomalous nuclear effect. It should be noted here that due to the heavy nuclear targets that are used, a small deviation from unity for α means a rather large difference between the (per nucleon) cross–section with the nuclear and the free nucleon target. The initial experiments [44] reported indeed very large values of α increasing with the p_T values reaching 1.8 at the highest $p_T = 6$ GeV. The rise was seen for both p and π^+ beams. However the jet–like character of these data were questionable. In case of the non–jet like data, a large nuclear enhancement can also be caused by final state multiscattering effects, which can conceivably be larger for nuclear targets as opposed to the nucleon target. Recently there has been more data on the nuclear dependence of large p_T jet production [46]. The data show the following features:

1. The data do show an anomalous nuclear enhancement, *i.e.*, α values bigger than unity. The enhancement increases with the transverse energy E_T, which is essentially a measure of the tranverse momentum p_T.

2. The 'jettier' events give smaller values of α than the non-jet-like events for the same value of E_T.

3. The 'jets' seen carry a large fraction of the available c.m. energy, $\sqrt{S}$.

The second feature above is consistent with the much larger values of α quoted by the earlier experiments. However, in the case of the newer data, due to better characterisation of the jet-like nature, multiscattering cannot be invoked to explain the anomalous nuclear enhancement and hence has to be interpreted as a reflection of the enhancement of F_2^{eA} over F_2^{ep} in certain x regions. Recently, some data on large p_T particle production has become available which shows only a modest rise of the cross–sections with the mass number A.

High statistics data on the A dependence of the J/ψ and Υ production [47, 48, 49] as well as on the large p_T DY ($\mu^+\mu^-$ pair) production [50] have become recently available. Experimental data indicate a nuclear suppression of the total cross–section. Differential cross–sections are studied in two variables: p_T and x_F. x_F is given by $2P_L/\sqrt{S}$, where P_L is the longitudinal momentum of the quarkonium or the $\mu^+\mu^-$ pair and $\sqrt{S}$ is the total c.m. energy. $\alpha(p_T)$ and $\alpha(x_F)$ show a modest p_T and x_F dependence respectively. For the large p_T DY pairs the α values are mostly in the vicinity of unity as opposed to the very large nuclear enhancement reported by earlier experiments [45].

As discussed in the earlier section, yet another interesting probe of the gluon densities is the photo- and electro-production of quarkonia. The early FNAL and SLAC data[51] reported suppression of the lepto-/photo-production cross–sections of the J/ψ as opposed to the EMC data[52] which reported an enhancement. The situation was clarified by the latest NMC experiments[53] where a modest A–dependence of the differential distributions of the J/ψ production has been reported.

Thus the experimental studies of the nuclear dependence of the different hard processes show quite different behaviours, viz.,

1. Large p_T jet production shows considerable nuclear enhancement.

2. Hadronic quarkonium production shows a suppression for nuclear targets.

3. The DY $\mu^+\mu^-$ pair production and large p_T particle production show a very modest (almost nil) nuclear dependence.

From our discussion in the earlier section we know that the dominant subprocesses in each of the above hard processes involve different initial state partons. Also as a result of the different kinematical conditions the different experiments probe parton densities at different values of x. Hence a demand that a given model of nuclear parton densities explain all these data on the nuclear dependence of the hard processes consistently can indeed constrain these models considerably and help us discriminate among them.

Table 5: Expected values of α in different models compared with the data [46].

Config.	Data Ref. [46]	Gas Ref. [33]	Six–Quark Ref. [38]	Hybrid Ref. [42]	Rescaling Ref. [39]
$E_T > 15$ GeV $P > 0.8$ (Jet–like) (with P cut)	1.14 ± 0.02	1.16	1.10	1.03	0.98
$E_T > 18$ GeV Jet–like without P cut	1.45 ± 0.01	1.47	1.16	1.06	0.98

5.4.2 Comparison of the model predictions with data

In this section we present some of the model predictions for the A–dependence of the different hard scattering processes and compare it with the abovementioned data. It should be added here that for consistency, one has to use different proton densities while calculating predictions of the different models of the EMC effect for the nuclear dependence of various hard scattering processes.

1) A dependence of jet production

The first process we consider is large p_T particle and jet production with nuclear targets. As mentioned in the description of the data, the interesting features of the data are that the high–p_T jets seen carry a rather large fraction of the total c.m. energy, $\sqrt{S}$. The large value of α implies therefore that the nuclear gluon distributions are somehow harder compared with that in a nucleon. This points towards cumulative or cluster effects which predict an extension of the nuclear parton densities beyond $x_{bj} > 1.0$ as opposed to the rescaling models. It can be proved on quite general grounds [54], using simply the experimental information on the signature of $d\rho^{EMC}/dx$, that $d\alpha/dp_T$ will always be less than 0 unless the nuclear structure functions extend into the region beyond $x_{bj} = 1$, the so called cumulative region. These qualitative expectations are indeed borne out by a comparison of the model predictions with the data. Table 5 shows the data along with the predictions of different models for the EMC effect.

A cut on the planarity P helps to choose jet–like events for events with lower E_T. For larger values of E_T the jet–like nature of the events is clear and no such cut is required. As one can see, the rescaling model fails to give an enhancement of the jet cross–sections. Furthermore, the predicted value of σ^A fails even to show the A^α behaviour. The cluster models [33, 38, 42] all give values of α bigger than 1. Stronger the cumulative effects higher are the values of α predicted. Here it is worth pointing out that we had used the older version of the six–quark bag model. We see that these data already seem to prefer cluster models over the rescaling type models of the EMC effect.

2) Nuclear dependence of the large p_T quarkonia and $\mu^+\mu^-$ pair production

Next we discuss a comparison of the data on hadroproduction of large p_T quarkonia (J/ψ and Υ) and $\mu^+\mu^-$ pairs with model predictions. We choose here FNAL data [48, 49, 50] for its high statistics, although comparisons with the earlier data with pion beams[47] do exist[55, 56].

The E772 experiment has provided data for the ratio

$$R^{J/\psi}(p_T) = \frac{d\sigma(pA \to J/\psi X)}{dp_T} \Big/ A\frac{d\sigma(pp \to J/\psi X)}{dp_T} \tag{5.13}$$

with an x_F-cut of $0.15 \leq x_F \leq 0.65$ on the J/ψ's, while for the Υ-production cross sections, they chose to present only $\alpha(p_T)$, where

$$\frac{d\sigma(pA \to \Upsilon X)}{dp_T} = A^{\alpha(p_T)}\frac{d\sigma(pp \to \Upsilon X)}{dp_T} \tag{5.14}$$

with a corresponding x_F-cut for Υ of $-0.2 \leq x_F \leq 0.6$. The nuclei used were carbon, calcium, iron and tungsten. One can compute each of the individual p_T-distributions in eqs. (5.13-5.14) incorporating these x_F-cuts. However one needs to use a specific model for the quarkonium formation. Fig. 26 exhibits the results for a specific model of hadronisation of the quarkonium, for all the four nuclei along with the corresponding data from the E772 collaboration. One sees that for the lighter nuclei both the two-component models, namely, the gas model and the six-quark cluster model, describe the data rather well. For the tungsten nucleus, however, *none* of the models seems to be in agreement with data.

Fig. 27 shows a comparison of the model predictions for α values for Υ-production with the E772-data. One sees a similar general agreement for the gas model and the six-quark cluster model as for J/ψ-production at moderate values of p_T. At the largest p_T, however, the E772-data rise too sharply compared to any model and could possibly indicate that these models tuned to earlier large x-data have to be better tuned to perform well in the small x-region.

The discrepancy at large p_T values for the Υ-data and for the tungsten target for the J/Ψ production do expose the inadequacy of all the three models of the EMC effect and the corresponding parametrisation of the nuclear parton densities considered here but the general agreement in other cases tells us that the data can indeed be described in terms of the structure function effects in general and the data are accurate enough to allow discrimination between different models of the nuclear structure functions.

The proton–induced dimuon pair production was studied over a wide range of x_F and p_T values. The data on the ratio of the *integrated* dimuon yield for different nuclei were compared with theoretical predictions, obtained by using the $q\bar{q}$ annihilation process, for various models of the EMC effect. It seemed[50] to rule out the

6-quark cluster model[35]. However, a later comparison[37] with an improved version of the model, showed that this model too can be consistent with the information on the ratio of the integrated dimuon yields.

Experimental information [50] is also available for the ratio

$$R^{DY} = \frac{d\sigma^{DY}}{dp_T}\,(pA \to \mu^+\mu^- X) \Bigg/ \frac{d\sigma^{DY}}{dp_T}\,(pp \to \mu^+\mu^- X) \ , \tag{5.15}$$

where $d\sigma^{DY}/dp_T$ is the differential DY cross section integrated over the continuum region (avoiding the resonances) $4 < M_{\mu^+\mu^-} < 9$ GeV and $M_{\mu^+\mu^-} \geq 11$ GeV, with $x_F > 0$.

Fig. 28 exhibits the results of a computation for the four different nuclei and the three different models with the corresponding data. Again we see that, similar to the case of resonance production, the general trends of the data are well described by the model predictions for the gas model and the 6-quark cluster model.

Thus in conclusion we see that already the available experimental information on quarkonia, dimuon and large p_T jet production indicate that two component models of the EMC effect seem to be preferred by the data. Further experiments with direct photon production with nuclear targets [57] or associated production of J/Ψ and γ[58] will help in this direction even more. But what is important to note that it is possible now to tune the nuclear gluon densities in different EMC models using the data already available. This should go a long way in helping us understand the physical origin of the EMC effect as well as help us estimate the QCD backgrounds to the 'hard' signals of QGP formation even better.

References

[1] M. Gell-Mann, Phys. Lett. **8**,214 (1964).

[2] For a summary of the early DIS results see, e.g., J.I. Friedman and H.K. Kendall, Ann. Rev. Nucl. Part. Sci. **22**, 203 (1972) and references therein; G.B. West, Phys. Rep. **18C**, 264 (1975) and references therein.

[3] J.D. Bjorken and E.A. Paschos, Phys. Rev. **185**, 1975 (1969).

[4] R.P. Feynman, Phys. Rev. Lett. **23**, 1415 (1969).

[5] D.J. Gross and F. Wilczek, Phys. Rev. Lett. **30**, 1343 (1973); Phys. Rev. **D8**, 3633 (1973);
H.D. Politzer, Phys. Rev. Lett. **30**, 1346 (1973).

[6] M. Gell-Mann, Acta Phys. Austriaca, Suppl.**IIX**, 733(1972);
M. Gell-Mann and H. Fritzsch, *XVI^{th} Int. Conf. on High Energy Physics*, Batavia, **II**, 135 (1972);
M. Gell-Mann, H. Fritzsch and H. Leutweyler, Phys. Lett. **B47**,365 91973);
For a recent review giving a persepective of QCD see, e.g., F. Wilczeck, Proceedings of workshop *QCD 20 years later*, **I**, 16, ed. P. Zerwas (Aachen 1992).

[7] For a review of how parton model can be understood in the context of pQCD see, e.g.,
G. Altarelli, Phys. Rep. **81C**, 1 (1982);
E. Reya, Phys. Rep. **69C**, 195 (1981).

[8] EMC Collaboration, J.J. Aubert et al., Phys. Lett **B 123**, 275 (1983).

[9] For a recent review of the data on and models of the EMC effect see, e.g., M. Arneodo, Nuclear Effects in Structure Functions, CERN-PPE/92-113.

[10] R.M. Godbole, in: Frontiers in particle physics, ed. Z. Ajduk, S. Pokorski and A.K. Wroblewski (World Scientific, Singapore, 1990) p.483.

[11] T. Kinoshita and W.B. lindquist, Phys. Rev. Lett. **47**,1573 (1981).

[12] Review of particle properties, part II, Phys. Rev. **45** (1992).

[13] E. Rutherford, Philos. Mag. **21**, 669 (1911).

[14] R. Hofstadter, Rev. Mod. Phys. **28**, 214 (1956) and references therein.

[15] P. N. Kirk et al., Phys. Rev. **D8**, 63 (1973).

[16] Field Theory in Particle Physics, B. de Wit and J. Smith, (North Holland).

[17] For a description of standard model and details see e.g. : Quarks and Leptons, F. Halzen and A. D. Martin, (John Wiley, New York, USA)

[18] For a review of the early developments in parton model see ,e.g., An Introduction to Quarks and Partons, F.E. Close, (Academic Press, London.)

[19] For a recent summary of the DIS data see, e.g, M. Virchaux, Proceedings of the workshop *QCD 20 years later* ed. P. Zerwas, (Aachen, 1992).

[20] For an early review see, e.g., C.H. Llewellyn Smith, Phys. Rep. **3 C**, 261(1972); For a more recent review of neutrino DIS see, e.g, Sanjib Mishra and F. Scuilli, Ann. Rev. Nucl. Part. Sci., **39**, 259 (1988).

[21] R.E. Taylor, Rev. Mod. Phys. **63**, 573 (1991);
H. W. Kendall, Rev. Mod. Phys. **63**, 597 (1991);
J.I. Friedman, Rev. Mod. Phys. **63**, 615 (1991).

[22] NMC, M. Arneodo et al, CERN-PPE/93-117, subm. to Rapid. Comm. in Phys. Rev.

[23] For a review see, e.g., H. Satz, in proceedings of the workshop *QCD 20 years later*, ed. P. Zerwas, (Aachen 1992).

[24] For a good summary of various subprocess cross–sections as calculated in perturbative QCD see, e.g., Applications of Perturbative QCD, R. D. Field,(Addison-Wesley, Redwood City, USA).

[25] For a good summary of early tests of perturbative QCD at the $p\bar{p}$ collider experiments see, e.g., G. Altarelli, Ann. Rev. of Nucl. and Part. Sci. **39**, 357 (1989).

[26] For an early discussion of electro- and photo-production of quarkonia see, e.g., the second of reference [7].

[27] For a compilation of the data on EMC effect see, e.g., Ref. [12].

[28] A. Krzywicki, Phys. Rev. **D 14**, 152 (1976);
R.M. Godbole and K.V.L. Sarma, Phys. Rev. **D25**,120 (1982).

[29] N.N. Nikolaev and B.G. Zakharov, Phys. Lett. **B260**, 414 (1991);

[30] C.H. Llewellyn Smith, Phys. Lett. **B128**, 107 (1983).

[31] S.V. Akulinichev et al, Phys. Rev. Lett. **55**, 2239 (1985).

[32] G.L. Li, K.F. Liu and G.E. Brown, Phys. Lett. **B213**, 531 (1988).

[33] S. Gupta and K. V. L. Sarma, Z. Phys. **C29**, 329 (1985).

[34] CDHS Collaboration, J. G. H. de Groot et al., Z. Phys. **C17**, 283 (1983).

[35] K.E. Lassila and U.P. Sukhatme, Phys. Lett. **B209**, 343 (1988); U.P. Sukhatme, G. Wilk and K.E. Lassila, Z. Phys. **C53**, 439 (1992).

[36] M. Sato, S. Coon, H. Pirner and J. Vary, Phys. Rev. **C33**, 1062 (1986).

[37] K.F. Lassila, U.P. Sukhatme, A. Harindranath and J. Vary, Phys. Rev. **C44**, 1188 (1991).

[38] C.E. Carlson and T.J. Havens, Phys. Rev. Lett. **51**, 261 (1983).

[39] F.E. Close, R.G. Roberts and G.G. Ross, Phys. Lett. **B129**, 346 (1993).

[40] S. Gupta, S. Banerjee and R.M. Godbole, Z. Phys. **C28**, 483 (1985).

[41] S. Gupta, Pramana **24**, 443 (1985).

[42] J. D. Deus, M. Pimenta and J. Varela, Phys. Rev. **D30**, 697 (1984).

[43] R.M. Godbole and S. Gupta, Phys. Lett. **B228**, 129 (1989).

[44] C. Bromberg et al., Phys. Rev. Lett. **42**, 1202 (1979).

[45] K. J. Anderson et al., Phys. Rev. Lett. **42**, 844 (1979)

[46] H.E. Miettinen et al., Phys. Lett. **B207**, 202 (1988).

[47] S. Kastanevas et al., Phys. Rev. Lett. **60**, (2121) (1988).

[48] D.M. Alde et al., Phys. Rev. Lett. **66**, 133 (1991).

[49] D.M. Alde et al., Phys. Rev. Lett. **66**, 2285 (1991).

[50] D.M. Alde et al., Phys. Rev. Lett. **64**, 2479 (1990).

[51] M. D. Sokoloff et al, Phys. Rev. Lett. **57**, 3003 (1986);
R.L. Anderson et al., Phys. Rev. Lett. **38**, 263 (1977).

[52] J.J. Aubert et al., Phys. Lett. **B152**, 1985.

[53] D. Allasia et al., Phys. Lett. **B258**, 493 (1991);
P. Amaudruz et al., Nucl. Phys. **B371**, 553 (1992).

[54] S. Gupta and R.M. Godbole, Phys. Rev. **D33**, (1986) 3453.

[55] R.M. Godbole and Sridhar K., Z. Phys. **C51**, 417 (1991).

[56] R.V. Gavai and S. Gupta, Z. Phys. **C49**, 663 (1991).

[57] S. Gupta and Sridhar K., Phys. Lett. **B197**, 259 (1987); Sridhar K., Z. Phys. **C55**, 401 (1992).

[58] R.V. Gavai, R.M. Godbole and Sridhar K., Phys. Lett. **B299**, 157 (1993).

[59] R.V. Gavai and R.M. Godbole, Nucl. Phys. **A566** (1994) 375 C, Proceedings of QM 93, Börlange, Sweden, June, 1993; BU-TH-93/5 (submitted for publication).

Figure Captions

Figure 1: Electromagnetic scattering process $eA \to eA$ along with the four momenta assignment for the various particles that are involved.

Figure 2: Kinematics of the elastic scattering process $eA \to eA$ in the laboratory frame.

Figure 3: Scattering of an electron e^- from the nuclear charge distribution.

Figure 4: Feynman diagram for the elastic scattering process $ep \to ep$ for a pointlike proton.

Figure 5: The form–factor $G_M^p(q^2)/\mu_p$ as measured in SLAC experiments as a function of q^2 (data taken from Ref. [15]). The solid line is the curve $\left(1 + q^2/0.71(\mathrm{GeV}^2)\right)^{-2}$.

Figure 6: Kinematics of the inelastic scattering process $ep \to eX$.

Figure 7: Allowed region in the $q^2 - \nu$ plane for the elastic, quasi–elastic and inclusive inelastic scattering.

Figure 8: $\tilde{W}_2 = M_p^2\, W_2(\nu, Q^2)/\pi\alpha$ as a function of $q^2$and ν (Figure taken from Ref. [16]). The details of the source of the data etc. to be found in Ref. [16].

Figure 9: Data on νW_2^{ep} as a function of q^2 for $\omega = 1/x = 1/4$ (taken from Ref. [2]). Different data correspond to different scattering angles as indicated in the figure.

Figure 10: Parton model picture of the DIS scattering of an e^- off a proton target.

Figure 11: DIS scattering of a ν off a proton target.

Figure 12: Schematic drawing of F_2 as a function of x for different values of q^2 in the range $0.01 - 200$ GeV^2 (taken from Ref. [18]).

Figure 13: Data on $e - \alpha$ scattering at $q^2 = 0.08$ GeV^2 (taken from Ref. [14]). A is the elastic peak for α particle while the elastic proton peak is shown by the dashed line which corresponds to $x = q^2/2m_\alpha\,\nu \simeq 0.25$. The portion BCDE indicates the momentum distribution of the nucleons in the α particle.

Figure 14: Parton model.

Figure 15: Impossibility of the backward scattering for the helicity preserving interaction.

Figure 16: Data on $\frac{F_2^{\mu n}(x)}{F_2^{\mu p}(x)}$ as a function of x (data taken from Ref. [22]).

Figure 17: The ratio R of eq. (4.23) as a function of q^2 (taken from Ref. [21]).

Figure 18: Ratio of $F_2^{\nu Fe}$ from the CCFR and CDHSW experiment to $F_2^{\mu A}$ from different μ DIS experiments. The μ structure functions are normalised so thata ratio of unity means mean square charge of $\frac{5}{18}$ (figure taken from Ref. [20]). See Ref.[20] for more details.

Figure 19: Data on $F_2^{\mu p} - F_2^{\mu n}$ from NMC as a function of x (data from Ref. [22]).

Figure 20: Parton model picture of the W^+ production in $p\bar{p}$ collisions.

Figure 21: DY process of production of a $\mu^+\mu^-$ pair production in hadronic collisions.

Figure 22: Impulse approximation for the nuclear DIS scattering.

Figure 23: Compilation of the data on EMC effect (from Ref. [12]). Details of the data available in Ref. [12]

Figure 24: Fits to the data on ρ_{EMC} in some models of the EMC effect [43].

Figure 25: The ratio ρ_g of eq. 5.2 as a function of x for some models of the EMC effect [58].

Figure 26: E772 data (inverted filled triangles) on the ratio $R^{J/\psi}$ of Eq. 5.13 compared with the predictions for the gas model (squares), six-quark cluster model (circles) and the rescaling model (open triangles) of the EMC effect [59].

Figure 27: E772 data on $\alpha(p_T)$ of eq. 5.14 compared with predictions of the three different models of the EMC effect mentioned in the text. Notation is same as in Fig. 26 [59].

Figure 28: E772 data on the ratio R^{DY} of Eq. 5.15 compared with predictions of the three models of the EMC effect. Notation is same as in Fig. 26 [59].

A Notations used

If A and B are two four vectors given by $A \equiv (\vec{a}, ia_o)$, $B \equiv (\vec{b}, ib_o)$ then,

$$A \cdot B = \vec{a} \cdot \vec{b} - a_0 b_0 = \vec{a} \cdot \vec{b} + a_4 b_4.$$

The γ matrix algebra is given as

$$\begin{aligned}
\{\gamma_\mu, \gamma_\nu\} &= 2\ \delta_{\mu\nu} \\
\gamma_\mu^\dagger &= \gamma_\mu \\
\gamma_\mu^2 &= 1 \\
\gamma_5 = \gamma_1\gamma_2\gamma_3\gamma_4 &= \frac{1}{4!}\epsilon_{\mu\nu\rho\sigma}\gamma_\mu\gamma_\nu\gamma_\rho\gamma_\sigma \\
\{\gamma_5, \gamma_\nu\} &= 0 \ \ ; \quad \mu = 1,2,3,4 \\
\gamma_5^\dagger &= \gamma_5 \\
\sigma_{\mu\nu} &= \frac{1}{2i}\left[\gamma_\mu, \gamma_\nu\right] = -i\ \gamma_\mu\gamma_\nu (\mu \neq \nu)
\end{aligned}$$

The Dirac equation for a particle of mass m is given in this metric by

$$(\gamma_\mu \partial_\mu + m)\Psi = 0.$$

In momentum space this becomes:

$$(i \not{P} + m)u(P) = 0 \ \text{ where } \ \not{P} = \gamma_\mu P_\mu$$

P stands for the four–momentum of the particle and $u(P)$ is the free particle spinor.

In this metric we have

$$\sum_s u_\alpha(P,s)\bar{u}_\beta(P,s) = (-i \not{P} + m)_{\alpha\beta}.$$

The trace theorems in this metric are given by:

$$\begin{aligned}
Tr(\gamma_\mu\gamma_\nu) &= 4\ \delta_{\mu\nu} \\
Tr\left[\gamma_\mu\gamma_\nu\gamma_\rho\gamma_\sigma\right] &= 4\ \left[\delta_{\mu\nu}\delta_{\rho\sigma} + \delta_{\mu\sigma}\delta_{\nu\rho} - \delta_{\mu\rho}\delta_{\nu\sigma}\right] \\
Tr\left[\gamma_\mu\gamma_\nu\gamma_\rho\gamma_\sigma\gamma_5\right] &= 4\ \epsilon_{\mu\nu\rho\sigma} \\
Tr\left[\gamma_1 \cdots \gamma_n\right] &= 0 \ \ (n \ \text{ odd}) \\
Tr\left[\gamma_5\gamma_\mu \cdots\right] &= 0 \ \text{ if } \ \gamma_\mu \cdots \ \text{ has less than 4 } \gamma \text{ matrices}
\end{aligned}$$

One more relation required in calculation of $(|\mathcal{M}|^2)$ is

$$\gamma_4[(\gamma \cdot A)(\gamma \cdot B)....(\gamma \cdot L)]^\dagger \gamma_4 = (-\gamma.L)\cdots(-\gamma \cdot A);$$

where $A, B \cdots L$ stand for four vectors representing four momenta.

B Problems

1 What is the energy of the probing e^- beam that will be required to probe the structure at a distance scale (say) $10^{-17}\ m$?

2 Calculate the form factors $F(Q^2)$ for the following charge distributions: 1) $\rho(R) = \delta^3(\vec{R})$, 2) $\rho(R) = \frac{m^2}{4\pi} exp(-mR)/R$ and 3)$\frac{m^3}{8\pi} exp(-mR)$ where $m > 0$.

3 Show by explicit calculation that the factor $\cos^2(\theta/2)$ in eq. (1.42) corresponds to the helicity non-flip amplitude.

4 Calculate the expression for the differential cross–section $\frac{d\sigma}{d\Omega}$ for the electromagnetic scattering of an electron incident on a target which is

1) spin 0, pointlike particle

2) spin 1/2, pointlike particle

3) spin 1/2 particle which is not pointlike.

The expression for the cross–section is given by eq. (2.4) The matrix element is given by eq. (2.3) where the electromagnetic current of the electron is given by eq. (2.9), and the electromagnetic current of the target in cases 2 and 3 are given by eqs. (2.9) and (2.14) respectively, whereas the electromagnetic current for the case 1) of a spinless, pointlike proton is given by

$$J^p_\mu = i q_p (P_4 + P_1)_\mu$$

where all the notations are as given in the lecture. The answers are given by eqs. (2.6),(2.5) and (2.17).

5 Show that considerations of gauge invariance and Lorentz invariance restrict the form of H'' as given by eq. (3.45), by following steps analogous to those used in deriving eq. (3.22). Using this, show that the expression for the differential cross–section is given by eq. (3.46).

6 Calculate the expressions for $F_2^{\nu p}$ and $F_3^{\nu p}$ given that,

$$\frac{d\sigma}{dy}(\nu_\ell\, q \to \ell\, q') = \frac{G_F^2 s}{\pi},$$
$$\frac{d\sigma}{dy}(\nu_\ell\, \bar{q} \to \ell\, \bar{q}') = \frac{G_F^2 s}{\pi}(1-y)^2.$$

Assume that the Callan-Gross relation given by eq. (4.5) is satisfied for the Neutrino structure functions $F_2^{\nu\, p}$ and $F_1^{\nu\, p}$ and use the expression for the double differential cross–section for the inelastic νp scattering given by eq. (3.47).

7 Show that in parton model, for an isoscalar nuclear target

$$\frac{F_2^{\ell A}}{F_2^{\nu A}} = \frac{5}{18} + \frac{1}{18}\left[\frac{4\,K(x)}{u_v(x) + d_v(x) + 4\,K(x)}\right],$$

where $K(x)$ is the SU(3) symmetric sea densities defined in eq. (4.15).

8 Derive eq. (4.25).