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Slow Relaxation in a Model with Many Conservation Laws : Deposition and Evaporation of Trimers on a Line

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Abstract

We study the slow decay of the steady-state autocorrelation function $C(t)$ in a stochastic model of deposition and evaporation of trimers on a line with infinitely many conservation laws and sectors. Simulations show that $C(t)$ decays as different powers of t, or as $\exp(-t^{1/2})$, depending on the sector. We explain this diversity by relating the problem to diffusion of hard core particles with conserved spin labels. The model embodies a matrix generalization of the Kardar-Parisi-Zhang model of interface roughening. In the sector which includes the empty line, the dynamical exponent z is 2.55 ± 0.15.

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It is well known that conservation laws strongly affect the decay in time of fluctuations in equilibrium. For example, in magnetic systems, the rate of decay of the spin autocorrelation function is quite different depending on whether or not the spin dynamics conserves magnetization [1]. What happens with more than one – indeed an infinity of – conservation laws? We address this question in this paper by studying the decay of autocorrelation functions in the steady state of a simple stochastic model of deposition and evaporation [2, 3]. This model has been shown to possess an infinite number of independent constants of motion [4]. Here we show that it exhibits a very rich variety of slow relaxations: depending on the initial conditions, we can get a very large number of power-law decays, or even stretched exponential decay. We present numerical evidence from Monte Carlo simulations for these different kinds of decay, and also provide an analytical understanding of this remarkable diversity in terms of the diffusive motion of hard core particles with spin. Our model can be viewed as a generalisation of the well-known Kardar-Parisi-Zhang (KPZ) problem [5] of roughening of a one dimensional interface, where the scalar-height variables in the KPZ model are replaced by a matrix-valued variables (say 2×2 complex matrices). In the steady state reached after starting from an empty lattice, we find that the density-density autocorrelation function decays as a power law in time, and the dynamical exponent $z \simeq 2.5$, very different from the KPZ value $z = 3/2$. The model thus includes a new universality class for nonlinear one-dimensional evolution models.

In the deposition-evaporation (DE) model under consideration, there is a site variable n_i, taking values 0 or 1, at each site i of a line, $1 \leq i \leq L$; it may be thought of as an occupation number variable of a lattice gas. We define pseudospin variables $S_i \equiv 2n_i - 1$. The time evolution is Markovian, specified by the following rates: In a small time dt, a triplet of spins at adjacent sites $i, i+1, i+2$ can flip simultaneously only if $S_i = S_{i+1} = S_{i+2}$. The rate is ϵ if the spins were originally -1, and ϵ' if they were originally +1. These rates satisfy the detailed balance condition corresponding to a non-interacting lattice gas Hamiltonian with chemical potential $\frac{1}{3}\ell n(\epsilon/\epsilon')$. However, the long-time steady states in the present model are nontrivial, as the dynamics is strongly non-ergodic. We have shown [4] that the total phase space of 2^L configurations breaks up into a large number N_L of mutually disconnected sectors, and N_L increases as $[(\sqrt{5}+1)/2]^L$ for large L.

This decomposition of phase space is a consequence of the existence of an infinity of conservation laws. A compact representation of these is provided by the "irreducible string"(IS), defined as follows: Regard the configuration as a string of up and down spins. Scan the string left to right, and stop at the first triplet of parallel spins encountered. If no such triplet is found the string is the IS. If a triplet is encountered, simply delete it, reducing length of string by 3, and repeat the process. The IS which finally results is a constant of motion, and different sectors of phase space correspond to different IS's [4].

Under time evolution, an initial configuration evolves into the steady state of the corresponding sector. For instance, if $\epsilon = \epsilon'$, in the steady state all configurations having the same IS as that of the initial configuration occur with equal probability. It is then interesting to ask how a dynamical quantity such as the steady state autocorrelation function

$$C_i(t) = \langle n_i(t+t_0) n_i(t_0) \rangle - \langle n_i \rangle^2 \tag{1}$$

varies from one sector to another. Figure 1 shows the behaviour of $C(t)$ obtained from Monte Carlo simulation studies in a number of different representative sectors. The data depicted in the figure pertain to four sectors in which the IS constitutes a finite fraction of all sites, and one sector in which the fraction is zero. These sectors correspond to (1) a random initial configuration (2) an IS formed by repeating $[\uparrow\downarrow\downarrow]$ $L/6$ times (3) an IS formed by repeating $[\uparrow\downarrow]$ $L/4$ times (4) an IS formed by repeating $[\uparrow\uparrow\downarrow\uparrow\downarrow\downarrow]$ $L/12$ times (5) the null sector (vanishing IS). Lattice sizes $L = 120,000$ were used and averages over $30-60$ different histories were taken. From the figure, we note the following points: (i) if the lattice is divided into 3 sublattices A, B, C, the autocorrelation function $C_i(t)$ depends, in general (e.g in Sectors 2 and 4), on the sublattice to which site i belongs. (ii) in several sectors and sublattices $C_i(t)$ decays as a power law $\sim t^{-\theta}$; the power θ is sector dependent. In Sector 1, $\theta \simeq 0.25$; in Sector 2 and on two sublattices of Sector 4, $\theta \simeq 0.5$; in Sector 5, $\theta \simeq 0.6$. (iii) in some sectors (e.g. Sector 3 and one sublattice in Sector 4), the decay is faster than a power law, and fits well to a stretched exponential form. Evidently, there is a very wide range of possible behaviours. Understanding the source and nature of this dynamical diversity is one of the main points of this Letter.

Consider time evolution in a sector labelled by an IS with elements $\{\alpha_n\} \equiv \{\alpha_1, \alpha_2, \cdots, \alpha_\ell\}$, where the length ℓ is a nonzero fraction $\rho = \ell/L$ of the total length L of the lattice. Let $\mathcal{C}(t)$ be the full string corresponding to the configuration at time t. If we apply the deletion algorithm to $\mathcal{C}(t)$, some characters are eventually deleted, others not. If $X_j(t)$ is the location of the $j^{\rm th}$ site (counting from the left) that remains undeleted, we may look upon the set $\{X_j(t)\}$ as the positions of ℓ interacting random walkers on a line; DE dynamics induces the time evolution of$\{X_j(t)\}$. The walkers cannot cross each other ($X_{j+1}(t) > X_j(t)$, for all j, all t) and they carry a conserved spin label $S_{X_j(t)} = \alpha_j$ for all t. Under time evolution $X_j(t)$ changes in jumps, by a multiple of 3 spaces at a time.

In the steady state, the joint probability distribution $Prob(\{X_j\})$ is proportional to the number of different configurations of the lattice, consistent with the positions of walkers being $\{X_j\}$. The substring between sites X_{j+1} and X_j should be a string reducible to a null string, and all such configurations are easily enumerated. The result is

$$Prob(\{X_j\}) = \mathcal{N} \prod_{j=0}^{\ell} g(X_{j+1} - X_j) \qquad (2)$$

where $\mathcal{N}$ is a normalization constant, and $X_0 = 0$, $X_{\ell+1} = L + 1$. The separation probability $g(r)$ can be computed using the generating function method of [4]. The result is $g(r) \sim r^{-3/2} exp(-\kappa r)$ for large r, where κ is the reduced pressure, which depends on on the density ℓ/L, and tends to zero as $(\ell/L)^2$ for small ℓ/L. We see that $\{X_j\}$ constitute the slow modes of the system as they evolve diffusively, and are linked to conserved quantities. If the typical relaxation time of a string of length λ reducible to the null string varies as λ^z, then for times $t \gg (L/\ell)^z$ we can assume that all the degrees of freedom other than $\{X_j\}$ are in instantaneous local equilibrium. Thus $g(r) \sim exp(-V(r))$, where the effective interaction $V(r)$ between walkers is attractive, and increases linearly with separation if $\kappa \neq 0$, and as $ln\ r$ if $\kappa = 0$.

While the logarithmic part of the interaction is crucial for the understanding of sectors in which the walker density $\ell/L \to 0$, it appears not to be very important if ℓ/L, and hence κ, is finite. In the latter case, we are led to consider a simpler system: ℓ random walkers on a line of length L, with each walker carrying a spin α_j which is unchanged under dynamics. Each walker jumps left or right by 3 steps only if no other walkers intervene. The point is that this simpler system of hard core random walkers with spin (HCRWS) has the exactly the same conservation laws as the original deposition evaporation model. Thus we are led to conjecture that the long-time behaviour of $C_i(t)$ in a particular sector of the DE model is essentially the same as the spin-spin autocorrelation function $D_i(t)$ in the HCRWS problem with the corresponding spin sequence.

It is then easy to understand the sublattice dependence of $C_i(t)$ observed in simulations. As each element α_n of the IS moves by multiples of 3 lattice spacings, it stays on the same sublattice, say A. The long-time behaviour of $C_i(t)$ for $i \in A$ is governed only by the subset $\{\alpha_{n'}\}_A$ on sublattice A, independent of $\{\alpha_{n'}\}_B$ and $\{\alpha_{n'}\}_C$, despite the fact that the elementary step of deposition or evaporation couples all sublattices strongly.

For the HCRWS model, mean squared fluctuations in the number of particles between points 0 and r grow as $t^{1/2}$ in the steady state [6]. To test for a similar behaviour in the DE system, we monitored $\sigma^2(t) \equiv \langle [N(r,t) - N(r,0)]^2 \rangle$ where $N(r,t)$ is the length of irreducible string between points 0 and r at time t. This quantity is roughly equal to number of walkers to the left of r, but remains well defined even in the sector where there are no walkers $\ell = 0$. Figure 2 shows Monte Carlo data for $\sigma^2(t)$ in a number of sectors. For all sectors with ℓ/L finite, $\sigma^2(t)$ is seen to grow as $t^{1/2}$, lending strong support to the conjecture of equivalence of the DE and HCRWS models. In the sector where the IS vanishes, on the other hand, σ^2 is found to grow as $t^{2\beta}$ with $\beta \simeq 0.19$. If the irreducible string is very short, then κ is very small, and fluctuations in the separation $(X_{i+1} - X_i)$ between near neighbours in the diffusing, interacting lattice gas are large. The low value of the exponent β implies that these fluctuations are slowly decaying.

In the HCRWS model, the spin-spin autocorrelation function in the steady state is defined as

$$D_i(t) = \langle S_i(t+t_0) S_i(t_0) \rangle - \langle S_i \rangle^2 \tag{3}$$

where $S_i(t)$ is spin at site i, and an unoccupied site is considered to have zero spin. $D_i(t)$ can be written in the form

$$D_i(t) = \sum_{m=-\infty}^{+\infty} G(m|t) \gamma_i(m) \tag{4}$$

Here, given that in the steady state site i occupied at some time t_0, $G(m|t)$ is the conditional probability that it is occupied at time $(t+t_0)$, and that the difference in the particle labels at these times is $3m$. This term is independent of the spin configuration $\{\alpha_i\}$ of the IS. The second term $\gamma_i(m)$ is the average value of $\alpha_k\alpha_{k+3m}$, averaged over different values of k in the IS consistent with the condition that particle k can occupy site i (i.e. $k = i$ mod 3). The different dependences of $D_i(t)$ on time in different sectors comes entirely from the different dependences of $\gamma_i(m)$ on m.

Using the known equivalence [7,8] of long-time HCRW dynamics to a stochastic harmonic model, $G(m|t)$ for large t is given by

$$G(m|t) \approx \frac{1}{\sqrt{2\pi\delta^2}} e^{-m^2/2\delta^2} \tag{5a}$$

where

$$\delta^2 = |m| erf\left(\frac{|m|}{\sqrt{2t}}\right) + \sqrt{\frac{2t}{\pi}} e^{-\frac{m^2}{2t}} \tag{5b}$$

Using eq. (4) and (5) we can determine the behaviour of $D_i(t)$ in the HCRWS model in different sectors.

Sector 1 (random initial condition): In this case $\gamma_i(m)$ is significant only for very small values of m. For m fixed, $G \sim t^{-1/4}$ for large t, whence $D_i(t)$ also varies as $t^{-1/4}$. Note that this result is also true for other sectors in which the correlations in the IS decay fast with distance.

Sector 2 [↑↓↓]: In this case $\alpha_k = \alpha_{k+3m}$ for all m. Hence $D_i(t)$ reduces to a density-density correlation which decreases as $t^{-1/2}$ for large t. The same behavior is expected in all periodic IS where the magnitization in each sublattice is nonzero.

Sector 3 [↑↓]: In this case $\gamma_i(m) = (-1)^m$, and this implies that $D_i(t)$ has the stretched exponential form $\exp[-(t/\tau)^{1/2}]$ at large time at all sites. The same behaviour would occur whenever the IS is periodic with zero net magnetization on each sublattice.

Sector 4 [↑↑↓↑↓↓]: On sublattices A and C we have $\alpha_k = \alpha_{k+3m}$, and thus as in sector 2, we have $D_i(t) \sim t^{-1/2}$. On sublattice B, $\gamma_i(m)$ is $(-1)^m$, and as in sector 3 we get stretched exponential decay.

If the sector is such that correlations in the IS imply that $\tilde{\gamma}_i(q)$, the Fourier transform of $\gamma_i(m)$, varies as $|q|^\phi$ for small q, then $D_i(t)$ would vary as $t^{-(1+\phi)/4}$.

It is evident that the HCRWS predictions for $D_i(t)$ accord very well with the Monte Carlo results for $C_i(t)$ shown in Fig. 1. The occurrence of stretched exponentials in the $[\uparrow\downarrow]$ sector, in particular, explains why earlier attempts [3] to fit finite size data to power laws yielded anomalously low values of the dynamical exponent z.

For any configuration $\{S_i\}$ of spins on the line, we define the matrix variable at site i by the equation

$$I_i(t) = \prod_{j=1}^{i} A(S_j(t)) \tag{6}$$

where $A(1)$ and $A(-1)$ are 2×2 complex matrices given by

$$A(1) = \begin{pmatrix} 1 & x \\ 0 & \omega \end{pmatrix} = A(-1)^\dagger \tag{7}$$

where $\omega \equiv exp(2\pi i/3)$ and x is a real parameter. Then as $A(1)^3 = A(-1)^3 = 1$, it follows that $I_L(t)$ is a constant of motion [4]. The evolution of the variables $\{I_i(t)\}$ is Markovian, governed by local transition rates. If we start with an empty line, $I_i(t = 0)$ take simple values, and get progressively disordered as time increases. For large x, the ratio $ln\ Tr\ I_r(t)/ln\ x$ behaves in qualitatively the same way as $N(r,t)$. At $t = 0$, the variance σ^2 of $N(r,t)$ is zero, and grows to a value proportional to r in the steady state. If $N(r,t)$ obeyed a stochastic evolution equation of the Kardar-Parisi-Zhang type [5], this variance would grow as $t^{2/3}$. Our numerical results (Fig. 2) clearly rule out this possibility. From Figs. 1 and 2 we see that the decay of the autocorrelation function is characterized by $\theta \simeq 0.6$, while correlations in the length of irreducible string up to a given point grow anomalously slowly, with $\beta \simeq 0.19$. Static correlation functions in this sector are characterized by power law decays as well. For instance, if we define an indecomposable substring as one which can be completely reduced, but which cannot be written as the concatenation of smaller substrings reducible to the null string [4], the probability of occurrence of an indecomposable substring of length r varies as $r^{-3/2}$ for large r. This is consistent with a random-walk picture of fluctuations in nearest-neighbour distance, and thus a static correlation exponent $\chi = 1/2$. The dynamical exponent $z = \chi/\beta$ is thus $\simeq 2.6$. A numerical diagonalization study [9] of the spectrum of the relaxation operator for finite systems with $L \leq 30$ yields a value $2.55 \pm .15$. This value of z is nonstandard, indicating that the dynamics in this sector is characterized by a new universality class. In this sense, our model is different then earlier attempt

[10] to generalize KPZ by allowing the height function to be an N component vector; it was argued that $z = 3/2$ independent of N in $d = 1$.

To summarize, we have shown that the deposition-evaporation model exhibits great diversity in the manner in which the autocorrelation function decays. There is strong evidence for the conjecture that in sectors with finite IS density ℓ/L, the dynamical behaviour is essentially the same as that of the spin correlation function of a system of hard core random walkers with the corresponding spin sequence. The diversity of relaxational behaviours in the DE model is linked to differences in correlations in spin patterns in the IS which labels that sector. In the sector composed of completely reducible configurations, the dynamics is characterized by a critical exponent $z \simeq 2.5$, indicative of a new universality class.

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Figure Captions

Figure 1: Variation of the decay of the autocorrelation function $C_i(t)$ in Sectors 1-5 (defined in text). Sector 1 (open triangles); Sector 2, sublattice A (+), sublattices B and C (filled triangles); Sector 3 (filled circles); Sector 4, sublattices A and C (open squares), sublattice B (open circles); Sector 5 (filled squares). The straight lines show power law decays $t^{-\theta}$ with $\theta = 0.25$ (top), 0.50 (middle), 0.62 (bottom); the curve shows a stretched exponential ($\sim \exp(-t^{1/2})$) decay.

Figure 2: Time-dependence of the mean squared fluctuations in the length of irreducible string $N(r,t)$ up to point r. The four sets of data (from top) pertain to Sectors 1-4 (shifted vertically for clarity) while the fifth pertains to Sector 5. The straight lines have slopes 0.50 and 0.38.