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Determining the CP properties of the Higgs boson

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The search and the probe of the fundamental properties of Higgs boson(s) and, in particular, the determination of their charge conjugation and parity (CP) quantum numbers, is one of the main tasks of future high-energy colliders. We demonstrate that the CP properties of a Standard Model-like Higgs particle can be unambiguously assessed by measuring just the total cross section and the top polarization in associated Higgs production with top quark pairs in e^+e^- collisions.

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We are at last entering the long awaited era, with the Large Hadron Collider (LHC) starting operation, of probing the mechanism by which the electroweak symmetry of the Standard Model (SM) of strong, weak and electromagnetic interactions is broken to provide masses for elementary particles. The SM makes use of one isodoublet complex scalar field which, after the spontaneous breaking of the $\mathrm{SU(2)_L \times U(1)_Y}$ symmetry, generates the weak gauge boson and the fermion masses and leads to the existence of one single spin–zero particle, the Higgs boson H, that is even under charge conjugation and parity (CP) [1, 2]. In extensions of the SM, the Higgs sector can be non-minimal and, for instance, the minimal supersymmetric extension (MSSM) is a constrained two–Higgs doublet model (2HDM), leading to a spectrum of five Higgs particles: two CP–even h and H, a CP–odd A and two charged $H^\pm$ bosons [2, 3].

Once a convincing signal for a Higgs boson has been established at the LHC, the next important step would be to determine its properties in all possible detail and to establish that it has the features that are predicted in the SM, that is: it is a spin–zero particle with the $\mathrm{J^{PC}} = 0^{++}$ assignments for parity and charge conjugation and that its couplings to fermions and gauge bosons are proportional to their masses. Ultimately, the scalar potential responsible for symmetry breaking should be reconstructed by measuring Higgs self–couplings. To achieve this goal, besides LHC preliminary analyses [4], the complementary high–precision measurements of the International Linear e^+e^- Collider (ILC) would be required [5, 6].

While the measurements of the spin, mass, decay width and couplings to fermions and gauge bosons of a SM–like Higgs boson are relatively straightforward [4, 5], the determination of its CP quantum numbers in an unambiguous way turns out to be somewhat problematic [7]. A plethora of observables that can be measured at the LHC and/or ILC, such as angular correlations in Higgs decays into $V = W, Z$ boson pairs [8, 9] or in Higgs production with or through these states [8, 10], are in principle sensitive to the Higgs spin–parity. However, if a Higgs boson is observed with substantial rates in these channels, it is very likely that it is CP–even since, even in the presence of CP violation, only the CP–even component of the HVV coupling is projected out. The VV couplings of a pure CP–odd A state are zero at tree–level and are generated only through tiny loop corrections.

The Higgs boson couplings to fermions provide a more democratic probe of its CP nature since, in this case, the CP–even and CP–odd components can have the same magnitude. One therefore needs to look at channels where the Higgs boson is produced and/or decays through these couplings. At the LHC, discarding the possibility of Higgs production in the main channel $gg \to H$ which proceeds through heavy quark loops followed by $H \to b\bar{b}, \tau^+\tau^-$ decays, that are subject to a rather large QCD background, one can only rely on Higgs production in association with top quarks, $pp \to t\bar{t}H$, followed by $H \to \gamma\gamma$ and $H \to b\bar{b}$. Techniques to discriminate between the CP–even or CP–odd state or a mixture, by exploiting the differences in the final state particle distributions in the production of the two states, have been suggested in Ref. [11]. However these channels are extremely difficult at the LHC: the CMS collaboration [4] has shown that the $H \to b\bar{b}$ signal cannot be extracted from the huge jet background while the decay channel $H \to \gamma\gamma$ is too rare and the two–photon decays from all production channels need to be combined to have a reasonably high signal significance [12].

In the clean environment of the ILC, the decay $H \to \tau^+\tau^-$ can be exploited [but only for $M_H \lesssim 140$ GeV when the branching ratio is significant] and the CP nature of the Higgs boson could be tested by studying the spin correlations between the the τ leptons [13, 14]. However, the Higgs has to be produced in the strahlung process $e^+e^- \to HZ$ and again, only the CP–even component of the HZZ coupling is projected out. The same argument holds for a heavy Higgs when the decay $H \to t\bar{t}$ is kinematically accessible.

One needs again to rely on Higgs production in the associated $e^+e^- \to t\bar{t}H$ process and in Ref. [15], it has been suggested to take advantage of the different phase space distributions for scalar and pseudoscalar Higgs production, to determine the CP nature of the $t\bar{t}H$ coupling and to probe CP violation when both CP components are present. The key point is to slice the phase space in configurations which are sensitive to the different CP components of the Higgs couplings and the latter are singled out, using appropriate weighting functions, with the additional requirement that the statistical error in the extraction of their coefficients is minimized. Besides the fact that it is not entirely clear whether this technique is experimentally feasible (as no detailed simulation has been attempted yet) and/or statistically costly (as the production cross section for the process is not very large), a simple physical interpretation of the difference between the behavior of a CP–even and CP–odd Higgs boson is lacking. Finally, let us recall that the determination of the Higgs CP quantum numbers can be performed unambiguously at the $\gamma\gamma$ version of the ILC [13, 16] but, unfortunately, this option seems very remote.

In this note, we propose a very simple and straightforward way to determine the CP nature of a SM–like Higgs boson. In the associated production process $e^+e^- \to t\bar{t}H$ [17, 18], the bulk of the cross section is generated when the Higgs is radiated off the heavy top quarks [18]. Besides allowing the determination of the important $Ht\bar{t}$ Yukawa coupling, we will show that the cross section, as well as the top quark polarization, behave in a radically different way for CP–even and CP–odd Higgs production. From the cross section measurement at two different energies and from the top quark polarization, one can exclude a CP–odd or a CP–even component of the Htt coupling with a very high confidence. A mixed CP state can be probed through simple CP–violating asymmetries for which we provide an example.

In the SM, associated production of Higgs bosons with a pair of top quarks, $e^+e^- \to t\bar{t}H$ [18], proceeds through two sets of diagrams: those where the Higgs boson is radiated off the $t, \bar{t}$ lines and a diagram where the Higgs boson is produced in association with a Z boson which then splits into an $t\bar{t}$ pair; Fig. 1. However, it has been shown that the latter contribution is very small, amounting for $\sqrt{s} \leq 1$ TeV to only a few percent [19]. In fact, since top quark pair production in e^+e^- collisions is known to be dominated by photon exchange, the bulk of the cross section is generated by the $e^+e^- \to \gamma^* \to t\bar{t}H$ subprocess. Detailed simulations have shown that the cross section can be measured with an accuracy of order 10% for masses up to $M_H \sim 200$ GeV [20].

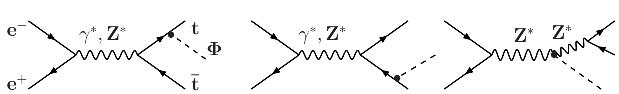

FIG. 1: Feynman diagrams for the associated production of Higgs bosons with a top quark pair.

For our discussion of a SM–like mixed CP Higgs state Φ, we use the following general form of the $t\bar{t}\Phi$ coupling

$$g_{\Phi tt} = -i \frac{e}{s_W} \frac{m_t}{2M_W} (a + ib\gamma_5) \qquad (1)$$

where the coefficients a and b are assumed to be real; $s_W \equiv \sin\theta_W = \sqrt{1 - c_W^2}$. One has $a = 1, b = 0$ in the SM and $a = 0, b \neq 0$ for a pure pseudoscalar. For the pseudoscalar case we take $b = 1$, consistent with a convenient normalization $a^2 + b^2 = 1$ chosen for the general case for a Higgs with an indefinite CP. Note that a non–zero value for the product ab will signal CP violation in the Higgs sector. For the $ZZ\Phi$ coupling, we will use the form, $g^{\mu\nu}_{ZZ\Phi} = -ic(eM_Z/s_W c_W) g^{\mu\nu}$ and for the numerical analysis we chose $c = a$ [15] as $c = 1(0)$ in the case of a CP–even (odd) Higgs boson. Thus, we will have only one free parameter b. Note, however, that this simple parameterization for a SM–like Higgs need not be true in, for instance, a general 2HDM, where a, b and c are three independent parameters.

We have calculated the cross section for the production of a mixed CP Higgs state in the process $e^+e^- \to t\bar{t}\Phi$, including the polarization dependence of the final state top quarks, using two independent methods: the helicity method in which the amplitudes are derived using the explicit form of the spinors and the Bouchiat–Michel method [21] in which the squared amplitudes are calculated with the trace technique. The lengthy results will be given elsewhere [22] and, for the unpolarized total cross section, they agree with those given in Ref. [18].

Neglecting the small contribution of the diagram involving the $ZZ\Phi$ vertex, the Dalitz density for the process, in terms of the energies $x_{1,2} = 2E_{t,\bar{t}}/\sqrt{s}$, reads

$$\frac{\mathrm{d}\sigma}{\mathrm{d}x_1 \mathrm{d}x_2} = \frac{3\alpha^2}{12\pi s} \left\{ \left[Q_e^2 Q_f^2 + \frac{(v_e^2 + a_e^2)(v_f^2 + a_f^2)}{(1-z)^2} + \frac{2 Q_e Q_f v_e v_f}{1 - z} \right] F_1^\Phi + \frac{v_e^2 + a_e^2}{(1-z)^2} a_f^2 F_2^\Phi \right\} |g_{\Phi tt}|^2 \qquad (2)$$

with $\alpha^{-1} = \alpha^{-1}(s) \sim 128$, $z = M_Z^2/s$ and $v_f = (2I_f^{3L} - 4Q_f s_W^2)/(4 s_W c_W)$, $a_f = 2I_f^{3L}/(4 s_W c_W)$ the usual Zff couplings given in terms of the charge Q_f and the isospin I_f^{3L}. The expressions of the form factors $F_{1,2}^\Phi$ for a scalar and pseudoscalar Higgs boson can be found in Ref. [18].

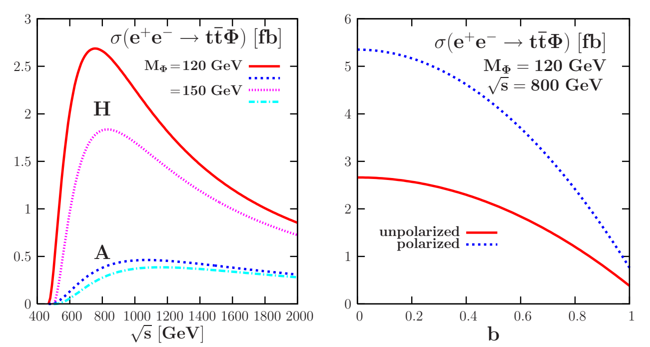

FIG. 2: The production cross sections $\sigma(e^+e^- \to t\bar{t}\Phi)$ for a scalar and a pseudoscalar Higgs boson as a function of $\sqrt{s}$ for two masses $M_\Phi = 120$ and 150 GeV (left) and for unpolarized and polarized $e^\pm$ beams as a function of the parameter b at $\sqrt{s} = 800$ GeV with $M_\Phi = 120$ GeV (right).

The left panel of Fig. 2 shows the production cross section $\sigma(e^+e^- \to t\bar{t}\Phi)$ (in which all contributions of the diagrams of Fig. 1 are included), for a pure scalar (H with $b = 0$) and a pseudoscalar (A with $b = 1$), as a function of the c.m. energy $\sqrt{s}$ for a Higgs mass of $M_\Phi = 120$ and also $M_\Phi = 150$ GeV for which the $\Phi \to \tau\tau$ decays are no longer effective. As can be seen, there is a striking difference in the threshold rise of the cross section in the scalar and pseudoscalar cases. In addition, for the same strength of the Φtt coupling, there is an order of magnitude difference between the H and A cross sections at moderate energies. It is only for very high energies, $\sqrt{s} \gg 1$ TeV, that one reaches the chiral limit where the two cross sections are equal, up to the small contribution of the diagram with the $ZZ\Phi$ coupling, as we have verified. Thus, these two features offer an extremely powerful discriminator of the CP properties of the spin–zero particle produced in association with the $t\bar{t}$ pair.

The very different behaviors of the cross sections near the production threshold can be understood in terms of simple angular momentum conservation arguments. Very close to the energy threshold, the simultaneous demand of angular momentum and parity conservation implies that, for scalar and pseudoscalar Higgs production, the orbital angular momentum of the overall $t\bar{t}\Phi$ system will be 0 and 1, respectively. Thus, in the A case there will be a softer dependence on the deviation from threshold, $\rho = 1 - 2m_t/\sqrt{s} - M_\Phi/\sqrt{s}$, and the rise is slower.

As a matter of fact, a look at the analytic expressions of the form factors $F^\Phi_{1,2}$, when expanded around threshold, gives for a light Higgs boson

$$\begin{aligned} F_1^H &= -F_2^H \simeq 12\left[m_t^2/(M_H\sqrt{s})\right]^{3/2} \ \rho^2 \\ F_1^A &= -F_2^A \simeq 4\left[m_t^4/(M_A s\sqrt{s})\right]^{1/2} \ \rho^3 \ . \end{aligned} \qquad (3)$$

The ρ^2 and ρ^3 dependence observed for the H and A case, respectively, is consistent with the above expectation. The difference in the threshold behavior of the cross sections is strong enough such that its measurement at just two different c.m. energies allows a clear determination of the CP properties of the Φ state. For instance, for $M_\Phi = 120$ GeV, the ratios of the cross sections measured at $\sqrt{s} = 800$ GeV and $\sqrt{s} = 500$ GeV is ~ 63 and ~ 7.5 respectively, for the pseudoscalar and scalar cases. It is worth noting that taking such a ratio will make the conclusion robust with respect to the effect of the top quark Yukawa coupling, the higher order radiative corrections or some systematic errors in the measurement.

For the case of a Higgs boson Φ with indefinite CP quantum numbers, it is instructive to study the b dependence of $\sigma(e^+e^- \to t\bar{t}\Phi)$ at a given energy and fixed M_Φ. It is clear that the total cross section being a CP–even quantity depends only on b^2. The right–hand panel of Fig. 2 illustrates the sensitivity to the parameter b, assuming $M_\Phi = 120$ GeV and $\sqrt{s} = 800$ GeV for unpolarized and polarized $e^\pm$ beams. For the latter, we assume the standard ILC values of $P_{e^-} = -0.8$ and $P_{e^+} = 0.6$ which lead to an increase of the total rate by a factor of two.

Due to its large decay width, $\Gamma_t \sim 1.5$ GeV, the top quark decays much before hadronization and its spin information is translated to the decay distribution before being contaminated by strong interaction effects. The lepton angular distribution in the decay $t \to bW \to b\ell\nu$ is independent of any non–standard effects in the decay vertex and is therefore a pure probe of the physics associated with the top quark production process [23]. Hence, it is interesting to see what probe of b is offered by the net polarization of the top quark; see also Ref. [24]. We have calculated the degree of t–quark polarization P_t which, for unpolarized and polarized beams, is given by

$$P_t = \frac{\sigma(t_L) - \sigma(t_R)}{\sigma(t_L) + \sigma(t_R)} \ . \qquad (4)$$

The left panel of Fig. 3 shows the expected polarization value as a function of $\sqrt{s}$ for the $H(b=0)$ and $A(b=1)$ cases, again for $M_\Phi = 120$ and 150 GeV. The degree of top polarization is also strikingly different in the two cases and has again a very different threshold dependence. Further, since P_t itself is constructed as a ratio of cross sections, the conclusions drawn from its value, will not be subject to the effect of the possibly model dependent normalization of the overall $t\bar{t}\Phi$ strength, higher order corrections, etc. P_t, a P–odd quantity, receives contributions from the interference between the γ and all Z–exchange diagrams; the one coming from the diagram involving the $ZZ\Phi$ vertex being small. Since the parity violating effect for the emission of a (pseudo)scalar is controlled by the (vector) axial–vector $Zt\bar{t}$ coupling, one expects the ratios of P_t values away from the threshold to be the ratio of the two couplings, $a_t/v_t \sim 3$. Indeed, at $\sqrt{s} = 800$ GeV this ratio is about a factor of three as seen from both the panels in Fig. 3. The use of polarized initial beams does not affect these relative values, but increases the absolute value of the top polarization by a factor of three in each case as expected.

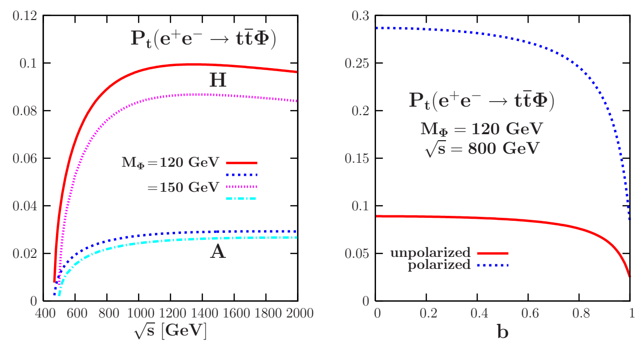

FIG. 3: The top quark polarization in the process $e^+e^- \to t\bar{t}\Phi$ for a scalar and a pseudoscalar Higgs boson as a function of $\sqrt{s}$ for two masses $M_\Phi = 120$ and 150 GeV (left) and with unpolarized and polarized $e^\pm$ beams as a function of the parameter b at $\sqrt{s} = 800$ GeV for $M_\Phi = 120$ GeV (right).

The discussions so far show us clearly that the threshold behavior of the cross section as well as the measurement of the top polarization will allow a clear discrimination between a scalar and pseudoscalar Higgs boson. The next natural question to ask is how these observables may be used to get information about the CP–mixing; i.e. the value of b. As can be seen from Figs. 2 and 3, the b–dependence of the cross section around $b = 0$ is much steeper than that of the polarization asymmetries.

Ignoring systematical errors, the sensitivity of the observable $O(b)$ to the parameter b at $b = b_0$ is Δb, if $|O(b) - O(b_0)| = \Delta O(b_0)$ for $|b - b_0| < \Delta b$, where $\Delta O(b_0)$ is the statistical fluctuation in O at an integrated luminosity $\mathcal{L}$. For the cross section σ and the polarization P_t, the statistical fluctuation at a level of confidence f are given by $\Delta\sigma = f\sqrt{\sigma/\mathcal{L}}$ and $\Delta P_t = f/\sqrt{\sigma\mathcal{L}} \times \sqrt{1 - P_t^2}$.

The sensitivity Δb from the measurement of the cross section is displayed in Fig. 4 (left) for $M_\Phi = 120$ GeV, at $\sqrt{s} = 800$ GeV with $\mathcal{L} = 500$ fb^{-1}. For polarized $e^\pm$ beams, it varies from 0.25 for $H(b=0)$ to 0.01 for $A(b=1)$. This is a rather precise determination obtained from a very simple measurement. To put this in perspective, one may note that the study of correlations in $\Phi \to \tau\tau$ decays

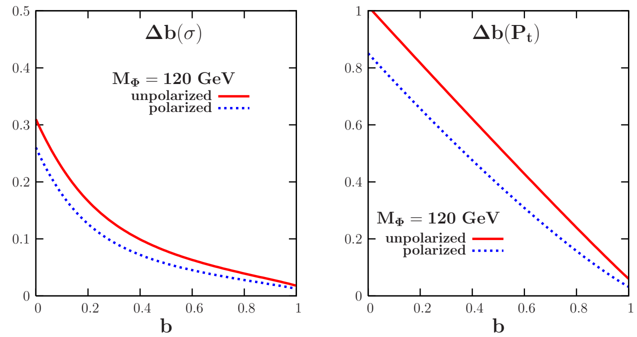

FIG. 4: The sensitivity of the cross section (left) and the top quark polarization (right) on the parameter b for $M_\Phi = 120$ at $\sqrt{s} = 800$ with $\mathcal{L} = 500$ fb^{-1}.

yields a $\sim 10\%$ measurement of b (which is systematics dominated) assuming SM production rates, i.e. $b = 0$. Further, in the $e^+e^- \to t\bar{t}\Phi$ case, the sensitivity is very good for $b=1$ while the $\Phi \to \tau\tau$ decays cannot be used anymore as A production through the AZZ coupling is strongly suppressed. The top polarization asymmetry is less sensitive to b and, for polarized initial beams, Δb varies from 0.8 near $b=0$ to 0.03 near $b=1$; Fig. 4 (right).

As mentioned before, the cross section and the degree of top polarization being CP–even, cannot depend linearly on b. On the other hand, observables depending on the sine of the azimuthal angle are linear in b and thus, can probe CP–violation directly. The up–down asymmetry of the antitop quark with respect to the top–electron plane is an example of such an observable. We have explicitly checked that this asymmetry is indeed linear in the parameter b and can reach values of order 5% for $M_\Phi = 120$ GeV at $\sqrt{s} = 800$ GeV. The non –zero value of the asymmetry is due to the presence of the channel involving the $ZZ\Phi$ coupling [25]. More details on the CP–odd asymmetries and the probe of CP–violation will be given elsewhere [22].

In summary: the total cross section and the top polarization asymmetry for associated Higgs production with top quark pairs in e^+e^- collisions, $e^+e^- \to t\bar{t}\Phi$, provide a very simple and unambiguous determination of the CP quantum numbers of a SM–like Higgs particle.

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