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FINITE TEMPERATURE PHASE TRANSITION IN SU(2) LATTICE GAUGE THEORY WITH EXTENDED ACTION

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ABSTRACT

We study the three dimensional fundamental-adjoint $SU(2)$ lattice gauge theory at finite temperature by Monte Carlo simulations. We find that the finite temperature deconfinement phase transition line joins the first order bulk phase transition line at its endpoint. Moreover, across the bulk transition line, the Polyakov loop undergoes a discontinuous jump implying the existence of both confining and deconfining phases on its two sides. Implications for universality and the nature of the confining-deconfining transition are discussed.

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1. **INTRODUCTION**

Quantum field theories are defined with an ultraviolet cut-off. The choice of this regulator is highly non-unique. In perturbation theory the renormalizability criterion ensures that the physical answers are independent of the cut-off parameter used. In fact, different choices of the regulators only change the scale in the theory, leaving physics unchanged.

A non-perturbative way to regulate a theory is to define it on a lattice, which has a built-in ultraviolet cut-off (the lattice spacing a). A regulated theory on the lattice corresponding to a particular theory in the continuum is non-unique due to the large freedom of choice of lattice action and the lattice type. This non-uniqueness is expected to be a lattice artifact and should disappear as the lattice spacing a tends to zero (or the lattice correlation length $\rightarrow \infty$), resuling in a universal continuum limit. The Wilson formulation [1] of lattice gauge theory is a particular choice of lattice action from a much more general class, given by

$$S = \sum_P \sum_R \beta_R \; Re \; Tr_R U_P \quad . \tag{1}$$

Here U_P, the plaquette action, is an ordered product of the four group elements U_{ij} ($\in SU(N)$) corresponding to the four links of an elementary plaquette. The sum over R extends over all the representations of the gauge group and the sum over P runs over all plaquettes. Tr_R denotes the trace over colour space in the representation R.

A variety of different forms of lattice actions have been employed in the past to check the scaling behaviour and the universality of the $SU(2)$ gauge theories. Dimensionless ratios of physical quantities, such as $M_G/\sqrt{\sigma}$[2], or $T_c/\sqrt{\sigma}$[3], where M_G is the lightest glueball mass, T_c is the deconfinement temperature and σ is the string tension, have been computed for the $SU(2)$ lattice gauge theory and shown to be almost independent of the lattice actions used. Even more stringently, functions of such dimensionless ratios, e.g., the energy density as a function of T/T_c, have been successfully used to demonstrate[4] universality. While these results have boosted our confidence in the existence of a non-perturbative continuum limit of the $SU(2)$ gauge theory, it is still not clear whether more qualitative aspects of the theory, such as the order of the deconfinement phase transition, also exhibit this universality. We investigate the confining-deconfining behaviour in the extended coupling plane of a particular class of theories belonging to eq. (1) and compare it with that of the usual Wilson action. The organization of this paper is as follows: In sect. 2 we introduce the model and the motivation to study it at non-zero temperatures. In sect. 3 we give a very brief review of the observables computed and their critical behaviour. In sect. 4 we discuss the data and its analysis. Sect. 5 consists of the observations and their implications.

2. THE MODEL

A well studied model belonging to the above class of $SU(2)$ gauge theory is described by the action

$$S = \sum_P \left(\beta \left(1 - \frac{1}{2} Tr_F U_P \right) + \beta_A \left(1 - \frac{1}{3} Tr_A U_P \right) \right) \quad . \tag{2}$$

Here F and A denote the fundamental and adjoint representations respectively. Comparing the naive classical continuum limit of eq. (2) with the standard $SU(2)$ Yang-Mills action, one obtains

$$\frac{1}{g_u^2} = \frac{\beta}{4} + \frac{\beta_A}{3} \quad . \tag{3}$$

Here g_u is the bare coupling constant of the continuum theory. One can also characterize the model with a pair of couplings (g_u, θ) with $\tan\theta = \beta_A/\beta$. In the non-perturbative continuum limit (i.e. $a \to 0$) each of these theories, characterized by a given θ, flow to the same critical fixed point, $g_u^c = 0$. The corresponding asymptotic scaling relation [5] is

$$a = \frac{1}{\Lambda(\theta)} \exp\left[-\frac{1}{2\beta_0 g_u^2} \right] \left[\beta_0 g_u^2 \right]^{\frac{-\beta_1}{2\beta_0^2}} \quad , \tag{4}$$

where

$$\log \frac{\Lambda(0)}{\Lambda(\theta)} = \frac{5\pi^2}{11} \frac{6\tan\theta}{(3 + 8\tan\theta)} \quad . \tag{5}$$

Here β_0 and β_1 are the usual first two coefficients of the β function for the $SU(2)$ gauge theory.

This model has been extensively studied after Bhanot and Creutz [6] found it to have a rich phase structure (fig.1). At $\beta_A = 0$, it reduces to the standard Wilson action. Along the $\beta = 0$ axis it describes the $SO(3)$ model which has a first order phase transition at $\beta_A^c \sim 2.5$. At $\beta_A = \infty$ it describes the Z_2 lattice gauge theory again with a first order phase transition at $\beta^c = \frac{1}{2}\ell n(1+\sqrt{2}) \approx 0.44$ [7]. Bhanot and Creutz found that these first order transitions extend into the (β,β_A) plane, ending at an apparent critical point located at (1.5,0.9). These transition lines are shown in fig.1 by continuous lines. The same phase diagram was also later produced qualitatively by mean field theory techniques [8]. The Λ-ratio was found to be smaller than its asymptotic value given by eq. (5) by a factor of $\sim$3.5 in a Monte Carlo determination[9] from the ratio of string tension evaluated on β_A =constant lines around (1.5, 0.9) and (2.2, 0.0) in the (β, β_A)-plane. It was later shown[4] that the θ dependance of this factor further differs rather strongly from eq. (5) on the $\beta_A = 0.9$ line. Thus in the vicinity of the endpoint of the first order line, the asymptotic scaling relation is known to be violated strongly. Ref. [9] also found that the string tension does not go through zero in this region, suggesting the endpoint to be a higher order phase transition, although the location of endpoint was obtained in Ref. [6] by extrapolating the discontinuity in average action to zero.

The motivation for Bhanot and Creutz to study this model by Monte Carlo simulation was to show that non-abelian lattice gauge theories need not be free of singularities to maintain confinement in the asymptotic scaling regime. As fig. 1 shows, one can go smoothly around the line of first order singularities without affecting confinement in the asymptotic regime. However, this is perhaps an incomplete picture of the phase diagram of the model. Since the numerical results, which established the phase diagram were obtained on small, N_σ^4, lattices for N_σ = 5-7, one must worry about a possible deconfinement phase transition on these lattices at sufficiently small coupling g_u. As all finite lattices are necessarily at finite temperature as well, the confining-deconfining transition line must exist in the extended plane on all lattices and one needs to understand its behaviour on larger lattices to draw conclusions about the zero temperature phase diagram of the theory and its possible implications to the continuum limit of the confining phase. The deconfinement phase transition has recently been extensively studied at

$\beta_A = 0$ [10, 11]. We follow this transition in the β_A direction and find that it merges with the bulk phase transition line found by Bhanot and Creutz. The phase diagram therefore splits into seperated confining and deconfining phases.

3. THE OBSERVABLES AND THEIR CRITICAL BEHAVIOUR

On a finite lattice of size $N_\sigma^3 \times N_\tau$ the volume and temperature are given by [12]

$$V = (N_\sigma a)^3 \text{ and } T = \frac{1}{N_\tau a} \quad . \tag{6}$$

Here a is the lattice spacing governed by the bare coupling g_u for sufficiently small a, as shown in eqns. (4-5). To avoid big finite spatial volume effects, one chooses $N_\sigma > N_\tau$ for finite temperature simulations. One basic observable which we studied is the average plaquette energy $\langle P \rangle$, where

$$P = \frac{1}{2} \frac{\sum_P Tr_F U_P}{6 N_\sigma^3 N_\tau} \tag{7}$$

Due to periodic boundary conditions in the timelike direction, the $SU(2)$ lattice gauge theories described by eq. (1) and in particular eq. (2) have a Z_2 invariance at finite temperature corresponding to the center of the group. Under this

$$U_0(\vec{n}, \tau_0) \rightarrow z U_0(\vec{n}, \tau_0) \;\; \forall n, \quad \tau_0 : \text{fixed} \; , \quad \text{and} \quad z \;\; \in Z_2 \quad . \tag{8}$$

Here $U_0(\vec{n}, \tau)$ is the timelike link at the lattice site $(\vec{n}, \tau)$. This transformation leaves the action invariant but the Polyakov loop, defined by,

$$L(\vec{n}) = \frac{1}{2} Tr \prod_{\tau=1}^{N_\tau} U_0(\vec{n}, \tau), \tag{9}$$

changes:

$$L \rightarrow zL \quad . \tag{10}$$

A nonvanishing value for $\langle L \rangle$ signals a spontaneous break-down of the global Z_2 symmetry. The thermal expectation value of the Polyakov loop or its average value $L = \frac{1}{N_\sigma^3} \sum_{\vec{n}} L(\vec{n})$ can also be shown to be the order parameter for the deconfinement phase transition since it is a measure of the free energy of an isolated free quark [12, 14].

It is been argued that the effective theory for this order parameter, obtained by integrating out all other degrees of freedom, is in the same universality class as the Ising model in 3-dimensions [12, 13]. Thus, if the deconfinement phase transition is of second order, then it will have the same critical exponents as those of the Ising model, provided the universality class does not have another second order phase transtion. On an infinite lattice, these exponents are β, γ and ν corresponding to the order parameter itself, its corresponding susceptibility and the correlation length:

$$\langle L \rangle \propto |T - T_c|^{\beta} \quad \text{for} \quad T \to T_c^+ \tag{11}$$

$$\chi \propto |T - T_c|^{-\gamma} \quad \text{for} \quad T \to T_c \tag{12}$$

$$\xi \propto |T - T_c|^{-\nu} \quad \text{for} \quad T \to T_c \quad . \tag{13}$$

For the 3-d Ising model $\beta \approx 0.325$, $\gamma \approx 1.24$ and $\nu \approx 0.63$. For the finite temperature $SU(2)$ gauge theory, one obtains these exponents from Monte Carlo simulations by simply fitting the order parameter[10, 14, 15] or by using the finite size scaling theory[16] for the susceptibilty. According to the latter, the susceptibility on a lattice of spatial extent N_σ is expected to grow like

$$\chi \propto N_\sigma^{\omega} \quad , \tag{14}$$

where $\omega = \gamma/\nu = 1.97$ according to the universality prediction above. If the phase transition were to be of first order instead, then one expects the exponent $\omega = 3$, corresponding to the dimensionality of the space [17]. In addition, of course, the order parameter is expected to exhibit a sharp, or even discontinuous, jump and the corresponding probability distribution should show a double peak structure. For $\beta_A = 0$, the universality prediction was verified by Monte Carlo simulation by Engels et. al. [10], who found $\omega = 1.93 \pm 0.03$.

4. DATA AND ANALAYSIS

Our Monte Carlo simulations were done on $8^3 \times 4$ and $10^3 \times 4$ lattices, using the Metropolis algorithm. Around $\beta_A = 1.1$ where the nature of phase transition changes from 2nd order to 1st order, we also simulated the model on a $12^3 \times 4$ lattice. The Ferrenberg-Swendsen [18] technique was used to extrapolate the data at neighbouring couplings to locate the critical point. Once located, much longer runs were performed at these points. (200,000, 150,000 and 50,000 sweeps on $8^3 \times 4$, $10^3 \times 4$ and $12^3 \times 4$ lattices respectively). Table 1 shows the values of the couplings (β,β_A) where simulations were performed along with the critical couplings (β_c) determined from the peak in susceptibility using Ferrenberg- Swendsen method. The last entry in the table (1) is the value of finite size scaling exponent (ω) calculated from the values of the susceptibility peak on $8^3 \times 4$ and $10^3 \times 4$. At β_A=0.9 and 1.1 the second value of ω is calculated from the susceptibility peaks on $8^3 \times 4$ and $12^3 \times 4$ lattices.

As the expectation value of the Polyakov loop is always zero on a finite lattice, we measure the absolute value of L after taking its lattice average. The values of average Polyakov loop $\langle|L|\rangle$, extrapolated by the Ferrenberg-Swendsen technique in the neighborhood of critical points are plotted in fig. 2-a,b,c,d for all β_A along with the actual values determined from the individual runs. The error bars shown were determined from binning. As seen from fig. 2, the slope of the Polyakov loop curve keeps increasing with increasing value of β_A. All the corresponding values of susceptibilities,

$$\chi = N_\sigma^3(\langle L^2\rangle - \langle|L|\rangle^2), \tag{15}$$

are plotted in fig. 3-a,b,c,d. We have also shown the first order and second order predictions for the susceptibility peaks on the $10^3 \times 4$ and $12^3 \times 4$ lattices based on the measured susceptibility peak on the $8^3 \times 4$ lattice and $\omega = 3.0$ and 1.97 respectively. As is clear from the susceptibility plots, the universality hypothesis is well supported by the data for all the β_A including and up to $\beta_A = 0.9$. Instead of using the critical exponents above, one can try to obtain them from the data by simply comparing the peak heights. The corresponding exponents from the data for susceptibility are given in table 1. They exhibit an excellent agreement with the universality prediction of

$\omega = 1.97$ for $\beta_A \leq 0.9$, confirming the second order nature of the phase transition.

At $\beta_A = 1.1$ there is evidence that the transition has become first order. The histograms of $|L|$ (fig. 4) on $N_\sigma = 10, 12$ show a definite two peak structure with the dips between the two peaks increasing with N_σ . The value of the susceptibility peak, however, lies below the first order predicted values but definitely outside the second order predicted region also. This is unlike the clear second order nature of the transition up to $\beta_A = 0.9$. It is possible that at $\beta_A = 1.1$, the corrections to the leading scaling prediction of eq. (14) are large and thus larger lattices are needed to ascertain the true nature of the phase transition. It has been found in simulations of Potts models with first order transitions, that much larger lattices are needed to see the correct finite size scaling exponents than are needed to see a two peak histogram [19]. To verify the onset of the first order nature of the phase transition for $\beta_A \geq 1.1$, we also simulated the model at $\beta_A = 1.5$, where we find it to be strongly first order. We performed 10^5 sweeps at $\beta = 1.05$, and 40,000 sweeps at $\beta = 1.02$ and 1.0535 (fig. 5-a,b,c). Due to the very large tunnelling time ($> 10^5$ sweeps), we were unable to locate the exact critical coupling and verify the first order scaling relation. But the evolution curves for both $|L|$ and P at $\beta = 1.02$, 1.05 and 1.0535 clearly indicate the co-existence of two phases seperated by a large barrier. The intermediate runs at $\beta = 1.03$ (50,000 sweeps) and at $\beta = 1.04$ (100,000 sweeps) also did not show any tunnelling. The data at $\beta_A = 1.1$ and 1.5 also indicates that the Polyakov loop has a discontinuity exactly across the bulk transition line, coincident with the discontinuity in the average value of the plaquette. The discontinuity in the values of average Polyakov loop (plaquette energy) at $\beta_A = 1.1$ and 1.5 is 0.23 (0.04) and 0.48 (0.25) respectively.

5. DISCUSSION

From fig. 1 and from the results presented in the previous section, it is clear that for $\beta_A \simeq 1.0$, the deconfinement phase transition joins the bulk transition line, and surprisingly remains apparently coincident for all values of $\beta_A \geq 1.0$. As far as, we can tell from our simulations, there is really no evidence for two transitions. The discontinuity in both the average plaquette and the average Polyakov loop is located at the same β and the latter jumps from an essentially zero value to a rather large value at the same point. The fluctuations in both the phases are rather small and are not suggestive of any superimposed second order phase transition, although the dominant first order nature could make it difficult to see them. The scaling behaviour of the finite temperature and bulk transitions is different for $N_\tau, N_\sigma \to \infty$. The β_c for the former should move to infinity, whereas it should remain anchored at finite value in the latter case. Therefore the joining together of these two transitions is a curious phenomenon, which, if it persists in this limit, would lead to a paradox. In the following, we discuss three possible scenarios and their implications as one approaches the continuum limit of the theory.

A] The most conservative possibility is that the joining of the two lines is accidental for $N_\tau = 4$ but they will eventually separate out for large enough N_τ. This scenario is consistent with Bhanot and Creutz's interpretation and the observed universality of the deconfinement phase transition from the equality of its scaling exponents with those of the 3-d Ising model. To explore this possibility a little further, we simulated the model at $\beta_A = 1.1$ and 1.5 on a $12^3 \times 6$ lattice also. Due to the large simulation time we could only bracket the critical coupling by looking at the peak position of the susceptibility and also the behaviour of the histograms. Our estimated β_c for β_A=1.1, $N_\tau = 6$ is 1.3425 ± 0.0025 which should be compared with the critical coupling on $8^3 \times 4$ lattice, $\beta_c = 1.3270 \pm 0.0008$, obtained from our data on the susceptibilty peaks[3]. Therefore, the shift in β_c in going from $8^3 \times 4$ to $12^3 \times 6$ lattice at $\beta_A = 1.1$ is definitely less than 0.0188. Moreover the transitions still remained coincident and first order. At $\beta_A = 1.5$, due to the large tunnelling time even on the $8^3 \times 4$ lattice, we ran a hysteresis cycle instead of runs at individual couplings. The forward hysteresis cycle at $\beta_A = 1.5$, on $12^3 \times 6$ shows $\beta_c < 1.065$. On the other hand, on the $8^3 \times 4$ lattice the evolution curves (fig. 5-a,b,c) show that the $\beta_c > 1.02$. Therefore the shift in β_c on going from $N_\tau = 4$ to $N_\tau = 6$ is also less than 0.045 here. Recall that at $\beta_A = 0$ this shift was observed to be 0.13[11], and if the asymptotic scaling relation were to be valid in this region, the expected shift in the critical coupling is 0.15. Of course, the scaling relations, eqs. (3-4) are known to be strongly violated in

[3]The errorbars given are upper limits based on the couplings of nearby runs which were seen to be definitely confined and deconfined.

this region. Nevertheless, the shifts are too tiny and at least suggest that much larger lattices are required to confirm this scenario if one is to move away from the bulk transition and see a separate second order deconfinement phase transition.

B] Taking the coincidence of finite temperature deconfinement phase transition and the bulk phase transition more seriously, one has basically two extreme alternatives. Either the interpretation of Bhanot and Creutz[6] of the bulk transition line was incorrect, and one has only a deconfinement transition to deal with, or the identification of the deconfinement transition at $\beta_A = 0$ needs to be investigated more, and one has only a bulk transition line on which the order of the transition changes from first to second at the point (1.5, 0.9). Both the alternatives have their own problems and inconsistencies with the published results. Thus, the nonzero string tension reported in Ref. [9] around this point suggests a higher order phase transition at that point while the results on the growth of susceptibility, reported both in this work, and Ref. [10] are indicative of a second order phase transition for $\beta_A \leq 0.9$. Indeed, if the entire phase transition line is bulk then all the evidence for the universality class of the deconfinement phase transition at $\beta_A = 0$ will have to be treated as accidental and a priori the global Z_2 center symmetry has also no unique role to play in that case. This is because for a four dimensional bulk system there is no particular reason to choose periodic boundary conditions, from which the Z_2 symmetry arises, whereas this is required for the finite temperature interpretation. However it should be pointed out that the use of an asymmetric lattice will mask the true scaling behavior of a 4-d bulk system, because the system will behave as a thin film or layer system which will have a three dimensional critical behavior in the limit $N_\sigma \to \infty$, N_τ finite [20]. From this point of view it is not surprising that the system exhibits a 3-d critical behavior, typified by the 3-d Ising model, in this limit, even if the underlying transition is a 4-d bulk transition which would presumably have a completely different critical behavior in the limit $N_\tau = N_\sigma \to \infty$. Of course, a crucial prediction of this hypothesis is that the entire transition line will persist at finite β_c as $N_\tau \to \infty$ for *all* β_A, and no connection of the confining phase of the pure gluonic theory with its asymptotically free phase will be possible. Therefore the continuum limit will be always deconfining, a possibility which has been raised previously in several contexts[21].

On the other hand, if the entire line is a deconfinement phase transition line, it will move towards the right hand side of the phase diagram as $N_\tau \to \infty$. However, our observation of an extremely small shift in β_c as N_τ went from 4 to 6 at $\beta_A = 1.1$ makes it unlikely to observe this shift for any N_τ used in any numerical simulations up to now. Moreover, the large discontinuity in the plaquette and the Polyakov loop then suggests the deconfinement phase transition to be of first order at β_A =1.1 and 1.5. Thus, the universality class seems to change with an apparently irrelevant coupling on these lattices. One can then only hope that the universality will be restored on larger lattices.

To summarize, we have studied the finite temperature deconfinement phase transition for the extended $SU(2)$ action on $N_\sigma^3 \times N_\tau$ lattices with $N_\sigma = 8$, 10, 12 and $N_\tau = 4$ and 6. We found that as β_A increases, the deconfinement phase transition moves towards smaller β and appears to join the previously known bulk transition line. The universality of the critical exponents for the $SU(2)$ deconfinement phase transition thus seems to be violated since the finite temperature deconfinement phase transition appears to change from being a second order one to a first order transition unless this behaviour is purely accidental on small lattices. Future simulations on larger lattices could, of course, yield a separation of the bulk transition from the deconfinement phase transition, however, no such hints could be obtained in the limited variations studied here. It will be interesting to study whether a similar phenomenon also takes place for the $SU(3)$ theory. We conclude that much larger lattices are required to settle the issues raised in this paper.

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FIGURE CAPTIONS

Fig.1 The phase diagram of the extended SU(2) lattice gauge theory. The solid lines are from simulations done on a 5^4 lattice by Bhanot and Creutz[6]. The broken line is the finite temperature deconfinement phase transition line based on the results discussed in the paper.

Fig.2 Average Polyakov loop ($\langle |L| \rangle$) at (a) β_A=0.5, (b) β_A=0.75, (c) β_A=0.9 and (d) β_A=1.1 on $8^3 \times 4$ and $10^3 \times 4$ lattices. At $\beta_A = 0.9$ and 1.1, results on $12^3 \times 4$ lattice are also shown. The points with error bars are results of simulations and the curves are extrapolations by the Ferrenberg-Swendsen technique from the run closest to the peak.

Fig.3 The susceptibility curves at (a)β_A=0.5, (b)β_A=0.75, (c)β_A =0.9 and (d)β_A= 1.1 on $8^3 \times 4$, $10^3 \times 4$ and $12^3 \times 4$ lattices. The first order (F.O) and second order (S.O) predictions, explained in the text, are explicitly shown. The higher (lower) predictions are for the $12^3 \times 4$ ($10^3 \times 4$) lattice. The points and curves are as in Fig. 2.

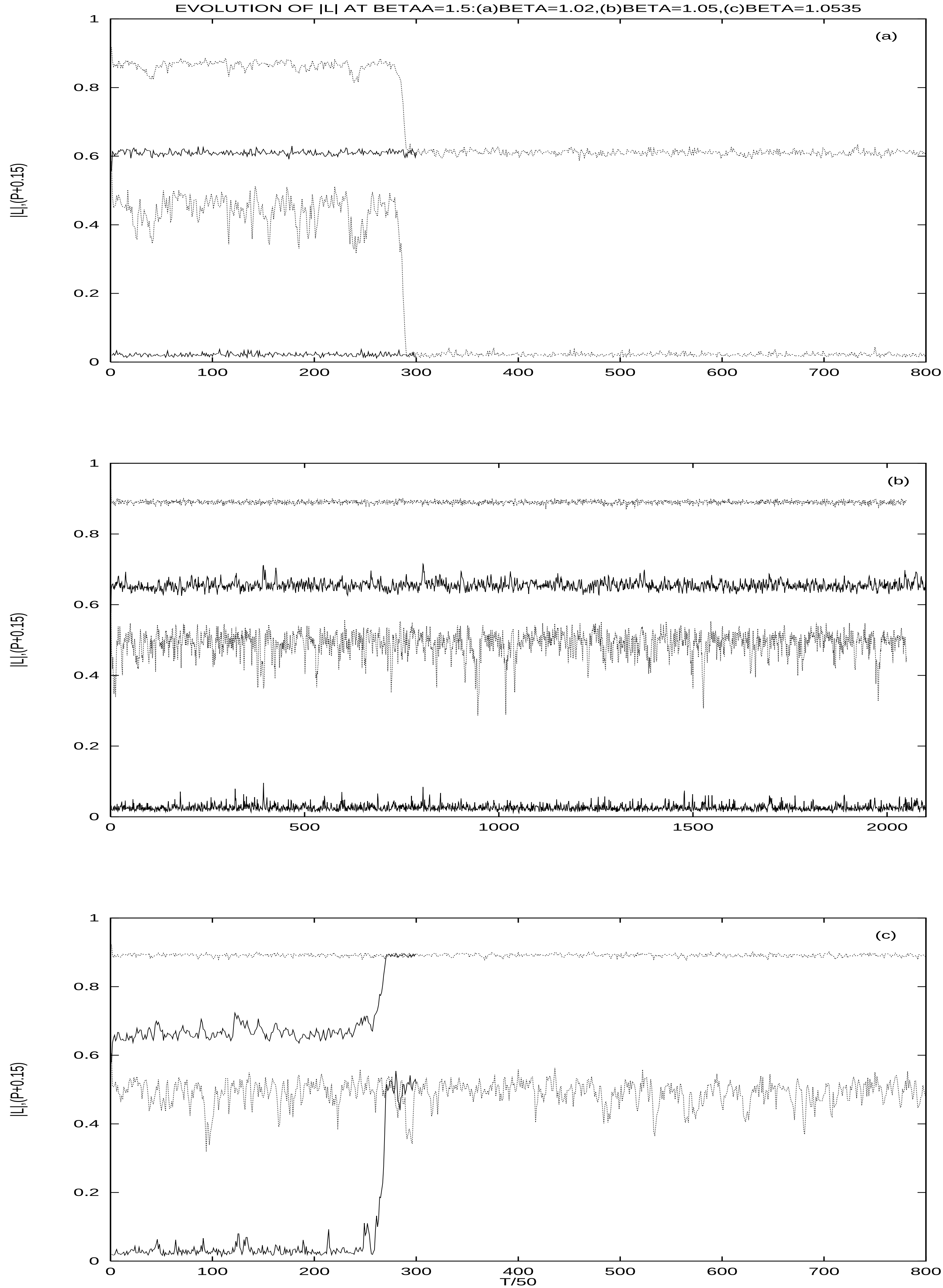
Fig.4 The probability density of $|L|$ at $\beta_A = 1.1$ on $8^3 \times 4$ ($\beta = 1.32635$), $10^3 \times 4$ ($\beta = 1.3265$) and $12^3 \times 4$ ($\beta = 1.32685$) lattices.

Fig.5 Evolution of $|L|$ and (P+0.15) on $8^3 \times 4$ lattice, at $\beta_A = 1.5$ and (a)$\beta = 1.02$, (b)$\beta = 1.05$, (c)$\beta = 1.0535$. The upper broken lines are cold starts and the lower solid lines are hot starts.

Table 1

The values of (β, β_A) at which simulations were performed, β_c and the finite size scaling exponent ω. The expected value for ω is 1.97 (3.0) if the deconfining phase transion is second order (first order).

β_A	β	β_c	ω
0.5	1.830 : $8^3 \times 4$	1.831	
	1.830 : $10^3 \times 4$	1.830	1.92(29)
0.75	1.610 : $8^3 \times 4$	1.613	
	1.610 : $10^3 \times 4$	1.609	1.53(32)
0.9	1.489 : $8^3 \times 4$	1.486	
	1.489 : $10^3 \times 4$	1.485	2.24(64)
	1.489 : $12^3 \times 4$	1.486	2.10(22)
1.1	1.32620 : $8^3 \times 4$	1.3276	
	1.32635 : $8^3 \times 4$	1.3275	
	1.32700 : $8^3 \times 4$	1.3270	
	1.32650 : $10^3 \times 4$	1.3273	
	1.32700 : $10^3 \times 4$	1.3271	2.31(28)
	1.32685 : $12^3 \times 4$	1.3270	2.34(15)



EVOLUTION OF |L| AT BETAA=1.5:(a)BETA=1.02,(b)BETA=1.05,(c)BETA=1.0535
(a)
(b)
(c)
|L|,(P+0.15)
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This figure "fig2-2.png" is available in "png" format from:

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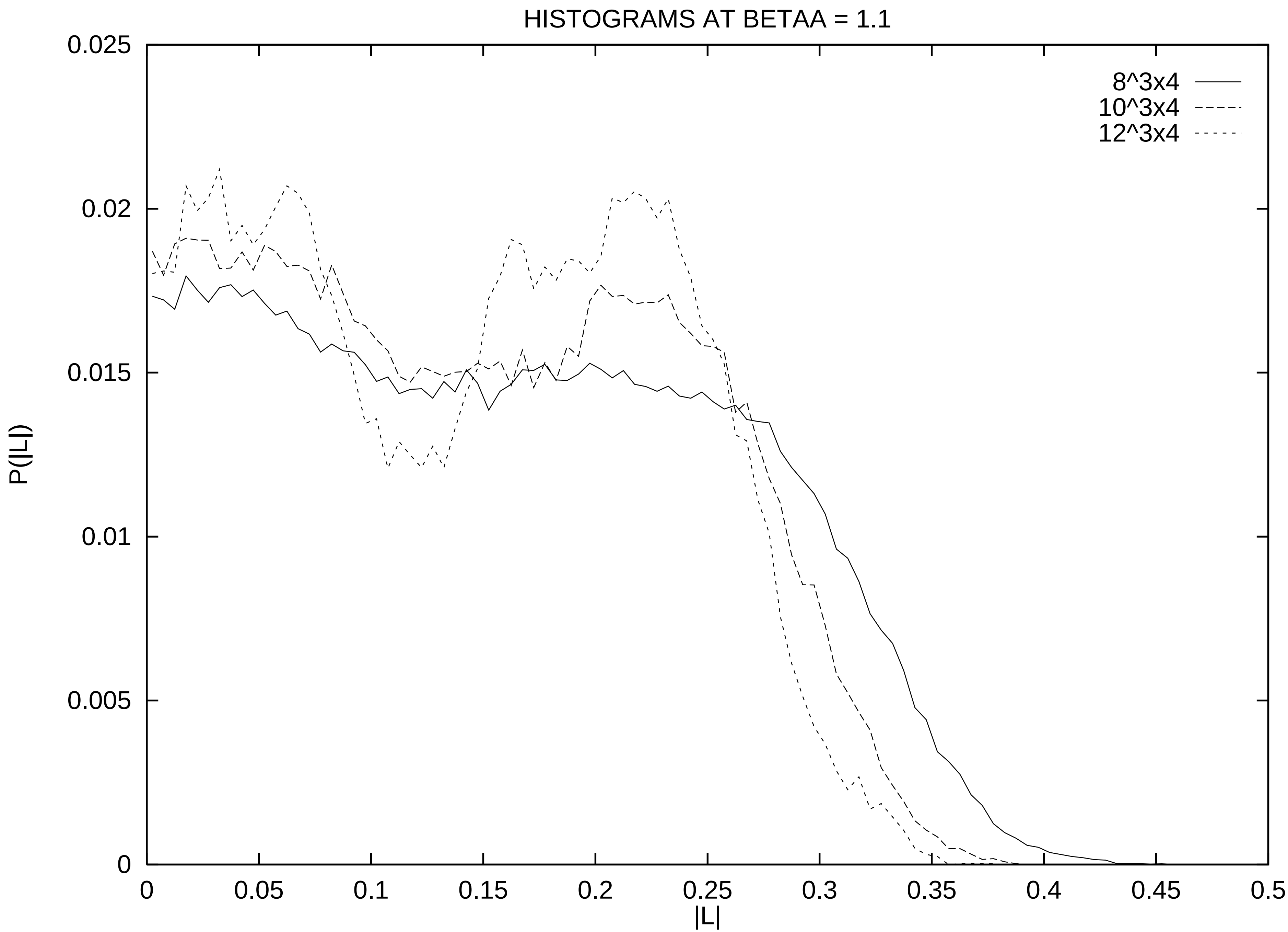
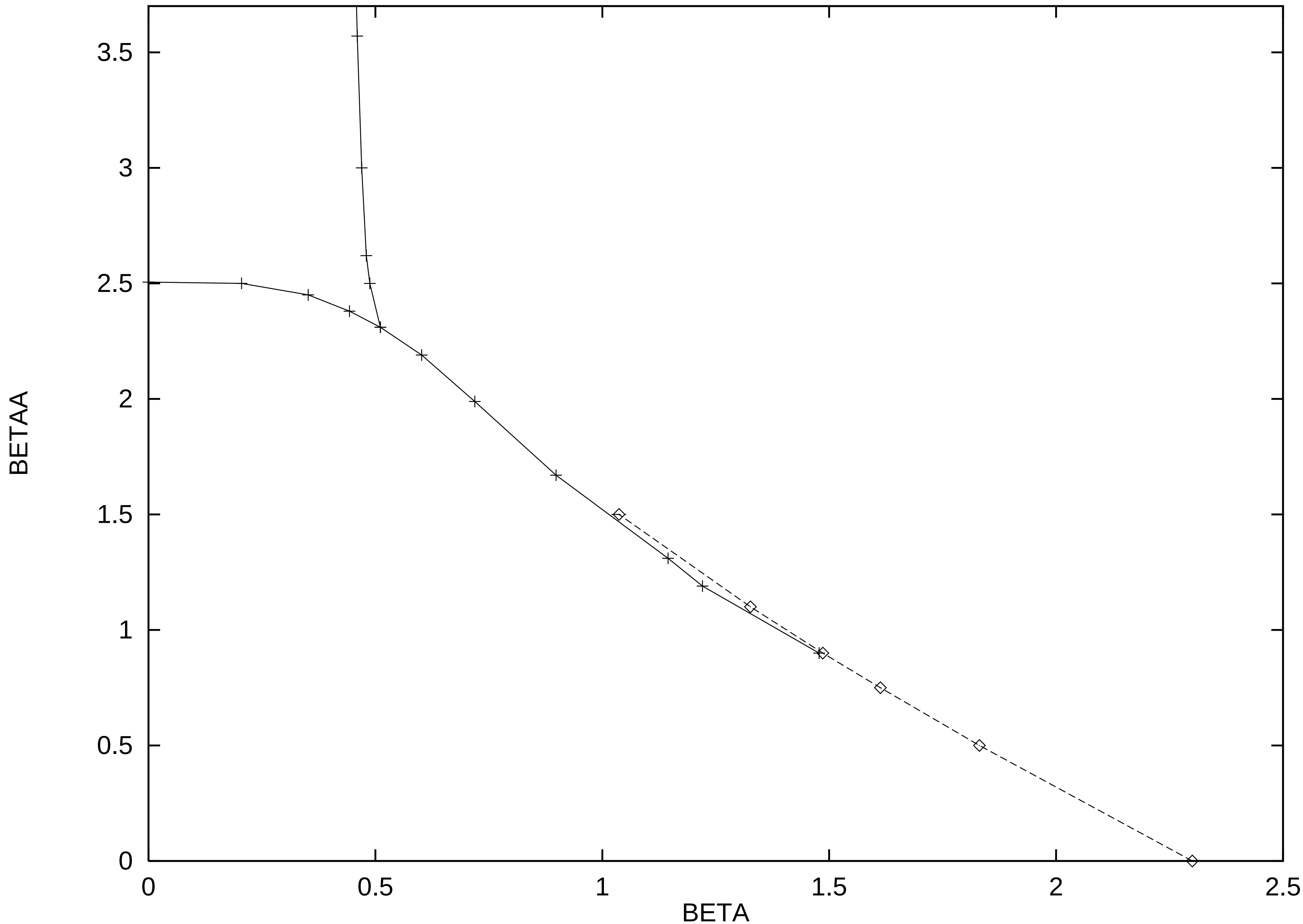
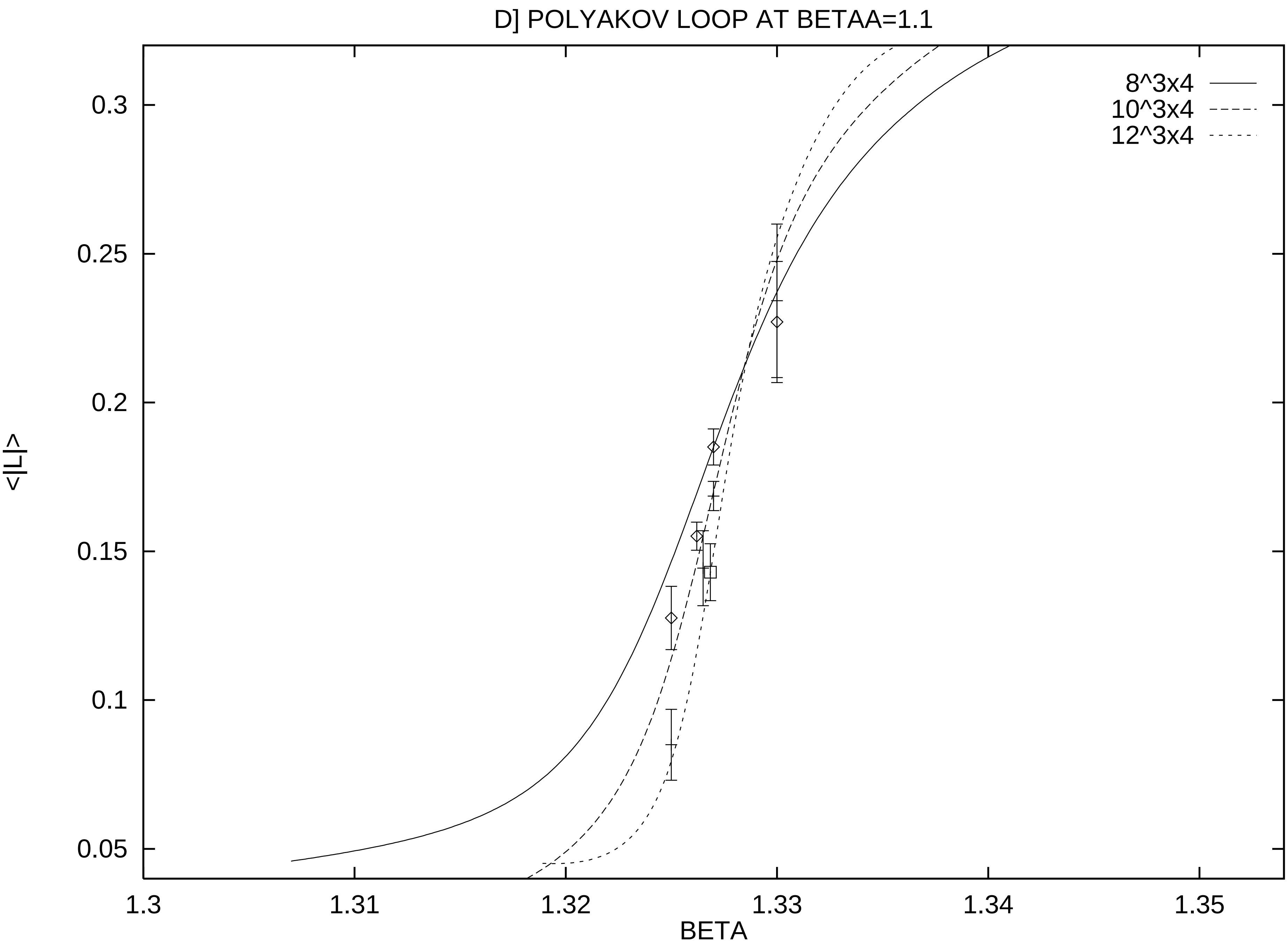
This figure "fig1-5.png" is available in "png" format from:

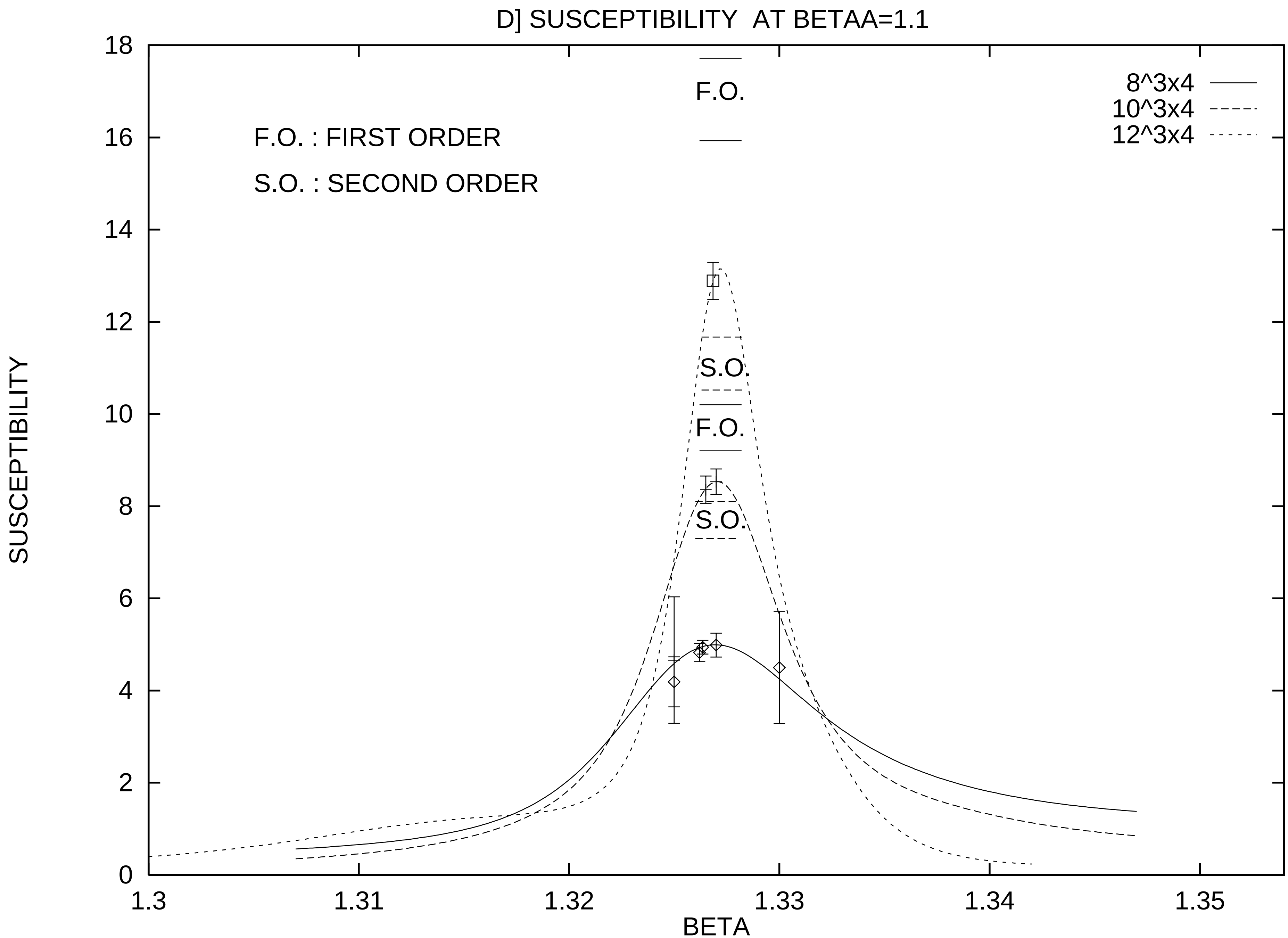
http://arXiv.org/ps/hep-lat/9403001v2



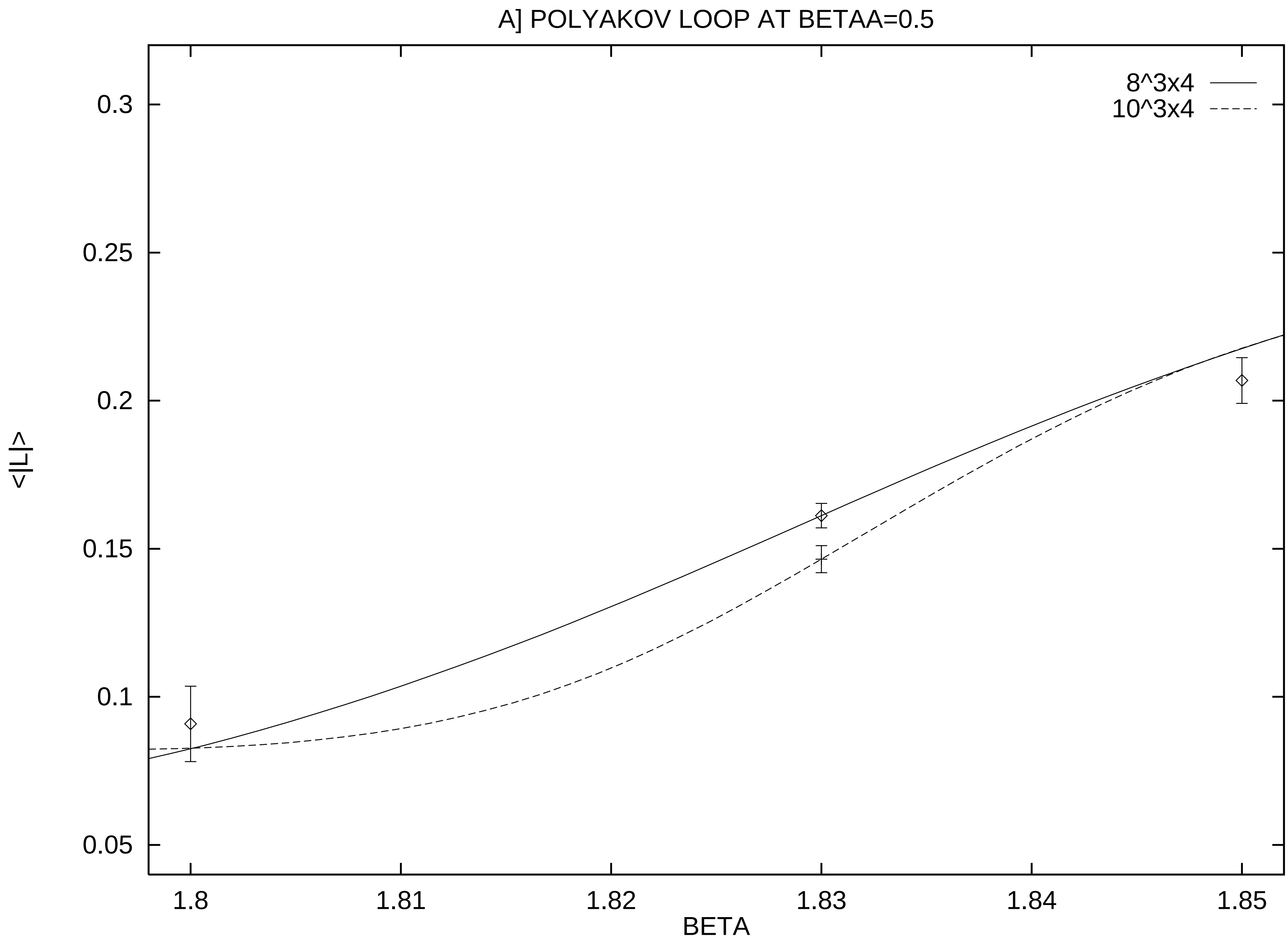
HISTOGRAMS AT BETAA = 1.1
0.025
0.02
0.015
0.01
0.005
0
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45
0.5
|L|
P(|L|)
8^3x4
10^3x4
12^3x4


3.5
3
2.5
2
1.5
1
0.5
0
BETAA
0
0.5
1
1.5
2
2.5
BETA

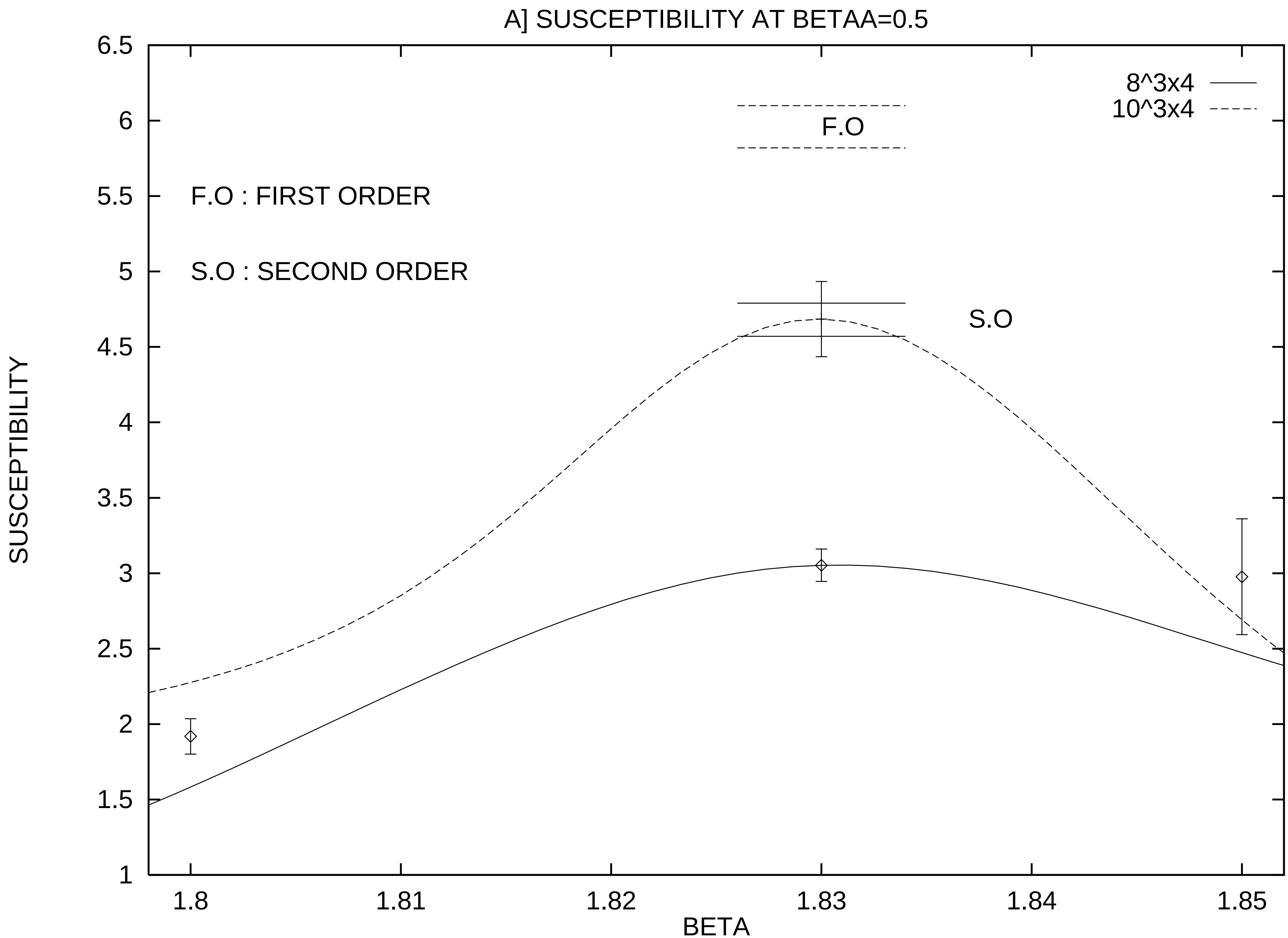

D] POLYAKOV LOOP AT BETAA=1.1
8^3x4
10^3x4
12^3x4
<|L|>
0.3
0.25
0.2
0.15
0.1
0.05
1.3
1.31
1.32
1.33
1.34
1.35
BETA

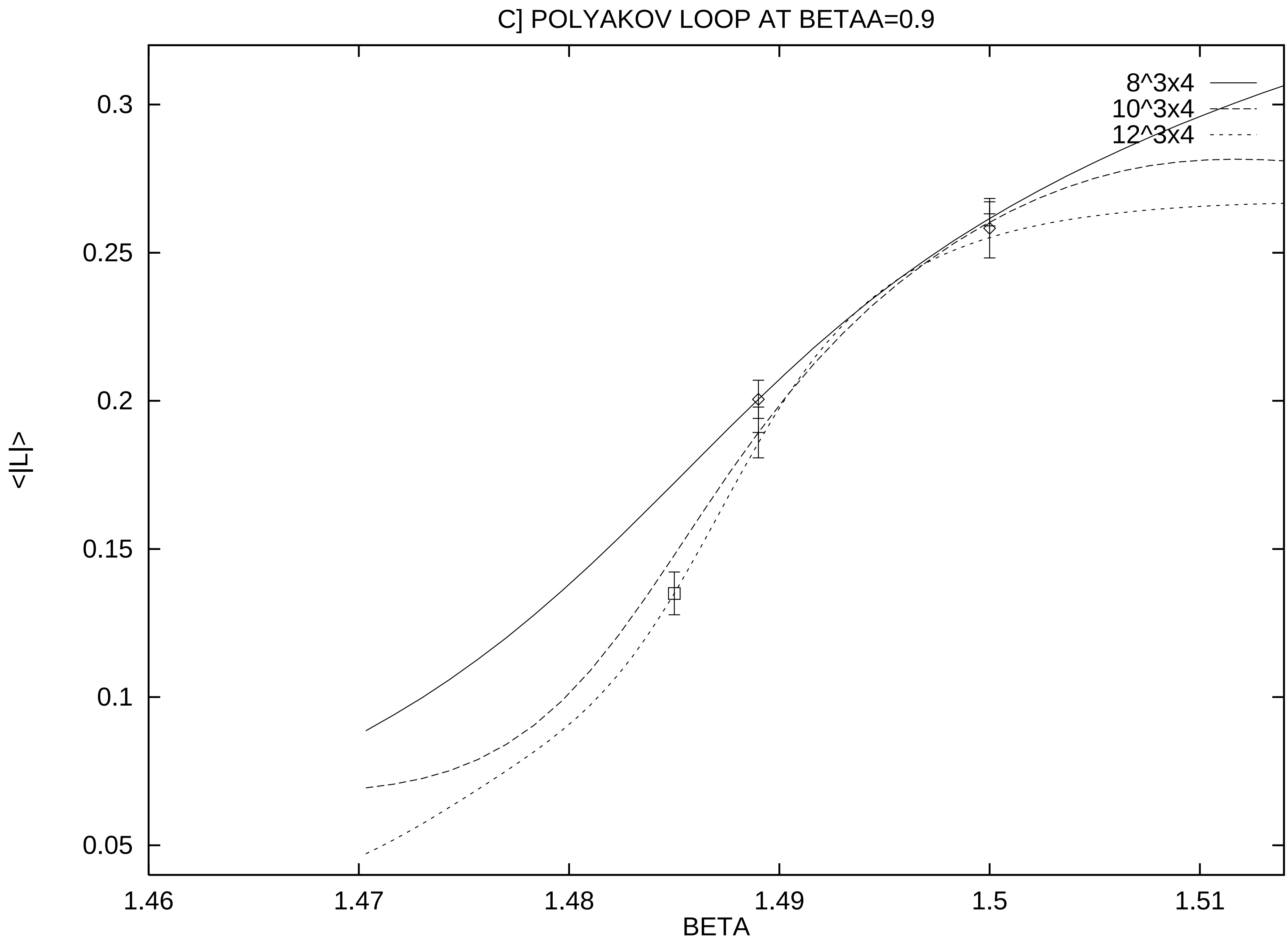


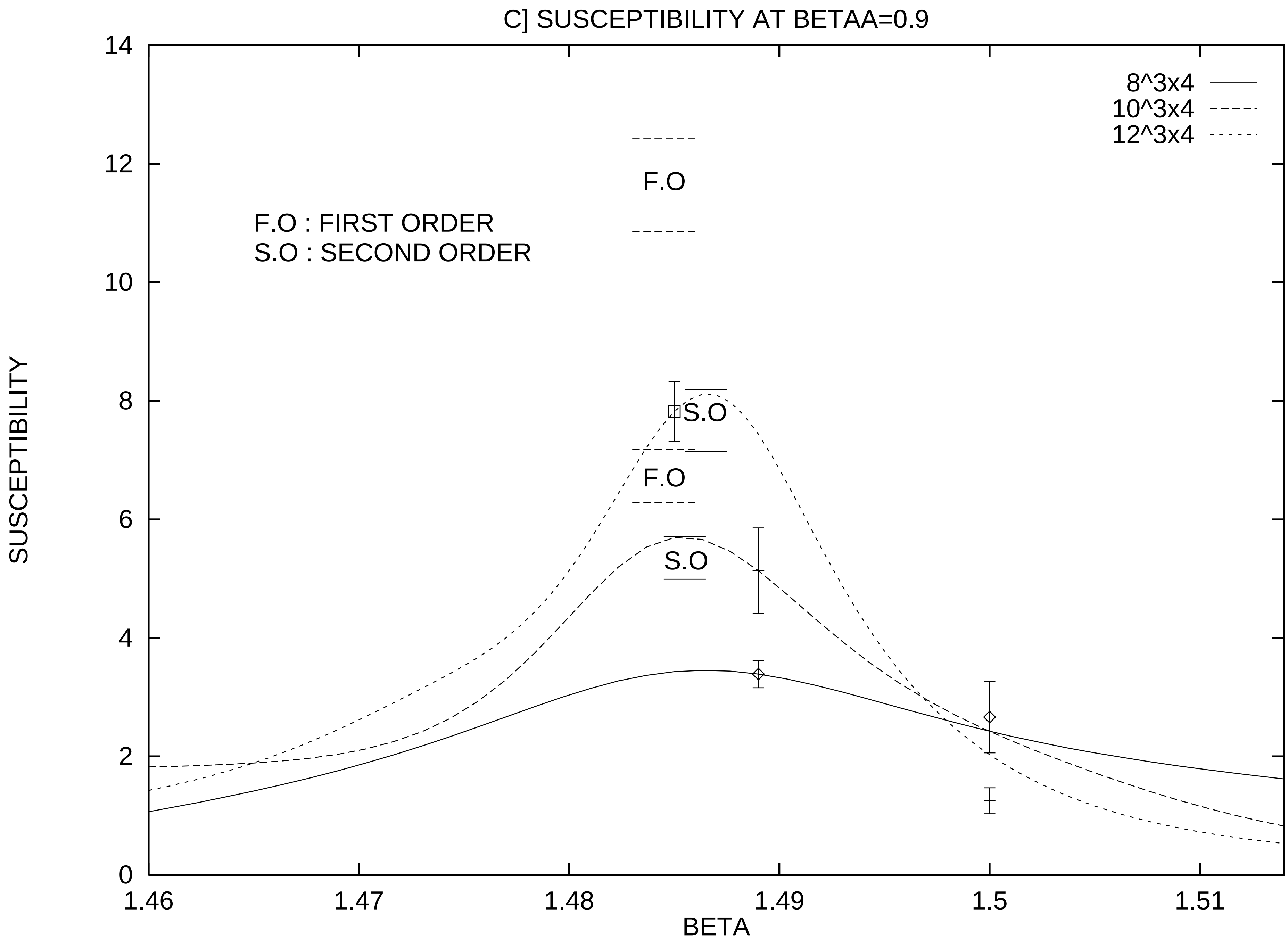
D] SUSCEPTIBILITY AT BETAA=1.1
F.O.
F.O. : FIRST ORDER
S.O. : SECOND ORDER
S.O.
F.O.
S.O.
8^3x4
10^3x4
12^3x4
SUSCEPTIBILITY
BETA
0
2
4
6
8
10
12
14
16
18
1.3
1.31
1.32
1.33
1.34
1.35

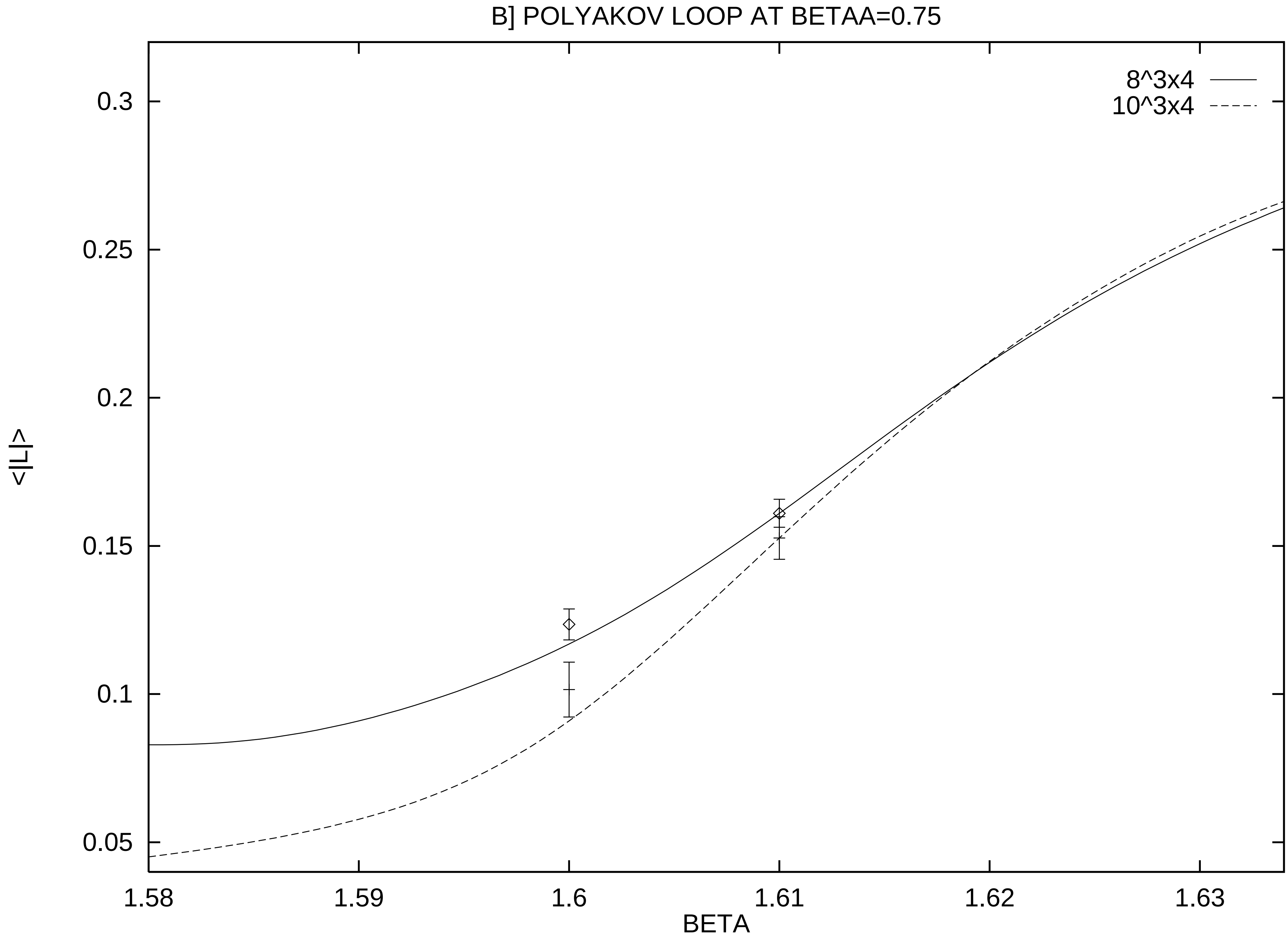


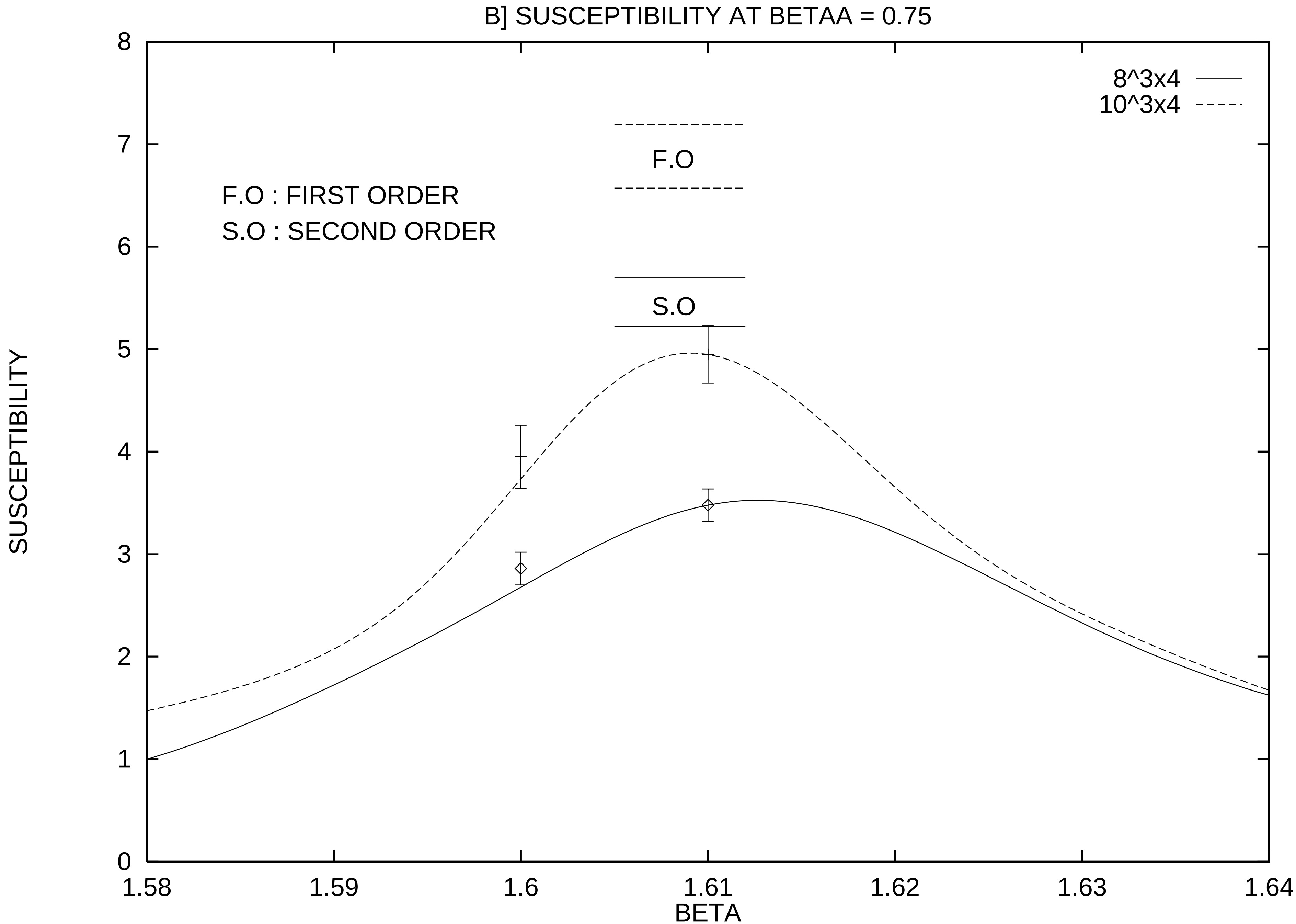
A] POLYAKOV LOOP AT BETAA=0.5
8^3x4
10^3x4
0.3
0.25
0.2
0.15
0.1
0.05
<|L|>
1.8
1.81
1.82
1.83
1.84
1.85
BETA


A] SUSCEPTIBILITY AT BETAA=0.5
SUSCEPTIBILITY
BETA
8^3x4
10^3x4
F.O
S.O
F.O : FIRST ORDER
S.O : SECOND ORDER
6.5
6
5.5
5
4.5
4
3.5
3
2.5
2
1.5
1
1.8
1.81
1.82
1.83
1.84
1.85


C] POLYAKOV LOOP AT BETAA=0.9
8^3x4
10^3x4
12^3x4
0.3
0.25
0.2
0.15
0.1
0.05
<|L|>
1.46
1.47
1.48
1.49
1.5
1.51
BETA


C] SUSCEPTIBILITY AT BETAA=0.9
SUSCEPTIBILITY
BETA
8^3x4
10^3x4
12^3x4
F.O : FIRST ORDER
S.O : SECOND ORDER
F.O
S.O
F.O
S.O
0
2
4
6
8
10
12
14
1.46
1.47
1.48
1.49
1.5
1.51


B] POLYAKOV LOOP AT BETAA=0.75
8^3x4
10^3x4
0.3
0.25
0.2
0.15
0.1
0.05
<|L|>
1.58
1.59
1.6
1.61
1.62
1.63
BETA


B] SUSCEPTIBILITY AT BETAA = 0.75
8^3x4
10^3x4
F.O
S.O
F.O : FIRST ORDER
S.O : SECOND ORDER
SUSCEPTIBILITY
BETA
0
1
2
3
4
5
6
7
8
1.58
1.59
1.6
1.61
1.62
1.63
1.64