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Supercooling and Nucleation in Phase Transitions
of the Early Universe

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Abstract

The three phase transitions – the GUT, the electro-weak and the quark-hadron, which the universe is assumed to have undergone produce very important physical effects if they are assumed to be of first order. It is also important that enough supercooling is produced at these transitions so that the rate of nucleation of the lower temperature phase out of the higher temperature phase is large. We argue on the basis of finite-size scaling theory that for the quark-hadron and the electro-weak phase transitions the universe does not supercool enough to give sizeable nucleation rates. Only for the GUT transition the nucleation probability seems to be significant.

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According to the standard big bang model, the universe started from a state of very high density and temperature. Due to the expansion of the universe the temperature falls and depending on the underlying theory of particle interactions, a sequence of phase transitions takes place. Physical consequences of the three such phase transitions have been extensively investigated. These are 1) the GUT phase transition at a $T_{c1} \sim O(10^{14} - 10^{16})$ GeV, where the symmetry between the strong and the electroweak interactions is spontaneously broken, 2) the electroweak (EW) phase transition at a $T_{c2} \sim O(10^2)$ GeV, where the electroweak symmetry is spontaneously broken, and finally 3) the chiral symmetry breaking and/or the confining QCD phase transition at a $T_{c3} \sim O(10^{-1})$ GeV. The value of T_{c1} is, of course, purely conjectural as there is no viable grand unified theory at the moment. However, non-perturbative results from simulations of corresponding lattice field theories indicate that the values of the other two critical temperatures, T_{c2} and T_{c3} are reasonably well determined.

It is natural to expect some cosmological and astrophysical consequences of these phase transitions. Indeed, the GUT phase transition leads to the formation of various topological structures– domain walls, cosmic strings, magnetic monopoles etc. [1]. It has been also exploited in the inflationary scenarios for the early universe. Recently there have been attempts to show that the baryon asymmetry of the universe can be generated at the electroweak phase transition [2,3]. There have been also speculations about the creation of initial density inhomogeneities necessary for large scale structure formation in the universe at this transition [4]. The quark-hadron transition has been shown [5,6] to lead to an alternative scenario for nucleosynthesis with substantially large baryon density contrast, Ω_B. In many of these applications, a (strong) first order phase transition with substantial supercooling has been assumed. The consequent bubble nucleation is then the mechanism responsible for the expected effects.

For the purposes of this article we will therefore assume that all the phase transitions to be of first order and examine critically the estimates of the corresponding nucleation rates. It is usually assumed that the high temperature symmetric phase, A, goes into a metastable state and is 'supercooled' before decaying into droplets of the less symmetric low

temperature phase, B. The universe passes through this series of equilibrium and metastable states only if the interactions necessary for particle distribution functions to adjust to the changing temperature are rapid compared to the expansion rate of the universe. A rough criterion that a reaction rate is fast enough for maintaining equilibrium is $\Gamma > H$ where Γ is the interaction rate per particle and H is the Hubble constant. If $\Gamma < H$ then the particles 'freeze out' and do not contribute to the maintenance of equilibrium [1].

The rate of nucleation of droplets of phase B has so far been calculated using the homogeneous nucleation theory [7]. It is assumed that the droplets of phase B arise through spontaneous thermodynamic fluctuations in phase A. For the formation of a spherical droplet of radius r the change in the free energy of the system is given by,

$$\Delta F = \frac{4\pi}{3}\left(p_A(T) - p_B(T)\right)r^3 + 4\pi r^2\sigma \tag{1}$$

where $p_A(T) - p_B(T)$ is the difference in pressure in the two phases at temperature T and σ is the surface tension of the interface of the phases. ΔF increases with r till a maximum value r_{cr} is reached where

$$r_{cr} = \frac{2\sigma}{p_B(T) - p_A(T)} \tag{2}$$

Droplets with $r < r_{cr}$ shrink and disappear while droplets with $r > r_{cr}$ grow. The rate of nucleation per unit volume is given by

$$I = I_0 exp\left(-\frac{\Delta F_{cr}}{T}\right) \tag{3}$$

where ΔF_{cr} is the value of ΔF for $r = r_{cr}$ and I_0 is the prefactor. For small supercooling, that is, for $\eta = (T_c - T)/T_c < 1$ Eq. (3) can be written as

$$I = I_0 exp\left(-\frac{16\pi}{3}\frac{\sigma^3}{T_c\Delta E^2\eta^2}\right) \tag{4}$$

where ΔE is the latent heat per unit volume. Thus the rate decreases exponentially, as supercooling becomes smaller, unless I_0 happens to be very large.

Since the early days of the nucleation theory much effort has gone into the calculation of I_0 [8]. The most well-known result is that due to Becker and Döring [9]. They calculated

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I_0 by considering explicitly the kinetics of the condensation process. In recent years Langer [10] has developed a more detailed theory of nucleation based on statistical mechanical considerations. He obtains an expression for I_0 very different from that of Becker and Döring. However the numerical values of I_0 do not differ much in the two theories [11].

For the quark-hadron phase transition, Fuller et al. [6] calculated the rate by setting $I_0 = T_c^4$. Recently Csernai and Kapusta [12] have calculated I_0 using Langer's theory. They obtain

$$I_0 = \frac{16}{3\pi} \left(\frac{\sigma}{3T} \right)^{3/2} \frac{\sigma \eta_A r_{cr}}{\xi_A^4 (\Delta w)^2} \tag{5}$$

where η_A and ξ_A are respectively the shear viscosity and a correlation length in the phase A and Δw is the difference in the enthalpy densities in the two phases. It turns out that the new values of I_0 are *smaller* than the prefactor T_c^4 for the quark-hadron phase transition [12]. Therefore, the crucial parameter which governs the nucleation rate in this case is indeed the possible amount of supercoooling the universe can undergo near T_c. It seems likely that these arguments apply to other, especially the elecroweak, phase transitions as well.

Before we address the question of the possible amount of supercooling the universe can undergo at any of these transitions, it is perhaps worthwhile to point out that even in a simple heterogeneous nucleation mechanism, it is still the supercooling which dominates the nucleation rate. As an example of heterogeneous nucleation we consider the condensation of light quarks on heavy quarks in the case of quark–hadron transition in analogy with the condensation of water vapour on ions [13]. The contribution to the change free energy of the system is now given by

$$\Delta F_{\text{impurity}} = \alpha \left(1 - \frac{\varepsilon_A}{\varepsilon_B} \right) \left(\frac{1}{r} - \frac{1}{r_0} \right) \tag{6}$$

where ε_A and ε_B are the dielectric constants in the A and B phases , r_0 is the effective radius of the impurity particles, in this case the heavy quarks and α is the QCD coupling constant. The Coulomb-like form of the potential in Eq. (6) is justified for a deconfined quark-gluon plasma at sufficiently high temperatures. Minimizing the full ΔF, one can obtain the corresponding r_{cr} and the modified nucleation rate. For small α, one obtains

$$r_{cr}^{het} = r_{cr} \left(1 - \frac{\alpha}{8\pi\sigma r_{cr}^3} \right). \tag{7}$$

From the above equation, one sees that the critical radius for the heterogeneous system is smaller than the r_{cr} for the homogeneous case, but it still is governed by η, the magnitude of supercooling, in the same way as before. Thus in the limit of vanishing supercooling the critical radius of the stable bubble is still too large to allow significant nucleation rates. Of course, for very small η the solution Eq. (7) is not valid but then it is easy to show that the full ΔF has no minimum at all.

This leads us to the central question which we wish to discuss in this paper: is it possible to estimate the amount of supercooling the universe can undergo at a phase transition ? We suggest that standard arguments from the finite size scaling theory [14,15] near a first order phase transition can be exploited to answer this question. If $\xi(T)$ is the correlation length in a given phase at a temperature T close to the transition point and L is the linear size of the system then clearly for $L \leq \xi$ one expects large finite size effects to cause rounding of a discontinuity and broadening of the transition region whereas for $L \gg \xi$, the system should behave as if it is in the thermodynamic limit. Indeed, while the system could remain trapped in a single, perhaps metastable, phase for the former case, it will be in a mixed phase of several domains of both phases for the latter. In fact, Challa et al [16] in their study of temperature-driven first order transitions have shown that

$$\frac{\Delta T}{T_c} \simeq \frac{T_c}{\Delta E L^3}. \tag{8}$$

This result can be expressed in terms of ξ and ΔE, by observing that ΔE, the latent heat per unit volume can be written as

$$\Delta E = \frac{c_1 T_c}{\xi^3}, \tag{9}$$

purely on dimensional grounds. Here c_1 is a constant. (Note that the above equation is consistent with the statement that a second order transition is the limit of a first order phase transition for infinite correlation length or vanishing latent heat.) Substituting Eq. (9) in Eq. (8) we get,

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$$\eta = A\left(\frac{\xi}{L}\right)^3$$
$$= A\left(\frac{\xi T_c}{L T_c}\right)^3, \tag{10}$$

where the constant of proportionality, A, should be typically $O(1)$. This relation has been verified for the quenched QCD and both A and ξT_c have been estimated [17] to be $O(1)$. For the electro-weak theory or any grand unified theory, no such test has so far been made and no estimate of ξT_c or A is available. On the other hand Eq. (9) has been verified for a large number of models in statistical mechanics [15]. It thus seems natural to assume that for electro-weak and GUT theories also both these dimensionless numbers are $O(1)$, although we will allow them to vary up to $O(10^2)$.

Thus for determining the amount of supercooling at a phase transition in the universe one needs the correlation length and the volume of the universe at the crtical temperature. The volume can be obtained from standard cosmology. Assuming an ideal gas equation of state for the matter in the early universe,

$$\rho = 3P = \frac{\pi^2}{30} N(T) T^4 \ , \tag{11}$$

where $N(T) = N_B(T) + 7/8 N_F(T)$ is the total number of bosonic (B) and fermionic (F) degrees of freedom. The age of the universe at temperature T is given by the relation [1]

$$t = \frac{1}{4\pi} \sqrt{\frac{45}{\pi N(T)}} \frac{M_P}{T^2} \ . \tag{12}$$

Here $M_P \sim 10^{19}$ GeV is the Planck mass. Since the radius of the universe as given by the particle horizon is $3t$, the volume, V, of the universe at temperature T is given by

$$V T^3 = \frac{405}{16} \sqrt{\frac{45}{\pi^7 N^3(T)}} \left(\frac{M_P}{T}\right)^3$$
$$= \frac{5.625 \times 10^{57}}{N^{3/2}(T) T^3} \tag{13}$$

Here T is expressed in GeV. Substituting Eq. (13) in Eq. (10), one finds that the super-cooling which the universe can undergo is

$$\eta = 1.78 \times 10^{-58} A(\xi T_c)^3 N^{3/2}(T_c) T^3 \ . \tag{14}$$

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As mentioned earlier, all quantities in Eq. (14) are known only for the quark-hadron phase transition. Using the data from lattice QCD for T_c and $\xi(T_c)T_c$ [17] and $N(T_c) = 51.25$ [corresponding to photons(2), gluons(16), electrons(4), muons(4), neutrinos(6) and two flavours of quarks(24)], one finds that the supercooling η is negligibly small, as shown in our earlier work [18] where all the caveats in using the lattice QCD data and their effects are also discussed. This result is not changed if we take the new results of Ref. 12 for I_0 or even the possibility of heterogeneous nucleation caused by heavy quarks.

Similar arguments as given above can be applied to *any* other phase transition occurring during the evolution of the universe. Thus, for the electroweak phase transition, $N(T_c)$ is 51.5 [corresponding to W and Z-bosons(6), taus(4), and four flavours of quarks(48)] and we obtain $T_c \sim 250$ GeV on simple dimensional grounds. One-loop perturbation theory yields [19] $\xi(T_c)T_c = 28.1$ and $T_c = 184$ GeV for a Higgs mass of 80 GeV, while lattice investigations [20] of the $O(4)$ model suggest $T_c = 370$ GeV for a Higgs mass close to its triviality bound of about 650 GeV. Choosing the estimates of Ref. 20, we find that

$$\eta_{\rm EW} = 1.57 \times 10^{-43} A \ , \tag{15}$$

which can be significant only for unnaturally large A. Note that the expected uncertainty of a factor of two in T_c and a correlation length which is 2-3 orders of magnitude larger than the one used above will not alter the conclusion at all. Furthermore, the ideal gas equation of state used here should be adequate, since the non-perturbative contributions to the energy density are unlikely to change $N(T_c)$ by more than an order of magnitude.

Finally, for the GUT phase transition, one sees that substituting $T_c = T_{c1}$, the small numerical factor in Eq. (14) is increased by 42-48 orders of magnitudes. Since $N(T)$ may increase by a further factor of 2 or so, the universe can undergo significant supercooling at this phase transition provided the correlation length is $\sim 100T_c$. Indeed it seems that only for theories which have either a transition temperature close to M_P or a very large (but finite) correlation length near T_c it is possible to get sufficient supercooling and hence interesting effects through bubble nucleation.

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REFERENCES

[1] S. Weinberg, *Gravitation and Cosmology* (Wiley, New York, 1972); E. W. Kolb and M. S. Turner, *The Early Universe* (Addison–Wesley, Redwood City, CA, 1990).

[2] A. G. Cohen, D. B. Kaplan and A. E. Nelson (to be published).

[3] G. R. Farrar and M. E. Shaposnikov, Preprint CERN-TH 6732/93.

[4] A. D. Dolgov, Phys. Rep. **222**, 309 (1992).

[5] J. H. Applegate, C. J. Hogan and R. Scherrer, Phys. Rev. D **35**, 1151 (1987).

[6] G. M. Fuller, G. J. Mathews and C. R. Alcock, Phys. Rev. D **37**, 1380 (1988).

[7] L. Landau and E. M. Lifshitz, *Statistical Physics* (Pergamon, Oxford, 1969).

[8] J. Frenkel, *Kinetic Theory of Liquids* (Clarendon Press, Oxford, 1946).

[9] R. Becker and W. Döring, Ann. Phys. (Leipzig) **24**, 719 (1935).

[10] J. S. Langer, Ann. Phys. (N.Y.) **54**, 258 (1969).

[11] J. S. Langer and L. A. Turski, Phys. Rev. A **8**, 3230 (1973).

[12] L. P. Csernai and J. I. Kapusta, Phys. Rev. D **46**, 1379 (1992)

[13] N. H. Fletcher, *The Physics of Rainclouds*, (Cambridge University Press, 1962), p. 48.

[14] M. N. Barber in *Phase Transitions*, Vol. 8 ed. C. Domb and J. Lebowitz (Academic Press, New York, 1983).

[15] K. Binder, Rep. Prog. of Phys. **50**, 783 (1987).

[16] M. S. S. Challa, D. P. Landau and K. Binder, Phys. Rev. B **34**, 1841 (1986).

[17] R. V. Gavai in *Quantum Fields on the Computer* (World Scientific, Singapore, 1992).

[18] B. Banerjee and R. V. Gavai, Phys. Lett. B **293**, 157 (1992).

[19] G. W. Anderson and L. J. Hall, Phys. Rev. D **45**, 2685 (1992); K. Enqvist, J. Ignatius, K. Kajantie and K. Rummukainen, Phys. Rev. D **45**, 3415 (1992).

[20] R. V. Gavai, U. M. Heller, F. Karsch, T. Neuhaus and B. Plache, Phys. Lett. B **294**, 84 (1992); K. Jansen and P. Seuferling, Nucl. Phys. B **343**, 507 (1990).