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UNIVERSALITY AND THE DECONFINEMENT PHASE TRANSITION IN SU(2) LATTICE GAUGE THEORY

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ABSTRACT

We study the three dimensional fundamental-adjoint $SU(2)$ lattice gauge theory at non-zero temperatures by Monte Carlo simulations. On an $8^3 \times 2$ lattice, at $\beta_A = 1.1$, where β_A is the adjoint coupling, we find no evidence of any transition at the location of a previously known bulk phase transition around $\beta = 1.33$. Moreover, the deconfinement transition at $\beta_A = 1.1$ occurs at $\beta = 1.20$ and is of first order for $\beta_A \geq 1.1$, thus implying a change of universality class from that of the Wilson action at $\beta_A = 0$. Computations of the plaquette susceptibility and the temporal and spatial Polyakov loops on $8^3 \times 4$ and $16^3 \times 8$ lattices at $\beta_A = 1.1$ further support these conclusions and suggest that the previously claimed bulk transition around $\beta = 1.33$ is, in fact, the first order deconfinement transition. Simulations at larger β_A and the measurements of the mass gaps from the correlation functions of temporal and spatial Polyakov loops also confirm the temperature dependent nature of the above transition. The consequences of our results on universality are discussed.

1. INTRODUCTION

One of the most outstanding problem in physics is to understand the mechanism for confinement and finite temperature deconfinement in quantum chromodynamics (QCD) or $SU(N)$ gauge theories in general. Colour confinement implies that the physical states in the Hilbert space are colour singlets. Non-abelian gauge theories at zero temperature are believed to have this confining property. Regulating a compact gauge theory on the lattice with cut-off a^{-1} , where a is the lattice spacing, confinement can be analytically established in the strong coupling region. Creutz[1] showed that no phase transition takes place at zero temperature for the usual Wilson action of the $SU(2)$ or $SU(3)$ lattice gauge theory as the continuum limit is approached, i.e., these theories have a smooth β function for all values of the cut-off, with a zero only when the cut-off goes to infinity. In contrast, the phase diagrams of lattice $SO(3)$, $SU(4)$ and $SU(5)$ models have bulk transitions separating the corresponding strong and weak coupling phases. However, Bhanot and Creutz[2] showed that the mere existence of a bulk phase transition in lattice theories does not necessarily imply a loss of confinement. As the lattice action of a given continuum theory is not unique, one could instead consider an extended lattice theory belonging to the same universality class. If the phase diagram of the theory in the extended coupling space allows a smooth continuation around the bulk singularities then the theory will retain its strong coupling confining property as the cut-off is removed. They illustrated this by considering an extended Wilson action defined by:

$$S = \sum_P \beta \left[1 - \frac{1}{2} \text{Tr}_F U_P \right] + \sum_P \beta_A \left[1 - \frac{1}{3} \text{Tr}_A U_P \right] \quad (1)$$

Here F, A denote fundamental and adjoint representations respectively, U_P is the ordered product of the four directed link variables $U_\mu(n) (\in SU(2))$ which form an elementary plaquette. The sum over P denotes the sum over all independent plaquettes of the lattice. As found in Ref. [2], this model has a rich phase structure, which is displayed in Fig. 1. At $\beta_A = 0$, it reduces to the usual Wilson action. Along the $\beta = 0$ axis, it describes the $SO(3)$ model with a first order phase transition at $\beta_A^c \sim 2.5$. At $\beta_A = \infty$, it describes the Z_2 lattice gauge theory with a first order phase transition

at $\beta^c = \frac{1}{2} \ln(1 + \sqrt{2}) \sim 0.44$. From the $\beta = 0$ and $\beta_A = \infty$ axes, the above two bulk transitions extend into the (β, β_A) plane, meet at a point **A** at (0.5, 2.4) and then continue as another line of bulk first order phase transitions. The fact that the latter line ends at a point **B** at (1.5, 0.9) in the phase diagram allows one to bypass all the bulk singularities without losing confinement in $SU(2)$ gauge theory. This model therefore retains its strong coupling confining property also in the continuum and is expected to be in the same universality class as the Wilson action ($\beta_A = 0$).

Comparing the naive classical continuum limit of Eq. (1) with the continuum $SU(2)$ Yang Mills action, one obtains

$$1/g_u^2 = \beta/4 + 2\beta_A/3 \quad (2)$$

Here g_u is the bare coupling constant of the continuum theory. It is convenient to characterise the model with g_u and θ , defined by $\theta = \tan^{-1} \beta_A/\beta$. The two loop β -function of this theory [3] tells us that theories characterised by different θ 's approach the universal critical fixed point $g_u^* = 0$ in the continuum limit, with the flow governed by the asymptotic scaling relation,

$$a = 1/\Lambda(\theta) \exp \left[-\frac{1}{2\beta_0 g_u^2} \right] [\beta_0 g_u^2]^{-\beta_1/2\beta_0^2}, \quad (3)$$

where

$$\frac{\Lambda(0)}{\Lambda(\theta)} = \frac{5\pi^2}{11} \frac{6 \tan \theta}{(3 + 8 \tan \theta)}. \quad (4)$$

Here β_0 and β_1 are the first two coefficients of β function for $SU(2)$ gauge theory. Eqs. (3) and (4) show that the continuum limit of the theory lies at $g_u^* = 0$ and that θ is an irrelevant coupling which changes only the scale of the theory. Up to two loops this illustrates that the extended Wilson action described by Eq. (1) is in the same universality class as the Wilson action.

Usually finite temperature lattice field theory simulations are carried out on $N_\sigma^3 \times N_\tau$ lattices, with $N_\sigma \gg N_\tau$ and the inverse physical temperature given by $N_\tau a(g_u)$. Here $a(g_u)$ is the lattice spacing at the coupling g_u , given in the asymptotic scaling region by Eqs. (3-4). The corresponding physical volume is $V = N_\sigma^3 a^3$. If T_c denotes the transition temperature of the deconfinement phase transition and is thus also a physical observable, one gets the scaling relation $N_\tau \propto a^{-1}$ in the limit $a \rightarrow 0$. Together with Eq. (3), this yields the scaling law for the critical coupling $g_u^c(N_\tau)$ for the lattice

with its temporal extent N_τ . If N_σ is not very large compared to N_τ , then one expects corrections to this scaling relation which are, however, small: The leading effect is still the exponential scaling of N_τ with the corresponding critical coupling $g_u^c(N_\tau)$. Furthermore, the relation between $\Lambda(\theta)$ and θ , given in Eq. (4), is obtained in leading order perturbation theory. Corrections to this relation are scheme dependent and not well known but they too must become vanishingly small for sufficiently large lattices. Note that as the temporal lattice size becomes infinitely large, i.e., as the continuum limit is approached, the critical coupling moves to zero: $g_u^c(N_\tau \rightarrow \infty) \rightarrow 0$. Thus in the language of finite size scaling theory one can characterise the deconfinement phase transition as one with a critical coupling $g_u^* = 0$ and an effectively logarithmic scaling of the critical coupling with the temporal lattice size N_τ .

The finite size scaling behaviour of the shift of the critical coupling (or temperature) of a bulk transition [4], on the other hand, is governed by

$$[g_u^c(N) - g_u^c(\infty)] \sim N^{-\delta} \quad , \quad (5a)$$

and the height of the peak of the corresponding specific heat, which in our case is the plaquette susceptibility, should scale according to

$$\chi_{max} \propto N^{\omega_s} \quad . \quad (5b)$$

Here δ is the shift exponent, N is the typical size of the system : $N = (N_\sigma^3 N_\tau)^{1/4}$. $g_u^c(N)$ and $g_u^c(\infty)$ are the critical couplings for the finite N^4 and the infinite lattices respectively and the location of χ_{max} can be used to define the critical coupling, $g_u^c(N)$. For a second order bulk phase transition, $\delta = 1/\nu$ and $\omega_s = \alpha/\nu$, where α and ν are the critical exponents of the plaquette susceptibility and the correlation length respectively. For a first order transition, ω is equal to the dimensionality of the lattice d ($=4$, in our case). The bulk transitions in the phase diagram in Fig. 1 must exhibit scaling relations similar to Eq. (5-a,b) for various θ on the transition line **AB**. As the lattice size is increased, the bulk transition should move towards the non-zero $g_u^c(\theta)$, with a rapid increase in the plaquette susceptibility, and remain anchored there as the lattice becomes infinitely large. Thus unlike the finite temperature deconfinement transition, the bulk transition is a spurious transition not seen by the continuum limit of the theory at $g_u \rightarrow 0$.

Recently we [5] argued that the phase diagram in Fig. 1 is incomplete. Since the diagram is largely based on Monte Carlo simulations on small

finite lattices N_σ^4 ($N_\sigma = 5-7$), it must necessarily have a deconfinement phase transition at sufficiently small coupling g_u . Monitoring the order parameter $\langle L \rangle$ for the deconfinement phase transition on $N_\sigma^3 \times N_\tau$ lattices, for $N_\sigma = 8-12$ and $N_\tau = 4$ and 6 , in order to understand the phase diagram better, we found that the known second order deconfinement transition at $\beta_A = 0.0$ moves into the phase diagram as β_A is increased and joins the first order bulk transition line around $\beta_A \simeq 1.0$ as shown by the dotted line **QR** in Fig. 1. This coincidence of the two transitions persists till $\beta_A = 1.5$ up to which the simulations were performed. Moreover, there was no evidence for two separate, bulk and deconfining, transitions. The fluctuations in Polyakov loops and plaquettes were correlated and their discontinuities occurred at the same coupling. Increasing the temporal size of the lattice resulted in a small shift in β_c and the transitions still remained coincident and discontinuous. While the critical exponents were in agreement with the Ising model, as predicted by universality[6], for $\beta_A \leq 0.9$, there was strong evidence that the transition is of first order for $\beta_A \geq 1.1$. The histograms of $|L|$ showed a definite two peak structure with the dip between the two peaks increasing with N_σ at $\beta_A = 1.1$ and a co-existence of two phases with a tunnelling time ($> 10^5$) at $\beta_A = 1.5$.

The above joining together of the deconfinement transition line with the previously claimed bulk transition line leads to a curious paradox because as discussed above, both their natures as well as scaling behaviours are different. We therefore proposed the following three possible scenarios in our previous paper to explain the paradox :

- A] The joining of the deconfinement transition line with the bulk transition line is accidental on the lattices used. They will decouple on larger (or smaller) lattices.
- B] The entire transition line corresponds to deconfinement phase transitions only and the previous identification of it as a bulk transition line is, in fact, incorrect.
- C] The whole transition line is due to bulk transitions, implying that the theory has no confinement in the continuum limit.

In this paper we try to resolve the above paradox, at least partially, by studying the model on $8^3 \times 2$, $8^3 \times 4$ and $16^3 \times 8$ lattices. The spread in N_τ

was thus maximised to accentuate the scaling behaviour of the deconfinement phase transition. On the $8^3 \times 2$ lattice at $\beta_A = 1.1$ we find a deconfining transition at $\beta = 1.20$ with no evidence of any transition around $\beta = 1.33$, where the bulk transition was expected. By also measuring the spatial Polyakov line $\langle L^\sigma \rangle$ in addition to the temporal Polyakov line $\langle L^\tau \rangle$, we attempted to increase the N_τ range further in the direction of asymptotic scaling region. The simultaneous measurements of these on the *same lattice* have a major consequence for the bulk phase transition hypothesis; since the volume and the geometry of the lattice is unchanged, one expects a bulk transition to occur at the same coupling β for both of them. We find that the transitions in L^τ and L^σ are located at *different* couplings for each of the lattices investigated. Although we do find the plaquette susceptibility to peak in a range of β close to the expected first order bulk transition at $\beta_A = 1.1$, the height of its peak is found to *decrease* as the lattice volume increases. Further, the deconfinement phase transition at $\beta_A = 1.1$ appears to be a first order transition, whose strength increases rapidly as one increases β_A : clear coexistence of two-states was seen for $\beta_A = 1.4$ and 1.5 . The scaling behaviour of the transition with N_τ as well as the computations of mass gaps obtained from our measurements of the correlation functions of the temporal and spatial Polyakov lines again confirm the deconfining and first order nature of the above transition.

The organisation of the paper is as follows: In section 2 we describe the observables used to monitor the phase transition. Section 3 is devoted to the description of our Monte Carlo data and its analysis. Our results and their consequences on universality are discussed in the next section. Finally we conclude the paper with a summary and a listing of the still unanswered questions.

2. THE OBSERVABLES

In order to understand the nature of the phase transitions around the transition points of the extended action, defined by Eq. (1), we study the expectation values of the temporal and spatial Polyakov loops and the behaviour of their correlators in the two phases. As discussed below, these observables and their correlators characterise the phases on the two sides of the deconfinement transition. We also studied the expectation value of the

plaquette, given by

$$\langle P \rangle = \frac{1}{2} \frac{\sum_p \langle \text{Tr}_F U_p \rangle}{6N_\sigma^3 N_\tau} \quad (6)$$

and the corresponding susceptibility,

$$\chi_P = 6N_\sigma^3 N_\tau (\langle P^2 \rangle - \langle P \rangle^2) \quad (7)$$

Here the expectation value of an observable O is defined by

$$\langle O \rangle = Z^{-1} \int \prod_{\mu, n} dU_\mu(n) O \exp -S \quad (8)$$

with S as the extended action given by Eq. (1) and Z as the corresponding partition function.

Due to periodic boundary conditions in the temporal direction, the $SU(2)$ gauge theory has a $Z(2)$ invariance corresponding to the center of the gauge group [7]. Under this symmetry

$$U_0(\vec{n}, \tau_0) \rightarrow z U_0(\vec{n}, \tau_0), \quad \forall \vec{n} \text{ \& fixed } \tau_0, \quad (9)$$

with $z \in Z(2)$. Here $U_0(\vec{n}, \tau)$ is the time like link at the lattice site (\vec{n}, τ) . Under the above transformation, the Polyakov loop, which is the order parameter for the deconfinement phase transition and is defined as

$$L^\tau \equiv \frac{1}{N_\sigma^3} \sum_{\vec{n}} L^\tau(\vec{n}) = \frac{1}{2N_\sigma^3} \sum_{\vec{n}} \text{Tr} \prod_{\tau=1}^{N_\tau} U_0(\vec{n}, \tau), \quad (10)$$

transforms as follows:

$$L^\tau \rightarrow z L^\tau. \quad (11)$$

The expectation value of the Polyakov loop can be shown[7, 8] to be a measure of the free energy of an isolated quark. Thus the deconfinement transition corresponds to spontaneous breaking of the $Z(2)$ symmetry, separating the $Z(2)$ symmetric low temperature (confined) phase from the high temperature broken (deconfined) phase. Since due to tunnellings between the two $Z(2)$ -vacua the expectation value of this order parameter on a finite lattice is zero, we chose, as usual, to look at its absolute value and the corresponding susceptibility, defined by

$$\chi = N_\sigma^3 (\langle L_\tau^2 \rangle - \langle |L^\tau| \rangle^2) \quad (12)$$

On an infinite lattice the critical behaviours of the Polyakov loop, its susceptibility and the correlation length of the theory are governed by the critical exponents β, γ and ν , defined as :

$$\langle L^\tau \rangle \propto |T - T_c|^\beta \quad \text{for } T \rightarrow T_c^+ \quad (13)$$

$$\chi \propto |T - T_c|^{-\gamma} \quad \text{for } T \rightarrow T_c \quad (14)$$

$$\xi \propto |T - T_c|^{-\nu} \quad \text{for } T \rightarrow T_c \quad (15)$$

One obtains these exponents from Monte Carlo simulations by simply fitting the order parameter [8, 9, 10] and/or by using the finite size scaling theory[4]. According to the latter, the maximum of the susceptibility, defined in Eq. (12), is expected to grow like

$$\chi_{\max} \propto N_\sigma^\omega, \quad (16)$$

where $\omega = \gamma/\nu$ for a second order transition and is equal to spatial dimension of the system (3 in our case) for a first order transition.

In addition, we also define averaged spatial Polyakov loops by

$$L^\sigma = \frac{1}{3} \sum_{i=1}^3 L_\sigma^i, \quad (17)$$

with

$$L_\sigma^i = \frac{1}{2(N_\sigma^2 N_\tau)} \sum_{\substack{n_0, \\ \vec{n} \perp \hat{i}}} Tr \prod_{n_i=1}^{N_\sigma} U_0(\vec{n}, n_0). \quad (18)$$

The summation over i is over the three spatial directions and \hat{i} are the corresponding unit vectors. Due to the periodic boundary conditions in the spatial directions also, L^σ can again be thought of as an order parameter for the deconfinement transition now for a lattice of spatial volume $N_\sigma^2 N_\tau$ and at an inverse temperature of $N_\sigma a$. Due to the larger effective “temporal” extent, the order parameter L^σ can be considered to probe a region closer to the asymptotic scaling region compared to that probed by L^τ . Of course, the effective 3-volume, $N_\sigma^2 N_\tau$, seen by L^σ , being smaller than the corresponding spatial volume, N_σ^3 , for L^τ , finite size effects are relatively more in this case. However, as discussed in the next section, on the lattices concerned these

corrections only introduce uncertainties of the order of a few per cent in the final results discussed in this paper. Therefore, for our purpose of exploring the transition region, spatial Polyakov loops are as good an order parameter as temporal Polyakov loops. Moreover, due to the averaging over the three spatial directions, for $N_\tau > \frac{N_\sigma}{3}$, the statistics for L^σ is also relatively better.

The most important advantage of measuring both the order parameter L^τ and L^σ turns out to be the ability to distinguish between the three scenarios mentioned earlier. Let us therefore consider them again in light of these observables.

- A] If both bulk and deconfining transitions are present and overlap for the lattice sizes investigated in Ref. [5], then the effective increase in the temporal extent by studying L^σ may allow us to see the deconfinement phase transition to shift away. L^τ and L^σ should show critical behaviours at β_c^τ and β_c^σ respectively with $\beta_c^\sigma > \beta_c^\tau$. At $\beta < \beta_c^\tau$ both L^τ and L^σ should show confinement. In the region $\beta_c^\tau < \beta < \beta_c^\sigma$, L^τ will be deconfined while L^σ should still be in the confined phase. For $\beta > \beta_c^\sigma$, both L^τ and L^σ should show deconfinement. A presence of a bulk phase transition may show up as a non-analyticity in L^σ or in the corresponding mass gap obtained from L^σ -correlations to be defined below.
- B] If there is only a deconfinement transition then the L^σ or the corresponding mass gap will be smooth at β_c^τ . Moreover, in the asymptotic scaling region the difference $\beta_c^\sigma - \beta_c^\tau$ should decrease and approach a constant as we increase the lattice size holding N_σ/N_τ constant while it should increase if the lattice size is increased only in the N_σ -directions.
- C] If there is only a bulk transition present then $\beta_c^\tau \approx \beta_c^\sigma$: both L^τ and L^σ should become significantly different from zero at about the same beta, especially since they are evaluated on a lattice with identical 4-volume and geometry. Also the average plaquette should show a critical behaviour roughly at the same coupling. Unlike scenario [B], the differences in the critical couplings should vanish in both the limits discussed there.

Another useful observable, mentioned already above, is the mass gap obtained from the correlations of the Polyakov loops. For this purpose, we

define sums of temporal and spatial Polyakov lines over the corresponding orthogonal planes:

$$\mathcal{L}^\tau(n_j) = \frac{1}{N_\sigma^2} \sum_{\vec{n} \perp \hat{j}} L^\tau(\vec{n}) \quad (19)$$

$$\mathcal{L}_i^\sigma(n_j) = \frac{1}{N_\sigma N_\tau} \sum_{\substack{n_0, \\ \vec{n} \perp \hat{i}, \vec{n} \perp \hat{j}}} L_i^\sigma(n_0, \vec{n}) \quad (20)$$

These can be thought of as the zero momentum projections of the Polyakov lines in 2+1 dimensions. In our simulations, we measure the zero momentum correlation functions of both the temporal and spatial Polyakov loops given by:

$$\Gamma^{\tau\tau(\sigma\sigma)}(|n|) = \langle \mathcal{L}^{\tau(\sigma)}(0) \mathcal{L}^{\tau(\sigma)}(n) \rangle \quad , \quad (21)$$

Here the indices on $\mathcal{L}^{(\tau)(\sigma)}$ have been suppressed. Using periodic boundary conditions in the temporal and spatial directions and introducing the eigenstates of the transfer matrix as the intermediate set of states one obtains

$$\Gamma^{\tau\tau(\sigma\sigma)}(|n|) = Z^{-1} \sum_{l,m} \langle l | \mathcal{L}^{\tau(\sigma)}(0) | m \rangle^2 \exp[-\mu_m^{\tau(\sigma)} |n|] \exp[-\mu_l^{\tau(\sigma)} (N_\sigma - |n|)] \quad . \quad (22)$$

In the large distance limit, ignoring the higher excited states of the transfer matrix and summing over only the lowest states, one can show that

$$\Gamma^{\tau\tau(\sigma\sigma)}(|n|) = v^{\tau(\sigma)^2} \left\{ \exp -\mu^{\tau(\sigma)} |n| + \exp -\mu^{\tau(\sigma)} (N_\sigma - |n|) \right\} \quad , \quad (23)$$

with $v^{\tau(\sigma)^2} \equiv Z^{-1} \langle 0 | \mathcal{L}^{\tau(\sigma)} | 1 \rangle^2$. $\mu^{\tau(\sigma)}$ are the mass gaps which can be shown[11, 12, 13, 14] to correspond to the physical mass, namely, string tension, and the tunnelling mass in the confined and deconfined phases respectively. This is because of the expectation of a linearly rising colour averaged potential between a heavy quark and antiquark in the confined phase. In the deconfined phase, the potential may become of the Debye-screened Coulomb form. However, on a finite lattice, the tunnelling between the two degenerate $Z(2)$ -vacua, $|0_+\rangle$ and $|0_-\rangle$ yields a symmetric $|0_s\rangle$ ground state and an antisymmetric first excited state $|0_a\rangle$, leading to a small “tunnelling mass gap”. In the infinite volume limit, the tunnelling mass goes to zero and the symmetry is spontaneously broken. Thus this mass gap also acts as an effective order parameter for the spontaneous breaking of the $Z(2)$ -symmetry

and deconfinement. Its main advantage over the Polyakov loop is its direct relation with the string tension in the confined phase. Thus its becoming vanishingly small across a phase transition is a clearer indication of it being a deconfinement phase transition. Of course, the string tension will also decrease across a bulk phase transition but its vanishing on the other side, especially as the lattice size grows to infinity, is very unlikely.

3. DATA AND ANALYSIS

Our Monte Carlo simulations were done mostly at $\beta_A = 1.1$ on $8^3 \times 2$, $8^3 \times 4$ and $16^3 \times 8$ lattices, using the Metropolis *et al.* algorithm. On the $8^3 \times 2$ lattice we also performed simulations at larger β_A to confirm the order of the phase transition and to estimate the shift in critical β . Additional simulations were also made on $10^3 \times 2$ and $12^3 \times 2$ lattices to look for the finite size scaling behaviour and the critical exponent ω , defined in section 2. Typically $1\text{--}2 \times 10^5$ iterations were done on all lattices with about a factor two less statistics on the biggest lattice. For the $8^3 \times 4$ and $16^3 \times 8$ lattices, we also measured the spatial Polyakov loops. Measurements of temporal and spatial zero momentum correlators were done on the $16^3 \times 8$ lattice both in the confined and deconfined regions. The distribution function of the Polyakov loop is a good tool for identifying the critical points to a reasonably good accuracy, especially for a first order transition on larger lattices. In the confined phase one expects this function to be a gaussian peaked around zero, while in the deconfined phase of the SU(2) gauge theory one expects two symmetric gaussian peaks located away from zero. At the transition point of a first order phase transition, one would have an additional peak of the same height located at zero. One can alternatively look at the order parameter, $|L|$, and correspondingly obtain a two peak structure. Even when the transition is of second order, or the precise location of the transition point is not found, these distribution functions can be used to set the lower and upper limits on β_c . For our semi-quantitative purposes of distinguishing the shifts due to the two different scaling behaviours of the deconfinement and the bulk phase transition on $8^3 \times 4$ and $16^3 \times 8$ lattices, it was usually enough to study the behaviour of the histograms of the temporal and spatial Polyakov loops at various β -values.

3.1 $N_\tau = 2$

Variation of the temporal size of the lattice should shift the deconfinement phase transition much more than the bulk phase transition, allowing one to test the scenario [A] of the previous section. Already in Ref. [5], a change from $N_\tau = 4$ to 6 was attempted but the results were inconclusive as the shift in the critical coupling was too small and only one transition was observed. From the scaling law for the deconfinement phase transition, one expects a much larger shift by going to *smaller* N_τ , although it usually takes one in the strong coupling region. Since our aim here is only to see whether there are two separate transitions, it is perhaps not so important that one does not work in the scaling region; hopefully, such qualitative aspects are not altered by the continuum limit.

Figs. 2a and 2c show the average temporal Polyakov line $\langle L_\tau \rangle$ and the average plaquette as a function of the coupling β for $\beta_A = 1.1$ on an $8^3 \times 2$ lattice. The later value was chosen because our earlier work[5] suggests a weak first order phase transition at $(\beta, \beta_A) = (1.33, 1.1)$ on an $8^3 \times 4$ lattice which could be coincident with the bulk transition seen in Ref. [2] . The corresponding susceptibilities are displayed in Figs. 2b and 2d. Both the Polyakov line and the plaquette have a sharp and large jump around $\beta = 1.20$ and correspondingly the respective susceptibilities exhibit a sharp peak there. The above behaviour of the Polyakov line and its susceptibility clearly indicates the presence of a deconfinement transition at $\beta_c = 1.20$. Note, however, that the behaviour of the Polyakov line and the plaquette is smooth in the region around $\beta = 1.33$ where the bulk transition was expected. For a better comparison a magnified view of the Polyakov line and the plaquette in the regions around the deconfinement phase transition and around $\beta = 1.33$ is shown in the Figs. 3a - 3d. In order to show a bigger region around $\beta = 1.33$, ranges of both x and y axis have been enlarged by a factor of four in Figs. 3b and 3d as compared to those of Figs. 3a and 3c. The remarkable flatness of the data in Figs. 3b and 3d, together with the absence of any structure in the susceptibilities in Figs. 2b and 2d, make it clear that there is no trace of any transition in the region around $\beta = 1.33$. In view of the different scaling laws for the deconfinement and bulk phase transitions and the additional results from Ref. [5], indicating a very small shift in β_c in going from an $8^3 \times 4$ lattice to a $12^3 \times 6$ lattice, this relatively large shift makes it

difficult to sustain the accidental coincidence hypothesis in scenario [A]. Of course, the lattices used may not be large enough for various corrections to the scaling behaviour to be small. Nevertheless, it would be remarkable, if not impossible, if they are similar in magnitude and sign for the two unrelated scaling forms.

The large shift in the critical coupling in going to $N_\tau = 2$ from $N_\tau = 4$ and 6, and the fact that the order parameter $\langle |L| \rangle$ shows a spontaneous breakdown of the $Z(2)$ symmetry at the transition point are indicative of the deconfining nature of the phase transition. Moreover, the histograms of L , on $8^3 \times 2$, $10^3 \times 2$ and $12^3 \times 2$ show a three peak structure at the transition point which, however, is not very pronounced at $\beta_A = 1.1$. Correspondingly, the determination of the critical exponent ω does not fix the order of the phase transition uniquely. This is similar to the $N_\tau = 4$ results in Ref. [5]. It was found there that the above transition was a relatively weak first order transition at $\beta_A = 1.1$ but for $\beta_A > 1.1$ the discontinuity increased with an increasing β_A . Having thus verified that the order of the phase transition does not change as N_τ is changed to 2 from 4, we simulated the theory at $\beta_A = 1.4$ and 1.5 on the above lattices with $N_\tau = 2$ to see if its strength increases. It does indeed increase and we were even able to confirm the first order nature by determining the critical exponent ω . The evolution curves for both the plaquette and the Polyakov loop in the respective critical regions at $\beta_A = 1.4$ are plotted in the Figs. 4a-4c and the corresponding histograms for the latter are shown in Fig. 5. One sees that they all suggest a clear first order deconfinement phase transition. One does observe a sharpening of the transition as the volume is increased. It may be interesting to note that the average plaquette also shows a discontinuity and fluctuations which are totally correlated to that of L , which is again in agreement with the similar observation on $N_\tau = 4$ lattice. Individual runs around the critical coupling were used to locate the maximum of the Polyakov susceptibility. This maximum value of χ was in turn used to compute the exponent ω , defined earlier by Eq. (16). On the $12^3 \times 2$ lattice, a large tunnelling time of ($\approx 30,000$ - $40,000$ sweeps) was observed at $\beta_A = 1.4$, and hence a relatively higher statistics run of 3×10^5 was made at the transition point to ensure a better sampling of both the phases. The critical couplings and the estimated values of the exponent ω are given in the Table 1. They are in excellent agreement with the finite size scaling prediction for a first order deconfinement phase transition. Finally, also the simulations at $\beta_A = 1.5$ support the above picture

and clear three-peak histograms are observed. The critical coupling for the $8^3 \times 2$ lattice for $\beta_A = 1.5$ is $\beta = 0.983 \pm 0.001$. Comparing this value with the corresponding $N_\tau = 4$ result, one sees that the shifts due to a change of N_τ from 4 to 2 become smaller as β_A grows larger, but even at $\beta_A = 1.5$ the shift is nevertheless significantly large enough to be sure that it is non-zero. The discontinuity in $|L|$ also appears to increase slightly on the smaller N_τ lattice when compared at the same β_A .

3.2 $N_\tau = 4$ and 8

The above results on the $N_\tau = 2$ lattices showed clearly the absence of any transition at the location of a previously claimed bulk phase transition and further showed a first order phase transition at a much smaller β . The latter can be consistently interpreted as a deconfinement phase transition. While they make the coincidence of a bulk transition unlikely, one would like to establish it as concretely as possible. With such a motivation, we simulated the extended action at $\beta_A = 1.1$ on $8^3 \times 4$ and $16^3 \times 8$ lattices. Now, we also monitored the L^σ since the corresponding effective 3-volume is substantial on these relatively bigger lattices.

The histograms for L^τ and L^σ on the $8^3 \times 4$ lattice at four values of the coupling $\beta = 1.31, 1.32635, 1.4$ and 1.65 are shown in Figs. 6a-6d. These couplings have been chosen such that the two intermediate couplings are close to the corresponding β_c^τ and β_c^σ . Due to the weakness of the first order transition, it was not possible to get a clear three-peak structure of the histogram. However, the reasonably flat behaviours of the L_τ and L_σ in the Figs. 6b and 6c show the onset of the deconfining transition and suggest that it is first order. Locating the critical point by the criterion of maximum susceptibility, β_c^τ was found in Ref. [5] to be 1.327 which is good agreement with the $\beta_c^\tau = 1.32635$ of Fig. 6c. Figs. 6a-6d, and our additional simulations (not shown in these figures) just below and above the critical couplings, show that these distributions follow the expected pattern of the *deconfinement transition* very neatly:

- a] For $\beta < \beta_c^\tau$, both L_τ and L_σ are confined,
- b] at $\beta = \beta_c^\tau = 1.32635$, L_τ is critical and L_σ is confined,
- c] at $\beta = \beta_c^\sigma = 1.4$, L_σ is critical and L_τ is deconfined,
- d] for $\beta > \beta_c^\sigma$, both L_τ and L_σ are deconfined.

A similarly chosen pattern of histograms on the $16^3 \times 8$ lattice at five values of the coupling $\beta=1.33, 1.35, 1.3508, 1.47$ and 1.57 is shown in Figs. 7a-7d. Again, we observe a confinement-deconfinement pattern in L_τ and L_σ like the one discussed above. Due to the narrower transition region, we were unable to get better histograms of L_τ and fix the critical coupling β_τ^c more accurately. However, as is clear from the Fig. 6b, the critical coupling $\beta_\tau^c(N_\tau = 8)$ is between 1.35 and 1.3508. We have also observed a clear signal for the coexistence of two phases at $\beta = 1.35$, with a discontinuity in the Polyakov line L_τ which is roughly 0.05. The corresponding value at the critical point on the $8^3 \times 4$ lattice was found to be 0.25. Due to the fluctuations in plaquette, its discontinuity could not be estimated but it appeared to be smaller than the one observed on the $8^3 \times 4$ lattice as well. Such a scaling of discontinuities is expected for a deconfinement transition but not a bulk transition. Fig. 8a illustrates the scaling of the Polyakov line and the deconfinement temperature at $\beta_A = 1.1$ as we increase the temporal lattice size and thus approach towards the continuum limit. One can clearly see the transition moving towards the continuum critical fixed point $g_u^*=0$ with a corresponding reduction in the discontinuity of L^τ . On the other hand the scaling behaviour of the plaquette susceptibility at $\beta_A = 1.1$ on the same lattices, namely on $8^3 \times 2$, $8^3 \times 4$ and $16^3 \times 8$ lattices, shown in Fig. 8b, does not show any bulk phase transition. It should be noted that on the largest lattice the peak of the susceptibility is indeed at a bit lower value of β compared to the location of the deconfinement transition hinted in Figs. 7-b and 8-a. Such a behaviour would be consistent with the scenario [A] where one expects the deconfinement phase transition to move away faster. However, the height of the peak of the susceptibility for the $16^3 \times 8$ lattice is seen to decrease in Fig. 8b although the bulk volume increases by a factor of 16. Combined with the expected finite size scaling behaviour of a bulk first order in Eq. (5b), this result clearly rules out the presence of bulk transition convincingly in this region too.

Comparing the critical value $\beta_c^\sigma = 1.4$ for $L^\sigma \equiv L(8)$ on the $8^3 \times 4$ with the more precise value $\beta_c^\tau = 1.3504 \pm 0.0004$ for $L^\tau \equiv L(8)$ on the $16^3 \times 8$ lattice, one finds that the former is slightly overestimated. If one uses the maximum of the susceptibility $\chi(|L_8|)$ to define the transition point on the $8^3 \times 4$ lattice on the other hand, then one obtains $\beta_c^\sigma=1.348$, where the Ferrenberg-Swendsen[15] histogramming method was used on the configurations generated at $\beta = 1.4$ to locate the peak. This relatively large difference

in the determination of β_c by two different methods for the $8^3 \times 4$ lattice is perhaps indicative of the smallness of the effective 3-volume for L_8 . Even on this small lattice thus the errors in determination of β_c by employing L_σ as the order parameter are $\approx 5\%$ and a quantitative estimate of β_c on an effectively larger temporal lattice is thus feasible. Since we are anyway only interested in seeing whether the transition indeed marches towards $g_u^* = 0$, such small shifts are perhaps tolerable to us, considering that it will allow us to probe $N_\tau = 16$ by studying L^σ on $16^3 \times 8$ lattice. With this caveat in mind, we see from Fig. 7c that $\beta_c \simeq 1.47$ for $N_\tau = 16$, as the L_{16} -distribution can be seen to become very flat at $L = 0$ on the $16^3 \times 8$ lattice at this coupling. Furthermore, using the configurations at $\beta = 1.47$ to locate the peak of the $|L|$ -susceptibility, one obtains $\beta_c^\sigma = 1.468$. Considering the increased effective 3-volume in this case, such an excellent agreement in these two determinations is in accord with the expectations. Allowing for a few per cent overestimation nevertheless due to the unusual geometry, one still sees that the transition is indeed slowly moving towards $g_u = 0$, although compared to the $\beta_A = 0.0$ results of Refs. [16, 17] these results are still far away from exhibiting a consistency with the two-loop scaling equations in Eq. (3-4).

The systematic behaviour of the histograms in both Figs. 6 and 7, suggesting that L^τ and L^σ start being non-zero at *different* locations with the former doing so always earlier, has important consequences for the scenarios [A] and [C] of sect. 2. Since the 4-volume for each figure is held constant, and so is the geometry of the lattice, merely one bulk phase transition would be inadequate to explain this behaviour of the Monte Carlo data. Thus, the scenario [C] is untenable. The strong correlation of the critical coupling with only the linear extent of the lattice along which the order parameter is defined, taken together with the decrease in the plaquette susceptibility at its peak in Fig. 8b and the results of previous subsection for $N_\tau = 2$ lattice, is also inconsistent with scenario [A], thus leaving the scenario B of a pure deconfinement phase transition as the only plausible one. Furthermore, the above mentioned decrease in the size of discontinuities in the lattice observables, namely the Polyakov line and the plaquette, again confirms the confining-deconfining nature of the transition. For a bulk transition these discontinuities would be expected to remain unaltered or increase, as the total volume of the lattice increases.

3.3 Polyakov Loop Correlations

As a final check of the nature of the transition along the line **QR**, which shifts strongly with the “temporal” extent of the lattice, as we saw above, we have investigated the behaviour of the Polyakov loop correlations and the corresponding mass gaps on the $16^3 \times 8$ lattice. From the discussion above and in sect. 2, one expects to see the mass gaps from the temporal and spatial Polyakov loop correlations to decrease strongly at the respective critical couplings. Moreover, they should roughly be the same, apart from finite size effects, in the phases where both the order parameters show the same behaviour.

The measurements of the zero momentum projected temporal and spatial correlation functions both above and below the transitions were done by recording their values after every 30 sweeps. The individual averages of temporal and spatial correlations were taken by summing $\Gamma^{\tau\tau}(n_j)$ over the three directions (j) and $\Gamma_i^{\sigma\sigma}(n_j)$ over (i,j) for fixed value of n_j . The discrete rotational covariance was ensured by plotting them for each of the three spatial directions (i=1,2,3) individually and ensuring that their differences lie within error bars. The above averaged values of the temporal and spatial correlators are plotted in the Figs. 9a-9b. A gradual flattening of the correlation functions is clearly seen in both Figs. 9a and 9b, as one increases the coupling β through the respective β_c . Below the transition they all fall rather steeply while a very flat behaviour is evident for very large β . The mass gaps in the confined and deconfined phases were obtained by fitting the correlation function with a single hyperbolic cosine function. Their values and the corresponding errors are given in Table 2 and are plotted in Fig. 10. The relatively large fluctuations suggest that much higher statistics may be needed for a precise and reliable determination of these mass gaps. Nevertheless, the small value of the mass gap above the respective phase transitions, indicated by the corresponding Polyakov line, is a clear signal of the $Z(2)$ symmetry being broken spontaneously in the infinite volume limit, leading to deconfinement of quarks. The relatively larger finite size effects for the spatial correlations are evident in the slower fall-off of the corresponding mass gaps, especially in the deconfinement region. The results are again in agreement with the conclusions of the previous subsections: one has deconfinement phase transitions at the respective β_c . Although the data

are somewhat noisy and thus not very conclusive, there is again no evidence of any extra first order bulk phase transition. It would have been nicer to have the spatial mass gaps at still lower β to confirm a continuity in them. However, we found a very rapid deterioration of the signal in the correlation functions in that region and large fluctuations in the correlation functions prevented us from arriving at any values.

4. DISCUSSION AND CONCLUSIONS

Our results in the previous section strongly suggest that the previously identified line of first order bulk phase transitions, shown in Fig. 1 by the line **QR**, is in fact a line of first order deconfinement phase transitions. No bulk phase transition seems to be present in that region. Since this bulk transition and its “shadow” on the $\beta_A = 0$ axis have been speculated to have an impact on the onset of confinement in the $SU(2)$ gauge theory at zero temperature, it is clearly necessary to investigate the phase diagram of Fig. 1 afresh on much bigger symmetric lattices than used in the past. Similar structure is also known to exist for other $SU(N)$ gauge theories, in particular for the $SU(3)$ theory as well. These may need to be investigated further to investigate their differences.

A first order deconfinement phase transition for $\beta_A \geq 1.1$ also implies a change of the universality class for the extended action at and above $\beta_A = 1.1$. It has been argued [6] that by integrating out the irrelevant degrees of freedom of the $SU(2)$ theory at finite temperature, one can obtain an effective theory of the order parameter L . If this theory and the Ising model in 3-dimensions, which too has a global $Z(2)$ symmetry for its order parameter, have only one fixed point in the space of couplings, then the two are in the same universality class. This leads to a prediction of various critical exponents for the former. For $\beta_A = 0$, these universality predictions were explicitly verified by high precision Monte Carlo simulations of Engels et al.[9]. They found $\frac{\beta}{\nu} = 0.545 \pm 0.030$, $\omega = \frac{\gamma}{\nu} = 1.93 \pm 0.03$, $\nu = 0.65 \pm 0.04$. The corresponding values for the Ising model are $\frac{\beta}{\nu} = 0.516 \pm 0.005$, $\frac{\gamma}{\nu} = 1.965 \pm 0.005$, $\nu = 0.63 \pm 0.003$. The change of the order of the transition (e.g at $\beta_A = 1.4, \omega \approx 3$) as a function of an apparently irrelevant coupling β_A shows a non-universal behaviour of the deconfining transition, which needs to be understood in the renormalisation group picture. It does cast a doubt on our understanding of the order of the deconfinement phase transitions from those in the corresponding spin models. Of course, one cannot rule out the possibility that this change of universality class is a purely strong coupling phenomenon; simulations on sufficiently larger lattices will show a second order deconfinement phase transition even at the large β_A used in this work. It would be very interesting, and illuminating, to check this since no such instance of a change of the order of the phase transition in

going over to the scaling regime is known so far. If confirmed, it would underline the importance of checking even such qualitative aspects of full QCD thermodynamics in the scaling or the asymptotic scaling regime.

A non-universal behaviour of the string tension depending on the lattice action used has been a question of debate in the past. In the SU(2) lattice gauge theory the Λ -ratios of the Manton and the Heat kernel action with that of the Wilson action were found to differ from their theoretically predicted values. However, these discrepancies were shown[18] to be due to higher order corrections to the theoretical values which were obtained in leading order perturbation theory. By taking the ratios of the physical observables in Monte Carlo simulations or by considering the effects of higher order terms the above actions were shown to be in the same universality class. In the case of the extended SU(2) action of Eq. (1), Bhanot and Dashen [19] found that at $\beta_A = 0.9$, the Λ -ratio, $\Lambda_{Extended}/\Lambda_{Wilson}$, obtained from the computations of string tension in Monte Carlo simulations, was roughly a factor of four higher than its perturbation theoretical value in leading order. Again, this was shown[18, 20] to be due to the strong coupling effects. In fact, the string tension data compared well with its strong coupling expansion [20] and correspondingly the 3-loop contribution to Λ ratio was quite large [18].

What we find in the change of the order of the deconfinement phase transition, however, is a loss of universality in qualitative features. and therefore should be considered much more seriously. Moreover, the line of deconfinement transition we investigated on $N_\tau = 4$ is remarkably close to the line of constant string tension, obtained from Monte Carlo simulation [19], at $\sigma a^2 = 0.14$. This line lies in the cross-over region[20]. Thus our results and conclusions could possibly be free from any strong coupling artifacts. In other words, one need not expect that by going to larger temporal lattices at $\beta_A > 1.0$ the scaling exponent ω will drop from 3 to its second order value 1.97, corresponding to the 3-dimensional Ising model. Taking seriously the above coincidence of the lines of the deconfinement transition and the constant string tension, along with the fact that up to 2-loops of the β function physics is independent of the value of θ , the above change of universality class indicates a possibility of a new fixed point and some non-perturbative effects. However, much more work is necessary to convincingly establish or rule out the above interesting possibilities.

In the case of pure Wilson action, Polonyi and Szlachanyi [21] showed the second order nature of the deconfining transition by computing the effective

action in terms of the order parameter L in the strong coupling limit. It will be interesting to compute the effect of adjoint coupling β_A and see how it changes the order of the transition. One expects a L^4 term with its coefficient becoming negative as the adjoint coupling increases above $\beta_A = 1.1$.

Let us summarise by stressing that the universality hypothesis in lattice gauge theories, in particular of extended Wilson action, should be looked at more critically. It will be interesting to study similar phenomenon with the corresponding extended SU(3) Wilson action also.

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FIGURE CAPTIONS

Fig.1 The phase diagram of the extended SU(2) lattice gauge theory. The solid lines are from simulations done on a 5^4 lattice by Bhanot and Creutz[19]. The broken lines **QR** is the finite temperature deconfinement phase transition line on the $8^3 \times 4$ lattice. The deconfinement line **ST** corresponds to $8^3 \times 2$ lattice.

Fig.2 (a)Average temporal Polyakov loop, (b) its susceptibility, (c) average plaquette, and (d) its susceptibility vs. β on an $8^3 \times 2$ lattice at $\beta_A = 1.1$.

Fig.3 Average temporal Polyakov loop and average plaquette as a function of β on an $8^3 \times 2$ lattice in the regions of the deconfinement phase transition (a, c) and the expected bulk transition (b, d).

Fig.4 Evolution of L_τ and $(P+0.35)$ $\beta_A = 1.4$ and on (a) $8^3 \times 2$ ($\beta_c = 1.03954$) (b) $10^3 \times 2$ ($\beta_c = 1.0398$) and (c) $12^3 \times 2$ ($\beta_c = 1.04007$)

Fig.5 Normalised probability distributions of L_τ on an $8^3 \times 2$, $10^3 \times 2$ and $12^3 \times 2$ lattices at $\beta_A = 1.4$ and $\beta_c=1.03954$, 1.0398 and 1.04007 respectively.

Fig. 6 Normalised probability distributions of L^τ and L^σ on $8^3 \times 4$ lattice at $\beta_A = 1.1$ and (a) $\beta = 1.31$, (b) $\beta = \beta_c^\tau = 1.32635$, (c) $\beta = \beta_c^\sigma \approx 1.4$ and (d) $\beta = 1.65$

Fig. 7 Same as Fig. 6 but for a $16^3 \times 8$ lattice and at (a) $\beta = 1.33$, (b) $\beta = \beta_c^\tau = 1.35, 1.3508$, (c) $\beta = \beta_c^\sigma \approx 1.47$ and (d) $\beta = 1.57$

Fig. 8 The nature of the first order transition at $\beta_A = 1.1$: (a) The scaling behaviour of L^τ on $8^3 \times 2$, $8^3 \times 4$ and $16^3 \times 8$ lattices in accordance with the deconfinement transition and (b) the scaling behaviour of the plaquette susceptibility, ruling out any bulk transition in this region.

Fig. 9 The (a) temporal and (b) spatial correlation functions on $16^3 \times 8$ lattices at $\beta_A = 1.1$ as a function of distance for various couplings β through the respective β_c .

Fig. 10 The temporal and spatial mass gaps μ^τ and μ^σ on $16^3 \times 8$ lattice at $\beta_A = 1.1$.

Table 1

The values of the critical couplings ($\beta_c, \beta_A = 1.4$) on $N_\tau=2$ lattices, the L^τ finite size scaling exponent ω . The expected value for ω is 3.0(1.97) if the deconfining phase transition is first order (second order).

N_σ	β_c	ω
8	1.039540	—
10	1.03980	3.246(243) [$8^3 \times 2 : 10^3 \times 2$]
12	1.040070	3.204(184) [$12^3 \times 2 : 8^3 \times 2$] 3.154(406) [$10^3 \times 2 : 12^3 \times 2$]

Table 2

The values of the temporal and spatial mass gaps, μ^τ and μ^σ , at $\beta_A = 1.1$ on $16^3 \times 8$ lattice.

β	μ^τ	μ^σ
1.34	0.33526(01368)	—
1.35	0.16618(00568)	—
1.3508	0.09530(00400)	—
1.3525	0.09327(00999)	—
1.355	0.07505(00930)	—
1.36	0.06538(00813)	—
1.41	0.03244(00472)	0.20908(00601)
1.44	0.02637(00398)	0.21849(00544)
1.45	0.03222(01499)	0.19063(01356)
1.47	0.02523(00419)	0.15654(00393)
1.5	0.02219(00720)	0.18009(00706)
1.54	0.01992(00621)	0.16590(00583)
1.57	0.02054(00390)	0.12904(00285)
1.59	0.02165(00452)	0.11620(00323)



















