A STUDY OF THE BULK PHASE TRANSITIONS OF THE SU(2) LATTICE GAUGE THEORY WITH MIXED ACTION

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ABSTRACT

Using the finite size scaling theory, we re-examine the nature of the bulk phase transition in the fundamental-adjoint coupling plane of the SU(2) lattice gauge theory at $\beta_A = 1.25$ where previous finite size scaling investigations of the deconfinement phase transition showed it to be of first order for temporal lattices with four sites. Our simulations on N^4 lattices with N=6, 8, 10, 12 and 16 show an absence of a first order bulk phase transition. We find the discontinuity in the average plaquette to decrease approximately linearly with N. Correspondingly, the plaquette susceptibility grows a lot slower with the 4-volume of the lattice than expected from a first order bulk phase transition.

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1. INTRODUCTION

The lattice regularization of quantum field theories is a gauge invariant non-perturbative tool to investigate long distance phenomenon, such as the confinement of quarks, and to extract the various low energy properties of quantum chromodynamics, such as the hadronic spectrum. As with other regularizations, there is a lot of freedom in defining a lattice field theory. In particular, a variety of different choices of the lattice action correspond to the same quantum field theory in the continuum. While most of the numerical simulations are performed for the Wilson action[1] for the gauge theories, other choices, some motivated by the desire to find a smoother continuum limit, have also been used. Indeed, since these actions differ merely by irrelevant terms in the parlance of the renormalization group, they must give rise to the same physical results. In view of the necessity of using finite lattices and the not-so-small lattice spacings in computer simulations, investigations with different actions can provide an independent check on the cut-off independence of the physical results.

From a more theoretical point of view, investigations with different lattice actions could provide clues in understanding the physics of confinement. Since confinement of quarks can be explicitly shown on the lattice in the strong coupling region, one naively expects a smooth passage to the weak coupling regime without any phase transitions in order for confinement to persist in the continuum limit. Bhanot and Creutz [2], extending the form of the action proposed by Wilson by adding an adjoint coupling term , showed that confinement could survive even though the phase diagram of the mixed action, shown in Fig. 1, has the so-called bulk phase transitions along the solid lines. The termination of the lower line at a finite adjoint coupling, β_A, in Fig. 1 allows a smooth path between the confining and the asymptotically free phases. The proximity of this end point to the $\beta_A = 0$ line, which defines the Wilson action, has commonly been held responsible for the abrupt change from the strong coupling region to the scaling region for the Wilson action. Its relative closeness to the $\beta_A = 0$ line for the $SU(2)$ theory compared to the $SU(3)$ theory has been thought of as a possible reason for the shallower dip in the corresponding non-perturbative β-function obtained[3] by Monte Carlo Renormalization Group methods. In view of the fact that rather small lattices were used to obtain the phase diagram in Fig. 1, it therefore appears

necessary to re-examine the phase diagram on bigger lattices and with better statistics. The work reported in this manuscript is a step in that direction.

Another major motivation for re-examining the phase diagram comes from our work[4, 5, 6] on the same mixed action at non-zero temperatures. Along the $\beta_A = 0.0$ axis, several finite temperature investigations have shown the presence of a second order deconfinement phase transition. Its critical exponents have been shown[7] to be in very good agreement with those of the three dimensional Ising model. Effective field theory arguments for the order parameter were used by Svetitsky and Yaffe[8] to conjecture the finite temperature $SU(2)$ gauge theory and the three dimensional Ising model to be in the same universality class. The verification of this universality conjecture thus strengthened our analytical understanding of the deconfinement phase transition. However, following the deconfinement phase transition into the extended coupling plane by simulating the extended action at finite temperature, we surprisingly found that:

a] The deconfinement transition was of second order, and in agreement with the universality conjectured exponents, only up to $\beta_A \approx 1.0$. It became definitely of first order for large enough $\beta_A (\geq 1.25)$.

b] There was no evidence of an another separate bulk transition at larger β_A, as suggested by Fig. 1.

Using asymmetric lattices, $N_\sigma^3 \times N_\beta$, with $N_\beta = 2$-8 and $N_\sigma = 8$-16, we obtained the following key results which lead us to the conclusions above:

a] Only one transition was found on all the lattices studied. The deconfinement order parameter acquired nonzero large value at the transition and showed a clear co-existence of both phases at the transition point for larger β_A.

b] The same critical exponent which established the transition to be in the Ising model universality class for $0.0 \leq \beta_A \leq 1.0$ became equal to the space dimensionality (=3), as a first order deconfinement phase transition would have, for larger β_A.

As argued in Ref. [4] already, the apparent coincidence of the trajectory of the deconfinement phase transition for $N_\beta = 4$ with the lower arm of

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the bulk phase diagram of Fig. 1 itself suggests a possible explanation of these results: The first order bulk transition overshadows the second order deconfinement phase transition. It is therefore mandatory to confirm the existence of the first order bulk transition on bigger symmetric lattices and with better statistics than that of Ref. [2]. In this paper, we undertake this task by simulating the mixed action on symmetric N^4 lattices, with $N = 6$-16. The organization of the paper is as follows: In section 2 we define the action we investigate and briefly recapitulate the definitions of various observables used and their scaling laws. We present the detailed results of our simulations in the next section and the last section contains a brief summary of our results and their discussion.

2. THE MODEL AND THE OBSERVABLES

The lattice action is constrained only by a) the gauge invariance and b) the limit of zero lattice spacing which must coincide with the continuum form of the action. Infinitely many different forms satisfying these criteria can be written down. Bhanot and Creutz extended the Wilson action to a form described by the action,

$$S = \sum_P \left(\beta \left(1 - \frac{1}{2} Tr_F U_P \right) + \beta_A \left(1 - \frac{1}{3} Tr_A U_P \right) \right) \quad . \tag{1}$$

Here U_P denotes the directed product of the basic link variables which describe the gauge fields, $U_\mu(x)$, around an elementary plaquette P. F and A denote that the respective traces are evaluated in the fundamental and adjoint representations respectively. We use the formula $Tr_A U = |Tr_F U|^2 - 1$.

Comparing the naive classical continuum limit of eq. (1) with the standard $SU(2)$ Yang-Mills action, one obtains

$$\frac{1}{g_u^2} = \frac{\beta}{4} + \frac{2\beta_A}{3} \quad . \tag{2}$$

Here g_u is the bare coupling constant of the continuum theory. Since the asymptotic scaling equation for the mixed action can be easily written down

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in terms of g_u with a Λ-parameter that depends on the ratio of β and β_A, it is clear that the introduction of a non-zero β_A does not affect the continuum limit: the theory for each β_A, including the usual Wilson theory for $\beta_A = 0.0$ flows to the same critical fixed point, $g_u^c = 0$, in the continuum limit and has the same scaling behavior near the critical point.

As mentioned already in the introduction, Bhanot and Creutz[2] found that the lattice theory defined by the extended action of eq.(1) has a rich phase structure (Fig. 1). Along the $\beta = 0$ axis, it describes the $SO(3)$ model which has a first order phase transition at $\beta_A^{crit} \sim 2.5$. At $\beta_A = \infty$, it describes the Z_2 lattice gauge theory again with a first order phase transition at $\beta^{crit} = \frac{1}{2}\ell n(1 + \sqrt{2}) \approx 0.44$ [9]. Ref. [2] found that these first order transitions extend into the (β, β_A) plane, ending at an apparent critical point located at (1.5,0.9). These transition lines are shown in Fig. 1 by continuous lines. The qualitative aspects of this phase diagram were also later reproduced by mean field theory [10] and large N[11] and strong coupling[12] expansions.

We simulated the mixed action above on N^4 lattices, with $N = 6$, 8, 10, 12, and 16. Periodic boundary conditions were used in all the four directions. The partition function is, as usual, given by,

$$Z = \int \prod_{x,\mu} dU_\mu(x) \quad \exp(-S) . \tag{3}$$

We used the simple Metropolis algorithm and tuned it to have an acceptance rate $\sim$ 30-40 %. The expectation values of the observables were recorded every 20 iterations to reduce the autocorrelations. Errors were determined by correcting for the autocorrelations and also by binning. The observables monitored were the average plaquette, P, defined as the average of $Tr_F U_P/2$ over all independent plaquettes, and the average of $L(\vec{n})$ over the three dimensional lattice spanned by $\vec{n}$, where L is defined by

$$L(\vec{n}) = \frac{1}{2}Tr_F \prod_{\tau=1}^{N} U_0(\vec{n}, \tau) . \tag{4}$$

Here $U_0(\vec{n}, \tau)$ is the timelike link at the lattice site $(\vec{n}, \tau)$ and due to the symmetry of our lattices any direction could be identified as the time direction. One sees that the L corresponds to the usual order parameter[13] for the deconfinement transition on a lattice with N temporal sites but with also

N spatial sites. We will comment later on the utility of such an observable on symmetric lattices. In order to monitor the nature of the bulk phase transition, we also define the plaquette susceptibility:

$$\chi_N = 6N^4(\langle P^2 \rangle - \langle P \rangle^2) \ . \tag{5}$$

According to the finite size scaling theory[14], the peak of the plaquette susceptibility at the location of the bulk transition we wish to study should grow on an N^d lattice like

$$\chi_N^{max} \propto N^\omega \ . \tag{6}$$

For a second order transition, $\omega = \alpha/\nu$, where α and ν characterize the growth of the plaquette susceptibility and the correlation length near the critical coupling (temperature) on an infinite lattice. If the phase transition were to be of first order instead, then one expects the exponent $\omega = d = 4$, corresponding to the dimensionality of the space [15]. In addition, of course, the average plaquette is expected to exhibit a sharp, or even discontinuous, jump and the corresponding probability distribution should show a double peak structure in case of a first order phase transition.

3. **RESULTS OF THE SIMULATIONS**

Our Monte Carlo simulations were done using the Metropolis algorithm on N^4 lattices with $N=$ 6, 8, 10, 12 and 16. The many different values of N were chosen to study the finite size scaling behavior of the theory and to compute the critical exponent ω. For verifying the nature of the lower arm of the phase diagram in Fig. 1, any value of β_A between 0.9 and 2.0 would be suitable. Considering, however, the results of Ref. [6], where the deconfinement transition at $\beta_A = 1.25$ was shown to be of first order using $N^3 \times 4$ lattices with $N = 8$, 10, and 12, we chose $\beta_A = 1.25$ although we also attempted additional simulations at $\beta_A = 1.5$. Histogramming techniques[16] were used to extrapolate to nearby β values for estimating the height and location of the peak of the plaquette susceptibility.

Figs. 2 and 3 display the distributions of the average plaquette P at $\beta_A = 1.25$ on 6^4, 8^4, 10^4 and 12^4, 16^4 lattices respectively. The values of the fundamental coupling β at which these runs were made are 1.2147, 1.2179,

1.2182, 1.2183 and 1.2184 respectively and the corresponding number of measurements, each separated by 20 iterations, are 135000, 107000, 109000, 32500 and 13250. Thus Fig. 2 is based on equally high statistics runs. While one observes a double peak structure on each of the lattices used, it is clear that the distance between the peaks, i.e., the discontinuity in the plaquette, ΔP, decreases with increasing lattice size. Furthermore, one can also conclude from Fig. 2 that the valley between the peaks becomes shallower with the increase in lattice size. Both these observations are, of course, precisely opposite of what one expects for a first order bulk phase transition. The results in Fig. 3 for the bigger lattices are also in accord with both these trends established in Fig. 2, although they are based on rather modest statistics. Indeed, we have actually displayed the results for two neighboring couplings $\beta = 1.2183$ and 1.2184 on the 16^4 lattice to highlight any possible double peak structures in these runs. If one disregarded the smaller peaks in these runs as statistically marginal, then the actual discontinuity in ΔP on the 16^4 is most likely smaller than that suggested by Fig. 3. In Fig. 4, we show the corresponding plaquette histograms for the finite temperature case of Ref. [6] where a first order deconfinement phase transition was established at the *same* $\beta_A = 1.25$. This was based on the determination of the critical exponent for the Polyakov loop (L) susceptibility which was found to be equal to the space dimensionality, three, in that case. Note that the relative increase in (spatial) volume in going from the smallest to the biggest lattice in Fig. 4 is comparable to that for the two smaller lattices in Fig. 2. However, one sees that the peaks in Fig. 4 hardly move and moreover, the valley structure deepens as the spatial volume is increased. Both these observations are in full accord with the expectations for a first order phase transition. Of course, the key difference in these figures is that the temporal extent is kept fixed for Fig. 4 whereas it too is increased along with the spatial extent in Figs. 2 and 3. Thus the deconfinement phase transition does exhibit the behavior expected of a first order phase transition while the bulk phase transition at the same β_A does not.

The above qualitative observation of a lack of a bulk first order-like behavior can be made quantitatively more firm by using conventional ideas and methods. Using smooth curves to approximate the peaks in Figs. 2-4, one can estimate the location of each peak and deduce the size of the discontinuity, ΔP, in the average plaquette in each case. These results are given in Table 1 and are also shown in Fig. 5 as a function of $1/N$, where the

simulations were done on an N^4 lattice. Also listed in Table 1 are the results for the finite temperature case of Fig. 4. The errors reflect the bin sizes in Figs. 2-4. While the ΔP in the finite temperature case is constant, the data on symmetric N^4 lattices are consistent with a linear fall off with $1/N$ and seem to suggest a zero discontinuity in the average plaquette on a finite but large lattice, of the $O(70^4)$. A linear extrapolation to infinite lattice predicts a $\Delta P(\infty) = -0.011 \pm 0.006$, which too suggests a lack of any transition although a second or higher order transition will be difficult to rule out. Fig. 6 exhibits the plaquette susceptibility as a function of β for the 6^4, 8^4 and 10^4 lattices. The data point in each case corresponds to the runs shown in Fig. 2 and the error bars reflect the increase in the autocorrelation length with lattice size. One can see the that our long runs on each of the lattices are indeed very close to the location of the peak. Thus minimal systematic errors are expected from the histogramming extrapolation in the location and the heights of the peaks listed in Table 2. Noting that the increase in the 4-volume, N^4, is respectively a factor of 3.16 and 7.72 compared to the smallest lattice, it is clear that the increase in plaquette susceptibility is far short of that needed for a first order bulk transition. A fit of the peak heights to eq. (6) yields an $\omega = 2.09 \pm 0.31$ which is to be contrasted with the space dimensionality, 4. The fitted value of ω is consistent with a linear decrease in the discontinuity ΔP seen in Fig. 5, as can be seen from eq. (5). One can use this exponent to predict the peak heights for the $N = 12$ and 16 lattices. As can be seen from Table 2, these values, 89 and 163 respectively, compare favorably with the Monte Carlo results. A better determination of these peak heights is computationally very hard due to both the large lattice sizes and the increase in autocorrelations. Nevertheless, it seems clear that the bulk transition at $\beta_A = 1.25$, if there is one, is not a first order transition. At the very least, this suggests the endpoint of the lower arm in Fig. 1 to be at $(\beta, \beta_A) = (1.2184, 1.25)$, although a still higher β_A and correspondingly smaller β seems more likely. In view of the results shown in Fig. 4, and discussed in more details in Ref. [6], a first order deconfinement phase transition does seem to exist at $\beta_A = 1.25$, indicating that the deconfinement transition (for $N_\beta = 4$ but infinite spatial volume) turns first order before β_A is large enough for a first order bulk phase transition to exist. One is therefore lead to conclude that the change in the order of the deconfinement phase transition is indeed a real finite temperature effect at large β_A.

We have also studied the histograms of the average Polyakov loop, L de-

fined above, to look for a possible deconfinement phase transition on these lattices and at $\beta_A = 1.25$. One expects large corrections due to finite volume since VT^3 is unity in stead of being very large compared to one. An expected consequence thus is significantly wider distributions for L, making the critical coupling, where the distribution develops a lot flatter peak or a multi-peak structure, shift. We found that the histograms for L were peaked at zero for all the β values discussed above. Increasing β a little, the peak flattened and developed a three peak structure which was however not as sharp as the plaquette histograms. As an example, let us quote that such a determination lead to $\beta_c(N = 6) \simeq 1.2179$, which should be compared with the β_c obtained from the peak of the plaquette susceptibility and given in Table 2. One may be tempted to interpret this as a sign that the deconfinement transition is splitting away from the bulk transition and moving to larger β. On the other hand, it is not uncommon for different observables and thus different definitions to yield different estimates for the location of the same transition on a *finite* lattice. Only in the thermodynamic limit is it necessary for all the estimates to coincide. As argued already, such differences are all the more natural since $VT^3 = 1$. It therefore appears to us that even on these symmetric lattices either the deconfinement phase transition is the only transition or at least it is coincident with a (second or higher order) bulk transition signaled by the rapid changes in the average action. It should be noted that a *decrease* in the plaquette discontinuity, as seen in Fig. 5, would be naively consistent with the first option since an increase in N for a constant critical temperature would amount to decreasing the lattice spacing in that case and a decrease in plaquette discontinuity could then be a way to hold the corresponding latent heat constant. In that case, the bulk finite size scaling arguments, involving a scaling with the 4-volume N^4 would not apply. In stead, it would be necessary to check the scaling by holding N_β fixed and with a large VT^3. Precisely such an exercise was carried out in Ref. [6] and it did establish a first order deconfinement phase transition.

We have also attempted to repeat the above exercise at a larger $\beta_A = 1.5$ to find out whether a first order bulk phase transition exists there. As the size of discontinuity on the smaller lattices, 6^4 and 8^4, also increases with the increase in β_A, it became difficult to perform any meaningful finite size scaling analysis. In particular, we found that the ordered and random starts at $\beta = 1.045$ on these lattices remained separate even after 50000 measurements, corresponding to 1 million sweeps. This calls for a use of better algorithms

which will encourage tunnelings between the two states and thus permit a reliable finite size scaling study. The plaquette discontinuities from our runs were found to be 0.22352 (15) and 0.22291 (12), showing a miniscule decrease of 0.00061 (19). While this is an encouraging sign for the existence of a first order bulk phase transition, the results of Fig. 7 show an amusing correlation of the deconfinement phase transition with it. Fig. 7 shows the hysteresis effects in the average plaquette P and the Polyakov loop L on N^4 lattices with $N = 4, 6, 8, 10$ and 12 at $\beta_A = 1.5$. Starting from a $\beta = 0.95$ with a disordered start for the gauge variables, 2000 iterations were performed at each β at an interval of $\delta\beta = 0.01$ to thermalize and then the observables were recorded over the next 8000 iterations. Similarly a run was begun from $\beta = 1.15$ with an ordered start to obtain the other branch. A good agreement between the two curves outside the metastable area is an indication that the metastabilities are real. In spite of the large fluctuations in L for larger β, it appears that both L and P on all the lattices undergo strong changes at about the same β. The size of plaquette discontinuity suggested by Fig. 7 is approximately the same as mentioned above and it seems to remain unchanged even on a 12^4 lattice. It should perhaps be noted that the decrease in the discontinuity in L is related to the well known observation of the decrease of L in the high temperature phase with N (or temporal lattice size). Unfortunately, the big metastable region in Fig. 7, related to the large discontinuity in P, makes it very hard to ascertain whether one is dealing with two transitions here or one and what finite size scaling properties they have (or it has).

4. SUMMARY AND DISCUSSION

The phase diagram of the mixed action of eq. (1) in the fundamental and adjoint couplings, β and β_A, has been a crucial input in understanding many properties of the $SU(2)$ and $SU(3)$ lattice theories and their continuum limit. The cross-over to the scaling region from the strong coupling region, as well as the dip in the non-perturbative β-function have been attributed to the location of the end point of the line of bulk first order phase transition. In fact, even the relative shallowness of the dip for the $SU(2)$ case compared to the $SU(3)$ case is thought to be due to the closeness of the corresponding end point to the $\beta_A = 0$ Wilson axis. The recently observed change of the order

of the deconfinement phase transition for the $SU(2)$ lattice gauge theory for lattices with four (and two) temporal sites could also be due to the seemingly puzzling coincidence of the bulk first order line with the deconfinement line for large enough β_A.

The phase diagram of Fig.1, taken from Ref. [2], was obtained on smaller lattices, 5^4-7^4, and with modest statistics. We simulated the extended action on N^4 lattices with $N = 6, 8, 10, 12,$ and 16 at $\beta_A = 1.25$ and 1.5. The choice of these adjoint couplings was based on the results of Ref. [4, 6] where the deconfinement transition was shown to be of first order for $N_\beta = 4$ using finite size scaling theory. In particular, the susceptibility for the order parameter, the Polyakov loop, was shown to increase linearly with spatial volume at β_A=1.25; it grew approximately as the two-third power of the spatial volume for small β_A which is similar to the Ising model in three dimensions which has a second order phase transition. We found that the plaquette distributions do exhibit a double peak structure on the symmetric lattices as well. However, the major difference was that the peaks appear to approach each other and the intervening valley appears to become shallower as the lattice size N is increased at the *same* $\beta_A = 1.25$. Quantitatively, this was reflected in a much slower increase in the plaquette susceptibility with the 4-volume than would be expected for a first order bulk phase transition and a linear decrease in the size of the discontinuity of the plaquette, ΔP, with N. The critical exponent ω, defined in eq. (6), was found to be 2.09 ± 0.31 in contrast to the expected value of 4 for a first order bulk phase transition. While a linear extrapolation of our data on ΔP suggests that it vanishes already on finite, $O(70^4)$ lattices, we are unable to rule out a second or higher order bulk phase transition at $\beta_A = 1.25$. We conclude that the endpoint of the bulk line in Fig. 1 is most likely at a $\beta_A \geq 1.25$ with a correspondingly smaller $\beta = 1.2184$ or lesser. It also seems therefore that the change of the order of the deconfinement phase transition for $N_\beta = 4$ at $\beta_A \simeq 1.25$ is unlikely to be influenced by any (first order) bulk phase transition. As argued in Ref. [6], strong coupling arguments do suggest precisely this, namely, the increase in β_A changes the effective potential for the order parameter to allow a first order deconfinement phase transition at sufficiently large β_A and thus the bulk dynamics need not play any role in such a change.

Our simulations at the larger $\beta_A = 1.5$ were inadequate to test in a similar manner using finite size scaling theory whether the transition there is a first order bulk phase transition or not. The large size of the plaquette

discontinuity on the smaller lattices meant that the Metropolis algorithm was inefficient in effectively sampling both the states, thus aborting our attempts to check whether $\omega = 4$. On the other hand, the decrease in ΔP with N appeared to be much smaller than at $\beta_A = 1.25$. Hysteresis runs on a variety of lattice sizes showed that the deconfinement order parameter too jumps at the transition rather abruptly, suggesting that the deconfinement transition is either very close to the bulk transition or even coincident. It would be very interesting to decipher the finite size scaling behavior at these larger β_A to distinguish the two very different transitions. Indeed, even in the case of the $SU(3)$ gauge theory, where simulations[17] with the mixed action have yielded a separation of the line of the deconfinement phase transitions from the line of bulk phase transitions with increasing N_β, a convincing demonstration of such a separation would really come from similar finite size scaling investigations.

6. ACKNOWLEDGMENTS

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FIGURE CAPTIONS

Fig. 1 The phase diagram of the extended SU(2) lattice gauge theory. Taken from Ref. [2].

Fig. 2 Probability distribution of the average Plaquette P at $\beta_A = 1.25$ on N^4 lattices with N=6, 8, and 10 The values of β for these runs are given in the text.

Fig. 3 Same as in Fig. 2 but for $N = 12$ and 16.

Fig. 4 Same as in Fig. 2 but for the asymmetric lattices $N^3 \times 4$ with $N = 8$, 10, and 12. lattices.

Fig. 5 The discontinuity in the average Plaquette, ΔP, on N^4 lattices as a function of $1/N$ for $\beta_A = 1.25$. The line denotes a simple linear fit.

Fig. 6 Plaquette susceptibility as a function of β on N^4 lattices with N=6, 8, and 10 for $\beta_A = 1.25$.

Fig. 7 The hysteresis in the average plaquette P and the order parameter for deconfinement, L, as a function of β. All the results were obtained on N^4 lattices for N=8, 10, and 12 and at $\beta_A = 1.25$.
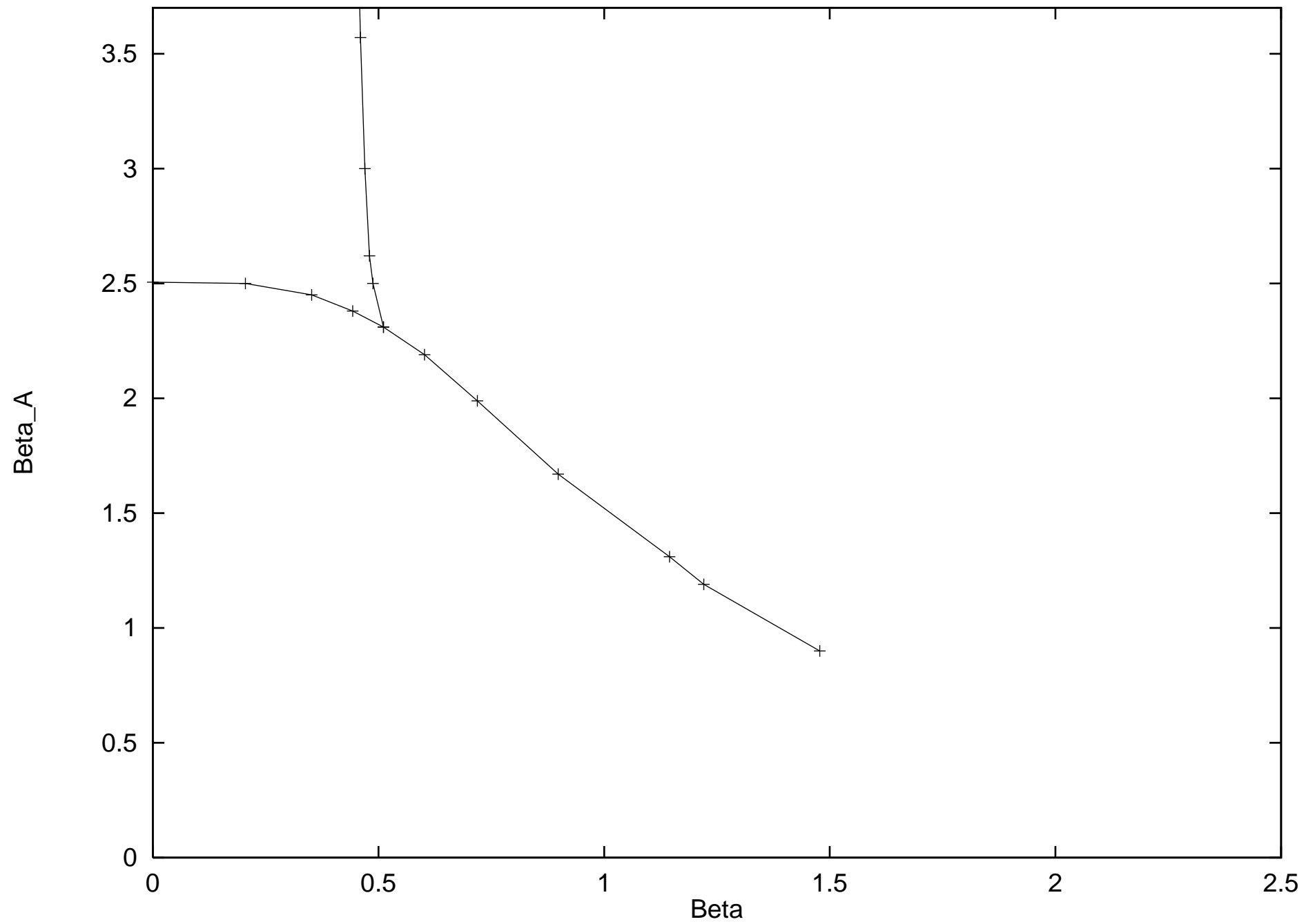
Table 1

The average values of the plaquette discontinuity ΔP
at $\beta_A = 1.25$ on symmetric N^4 lattices and asymmetric
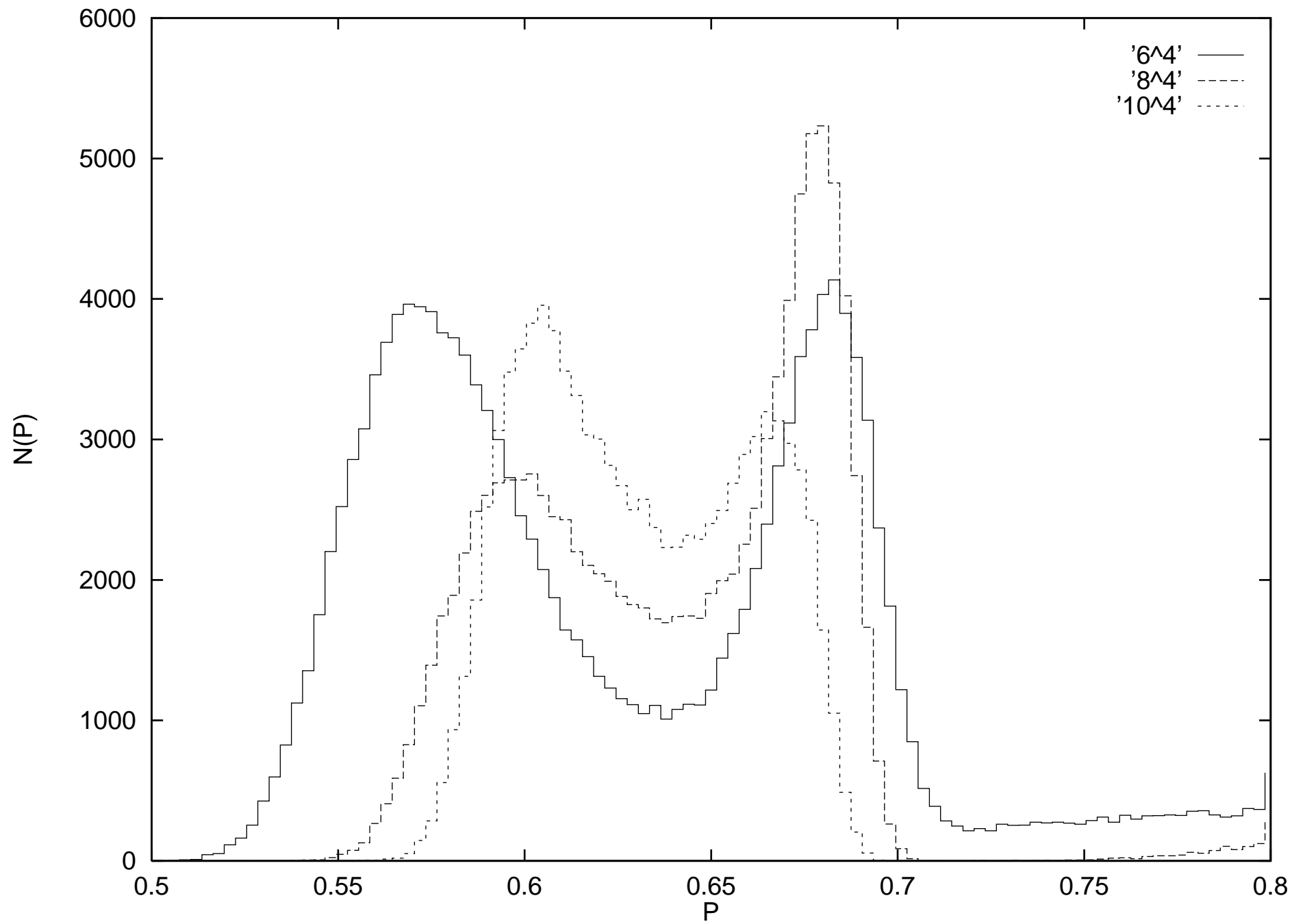$N^3 \times 4$ lattices. The data for latter are taken
from Ref. [6]

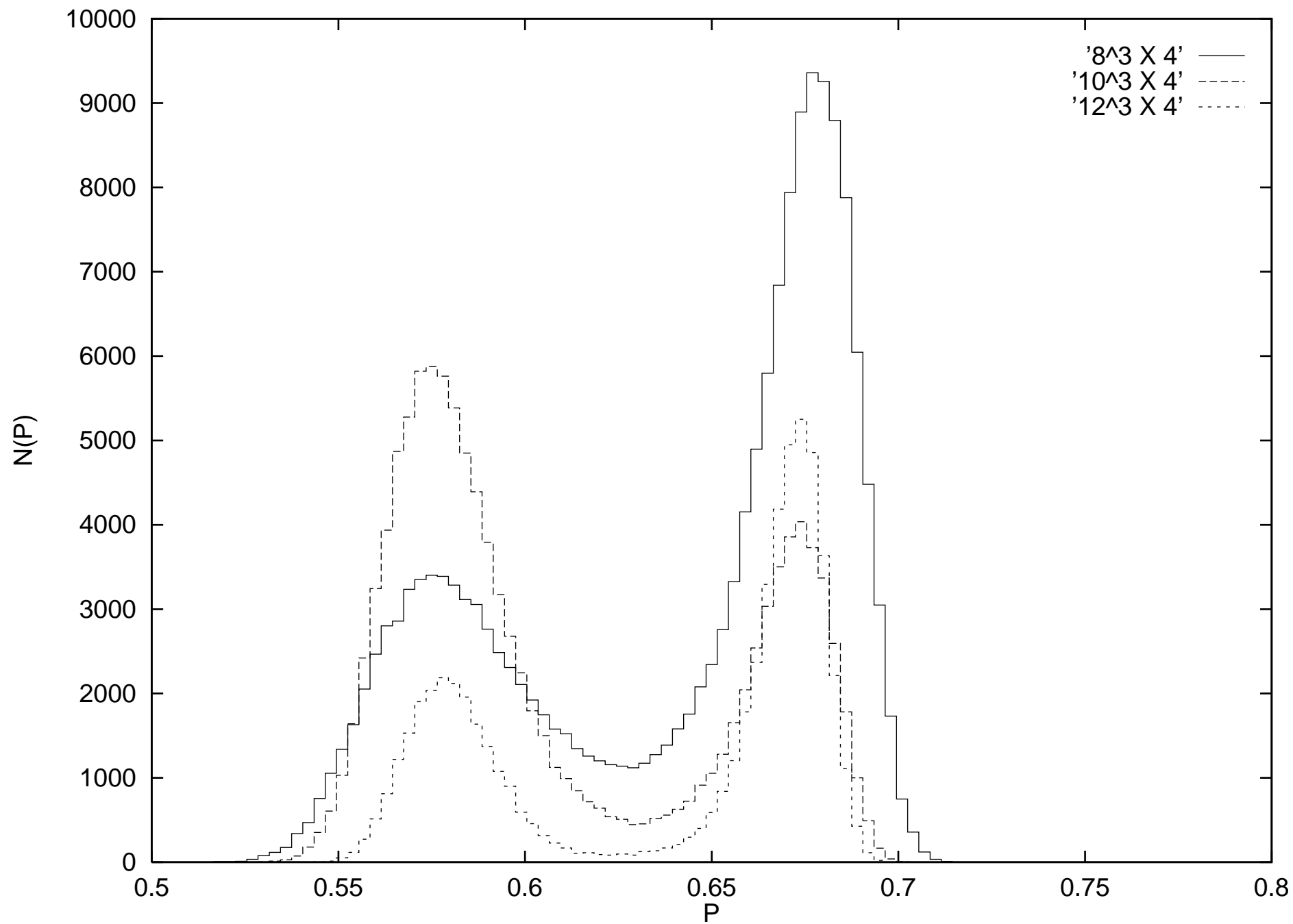| Lattice Size	ΔP
6^4	0.111(4)
8^4	0.081(4)
10^4	0.060(4)
12^4	0.0495(40)
16^4	0.036(4)
$8^3 \times 4$	0.102(4)
$10^3 \times 4$	0.099(4)
$12^3 \times 4$	0.096(4)

<center>Table 2</center>

The values of β at which simulations were performed
on N^4 lattices at $\beta_A = 1.25$, $\beta^{crit.}$ and the
height of the plaquette susceptibility peak, χ_N^{max}.
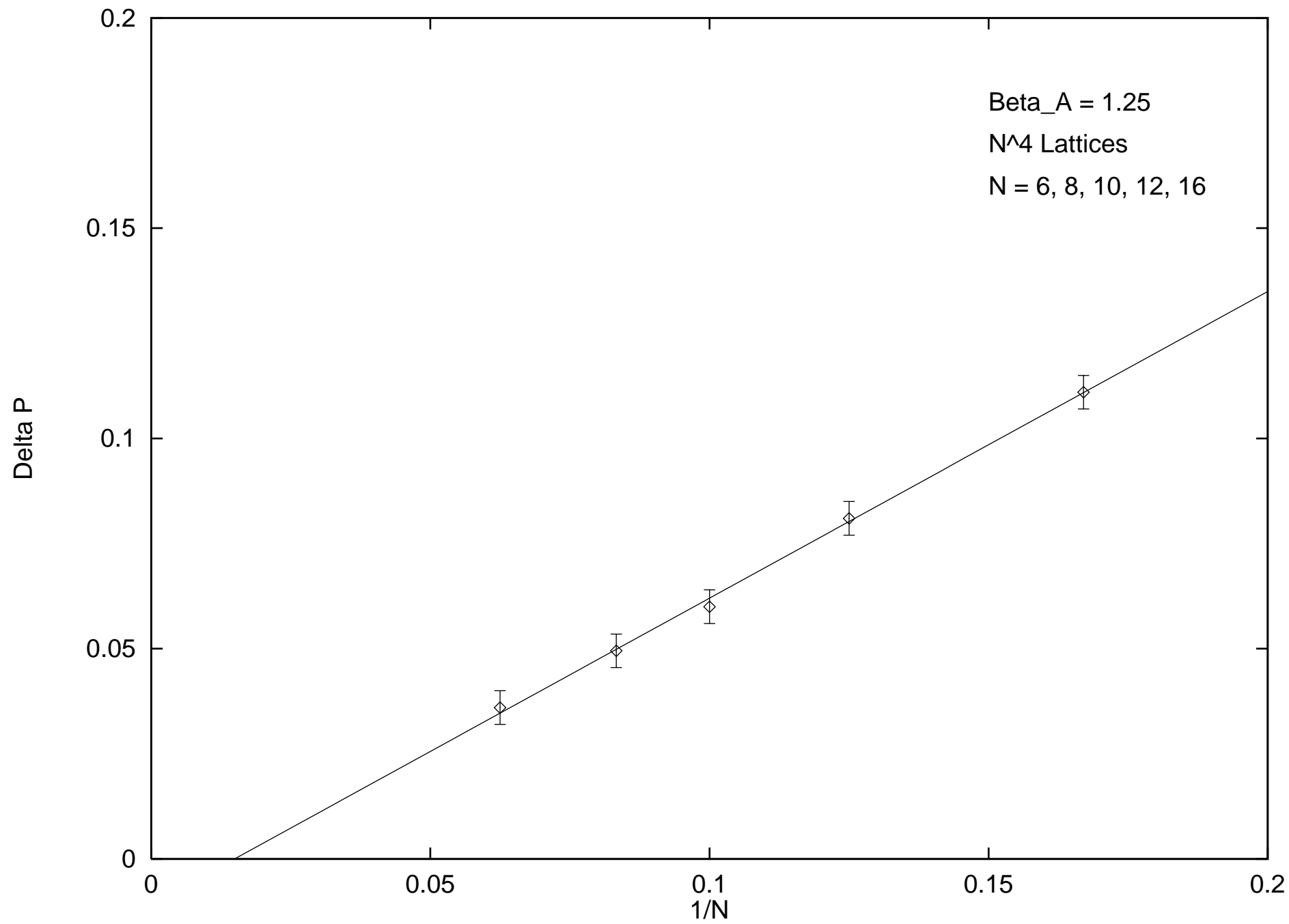
Lattice Size	β	β_c	χ_N^{max}
6^4	1.2147	1.21497	20.73(1.14)
8^4	1.2179	1.21783	41.78(4.90)
10^4	1.2182	1.21826	51.70(11.42)
12^4	1.2183	1.2183	~ 74
16^4	1.2184	1.21836	~ 134

Beta_A = 1.5; N^4 lattices