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Stability of the Bulk Phase Diagram
of the SU(2) Lattice Gauge Theory
with Fundamental - Adjoint Action

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Abstract

Using improved mean field and strong coupling expansions we re-analyse the bulk phase diagram of the fundamental-adjoint action of the SU(2) Lattice Gauge Theory. We find that the qualitative features of the bulk phase diagram are robust and unchanged by the inclusion of higher order terms. On the other hand, some of the quantitative features, such as the location of the endpoint of the line of bulk phase transitions, seem to be strongly dependent on the higher order terms of the strong coupling expansion.

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Lattice regularization of a continuum action is not unique. For non-abelian gauge theories the Wilson action [1] is the most popular one, but other actions have been studied in the literature. In particular, lately there has been a resurgence of interest in the Bhanot-Creutz action[2]

$$S = \sum_p \left\{ \beta_f (1 - \frac{1}{2} Tr_F U_p) + \beta_a (1 - \frac{1}{3} Tr_A U_p) \right\}. \tag{1}$$

Here F and A denote the fundamental and adjoint representations respectively. The Wilson action is a special case of (1), corresponding to $\beta_a = 0$. The action (1) was first studied by Bhanot and Creutz for SU(2) gauge theories in order to understand the bulk phase transition found in numerical studies of some non-abelian gauge theories (SO(3), SU(4) etc.) and the role they play in the physics of confinement. They found a line of first order transition in the β_f - β_a plane (see Fig.1) that ended at a finite β_a. Since the location of the peak in the plaquette susceptibility for the Wilson action corresponds to the interception of the extrapolation of this line with the $\beta_a = 0$ axis, it has been considered as a possible source for the observed crossover in the string tension. It is thus a possibly important key in our understanding of the confinement phenomenon.

However, recent [3, 4] finite temperature investigations of this action have cast some doubt on the nature of the phase transition line seen in [2]. It was found that switching on a nonzero β_a, the known finite temperature phase transition of the Wilson action becomes a line and joins the above mentioned bulk transition line. Moreover, the order of the deconfinement transition changes from second to first order at $\beta_a \geq 1.5$. No indications of two separate transitions were found at any β_a. After considering various possibilities, it was concluded [4]that the transition line is not a bulk one, but the deconfinement transition line. Since the study in [2] was done on small lattices, which were at relatively high temperature, such a misinterpretation is possible.

A finite size scaling-based analysis of the bulk transition has also been done [6] to determine the nature of the phase transition. It was found that the line of the phase transition ends at a somewhat higher β_a than what was found in [2]. Simulations at a still higher value of β_a (=1.5) suggested the presence of a 1st order bulk phase transition, but the Polyakov line, which is the order parameter of the finite temperature phase transition, was also found to jump across this transition at $\beta_a = 1.5$, thus further making a distinction between the zero temperature bulk transition and the finite temperature deconfinement transition very difficult.

Numerical simulations thus seem to give conflicting signals. The deconfinement order parameter acquires non-zero vacuum expectation value at these transitions for all β_a. However, the shift of the transition point with N_t - the temporal lattice size - decreases and becomes negligibly small. The latter is suggestive of a bulk transition unless this behaviour changes on

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very large lattices. In the absence of such large lattice simulations it may be instructive to look for guidance by conventional analytical techniques.

The existence of a deconfinement phase transition for $\beta_a = 0$ has been rigorously proven [7] in the strong coupling limit. In [5] a leading order strong coupling analysis of the action in (1) was done at finite temperature. It yielded a deconfinement phase transition line in the (β_f, β_a) plane, along with a change in order of the phase transition for large β_a, as seen in [3].

A mean-field analysis of (1) at zero temperature was done in Ref [8] and the results of [2] were supported. In view of the results of [6], however, it seems to be necessary to reexamine the results of Ref [8]. In this note, we have attempted to check how stable the results of Ref [8] are, both qualitatively and quantitatively, by improving and extending their work to higher orders in a consistent manner. We find that our results still predict qualitatively the same bulk phase diagram as in [8] but the location of the endpoint of the transition line is very sensitive to higher orders and cannot be precisely obtained by our study.

The mean field analysis proceeds by writing the partition function

$$Z(\beta_f, \beta_a) = \int \prod_l dU_l \exp(-S) \tag{2}$$

in the axial gauge by fixing all the links in time direction equal to 1. The SU(2) elements are parametrized as $U = u^o + i\mathbf{u}.\sigma$, for real numbers $(u_0, \mathbf{u})$ satisfying $u_0^2 + u_1^2 + u_2^2 + u_3^2 = 1$. The standard technique [9, 10] of Fourier transforming the measure and the action gives

$$Z(\beta_f, \beta_a) = \int \prod_l dv_l \int \prod_l \frac{d\alpha_l}{(2\pi i)^4} \exp(-S(\beta_f, \beta_a, v_l) + \sum_l (w(\alpha) - \alpha_l^o v_l^o - \alpha_l.\mathbf{v}_l)) \tag{3}$$

where

$$w(\alpha) = \ln \int d\Omega \exp(\alpha^o u^o + \alpha.\mathbf{u}). \tag{4}$$

One then looks for translationally invariant saddle points of (3) of the form [9]

$$v_l = (v, \mathbf{0}), \alpha_l = (\alpha, \mathbf{0}). \tag{5}$$

For $\beta_f < 2$ one gets the stable solution $v = 0, \alpha = 0$ [8]. An expansion of the free energy around this saddle point gives just the strong coupling expansion. The strong coupling expansion for the Bhanot-Creutz action has been obtained upto terms of order sixteen by Dashen et al [11]. In our notation the series for free energy as obtained by them is

$$\begin{aligned} F_I(\beta_f, \beta_a) = \quad & 6\ln b_0 + 4(4b_1^6 + 9b_2^6) + 36(4b_1^{10} + 9b_2^{10}) \\ & + 36(12b_1^{10}b_2 + 24b_1^6 b_2^5 + 27b_2^{11} - 224b_1^{12} - 1368b_1^6 b_2^6 - 1359b_2^{12}) \\ & + 4(432b_1^5 b_2^5 b_3 + 405b_2^{10}b_4 + 16b_3^6 + 25b_4^6) \end{aligned} \tag{6}$$

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and the characters b_0 and b_j, j=1,2,3,4, are evaluated as

$$b_0 = \int_0^{4\pi} \frac{d\theta}{2\pi} \sin^2 \frac{\theta}{2} \exp\left(-\beta_f(1 - \cos\frac{\theta}{2}) - \frac{2}{3}\beta_a(1 - \cos\theta)\right), \tag{7}$$

$$b_j = \frac{1}{(j+1)b_0} \int_0^{4\pi} \frac{d\theta}{2\pi} \sin\frac{\theta}{2} \sin\frac{(j+1)\theta}{2} \exp\left(-\beta_f(1 - \cos\frac{\theta}{2}) - \frac{2}{3}\beta_a(1 - \cos\theta)\right). \tag{8}$$

The characters are now expanded in a series of β_f and β_a to rewrite F_I explicitly as an expansion in powers of β_f, β_a. We checked that for our region of interest, the difference between eqn(6) and the explicit series is negligible. Also the convergence of the strong coupling series was checked by looking at the series terminated at differet orders. It was found that the strong coupling series converges very slowly near the transition line, specially for smaller β_a values.

For regions of large β_a and β_f there are stable solutions of (3) for nonzero v, α satisfying the equations [8]

$$v = \frac{I_2(\alpha)}{I_1(\alpha)}, \tag{9}$$

$$\alpha = \beta_f 2v(1 + (d-2)v^2) + \beta_a \frac{16}{3}v^3(1 + (d-2)v^4), \tag{10}$$

where I_2, I_1 are the modified Bessel functions; d, the space-time dimensionality, will be 4 in all our calculations.

Expanding near this saddle point, the partition function, or equivalently, the free energy per site, is

$$F_{II}(\beta_f, \beta_a) = \frac{1}{N} \ln Z(\beta_f, \beta_a) = F_{tree} + F_{1-loop} + F_{2-loop} + ... \tag{11}$$

The first two terms of this series were calculated in Ref [8] and are

$$F_{tree} = 3\beta_f(v^2 + v^4 - 2) + 4\beta_a(v^4 + v^8 - 2) + 3\left(\ln\frac{2I_1(\alpha)}{\alpha} - \alpha v\right), \tag{12}$$

$$F_{1-loop} = \frac{3}{2}\left(\ln 2 + 3\ln\frac{\alpha}{2v} - \ln(\beta_f + \frac{8}{3}\beta_a v^2)\right) - 3\ln\left(\beta_f(1 + 3v^2) + \frac{8}{3}\beta_a v^2(1 + 3v^4)\right)$$
$$+ 3K_1 \tag{13}$$

where

$$K_1 = -\int_{-\pi}^{\pi} \frac{d^4p}{(2\pi)^4} \ln(1 - C_1 \cos p_o - C_2(\cos p_1 + \cos p_2 + \cos p_3)) \tag{14}$$

with

$$C_1 = (1 + 3c)^{-1}, C_2 = (3 + \frac{1}{c})^{-1} \tag{15}$$

and

$$c = \frac{3\beta_f v^2 + 8\beta_a v^6}{3\beta_f + 8\beta_a v^2}. \tag{16}$$

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A comparison of magnitude of F_{tree} and F_{1-loop} near the phase transition line shows that they are of the same order of magnitude, and so care should be taken in using (11). The loop expansion in (11) is justified by saying that the higher loop corrections are suppressed by inverse powers of α [10]. However, the lower loop terms might have a large contribution from higher powers of $1/\alpha$. Reexpanding $F_{tree} + F_{1-loop}$ in (11) we found that the term proportional to $1/\alpha$ comes with a coefficient $\sim O(10)$. So for consistency, one should explicitly write the series as a series in inverse powers of α. Both due to the sensitivity of the location of the endpoint of the phase transition line, and to check the convergence of the series in $1/\alpha$, it is necessary to evaluate the series up to the term proportional to $1/\alpha$. This can be done [14] by a comparison with the weak coupling perturbation series. Using the weak coupling series of Ref [11], one gets the following expression for free energy around this saddle point :

$$F_{II} = \frac{2 + \frac{3r}{2}}{2 + 4r}\alpha - \frac{9}{2}\ln\alpha + c_0 + \frac{c_1}{\alpha} + O(\frac{1}{\alpha^2}) \tag{17}$$

where $r = 4\beta_a/3\beta_f$, and the coefficients are

$$c_0 = -\frac{3}{2}\ln\pi + \frac{9}{2}\ln 3 - 3\ln 2 + \frac{(28 + 157r + 102r^2)}{8(1 + 2r)^2} + 3K_2, \tag{18}$$

$$c_1 = 5.4 - \frac{68 + 1115r + 804r^2 + 1692r^3}{32(1 + 2r)^3} - \frac{9(3 + r)}{4(1 + 2r)} \tag{19}$$

and

$$K_2 = -\int_{-\pi}^{\pi}\frac{d^4p}{(2\pi)^4}\ln\left(1 - \frac{1}{4}\sum_{\nu=0}^{3}\cos p_\nu\right) \tag{20}$$

$$\approx .0798. \tag{21}$$

It is assuring to note that the coefficient of the $O(1/\alpha)$ term is $\sim O(1)$ - large contributions from F_{1-loop} and F_{2-loop} cancel to give a well-behaved series. In the following, we will use eqn(17) for the free energy in this region.

Pure Yang-Mills theory has other saddle point solutions[12] : the "fluxon" configurations, which are topological excitations corresponding to center of the gauge group. These are stable for $\beta_a > \frac{3}{8}\beta_f$ [13]. For the region of large β_f, contributions of these maxima are much smaller than the higher loop terms and so we will neglect these configurations.

Since the term proportional to β_a in (1) is blind to the center of the group, such configurations are not suppressed on or near the $\beta_f = 0$ axis, and will have to be taken into account. This is done [8] by expanding the partition function in a series in β_f, to obtain

$$F_{III}(\beta_f, \beta_a) = -6\beta_f(1 - \frac{1}{2}\beta_f) + F_{SO(3)}(\beta_a + \frac{3}{8}\beta_f^2) + O(\beta_f^4) \tag{22}$$

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where, up to one loop,

$$F_{SO(3)}(\beta_a) \approx 3\Big(\beta_a(v^2 + v^4 - 2) + \ln(I_0(\alpha) - I_1(\alpha)) + \frac{\alpha}{2} - \frac{3}{2}\alpha v\Big)$$
$$+ \frac{3}{2}\Big(\ln 2 + 3\ln(1 + 2v^2) - 2\ln(1 + 3v^2) + 2K_1\Big). \tag{23}$$

Here K_1 is given by eqn(14) with a substitution of $c = v^4$ in (15). α, v are solutions of

$$v = \frac{1}{3}\frac{I_1(\alpha) - I_2(\alpha)}{I_0(\alpha) - I_1(\alpha)}, \tag{24}$$

$$\alpha = \frac{4}{3}\beta_a v(1 + 2v^2). \tag{25}$$

One can check that coefficient of the term proportional to $1/\alpha$ in $F_{SO(3)}$ is $\sim O(1)$ - the loop expansion does not differ substantially from an expansion in $1/\alpha$. Figs. 1 and 2 display a comparison of the Monte Carlo data [2, 6] with the predictions for phase transition lines obtained by comparing the free energies F_I, F_{II}, F_{III} in the different regions.
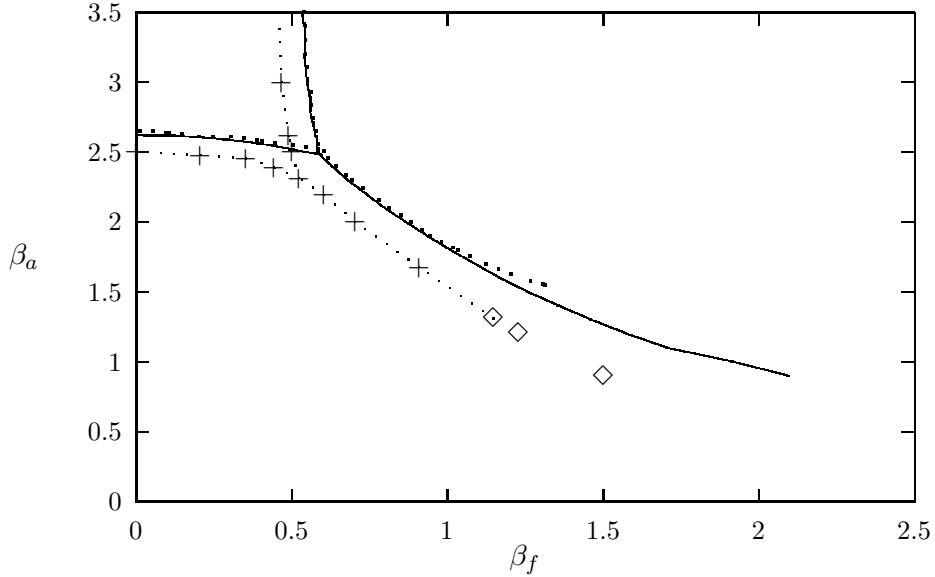

Fig.1 : The points (joined by thin dots) are the Monte Carlo data of [2], with open diamonds denoting the points where a first order bulk phase transition was ruled out by [6]. The full line is the curve obtained in [8], thick dotted line is obtained taking upto 10th order in eqn(6).

In Fig.1 the Monte Carlo data are shown along with the curve obtained in [8] by comparing F_I upto 10th order in eqn(6)and F_{II}, F_{III} upto one loop order in eqns (11), (22). Also shown

is the curve obtained by taking the strong coupling series upto 16th order. While taking the higher order terms in strong coupling expansion changes the location of the endpoint of the phase transition line drastically, moving it closer to the Monte Carlo data, the location of the transition line itself changes very little for $\beta_a \geq 1.8$.
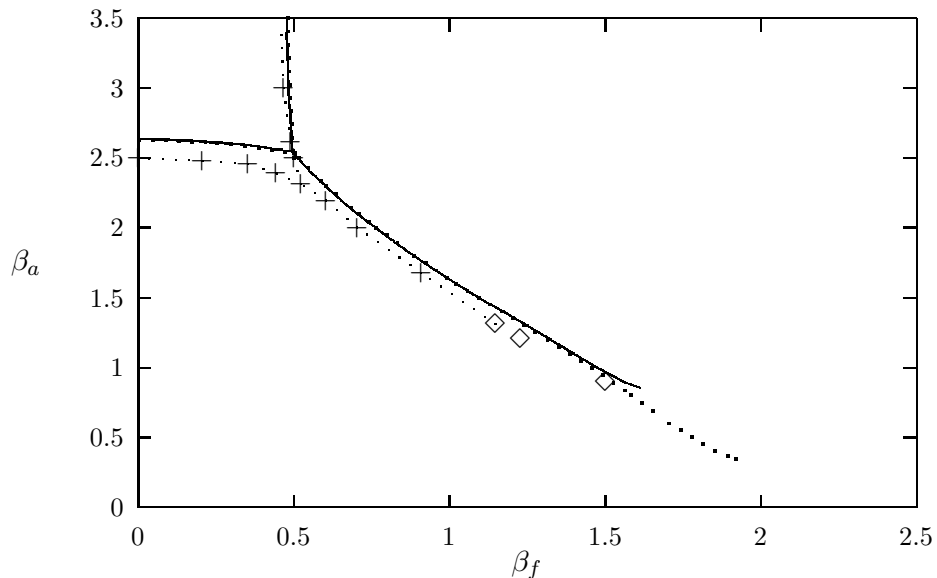

Fig.2 : The Monte Carlo data are shown against prediction using 16th order strong coupling expansion and eqns (17),(22) (full line). The thick dotted line corresponds to 10th order in strong coupling series.

In Fig.2 we show the prediction for phase transition curve obtained by comparing the improved mean field series of eqn(17), the strong coupling series upto 16th order and eqn(22). Also shown is the curve for taking strong coupling series upto 10th order. The same feature, namely, the extreme sensitivity of the endpoint and robustness of the upper part of the curve is noted. Also, a comparison of Figs. 1 and 2 reveals that the improvement in the mean field series leads to a curve that is considerably closer to the Monte Carlo results. The convergence of the mean field series was checked by checking that on taking just the first two terms in (17), the transition line is left unchanged. The endpoint of the transition line, however, is still in disagreement with [6]. Due to the extreme sensitivity of the endpoint, it seems that very high orders in strong coupling series may be needed in order to pinpoint it precisely. However, since the strong coupling expansion seems to be well behaved near the phase transition line for $\beta_a \geq 1.5$, the existence of the bulk transition at least upto this region seems to be confirmed.

The only caveat for this conclusion is the choice of gauge fixing. It is always advisable to check for possible gauge dependence of a result obtained by gauge fixing. In this case, it is even more so since the axial gauge constrains the deconfinement order parameter to be nonzero in region II whereas it is zero (or small) in the strong coupling region I. It would be interesting to confirm the bulk phase diagram in Fig.2 by using another gauge condition to establish the bulk transition for $\beta_a \sim 1.5$ beyond doubt.

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