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Phase Diagram of SO(3) Lattice Gauge Theory at Finite Temperature

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The phase diagram of $SO(3)$ lattice gauge theory at finite temperature is investigated by Monte Carlo techniques with a view i) to understand the relationship between the deconfinement phase transitions in the $SU(2)$ and $SO(3)$ lattice gauge theories and ii) to resolve the current ambiguity of the nature of the high temperature phases of the latter. Phases with positive and negative adjoint Polyakov loop, L_a, are shown to have the same physics. A first order deconfining phase transition is found for $N_t = 4$.

1. Introduction

Since the continuum limit of a lattice gauge theory is governed by its 2-loop β-function, one expects the physics of confinement and deconfinement for pure $SU(2)$ gauge theory to be identical to that of pure $SO(3)$ gauge theory. On the other hand, $SO(3)$ does not have the $Z(2)$ center symmetry whose spontaneous breakdown in the case of the $SU(2)$ theory indicates its deconfinement transition. This makes the investigation of the phase diagram of the $SO(3)$ gauge theory especially interesting and important. It has been argued[1] that the deconfinement transition for the $SO(3)$ lattice gauge theory may show up as a cross over which sharpens in the continuum limit to give an Ising-like second order phase transition.

Another reason for investigating the finite temperature transition in $SO(3)$ gauge theory is that it is supposed[2] to have a bulk phase transition and may thus provide a test case for studying the interplay between these different types of phase transitions. Recently, simulations of the Bhanot-Creutz action for SU(2) gauge theory[2],

$$S = \sum_p \left\{\beta_f(1-\frac{1}{2}\mathrm{Tr}_f\ U_p)+\beta_a(1-\frac{1}{3}\mathrm{Tr}_a\ U_p)\right\}, \quad (1)$$

at finite temperature revealed[3] that the known deconfinement transition point in usual Wilson action becomes a line in the β_f-β_a plane and joins the bulk transition line seen in [2]. The order of the deconfinement transition was also seen to change from second to first for $\beta_a \geq 1.25$. The studies in [3] were all done for a relatively small β_a, i.e., close to the Wilson action. $SO(3)$ gauge theory provides an example far away from it.

A study of finite temperature SO(3) gauge theory was carried out in [4] and a deconfining transition for this theory was found. However, there was some ambiguity about the nature of the high temperature phase and the order of the phase transition in [4]. In this work, we attempt to clarify these ambiguities.

2. Actions and Observables

The Wilson action for SO(3) gauge theory is

$$S = \beta \sum_p (1 - \frac{1}{3}\mathrm{Tr}\ U_p) \quad , \quad (2)$$

where U_p denotes the directed product of the link variables, $U_\mu(x) \in SO(3)$, around an elementary plaquette p. The action (1) for β_f=0 also corresponds to an $SO(3)$ gauge theory which was found in [2] to have a first order bulk transition at $\beta_a \sim 2.5$. A third action we used is the Halliday-Schwimmer action [5]

$$S = \beta_v \sum_p (1 - \frac{1}{2}\sigma_p \mathrm{Tr}_f\ U_p) \quad . \quad (3)$$

Here the link variables $U_\mu(x) \in SU(2)$ and $\sigma_p = \pm 1$. Besides the integration over the link vari-

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ables, the partition function in this case also contains a summation over all possible values of $\{\sigma_p\}$. It too shows [5] a first order bulk phase transition at $\beta_v \sim 4.5$. The chief advantage of this action is that both the link variables U_μ and σ_p can be updated using heat-bath algorithms.

We studied the adjoint plaquette P, defined as the average of $\frac{1}{3}\mathrm{Tr}_a U_p$ $[\sigma_p \mathrm{Tr}_f U_p]$ over all plaquettes for actions (1) and (2) [action (3)]. We also measured $\langle L_a \rangle$, which is the average over all spatial sites of the adjoint Polyakov loop, defined by $L_a(\vec{r}) = \mathrm{Tr_a} \prod_{\mathrm{i=1}}^{\mathrm{N_t}} \mathrm{U_t}(\vec{\mathrm{r}}, \mathrm{i})$. Note that $\langle L_a \rangle$ is not an order parameter. Since $\langle L_a \rangle$ can be thought of as a measure of the free energy of an adjoint quark which can be screened by gluons created from the vacuum, it is not constrained to be zero in the confined phase. Similarly an adjoint Wilson loop is not supposed to show area law in the confining phase. However, creation of gluon pairs from vacuum costs a considerable amount of energy as glueballs are heavy. It may therefore be favourable for adjoint quarks also to have a string between them, at least when they are not too far separated. Using the action (1) for $\beta_f = 0$ on a $7^3 \times 3$ lattice, Ref. [4] found that $\langle L_a \rangle$ was consistent with zero till $\beta_a \sim 2.5$, after which it became nonzero, indicating a deconfinement transition around this value of β_a.

3. The High Temperature Phase

An unexpected and curious result of Ref. [4] was that after becoming nonzero in the high temperature phase, $\langle L_a \rangle$ settles into either a positive value ($\to 3$ as $\beta_a \to \infty$), or a negative value ($\to -1$ as $\beta_a \to \infty$), the two states being degenerate in free energy. In [4] the negative $\langle L_a \rangle$ state was interpreted as the manifestation of another zero temperature confined phase.

We have carried out a number of tests in order to understand the nature of the negative $\langle L_a \rangle$ state. First, it was checked that the appearance of this phase is not due to any algorithmic problem by observing that it appears for all the three actions discussed above. We then checked that the value of $\langle L_a \rangle$ is quite stable against changes in spatial lattice sizes from $9^3 \times 3$ to $18^3 \times 3$ lattices. Next, we introduced a polarising 'magnetic

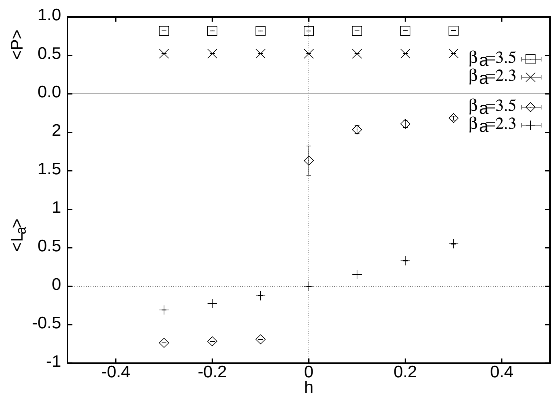

Figure 1. $\langle P \rangle$ and $\langle L_a \rangle$ as a function of magnetic field h.

field' by adding a term $h \sum_{\vec{x}} L_a(\vec{x})$ to the action. As shown in the upper half of Fig. 1, the average plaquette P on a $7^3 \times 3$ lattice is not affected strongly by this term either below the transition ($\beta_a = 2.3$) or above the transition ($\beta_a = 3.5$). However, $\langle L_a \rangle$ is. Irrespective of the start, it converges to a unique value whose sign is determined by that of h. This is similar to the $SU(2)$ case and suggests strongly that the high temperature phase of the $SO(3)$ gauge theory also manifests itself in two ways corresponding to positive and negative $\langle L_a \rangle$. Also note in Fig. 1 that the extrapolation of $\langle L_a \rangle$ to vanishing h yields a value consistent with zero below the phase transition.

A further test of whether the physics of these two phases is the same is the equality of the correlation lengths in these phases. We measured the correlation function, defined by

$$\Gamma(r) = \sum_i \sum_{\vec{x}} \langle L_a(\vec{x} + r e_i)\ L_a(\vec{x}) \rangle \quad , \tag{4}$$

on an $8^3 \times 4$ lattice for $\beta_a = 2.3$ and for the positive and negative $\langle L_a \rangle$ states at $\beta_a = 2.6$ and 3.5. It was found that i) at $\beta_a = 2.3$, which is below the transition, the correlator vanishes rapidly with r, ii) it approaches a constant above the phase transition and iii) the constant is bigger for larger β_a and bigger in the positive $\langle L_a \rangle$-

phase for the same β_a. The mass gap, obtained from the connected parts of the correlator above or from their zero momentum projected versions, was similar for *both* the positive and negative $\langle L_a \rangle$ states corresponding to both $\beta_a = 2.6$ and 3.5, as expected for states with same physics. It is, however, considerably different for $\beta_a = 2.3$.

4. Order and Nature of the Transition

In simulations on $4^3 \times 4$, $6^3 \times 4$ and $8^3 \times 4$ lattices with the actions (1) and (3), long metastable states were observed on all lattices near the transition region, signaling a possible first order transition. $\langle L_a \rangle$ was seen to tunnel between all the three states, two of which correspond to the same value of the action. Runs on smaller lattices show more tunnellings and larger fluctuations in the positive L_a-phase. The estimated transition points for $4^3 \times 4$, $6^3 \times 4$ and $8^3 \times 4$ lattices are $\beta_{vc} = 4.43 \pm 0.02$, 4.45 ± 0.01 and 4.45 ± 0.01 respectively.

Fig. 2 displays distributions of L_a from the runs made at the critical couplings but from different starts. We performed about 100K-400K heat-bath sweeps depending on the size of the lattice. While the frequent tunnelling smoothens the peak structure for the $4^3 \times 4$ lattice considerably, a clear three-peak structure is seen for both the $6^3 \times 4$ and the $8^3 \times 4$ lattices. The stability of these peaks under changes in spatial volume suggests the phase transition to be of first order. The estimates of the discontinuities in the plaquette, $\langle L_a \rangle_+$ and $\langle L_a \rangle_-$ are 0.0575 ± 0.0030, 0.87 ± 0.04 and 0.28 ± 0.04 respectively. It is also interesting to note that i) the peak for the confined phase is almost precisely at zero and ii) normalising by the maximum allowed $\langle L_a \rangle$ in each phase, the discontinuities for both the positive and negative phases are equal, being 0.29 ± 0.01 and 0.28 ± 0.04 respectively.

We also studied the theory on $8^3 \times 2$, 4^4, 6^4 and 8^4 lattices. On all these lattices, only one transition point was found, where both the plaquette and $\langle L_a \rangle$ show a discontinuity. A clear shift in β_c was found in going from $N_t = 2$ to $N_t = 4$ but no perceptible change in β_c was found in going from $N_t = 4$ to 6 and 8 for both actions (1) and (3).

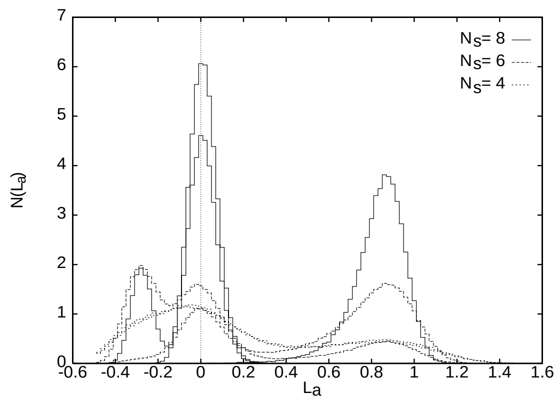

Figure 2. The distribution of L_a on $N_s^3 \times 4$ lattices at their critical couplings.

This is in sharp contrast to the SU(2) case, and is also unexpected for a deconfinement transition.

5. Summary

Our simulations with a variety of actions showed the negative $\langle L_a \rangle$-state to be present for all of them. However, using a 'magnetic field' term to polarise, we found a unique $\langle L_a \rangle$ state depending on the sign of the field. The correlation function measurements in both the phases of positive and negative $\langle L_a \rangle$ indicated that the two states are physically identical high temperature deconfined phases of SO(3) gauge theory. Although a shift in β_c was observed in changing N_t from to 2 to 4, no further shift was seen for N_t =6 and 8 which is characteristic of a bulk phase transition.

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