

Z_2 Monopoles, Vortices and the Universality of the $SU(2)$ Deconfinement Transition

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Abstract

We investigate the effect of Z_2 magnetic monopoles and vortices on the finite temperature deconfinement phase transition in the fundamental - adjoint $SU(2)$ lattice gauge theory. In the limit of complete suppression of the Z_2 monopoles, the mixed action for the $SU(2)$ theory in its Villain form is shown to be self-dual under the exchange of the fundamental and adjoint couplings. By further suppressing the Z_2 vortices we show that the extended model reduces to the Wilson action with a modified coupling. The universality of the $SU(2)$ deconfinement phase transition with the Ising model is therefore expected to remain intact in the entire plane of the fundamental-adjoint couplings in the continuum limit. The self-duality arguments related to the suppression of Z_2 monopoles are also applicable to the Villain form of mixed action for the $SU(N)$ theory with Z_N magnetic monopoles.

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1. INTRODUCTION

Confinement of non-abelian color degrees of freedom is widely regarded as an outstanding problem in physics. Condensation [1] of certain magnetic monopoles is a plausible mechanism to explain it. Quantizing the theory on a discrete space-time lattice, one hopes to shed more light on the inherently non-perturbative phenomenon of quark confinement. Although, this quantization procedure is not unique, different ways of defining the gauge theories on lattice are expected to lead to the same physics when the continuum limit of vanishing lattice spacing is taken. In the strong coupling region, however, where the coupling constant and lattice spacing are relatively large, it is expected that lattice artifacts will usually affect the physics. Often such changes are purely quantitative in nature: the string tension or the hadron spectra exist in the strong coupling expansion but the dependence of their dimensionless ratios on the coupling changes as one goes in to the asymptotic scaling region. By incorporating higher order corrections in the coupling constant, one can hope for a larger scaling region where the quantitative differences due to artifacts disappear with decreasing lattice spacing. However, the lattice artifacts may affect physics even **qualitatively**. The simplest known and well understood example is that of compact U(1) lattice gauge theory. It has magnetic monopoles which arise due to the compact nature of the gauge fields. In the strong coupling region their condensation gives rise to a confining phase with a linearly rising potential between electric charges: $V(r) \propto r$. This is well known as the dual Meissner effect. The theory undergoes a first order phase transition at intermediate value of electric charge. Beyond this transition, the magnetic monopoles become irrelevant and the standard Coulomb potential, $V(r) \propto r^{-2}$, is recovered. Thus physical laws with a large lattice spacing can be drastically different in nature than in the continuum limit. To the best of our knowledge, no such example exists in non-abelian lattice gauge theories. We will discuss below a similar phenomenon in SU(2) lattice gauge theory which is due to Z_2 magnetic monopoles and Z_2 electric vortices.

In non-abelian lattice gauge theories, many quantitative tests of the universality of the continuum limit have been carried out in the past. In particular, the fundamental-adjoint SU(2) action[6] has been extensively studied [2, 3, 4, 5] in this context due to its rich phase structure, shown in Fig. 1 by solid lines of bulk phase transitions [6, 7]. We argued in Ref. [8] that the phase diagram in Fig. 1 with only bulk boundaries was incomplete: it must have lines of deconfinement phase transition for different N_τ . Subsequently, we located the deconfinement transition lines[8, 9, 10] in Fig. 1 using the behavior of the corresponding order parameter. Along the Wilson axis ($\beta_A = 0.0$), the deconfinement transition has been very well studied. Monte Carlo simulations have established *i*) a scaling behavior[11] for the critical coupling, and thus an approach to the continuum limit, and *ii*) a clear second order deconfinement phase transition with critical exponents[12] in very good agreement with those of the three dimensional Ising model. These provided an explicit verification of the Svetitsky and Yaffe[13] universality conjecture.

The continuum physics should not depend upon the group representation chosen to describe the dynamical variables. Therefore it is expected that the adjoint SO(3) coupling (β_A) (see Section 2) will not alter the continuum limit of the mixed SU(2)-SO(3) action. From our work[8, 9, 10] on the mixed action at non-zero temperatures, we, however, found the following surprising results:

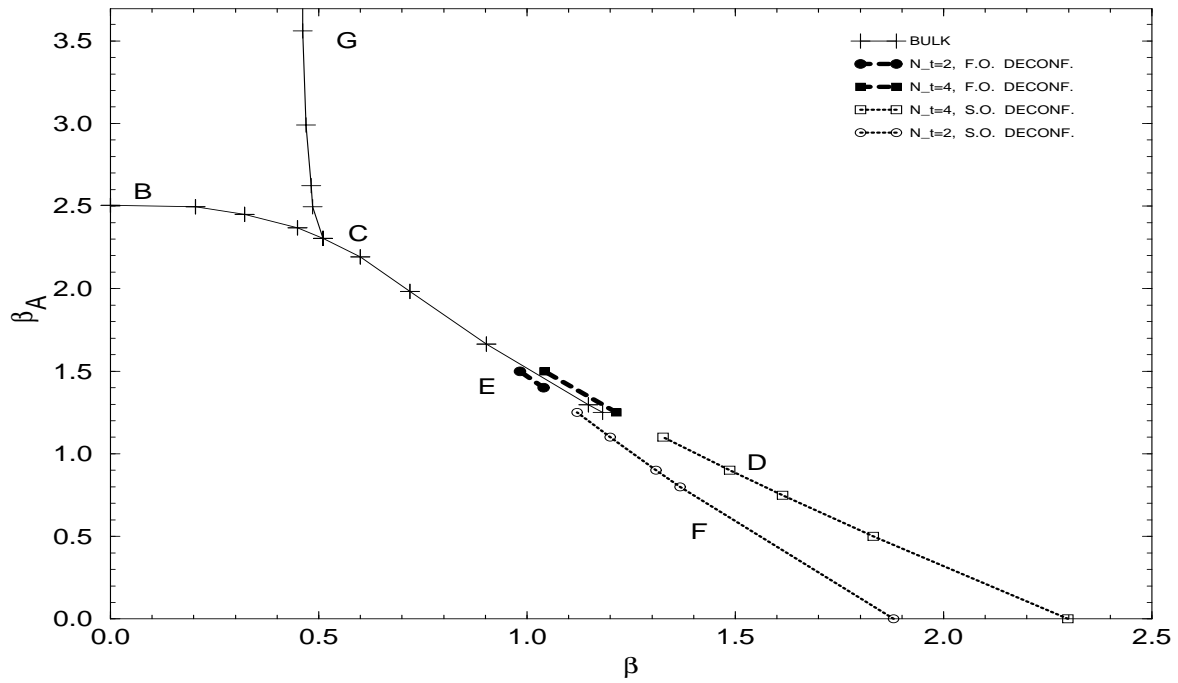


Figure 1: The phase diagram of the mixed $SU(2)$ lattice gauge theory. The solid lines are from Ref. [6, 7]. The dotted(thick-dashed) lines with hollow(filled) symbols are the second(first) order deconfinement phase transition lines[8, 9, 10] on $N_\tau = 2$ (circles) and 4 (squares) lattices.

- a] The transition remained second order in agreement with the Ising model exponents up to $\beta_A \approx 1.0$ but it became definitely first order for larger β_A . The order parameter for the deconfinement phase transition became nonzero discontinuously at this transition for larger β_A .
- b] There was no evidence of a second separate bulk transition.

Similar results, indicative of apparent violation of universality, were also obtained with a variant Villain form for the $SU(2)$ mixed action[14] and for the $SO(3)$ lattice gauge theory[15]. Such qualitative violations *must be distinguished* from the quantitative violations of universality found earlier in[17]. The latter disappear as the lattice spacing is reduced to zero and higher order corrections in g_u^2 are included[18, 19, 20]. This will obviously not be the case for any qualitative disagreements like the one mentioned above for the universality of the $SU(2)$ deconfinement transition with the Ising model. While the change in the order of the deconfinement phase transition was found on lattices with a variety of different temporal sizes, only one transition was found in each case at which the deconfinement order parameter became non-zero. This led to the suspicion that the bulk line was misidentified and was actually a deconfinement line. On the other hand, the transition at larger β_A shifted very little, even as the temporal lattice size was increased up to 16. One expects the deconfinement transition to move to larger couplings as N_τ , i.e the temporal lattice size, becomes larger. A lack of shift was therefore more in tune with it being a bulk transition. Direct numerical[7] and analytical[21] evidences for the presence of the bulk transition were also obtained. A possible way out was to blame the small temporal extent of the lattices used. Nevertheless, if a universal result exists in the $a \rightarrow 0$ limit, then the tricritical point T, where the deconfinement phase transition changes its nature from second order (thin dotted lines in Fig. 1) to first order (thick dotted lines in Fig. 1), must not be seen in the continuum limit. Therefore, one expects that the tricritical point will move rapidly towards the upper right corner ($\beta \rightarrow \infty, \beta_A \rightarrow \infty$) of Fig. 1 as the lattice size increases. In the continuum limit, the deconfinement transition will then be second order and its universality with Ising model will be recovered. However, in [10] we found that on going from $N_\tau = 2$ to 4 the tricritical point moved towards lower values of the adjoint coupling. Studying the shapes of the histogram and invoking Polyakov loop effective potential arguments[10], we concluded that the tricritical point moved slowly upwards, if at all, on going from $N_\tau = 4 \rightarrow 6 \rightarrow 8$ lattices³. These results are paradoxical and need explanation. In view of the above, i.e, the change in the nature of the deconfinement transition from second order to first order, a qualitative violation in universality is a real possibility.

In this paper, we take a different approach in an attempt to resolve these paradoxes. While larger temporal lattices could perhaps be used to solve them by brute force, a physical understanding of the universality of the continuum limit of the $SU(2)$ lattice gauge theory clearly requires an insight into the origin of the first order deconfinement transition and the tricritical point. In particular, one needs to a) identify the degrees of freedom which cause this unexpected change in the order

³Such shifts of the tricritical points first towards lower values of β_A on going from $N_\tau=2$ to 4 and then towards higher values for $N_\tau=6, 8$ were also observed in [14] where the Villain form (1) was simulated.

of the deconfinement transition, and b) study whether these degrees of freedom are relevant or irrelevant in the continuum limit. Towards this end, we investigate the effect of the topological Z_2 degrees of freedom associated with the $SO(3)$ part of the action (i.e, the term proportional to β_A in (1)) on the deconfinement transition line in the extended (β, β_A) coupling plane. It is important to note that these Z_2 degrees of freedom defined below are different from those defined for $SU(2)$ Wilson theory ($\beta_A = 0$) [23, 25, 26]. By adding chemical potential terms for the Z_2 monopoles and vortices, believed[23] to be irrelevant in the continuum limit, we show that no first order deconfinement phase transition exists if they both are suppressed. Thus the onset of the first order deconfinement seems to be due to the Z_2 topological degrees of freedom. In the past these Z_2 degrees of freedom have been studied in the context of bulk transitions in the extended model and cross over phenomenon along the Wilson axis ($\beta_A = 0$) [26, 22, 24, 25], the bulk transition along the $SO(3)$ gauge theory [2] and the Z_2 vortex transition along the $\beta_A = \infty$ axis. We propose to use them for the study of deconfinement phase transition and for the entire (β, β_A) plane.

We employ the Villain form of the mixed $SU(2)$ action for our studies below and find :

1. The theory defined by mixed action is self-dual under the exchange of the fundamental and adjoint couplings in the absence of Z_2 magnetic monopoles.
2. Further suppressing the Z_2 vortices completely reduces it to the standard Wilson action but with modified coupling; it thus can have only a second order deconfinement transition.

2. THE MODEL

The Villain form of the $SU(2)$ - $SO(3)$ mixed action is defined by

$$Z = \prod_{n,\mu} \sum_{\sigma_p(n)=\pm 1} \int dU_\mu(n) \exp \left(\frac{1}{2} \sum_p (\beta + \beta_A \sigma_p) \text{Tr}_F U_p \right) . \quad (1)$$

In (1) $U_\mu(n)$ are the $SU(2)$ link variables, $\sigma_p(n)$ are the Z_2 plaquette fields. The first term in (1) describes the standard $SU(2)$ Wilson action and the second term describes the $SO(3)$ action. As mentioned earlier, this form was used in [14] for studies similar to ours. Indeed, apart from changes in numerical values of the couplings, the results were identical, leading to essentially the same phase diagram as in Fig. 1.

The main advantage of the Villain form is that the dynamical excitations corresponding to the Z_2 topological degrees of freedom, both magnetic and electric in nature, become visible[2] in the partition function. In particular, it provides a better control over them and their contributions to any physical phenomenon.

The Z_2 magnetic monopoles and Z_2 electric vortex charges are obtained through the plaquette field $\sigma_p \equiv \sigma_{\mu\nu}$ as follows. We first define the Z_2 field strength tensor $F_{\mu\nu}$ by

$$\sigma_{\mu\nu}(n) \equiv \exp(i\pi F_{\mu\nu}) \quad (2)$$

The Z_2 magnetic (electric) currents ρ_c (ρ_l) over a cube c (link l) located at a lattice site n and oriented along (μ, ν, λ) (μ) directions are defined as follows

$$\rho_c(n) \equiv \prod_{p \in \partial c} \sigma_p, \quad \rho_l(n) \equiv \prod_{p \in \tilde{\partial} l} \sigma_p. \quad (3)$$

In (3) ∂ and $\tilde{\partial}$ stand for boundaries and co-boundaries of the cube c and the link l respectively. We extract the standard electric and magnetic currents by the identification:

$$\rho_l(n) \equiv \exp(i\pi J_\mu(n)), \quad \rho_c(n) \equiv \exp(i\pi M_\mu(n)), \quad (4)$$

In (4) the electric current $J_\mu(n)$ are on the link l and the magnetic current $M_\mu(n)$ is defined on the link μ which is orthogonal to the cube c . Using (2), (3) and (4), we recover the Z_2 Maxwell equations in the standard form:

$$\Delta_\mu F_{\mu\nu}(n) = J_\nu(n), \quad \Delta_\mu \tilde{F}_{\mu\nu}(n) = M_\nu(n), \quad (5)$$

where, $\Delta_\mu(n)$ is the lattice difference operator in the μ^{th} direction. It is clear from (3) that the Z_2 magnetic and electric charge definitions are dual to each other. Note that the Z_2 monopoles defined by the second equation in (4) are different from those obtained by an abelian projection [1] with respect to an $SU(2)$ adjoint Higgs⁴ field $\vec{\phi}$ with the abelian field strength tensor defined by:

$$F_{\mu\nu}^{U(1)} \equiv \frac{\vec{\phi} \cdot \vec{F}_{\mu\nu}}{|\vec{\phi}|} - \frac{\vec{\phi}}{|\vec{\phi}|^3} (D_\mu \vec{\phi} \times D_\nu \vec{\phi}). \quad (6)$$

Here $\vec{F}_{\mu\nu}$ is the $SU(2)$ field strength tensor derived from $Tr U_p$. The $U(1)$ topological magnetic current, derived from (6), and the corresponding $U(1)$ magnetic monopoles are clearly different from the Z_2 monopoles described by (3). It has been suggested [1] that the condensation of the former is perhaps responsible for confinement of color. On the other hand, the Z_2 topological degrees of freedom defined in (3) are the lattice artifacts. We will show that they are responsible for changing the order of the deconfinement transition in the extended (β, β_A) coupling plane.

In order to control the effects of the Z_2 topological degrees of freedom we add two terms proportional to their Z_2 magnetic and electric vortex charges in (1) :

⁴e.g in the pure gauge theory one could choose [1] $\vec{\phi} \equiv \vec{F}_{12}$.

$$Z_{\lambda,\gamma} = \prod_{n,\mu} \sum_{\sigma_p(n)=\pm 1} \int dU_\mu(n) \exp \left(\frac{1}{2} \sum_p (\beta + \beta_A \sigma_p) \text{Tr}_F U_p + \lambda \sum_c \rho_c + \gamma \sum_l \rho_l \right) . \quad (7)$$

Note that the formal continuum limit is expected to be left unchanged by the addition of λ and γ terms for all values of λ and γ . We discuss below the following cases:

1. $\gamma = 0, \lambda = \infty$, i.e, a complete suppression of the Z_2 monopoles without affecting the Z_2 electric vortices.
2. $\gamma = \infty, \lambda = 0$, i.e, a complete suppression of Z_2 electric vortices without affecting the Z_2 magnetic monopoles.
3. $\gamma = \infty, \lambda = \infty$, i.e, complete suppression of all the Z_2 topological degrees of freedom.

Case 1.

In the limit $\lambda = \infty$ at $\gamma = 0$, one obtains the following constraint:

$$\sigma_c = 1 \quad \forall c \quad (8)$$

This amounts to complete suppression of point like magnetic monopoles. Note that these local constraints can still allow the global Z_2 magnetic fluxes arising due to the periodic boundary conditions. While such boundary conditions in the spatial directions are not obligatory, they have to be enforced in the temporal directions. However, in what follows these global fluxes are irrelevant for our results. The constraints (8) are the Bianchi identities in the absence of magnetic charges (see (4) and (5)). They can be trivially solved by introducing Z_2 link fields $\sigma_\mu(n)$, such that

$$\sigma_p \equiv \sigma_{\mu\nu}(n) = \sigma_\mu(n) \sigma_\nu(n + \mu) \sigma_\mu^\dagger(n + \nu) \sigma_\nu^\dagger(n) \quad (9)$$

The partition function in this limit therefore becomes

$$Z_{\lambda \rightarrow \infty, \gamma} = \prod_{n,\mu} \sum_{\sigma_\mu(n)=\pm 1} \int dU_\mu(n) \exp \left(\frac{1}{2} \sum_p (\beta + \beta_A (\sigma_\mu(n) \sigma_\nu(n + \mu) \sigma_\mu(n + \nu) \sigma_\nu(n))) \text{Tr}_F U_p \right) . \quad (10)$$

It has a local $SU(2) \otimes Z_2$ gauge invariance:

$$\begin{aligned} U_\mu(n) &\rightarrow G(n)U_\mu(n)G^{-1}(n+\mu) & G(n) \in SU(2) \\ \sigma_\mu(n) &\rightarrow \sigma_\mu(n) \end{aligned} \quad (11)$$

$$\begin{aligned} \sigma_\mu(n) &\rightarrow z(n)\sigma_\mu(n)z^{-1}(n+\mu) & z(n) \in Z_2 \\ U_\mu(n) &\rightarrow U_\mu(n) \end{aligned} \quad (12)$$

We now define a new SU(2) field on each link:

$$\tilde{U}_\mu(n) \equiv \sigma_\mu(n)U_\mu(n) \quad (13)$$

Exploiting the SU(2) group invariance of the Haar measure and the Z_2 nature of σ -variables the partition function (10) in terms of new variables is:

$$\begin{aligned} Z_{\lambda \rightarrow \infty, \gamma} = \prod_{n, \mu} \sum_{\sigma_\mu(n) = \pm 1} \int dU_\mu(n) \exp \left(\frac{1}{2} \sum_p \left(\beta_A + \right. \right. \\ \left. \left. \beta \left(\sigma_\mu(n)\sigma_\nu(n+\mu)\sigma_\mu(n+\nu)\sigma_\nu(n) \right) \right) \right) Tr_F U_p \quad . \end{aligned} \quad (14)$$

Eqs. (10) and (14) have exactly the same form with $\beta \leftrightarrow \beta_A$. Therefore, in the absence of Z_2 magnetic monopoles the physics of the extended model is self-dual under the exchange of the fundamental and adjoint couplings.

Note that the self-duality arguments also go through for the SU(N) ($\forall N$) extended Villain actions. The plaquette and the link fields $\sigma_{\mu\nu}$ and σ_μ take values in the Z_N group in that case. ρ_c is clearly then complex and the action in (7) will have to be modified by including its real part there. However, rest of the arguments are exactly the same as for the SU(2) case and again, in the limit of total suppression of Z_N monopoles, the physics will be self-dual under $\beta \leftrightarrow \beta_A$.

Case 2.

The limit $\gamma = \infty, \lambda = 0$ corresponds to solving the constraint:

$$\rho_l = 1 \quad \forall l \quad . \quad (15)$$

This limit suppresses all the Z_2 electric vortices. It is therefore dual to the $\lambda = \infty$ limit discussed above. Again, as discussed in Case 1 above the local constraints (15) leave the global Z_2 electric fluxes due to the periodic boundary conditions unaffected. The solution of (15) corresponds to solving $\Delta_\mu F_{\mu\nu} = 0$ in terms of the dual vector potentials and can be written as:

$$\sigma_{\mu\nu}(n) = \exp i \epsilon_{\mu\nu\rho\delta} \Delta_\rho \sigma_\delta(n) \quad (16)$$

In (16) $\sigma_\delta(n)$ describes the dual vector potential on the link δ at lattice site n . Unlike in the case of the suppression of Z_2 magnetic charges suppression above, the suppression of Z_2 electric vortices does not lead to a self-dual model.

Case 3.

The $\lambda = \gamma = \infty$ limit corresponds to complete suppression of both magnetic as well as electric Z_2 topological degrees of freedoms. The Maxwell eqns. in (5) are now without electric or magnetic sources:

$$\Delta_\mu F_{\mu\nu}(\sigma) = 0, \quad \Delta_\mu \tilde{F}_{\mu\nu}(\sigma) = 0. \quad (17)$$

The solution of the second of equation above, involving the dual fields, is still given by eqn. (9)). One thus has to solve for the remaining electric equation with the additional constraint of gauge invariance of the Z_2 link fields $\sigma_\mu(n)$ which define the $F_{\mu\nu}$. Fixing the gauge symmetry (eqn. (12)) such that $\Delta_\mu A_\mu = 0$, where the Z_2 vector potential A_μ is related to σ_μ as $\sigma_\mu = \exp(iA_\mu)$, one can easily show that the gauge potential A_μ satisfies the following equation:

$$\square A_\mu(n) = 0 \quad . \quad (18)$$

It can be solved by a Fourier transform, leading to a solution $\sigma_\mu(n) = \text{constant}$, which can be $+1$ or -1 leading to $\sigma_p = 1$. Substituting back in (7), the extended action in this limit reduces to the standard Wilson action with coupling $\beta + \beta_A$. It must therefore have a second order deconfinement transition along the line $\beta + \beta_A = \text{constant}$, as known from the results for $\beta_A = 0$.

3. SUMMARY and DISCUSSION

Assuming the Z_2 topological degrees of freedom to be irrelevant in the continuum, as suggested by perturbative arguments, we have shown that the *qualitative* changes in the SU(2) deconfinement transition found earlier both by Monte Carlo as well as strong coupling expansions are likely to be due to the above degrees of freedom and hence lattice artifacts; suppressing them completely leads to the same physical result for the deconfinement phase transition for all β_A , including large β_A . These qualitative changes in the SU(2) gauge theory on lattice are perhaps similar in spirit to the unphysical *confining* phase of U(1) compact lattice gauge theory. The latter is also due to the topological U(1) magnetic monopoles which are irrelevant in the continuum. In order for the above universal results for the SU(2) deconfinement phase transition to be valid for the mixed action it is important to establish non-perturbatively that the Z_2 degrees of freedom are indeed irrelevant in the continuum limit. In the context of pure SU(2) lattice gauge theory it has been shown[23] that the probability for the Z_2 monopoles excitation on a set of C cubes:

$$\langle \prod_{c \in C} \theta(-\sigma_c) \rangle_{\text{Limit } \beta \rightarrow \infty} \leq L_1 \exp(-L_2 \beta C) \quad (19)$$

In (19) $\theta(x) \equiv 1(0)$ if $x \geq 0$ (otherwise) and L_1, L_2 are constants. Depending on the value of L_2 , the Z_2 monopoles could be extremely rare. Also, the Z_2 electric vortices are known to condense and give rise to a first order bulk transition shown by line CG in Fig. 1. These arguments suggest that the original theory, i.e., $\gamma = \lambda = 0$, should also exhibit universality for very large N_τ . Of course, it is still necessary to extend the argument of Ref. [23] to the entire (β, β_A) plane for both the Z_2 monopoles and vortices to be sure that this is indeed the case. Moreover, the curious coincidence of the bulk and deconfinement transition for the case of $\gamma = \lambda = 0$ for many different sizes of temporal lattice sizes still remains unexplained, although one can now be more hopeful that for *very* large lattices at least the universality will be respected, as it should be.

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