

# $Z_2$ Monopoles, Vortices and the Universality of the $SU(2)$ Deconfinement Transition

Rajiv V. Gavai<sup>a,1</sup> and Manu Mathur<sup>b,2</sup>

<sup>a</sup> *Theoretical Physics Department, Tata Institute of Fundamental Research,  
Homi Bhabha Road, Mumbai 400005, India*

<sup>b</sup> *S. N. Bose National Centre for Basic Sciences, JD Block,  
Sector III, Salt Lake City, Calcutta 700091, India.*

## Abstract

We investigate the effect of  $Z_2$  magnetic monopoles and vortices on the finite temperature deconfinement phase transition in the fundamental - adjoint  $SU(2)$  lattice gauge theory. In the limit of complete suppression of the  $Z_2$  monopoles, the mixed action for the  $SU(2)$  theory in its Villain form is shown to be self-dual under the exchange of the fundamental and adjoint couplings. By further suppressing the  $Z_2$  vortices we show that the extended model reduces to the Wilson action with a modified coupling. The universality of the  $SU(2)$  deconfinement phase transition with the Ising model is therefore expected to remain intact in the entire plane of the fundamental-adjoint couplings in the continuum limit. The self-duality arguments related to the suppression of  $Z_2$  monopoles are also applicable to the Villain form of mixed action for the  $SU(N)$  theory with  $Z_N$  magnetic monopoles.

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<sup>1</sup>E-mail:gavai@tifr.res.in

<sup>2</sup>E-mail:manu@boson.bose.res.in

## 1. INTRODUCTION

Confinement of non-abelian color degrees of freedom is widely regarded as an outstanding problem in physics. Condensation [1] of certain magnetic monopoles is a plausible mechanism to explain it. Quantizing the theory on a discrete space-time lattice, one hopes to shed more light on the inherently non-perturbative phenomenon of quark confinement. Although, this quantization procedure is not unique, different ways of defining the gauge theories on lattice are expected to lead to the same physics when the continuum limit of vanishing lattice spacing is taken. In the strong coupling region, however, where the coupling constant and lattice spacing are relatively large, it is expected that lattice artifacts will usually affect the physics. Often such changes are purely quantitative in nature: the string tension or the hadron spectra exist in the strong coupling expansion but the dependence of their dimensionless ratios on the coupling changes as one goes in to the asymptotic scaling region. By incorporating higher order corrections in the coupling constant, one can hope for a larger scaling region where the quantitative differences due to artifacts disappear with decreasing lattice spacing. However, the lattice artifacts may affect physics even **qualitatively**. The simplest known and well understood example is that of compact  $U(1)$  lattice gauge theory. It has magnetic monopoles which arise due to the compact nature of the gauge fields. In the strong coupling region their condensation gives rise to a confining phase with a linearly rising potential between electric charges:  $V(r) \propto r$ . This is well known as the dual Meissner effect. The theory undergoes a first order phase transition at intermediate value of electric charge. Beyond this transition, the magnetic monopoles become irrelevant and the standard Coulomb potential,  $V(r) \propto r^{-2}$ , is recovered. Thus physical laws with a large lattice spacing can be drastically different in nature than in the continuum limit. To the best of our knowledge, no such example exists in non-abelian lattice gauge theories. We will discuss below a similar phenomenon in  $SU(2)$  lattice gauge theory which is due to  $Z_2$  magnetic monopoles and  $Z_2$  electric vortices.

In non-abelian lattice gauge theories, many quantitative tests of the universality of the continuum limit have been carried out in the past. In particular, the fundamental-adjoint  $SU(2)$  action[6] has been extensively studied [2, 3, 4, 5] in this context due to its rich phase structure, shown in Fig. 1 by solid lines of bulk phase transitions [6, 7]. We argued in Ref. [8] that the phase diagram in Fig. 1 with only bulk boundaries was incomplete: it must have lines of deconfinement phase transition for different  $N_\tau$ . Subsequently, we located the the deconfinement transition lines[8, 9, 10] in Fig. 1 using the behavior of the corresponding order parameter. Along the Wilson axis ( $\beta_A = 0.0$ ), the deconfinement transition has been very well studied. Monte Carlo simulations have established *i*) a scaling behavior[11] for the critical coupling, and thus an approach to the continuum limit, and *ii*) a clear second order deconfinement phase transition with critical exponents[12] in very good agreement with those of the three dimensional Ising model. These provided an explicit verification of the Svetitsky and Yaffe[13] universality conjecture.

The continuum physics should not depend upon the group representation chosen to describe the dynamical variables. Therefore it is expected that the adjoint  $SO(3)$  coupling ( $\beta_A$ ) (see Section 2) will not alter the continuum limit of the mixed  $SU(2)$ - $SO(3)$  action. From our work[8, 9, 10] on the mixed action at non-zero temperatures, we, however, found the following surprising results:

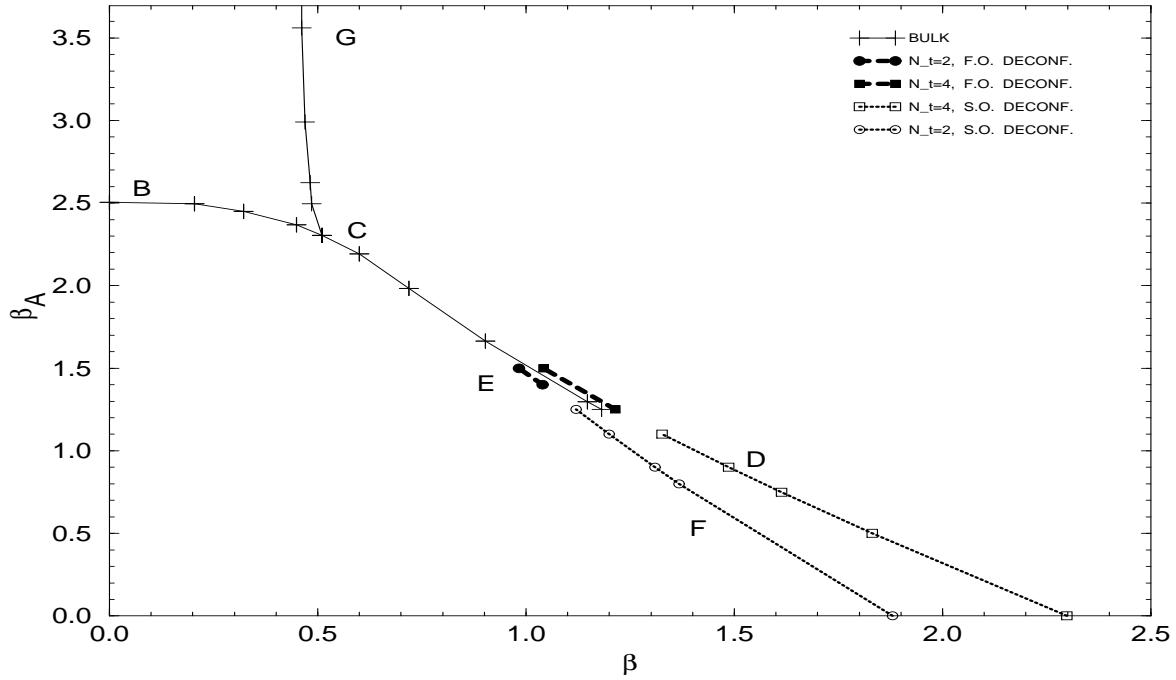


Figure 1: The phase diagram of the mixed  $SU(2)$  lattice gauge theory. The solid lines are from Ref. [6, 7]. The dotted(thick-dashed) lines with hollow(filled) symbols are the second(first) order deconfinement phase transition lines[8, 9, 10] on  $N_\tau = 2$  (circles) and 4 (squares) lattices.

- a] The transition remained second order in agreement with the Ising model exponents up to  $\beta_A \approx 1.0$  but it became definitely first order for larger  $\beta_A$ . The order parameter for the deconfinement phase transition became nonzero discontinuously at this transition for larger  $\beta_A$ .
- b] There was no evidence of a second separate bulk transition.

Similar results, indicative of apparent violation of universality, were also obtained with a variant Villain form for the  $SU(2)$  mixed action[14] and for the  $SO(3)$  lattice gauge theory[15]. Such qualitative violations *must be distinguished* from the quantitative violations of universality found earlier in[17]. The latter disappear as the lattice spacing is reduced to zero and higher order corrections in  $g_u^2$  are included[18, 19, 20]. This will obviously not be the case for any qualitative disagreements like the one mentioned above for the universality of the  $SU(2)$  deconfinement transition with the Ising model. While the change in the order of the deconfinement phase transition was found on lattices with a variety of different temporal sizes, only one transition was found in each case at which the deconfinement order parameter became non-zero. This led to the suspicion that the bulk line was misidentified and was actually a deconfinement line. On the other hand, the transition at larger  $\beta_A$  shifted very little, even as the temporal lattice size was increased up to 16. One expects the deconfinement transition to move to larger couplings as  $N_\tau$ , i.e the temporal lattice size, becomes larger. A lack of shift was therefore more in tune with it being a bulk transition. Direct numerical[7] and analytical[21] evidences for the presence of the bulk transition were also obtained. A possible way out was to blame the small temporal extent of the lattices used. Nevertheless, if a universal result exists in the  $a \rightarrow 0$  limit, then the tricritical point  $T$ , where the deconfinement phase transition changes its nature from second order (thin dotted lines in Fig. 1) to first order (thick dotted lines in Fig. 1), must not be seen in the continuum limit. Therefore, one expects that the tricritical point will move rapidly towards the upper right corner ( $\beta \rightarrow \infty, \beta_A \rightarrow \infty$ ) of Fig. 1 as the lattice size increases. In the continuum limit, the deconfinement transition will then be second order and its universality with Ising model will be recovered. However, in [10] we found that on going from  $N_\tau = 2$  to 4 the tricritical point moved towards lower values of the adjoint coupling. Studying the shapes of the histogram and invoking Polyakov loop effective potential arguments[10], we concluded that the tricritical point moved slowly upwards, if at all, on going from  $N_\tau = 4 \rightarrow 6 \rightarrow 8$  lattices<sup>3</sup>. These results are paradoxical and need explanation. In view of the above, i.e, the change in the nature of the deconfinement transition from second order to first order, a qualitative violation in universality is a real possibility.

In this paper, we take a different approach in an attempt to resolve these paradoxes. While larger temporal lattices could perhaps be used to solve them by brute force, a physical understanding of the universality of the continuum limit of the  $SU(2)$  lattice gauge theory clearly requires an insight into the origin of the first order deconfinement transition and the tricritical point. In particular, one needs to a) identify the degrees of freedom which cause this unexpected change in the order

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<sup>3</sup>Such shifts of the tricritical points first towards lower values of  $\beta_A$  on going from  $N_\tau = 2$  to 4 and then towards higher values for  $N_\tau = 6, 8$  were also observed in [14] where the Villain form (1) was simulated.

of the deconfinement transition, and b) study whether these degrees of freedom are relevant or irrelevant in the continuum limit. Towards this end, we investigate the effect of the topological  $Z_2$  degrees of freedom associated with the  $SO(3)$  part of the action (i.e, the term proportional to  $\beta_A$  in (1)) on the deconfinement transition line in the extended  $(\beta, \beta_A)$  coupling plane. It is important to note that these  $Z_2$  degrees of freedom defined below are different from those defined for  $SU(2)$  Wilson theory ( $\beta_A = 0$ ) [23, 25, 26]. By adding chemical potential terms for the  $Z_2$  monopoles and vortices, believed[23] to be irrelevant in the continuum limit, we show that no first order deconfinement phase transition exists if they both are suppressed. Thus the onset of the first order deconfinement seems to be due to the  $Z_2$  topological degrees of freedom. In the past these  $Z_2$  degrees of freedom have been studied in the context of bulk transitions in the extended model and cross over phenomenon along the Wilson axis ( $\beta_A = 0$ ) [26, 22, 24, 25], the bulk transition along the  $SO(3)$  gauge theory [2] and the  $Z_2$  vortex transition along the  $\beta_A = \infty$  axis. We propose to use them for the study of deconfinement phase transition and for the entire  $(\beta, \beta_A)$  plane.

We employ the Villain form of the mixed  $SU(2)$  action for our studies below and find :

1. The theory defined by mixed action is self-dual under the exchange of the fundamental and adjoint couplings in the absence of  $Z_2$  magnetic monopoles.
2. Further suppressing the  $Z_2$  vortices completely reduces it to the standard Wilson action but with modified coupling; it thus can have only a second order deconfinement transition.

## 2. THE MODEL

The Villain form of the  $SU(2)$ - $SO(3)$  mixed action is defined by

$$Z = \prod_{n,\mu} \sum_{\sigma_p(n)=\pm 1} \int dU_\mu(n) \exp \left( \frac{1}{2} \sum_p (\beta + \beta_A \sigma_p) \text{Tr}_F U_p \right) . \quad (1)$$

In (1)  $U_\mu(n)$  are the  $SU(2)$  link variables,  $\sigma_p(n)$  are the  $Z_2$  plaquette fields. The first term in (1) describes the standard  $SU(2)$  Wilson action and the second term describes the  $SO(3)$  action. As mentioned earlier, this form was used in [14] for studies similar to ours. Indeed, apart from changes in numerical values of the couplings, the results were identical, leading to essentially the same phase diagram as in Fig. 1.

The main advantage of the Villain form is that the dynamical excitations corresponding to the  $Z_2$  topological degrees of freedom, both magnetic and electric in nature, become visible[2] in the partition function. In particular, it provides a better control over them and their contributions to any physical phenomenon.

The  $Z_2$  magnetic monopoles and  $Z_2$  electric vortex charges are obtained through the plaquette field  $\sigma_p \equiv \sigma_{\mu\nu}$  as follows. We first define the  $Z_2$  field strength tensor  $F_{\mu\nu}$  by

$$\sigma_{\mu\nu}(n) \equiv \exp(i\pi F_{\mu\nu}) \quad (2)$$

The  $Z_2$  magnetic (electric) currents  $\rho_c$  ( $\rho_l$ ) over a cube  $c$  (link  $l$ ) located at a lattice site  $n$  and oriented along  $(\mu, \nu, \lambda)$  ( $\mu$ ) directions are defined as follows

$$\rho_c(n) \equiv \prod_{p \in \partial c} \sigma_p, \quad \rho_l(n) \equiv \prod_{p \in \tilde{\partial} l} \sigma_p. \quad (3)$$

In (3)  $\partial$  and  $\tilde{\partial}$  stand for boundaries and co-boundaries of the cube  $c$  and the link  $l$  respectively. We extract the standard electric and magnetic currents by the identification:

$$\rho_l(n) \equiv \exp(i\pi J_\mu(n)), \quad \rho_c(n) \equiv \exp(i\pi M_\mu(n)), \quad (4)$$

In (4) the electric current  $J_\mu(n)$  are on the link  $l$  and the magnetic current  $M_\mu(n)$  is defined on the link  $\mu$  which is orthogonal to the cube  $c$ . Using (2), (3) and (4), we recover the  $Z_2$  Maxwell equations in the standard form:

$$\Delta_\mu F_{\mu\nu}(n) = J_\nu(n), \quad \Delta_\mu \tilde{F}_{\mu\nu}(n) = M_\nu(n), \quad (5)$$

where,  $\Delta_\mu(n)$  is the lattice difference operator in the  $\mu^{th}$  direction. It is clear from (3) that the  $Z_2$  magnetic and electric charge definitions are dual to each other. Note that the  $Z_2$  monopoles defined by the second equation in (4) are different from those obtained by an abelian projection [1] with respect to an  $SU(2)$  adjoint Higgs<sup>4</sup> field  $\vec{\phi}$  with the abelian field strength tensor defined by:

$$F_{\mu\nu}^{U(1)} \equiv \frac{\vec{\phi} \cdot \vec{F}_{\mu\nu}}{|\vec{\phi}|} - \frac{\vec{\phi}}{|\vec{\phi}|^3} (D_\mu \vec{\phi} \times D_\nu \vec{\phi}). \quad (6)$$

Here  $\vec{F}_{\mu\nu}$  is the  $SU(2)$  field strength tensor derived from  $Tr U_p$ . The  $U(1)$  topological magnetic current, derived from (6), and the corresponding  $U(1)$  magnetic monopoles are clearly different from the  $Z_2$  monopoles described by (3). It has been suggested [1] that the condensation of the former is perhaps responsible for confinement of color. On the other hand, the  $Z_2$  topological degrees of freedom defined in (3) are the lattice artifacts. We will show that they are responsible for changing the order of the deconfinement transition in the extended  $(\beta, \beta_A)$  coupling plane.

In order to control the effects of the  $Z_2$  topological degrees of freedom we add two terms proportional to their  $Z_2$  magnetic and electric vortex charges in (1) :

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<sup>4</sup>e.g in the pure gauge theory one could choose [1]  $\vec{\phi} \equiv \vec{F}_{12}$ .

$$Z_{\lambda,\gamma} = \prod_{n,\mu} \sum_{\sigma_p(n)=\pm 1} \int dU_\mu(n) \exp \left( \frac{1}{2} \sum_p (\beta + \beta_A \sigma_p) Tr_F U_p + \lambda \sum_c \rho_c + \gamma \sum_l \rho_l \right) . \quad (7)$$

Note that the formal continuum limit is expected to be left unchanged by the addition of  $\lambda$  and  $\gamma$  terms for all values of  $\lambda$  and  $\gamma$ . We discuss below the following cases:

1.  $\gamma = 0, \lambda = \infty$ , i.e, a complete suppression of the  $Z_2$  monopoles without affecting the  $Z_2$  electric vortices.
2.  $\gamma = \infty, \lambda = 0$ , i.e, a complete suppression of  $Z_2$  electric vortices without affecting the  $Z_2$  magnetic monopoles.
3.  $\gamma = \infty, \lambda = \infty$ , i.e, complete suppression of all the  $Z_2$  topological degrees of freedom.

### Case 1.

In the limit  $\lambda = \infty$  at  $\gamma = 0$ , one obtains the following constraint:

$$\sigma_c = 1 \quad \forall c \quad (8)$$

This amounts to complete suppression of point like magnetic monopoles. Note that these local constraints can still allow the global  $Z_2$  magnetic fluxes arising due to the periodic boundary conditions. While such boundary conditions in the spatial directions are not obligatory, they have to be enforced in the temporal directions. However, in what follows these global fluxes are irrelevant for our results. The constraints (8) are the Bianchi identities in the absence of magnetic charges (see (4) and (5)). They can be trivially solved by introducing  $Z_2$  link fields  $\sigma_\mu(n)$ , such that

$$\sigma_p \equiv \sigma_{\mu\nu}(n) = \sigma_\mu(n) \sigma_\nu(n + \mu) \sigma_\mu^\dagger(n + \nu) \sigma_\nu^\dagger(n) \quad (9)$$

The partition function in this limit therefore becomes

$$Z_{\lambda \rightarrow \infty, \gamma} = \prod_{n,\mu} \sum_{\sigma_\mu(n)=\pm 1} \int dU_\mu(n) \exp \left( \frac{1}{2} \sum_p (\beta + \beta_A (\sigma_\mu(n) \sigma_\nu(n + \mu) \sigma_\mu(n + \nu) \sigma_\nu(n))) Tr_F U_p \right) . \quad (10)$$

It has a local  $SU(2) \otimes Z_2$  gauge invariance:

$$\begin{aligned} U_\mu(n) &\rightarrow G(n)U_\mu(n)G^{-1}(n+\mu) & G(n) \in SU(2) \\ \sigma_\mu(n) &\rightarrow \sigma_\mu(n) \end{aligned} \quad (11)$$

$$\begin{aligned} \sigma_\mu(n) &\rightarrow z(n)\sigma_\mu(n)z^{-1}(n+\mu) & z(n) \in Z_2 \\ U_\mu(n) &\rightarrow U_\mu(n) \end{aligned} \quad (12)$$

We now define a new SU(2) field on each link:

$$\tilde{U}_\mu(n) \equiv \sigma_\mu(n)U_\mu(n) \quad (13)$$

Exploiting the SU(2) group invariance of the Haar measure and the  $Z_2$  nature of  $\sigma$ -variables the partition function (10) in terms of new variables is:

$$\begin{aligned} Z_{\lambda \rightarrow \infty, \gamma} = \prod_{n, \mu} \sum_{\sigma_\mu(n)=\pm 1} \int dU_\mu(n) \exp \left( \frac{1}{2} \sum_p (\beta_A + \right. \right. \\ \left. \left. \beta (\sigma_\mu(n)\sigma_\nu(n+\mu)\sigma_\mu(n+\nu)\sigma_\nu(n)) \right) Tr_F U_p \right) . \quad (14) \end{aligned}$$

Eqs. (10) and (14) have exactly the same form with  $\beta \leftrightarrow \beta_A$ . Therefore, in the absence of  $Z_2$  magnetic monopoles the physics of the extended model is self-dual under the exchange of the fundamental and adjoint couplings.

Note that the self-duality arguments also go through for the SU(N) ( $\forall N$ ) extended Villain actions. The plaquette and the link fields  $\sigma_{\mu\nu}$  and  $\sigma_\mu$  take values in the  $Z_N$  group in that case.  $\rho_c$  is clearly then complex and the action in (7) will have to be modified by including its real part there. However, rest of the arguments are exactly the same as for the SU(2) case and again, in the limit of total suppression of  $Z_N$  monopoles, the physics will be self-dual under  $\beta \leftrightarrow \beta_A$ .

### Case 2.

The limit  $\gamma = \infty, \lambda = 0$  corresponds to solving the constraint:

$$\rho_l = 1 \quad \forall l . \quad (15)$$

This limit suppresses all the  $Z_2$  electric vortices. It is therefore dual to the  $\lambda = \infty$  limit discussed above. Again, as discussed in Case 1 above the local constraints (15) leave the global  $Z_2$  electric fluxes due to the periodic boundary conditions unaffected. The solution of (15) corresponds to solving  $\Delta_\mu F_{\mu\nu} = 0$  in terms of the dual vector potentials and can be written as:

$$\sigma_{\mu\nu}(n) = \exp i \epsilon_{\mu\nu\rho\delta} \Delta_\rho \sigma_\delta(n) \quad (16)$$

In (16)  $\sigma_\delta(n)$  describes the dual vector potential on the link  $\delta$  at lattice site  $n$ . Unlike in the case of the suppression of  $Z_2$  magnetic charges suppression above, the suppression of  $Z_2$  electric vortices does not lead to a self-dual model.

### Case 3.

The  $\lambda = \gamma = \infty$  limit corresponds to complete suppression of both magnetic as well as electric  $Z_2$  topological degrees of freedoms. The Maxwell eqns. in (5) are now without electric or magnetic sources:

$$\Delta_\mu F_{\mu\nu}(\sigma) = 0, \quad \Delta_\mu \tilde{F}_{\mu\nu}(\sigma) = 0. \quad (17)$$

The solution of the second of equation above, involving the dual fields, is still given by eqn. (9)). One thus has to solve for the remaining electric equation with the additional constraint of gauge invariance of the  $Z_2$  link fields  $\sigma_\mu(n)$  which define the  $F_{\mu\nu}$ . Fixing the gauge symmetry (eqn. (12)) such that  $\Delta_\mu A_\mu = 0$ , where the  $Z_2$  vector potential  $A_\mu$  is related to  $\sigma_\mu$  as  $\sigma_\mu = \exp(iA_\mu)$ , one can easily show that the gauge potential  $A_\mu$  satisfies the following equation:

$$\square A_\mu(n) = 0 . \quad (18)$$

It can be solved by a Fourier transform, leading to a solution  $\sigma_\mu(n) = \text{constant}$ , which can be  $+1$  or  $-1$  leading to  $\sigma_p = 1$ . Substituting back in (7), the extended action in this limit reduces to the standard Wilson action with coupling  $\beta + \beta_A$ . It must therefore have a second order deconfinement transition along the line  $\beta + \beta_A = \text{constant}$ , as known from the results for  $\beta_A = 0$ .

## 3. SUMMARY and DISCUSSION

Assuming the  $Z_2$  topological degrees of freedom to be irrelevant in the continuum, as suggested by perturbative arguments, we have shown that the *qualitative* changes in the  $SU(2)$  deconfinement transition found earlier both by Monte Carlo as well as strong coupling expansions are likely to be due to the above degrees of freedom and hence lattice artifacts; suppressing them completely leads to the same physical result for the deconfinement phase transition for all  $\beta_A$ , including large  $\beta_A$ . These qualitative changes in the  $SU(2)$  gauge theory on lattice are perhaps similar in spirit to the unphysical *confining* phase of  $U(1)$  compact lattice gauge theory. The latter is also due to the topological  $U(1)$  magnetic monopoles which are irrelevant in the continuum. In order for the above universal results for the  $SU(2)$  deconfinement phase transition to be valid for the mixed action it is important to establish non-perturbatively that the  $Z_2$  degrees of freedom are indeed irrelevant in the continuum limit. In the context of pure  $SU(2)$  lattice gauge theory it has been shown[23] that the probability for the  $Z_2$  monopoles excitation on a set of  $C$  cubes:

$$< \prod_{c \in C} \theta(-\sigma_c) >_{\text{Limit } \beta \rightarrow \infty} \leq L_1 \exp(-L_2 \beta C) \quad (19)$$

In (19)  $\theta(x) \equiv 1(0)$  if  $x \geq 0$  (otherwise) and  $L_1, L_2$  are constants. Depending on the value of  $L_2$ , the  $Z_2$  monopoles could be extremely rare. Also, the  $Z_2$  electric vortices are known to condense and give rise to a first order bulk transition shown by line  $CG$  in Fig. 1. These arguments suggest that the original theory, i.e.,  $\gamma = \lambda = 0$ , should also exhibit universality for very large  $N_\tau$ . Of course, it is still necessary to extend the argument of Ref. [23] to the entire  $(\beta, \beta_A)$  plane for both the  $Z_2$  monopoles and vortices to be sure that this is indeed the case. Moreover, the curious coincidence of the bulk and deconfinement transition for the case of  $\gamma = \lambda = 0$  for many different sizes of temporal lattice sizes still remains unexplained, although one can now be more hopeful that for *very* large lattices at least the universality will be respected, as it should be.

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