Renormalized Polyakov loop in the Fixed Scale Approach

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Introduction

Results

Summary

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- Polyakov loop $L(\vec{x})$ — Deconfinement Order Parameter (Spontaneous Breaking of $Z(N)$) (McLerran & Svetitsky, PRD 1981)

- One hopes to construct effective theories (Pisarski, PRD 2006) of $L$ for investigations of deconfinement phase transitions and many models employ $L$. 


Introduction

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- One hopes to construct effective theories (Pisarski, PRD 2006) of \( L \) for investigations of deconfinement phase transitions and many models employ \( L \).

- On an Euclidean \( N_\sigma^3 \times N_\tau \) lattice \( L(\vec{x}) \) is defined at a site \( \vec{x} \) as
  \[
  L(\vec{x}) = \frac{1}{N_c} \text{Tr} \prod_{x_0=1}^{N_\tau} U^4(\vec{x}, x_0).
  \]

- No SSB on finite lattices/volumes. Usually one defines \( \bar{L} = \sum_{\vec{x}} L(\vec{x}) / N_\sigma^3 \), and employs \( \langle |\bar{L}| \rangle \), or its susceptibility, to locate the deconfinement phase transition.

- \( \langle |\bar{L}| \rangle \to 0 \) as \( 1/\text{Volume} \) in the confined phase, and \( \langle |\bar{L}| \rangle \neq 0 \) in the deconfined phase.
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• Like any Wilson loop, Polyakov loop needs to be renormalized.

• More so, since as an order parameter it seeks to label phases by being zero or nonzero.
Earlier Work

♣ The physical interpretation of $L$ as relate to the free energy of a single static quark offers a clue.

♠ The single quark free energy $F_b(N_\tau, a)$ is obtained from

$$\ln \langle |\tilde{L}| \rangle = -F_b(T)/T = -aN_\tau F_b(N_\tau, a).$$
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- Use of $N_\tau$-grids and fits to $L$ (Dumitru et al. PRD 2004)

- Use of renormalization group iteratively (S. Gupta et al. PRD 2008)
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Fixed Scale Approach

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Let $\beta_c$, corresponding to the position of the peak of the $|L|$-susceptibility for some fixed $N_{\tau,c}$, be the choice of the fixed scale $a_c$.

Further, let it lie in the scaling region, then in the fixed scale approach $T/T_c = N_{\tau,c}/N_\tau$. 
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Further, let it lie in the scaling region, then in the fixed scale approach $T/T_c = N_{\tau,c}/N_\tau$.

Write the single quark free energy as a sum of a would-be divergent and a regular contribution,

$$F_b(T, a_c) = F(T, a_c) - A(a_c),$$

where $A$ is the would-be divergent free energy in physical units.
Since

\[
\frac{T}{T_c} \ln \langle |\bar{L}| \rangle = -\frac{F(T, a_c)}{T_c} + \frac{A(a_c)}{T_c},
\]

the free energy at any two different scales, \(a_{c1}\) and \(a_{c2}\), differs by the same constant at all \(T\).

◊ Use \(\langle |L| \rangle\) at just one temperature to eliminate the relative shift \(\Rightarrow\) All cut-off dependence of the order parameter is gone in the entire \(T\)-range.
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In the following, I consider the simple case of \( SU(2) \) to demonstrate how well it works. It should work similarly for any \( N_c \) or QCD.

I employ the critical \( \beta \) for \( N_\tau = 4, 6, 8 \) and 12 from the table of Velytsky, IJMP C19, (2008), 1079, which agree with earlier results where available.
4 different scales: Tc4, Tc6, Tc8 and Tc12 with \( a \to 0 \) progressively. Increasing Spatial Volume leads to decrease in \( L \) for \( T < T_c \).
• Illustrate for two scales: Different behaviour in $T$ for the Free Energy. Shift $F$ by $\Delta F(2T_c)$.
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– I chose 3 constants to shift all the data to the Tc4 scale: The Tc6, Tc8, Tc12 results have simply jumped to their appropriate place on the \( \langle |L| \rangle \) for it.
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– Does the renormalized \( L \) then climb to unity slowly?
High Temperature Perturbation Theory (Gava-Jengo, PLB 1981) tells us that \( L \to 1 \) from above at very large \( T \) : 
\[
L = 1 + C_3 g^3 + \mathcal{O}(g^4),
\]
where \( c_3(N_c) > 0 \) is a constant.
• High Temperature Perturbation Theory (Gava-Jengo, PLB 1981) tells us that $L \to 1$ from above at very large $T$: $L = 1 + C_3 g^3 + O(g^4)$, where $c_3(N_c) > 0$ is a constant.

• Instead of shifts at $2T_c$ for varying scales, try a fit

$$-\ln \langle |\bar{L}_j| \rangle = F(2T_c)/2T_c + B \cdot N_{\tau j}/2.$$
Eliminating the $B$-dependent divergent term for the Tc4-scale in addition to the shifts, one has,

$L$ now does go to unity from above at large $T$. Large volumes, aspect ratio of $\sim 10$, needed for $L \approx 0$ for low $T$. 

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R. V. Gavai
Summary

• I showed that the fixed scale approach leads to a natural definition of a physical, $N_T$-independent, order parameter which is defined in both the confined and the deconfined phases.

• It does not need two-point correlations, and works for even coarse lattices ($a \leq 1/4T_c$).
Summary

• I showed that the fixed scale approach leads to a natural definition of a physical, $N_T$-independent, order parameter which is defined in both the confined and the deconfined phases.

• It does not need two-point correlations, and works for even coarse lattices ($a \leq 1/4T_c$).

• The definition itself does not depend on any lattice artifacts or the lattice size in the deconfined phase.

• It displays the expected behaviour in both the phases, i.e., volume dependence in the low $T$-phase and approach to unity from above in high $T$-phase.