Blind Subspace DOA Estimation in Multipath DS/CDMA Channels

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ABSTRACT

In this paper, we consider the problem of blind estimation of the directions of arrival (DOA's) of users' paths in a multipath DS/CDMA channel. Making use of the signal that is sampled at multiple antenna elements and using a subspace based MUSIC-like technique, we show the possibility of DOA estimation using two search methods. The first provides path delays and DOAs simultaneously, and the second provides only DOAs. Knowledge of the chip waveform is used in the first method. It is seen that the two methods exhibit good estimation accuracy, besides being extremely near –far resistant.

1. INTRODUCTION

Channel parameter estimation in DS/CDMA systems, generally, is an important issue for multiuser detection. When antenna arrays are used at the receiver, for capacity and performance improvement, the directions of arrivals (DOA's) of various signals become an important parameter needed for beamforming toward the desired signals and implementing space-time multiuser receivers [1,2,3]. However, most of the available techniques for DOA estimation do not exploit the structure of the DS/CDMA signal and the multiuser interference (MUI) [4] and therefore require an impractically large number of antennas. Recently, a few techniques have been proposed for DOA estimation in DS/CDMA systems such as [2,5,6].

In [2], the DOA response vector of the desired signal was approximated as the dominant eigenvector of the difference between the *pre-* and the *post-correlation* spatial matrices. A problem with this approach is that it relies on the large process gain of the system, and does not exploit the code periodicity (assuming use of short codes); hence its performance degrades for strong MUI. However, it should be mentioned that this method has the advantage of handling aperiodic codes. In [5], a procedure is presented for approximate maximum likelihood delay/DOA estimation. However, the method relies on the use of a known training sequence for the desired user, causing a reduction in the data throughput. The overhead would be specially large in fast time varying channels where this training would be needed more often. In [6], a DOA estimation method was proposed making use of the pre- and post- correlation matrices of the input signal and making use of the code periodicity for suppressing MUI, but has the disadvantage of requiring knowledge of the path delays and attenuations for all the users.

In this paper, we propose a blind subspace procedure for DOA estimation of DS/CDMA signals in a multipath scenario. Using an antenna array with a certain number of elements, we develop two search methods for DOA estimation: the first provides joint delay/DOA estimation through a delay search that makes use of the knowledge of the chip waveform, and the second provides only the DOA estimates without using this knowledge. The system considered is a quasi-synchronous system for simplicity of presentation.

2. THE SIGNAL MODEL

We consider here a scenario of *K*-users in a quasi-synchronous DS/CDMA system using binary phase shift keying (BPSK) modulation, and a uniform linear antenna array of M-elements with $\lambda/2$ spacing for signal reception. A short spreading code is used, i.e., the code period P' is taken to be one bit or symbol period. Without loss of generality, it is assumed that each user signal arrives from L paths. The assumption of quasi-stationarity is meant here to imply that all the multipath signals arrive within a few chip durations (i.e. a maximum of q-chips with $q < P'$). Thus all the user signals are synchronized to within the tolerance of a few chips only. Hence, it will be possible (with negligible loss in performance) to carry out parameter estimation using only those code chips that are free of intersymbol interference (ISI). The number of ISI-free chips shall be denoted by P. After down conversion, the received signal is chip matched filtered and sampled at the chip rate to give (in the absence of noise)

$$x_m(n,p) = \sum_{k=1}^{K} s_k(n) \sum_{l=1}^{L} a_{l,k}\ \rho_{m,k,l}\ c_{k,l}(p) \quad (1)$$

where

$a_{l,k}$: is the complex gain of the l^{th}-path of the k^{th}-user channel at the first antenna.

$\rho_{m,k,l}$: is the array response of the l^{th}-path of the k^{th}-user at the m^{th}- antenna, expressed as $\exp(-j\pi(m-1)\sin(\theta_{l,k}))$ where $\theta_{l,k}$: is the DOA of this path. $c_{k,l}(p)$ denotes the p^{th}-chip of the k^{th}-user's code arriving through the l^{th}-path, taking into cosideration the code delay in that path and the pulse shape used. $s_k(n)$ denotes the k^{th} user symbol at time instant n. P denotes the process gain considering ISI-free chips only.

We start by constructing a signal matrix $\mathbf{X_m}$ for the samples received at the m^{th}- antenna in an N-symbol block interval as

$$\mathbf{X_m} = \begin{bmatrix} x_m(1,1) & x_m(1,2) & \cdots & x_m(1,P) \\ \vdots & \vdots & & \vdots \\ x_m(N,1) & & \cdots\cdots\cdots & x_m(N,P) \end{bmatrix} \quad (2)$$

It is easy to see that this matrix can be expressed in terms of the symbol and code matrices as follows

$$\mathbf{X_m} = \mathbf{S}\,\mathbf{H_m} \quad (3a)$$

where

$$\mathbf{S} = \begin{bmatrix} s_1(1) & s_2(1) & \cdots & s_K(1) \\ \vdots & \vdots & & \vdots \\ s_1(N) & \cdots\cdots\cdots\cdots & & s_K(N) \end{bmatrix} \quad (3b)$$

$$\mathbf{H_m} = \begin{bmatrix} \mathbf{h}_{m,1} & \mathbf{h}_{m,2} & \cdots\cdots & \mathbf{h}_{m,K} \end{bmatrix}^{\mathbf{T}} \quad (3c)$$

$$\mathbf{h}_{m,k} = \mathbf{C}_k \mathbf{g}_{m,k} \quad (4)$$

$$\mathbf{C}_k = \begin{bmatrix} \mathbf{c}_{k,\mathbf{1}} & \mathbf{c}_{k,\mathbf{2}} & \cdots\cdots & \mathbf{c}_{k,L} \end{bmatrix} \quad (5)$$

being the code- matrix of the k^{th} user as seen from the L-paths of its arrival, with

$$\mathbf{c}_{k,l} = \begin{bmatrix} c_{k,l}(1) & c_{k,l}(2) & \cdots\cdots & c_{k,l}(P) \end{bmatrix}^{\mathbf{T}} \quad (6)$$

The vector $\mathbf{g_{m,k}}$, given by

$$\mathbf{g}_{m,k} = \begin{bmatrix} a_{1,k}\rho_{m,k,1} & a_{2,k}\rho_{m,k,2} & \cdots a_{L,k}\rho_{m,k,L} \end{bmatrix}^{\mathbf{T}} \quad (7)$$

is the multipath parameter vector for user-k at the m^{th}- antenna taking into account the path attenuation as well as the array response to signals from various paths. Further we define the matrix $\mathbf{Y_m}$ as

$$\mathbf{Y_m} = \begin{bmatrix} \mathbf{X_m} & \mathbf{X_{m+1}} \end{bmatrix} \quad \text{for } m=1 \text{ to } M\text{-}1. \quad (8)$$

It is easy to see that in the absence of noise we can write the following matrix relation

$$\mathbf{Y_m} = \mathbf{S}\,\mathbf{H_m}' \quad (9)$$

where

$$\mathbf{H}_m' = \begin{bmatrix} \mathbf{h}'_{m,1} & \cdots\cdots & \mathbf{h}'_{m,K} \end{bmatrix}^{\mathbf{T}} \quad (10)$$

$$\mathbf{h}'_{m,k} = \mathbf{C}'_k \mathbf{g}_{m,k} \quad (11)$$

$$\mathbf{C}'_k = \begin{bmatrix} \mathbf{c}_{k,1} \cdots & \mathbf{c}_{k,L} \\ \rho_{2,k,1}\mathbf{c}_{k,1} \cdots & \rho_{2,k,L}\mathbf{c}_{k,L} \end{bmatrix} \stackrel{\Delta}{=} \begin{bmatrix} \mathbf{c}'_{k,1} & \cdots\cdots & \mathbf{c}'_{k,L} \end{bmatrix} \quad (12)$$

This matrix definition of the input signal samples is central to the formulation in the coming section.

3. THE DOA ESTIMATION

The approach adopted here for DOA estimation is a subspace approach that is based on resolving the desired user effective code $\mathbf{h}'_{m,k}$ in terms of path codes $\mathbf{c}'_{k,l}$. The advantage of this approach is that if the effective code ($\mathbf{h}'_{m,k}$), over two or more antennas, is properly expressed in terms of path codes then this can easily lead to DOA estimation of all paths, simply from estimates of ρ - values in (12). The proposed method will make use of the channel estimation procedure of [7], besides an antenna array at the receiver for achieving this goal.

In the MUSIC search, the vector space of the received signal is decomposed into a signal subspace (spanning all users' signals) and its orthogonal complement, which is called the noise subspace. In case of a single path channel, the signal subspace is comprised of the span of all the user path codes. However, in a multipath channel, the signal space is comprised of the span of the effective codes of the various users. Consequently, such a subspace decomposition in the multipath situation can not help to resolve the signal (through the MUSIC search) into individual path codes.

To resolve the problem, we propose here, finding a subspace that is comprised of the span of the codes of individual paths for a specific (desired) user. Once this subspace is found, we can make use of its orthogonal complement in a

MUSIC-like search procedure to find delays/DOAs or only DOAs. Fortunately, as shown in this paper, this can be easily done by using an antenna array with number of elements more than the number of paths /user, and by making use of the properties of a certain matrix **A** which is constructed for the specific desired user (say the k^{th}-user) as

$$\mathbf{A} = \left[\mathbf{h}'_{1,k} \quad \mathbf{h}'_{2,k} \quad \cdots\cdots \quad \mathbf{h}'_{M-1,k}\right] \qquad (13)$$

A useful property for this matrix is given in the following proposition:

<u>*Proposition:*</u>

If M-antennas are used at the receiver, and M> the number of paths of the desired (k^{th}) user and if the DOA's of these paths are distinct, then:

1. The left null space of **A** is orthogonal to all path codes of the k^{th}-user, i.e., to $\{\ \mathbf{c}'_{k,1},\ \mathbf{c}'_{k,2}, \cdots\cdots \mathbf{c}'_{k,L}\ \}$.

2 Rank(**A**)= number of paths for the k^{th} –user.

Proof:

From (11) and (12) we get

$$\mathbf{A} = \mathbf{C}'_k \mathbf{G}_k \qquad (14)$$

Where

$$\mathbf{G}_k = \left[\mathbf{g}_{1,k} \quad \mathbf{g}_{2,k} \quad \cdots\cdots \mathbf{g}_{M-1,k}\right] \qquad (15)$$

If a vector **v** lies in the left null space of **A**, this implies

$$\mathbf{v}^{\mathbf{H}} \mathbf{C}'_k \mathbf{G}_k = \mathbf{0} \qquad (16)$$

Let

$$\mathbf{v}^{\mathbf{H}} \mathbf{C}'_k = \left[b_1 \quad b_2 \quad \cdots\cdots b_L\right] \qquad (17)$$

Then

$$\mathbf{G}_k^{\mathbf{H}} \left[b_1^* \cdots\cdots b_L^*\right]^{\mathbf{T}} = \mathbf{0} \qquad (18)$$

But $\mathbf{G}_k^{\mathbf{H}}$ is a vandermonde matrix with distinct entries, hence it is full rank, and there is no solution for (18) except the trivial solution viz., b_l=0, for l=1 to L. Hence, any vector in the left null space of **A** should be orthogonal to all user's path codes. It is also obvious that **A** has a rank of L , since both $\mathbf{C}'_k$, and $\mathbf{G}_k$,in (14), are full rank matrices with rank=L.

Hence proved.

In the presence of noise, **A** can be estimated using the subspace approach of [7], applied to outputs from a pair of antenna elements as given in ($\mathbf{Y}_m$), to estimate $\mathbf{h}'_{m,k}$, for m=1 to M-1. It is easy to argue that the effective codes are those linear combinations of the path codes which are orthogonal to the noise subspace[1] of $\mathbf{Y}_m^{\mathbf{T}}$. Taking a clue from (4) and (11) and using this argument, we can construct estimates for the effective code of the k^{th} user at the m^{th} antenna as

$$\hat{\mathbf{h}}'_{m,k} = \mathbf{B}'_k \, \mathbf{w}_{\mathbf{m}} \qquad (19)$$

where

$$\mathbf{w}_{\mathbf{m}} = \arg\left\{ \min_{\mathbf{w}} (\mathbf{w}^{\mathbf{H}} \mathbf{B}'^{\mathbf{H}}_k \mathbf{Q}_m \mathbf{Q}_m^{\mathbf{H}} \mathbf{B}'_k \mathbf{w}) \right\}, \qquad (20)$$

subject to $\mathbf{w}^{\mathbf{H}}\mathbf{w}=1$

$$\mathbf{B}'_k = \begin{bmatrix} \mathbf{B}_k & \mathbf{0} \\ \mathbf{0} & \mathbf{B}_k \end{bmatrix} \qquad (21)$$

and

$$\mathbf{B}_k = \begin{bmatrix} b_k(1) & b_k(2) & \cdots\cdots & b_k(q) \\ \vdots & \vdots & & \vdots \\ b_k(P) & \cdots\cdots\cdots\cdots & & b_k(P+q-1) \end{bmatrix} \qquad (22)$$

where $\mathbf{b}_k$ is the binary Gold-code assigned to user-k and $\mathbf{Q}_m$ is the noise subspace of $\mathbf{Y}_m^{\mathbf{T}}$. Hence

$$\hat{\mathbf{A}} = \left[\hat{\mathbf{h}}'_{1,k} \quad \hat{\mathbf{h}}'_{2,k} \quad \cdots\cdots \quad \hat{\mathbf{h}}'_{M-1,k}\right] \qquad (23)$$

Now, if we perform singular value decomposition for $\hat{\mathbf{A}}$ as ($\hat{\mathbf{A}} = \mathbf{U} \sum \mathbf{V}$)then the number of paths $\hat{L}$ can be estimated as the number of significant singular values of $\hat{\mathbf{A}}$, and the left null space of **A** can be estimated as the span of the left singular vectors of $\hat{\mathbf{A}}$ that are associated with the ($2P$-$\hat{L}$) least significant singular values, i.e.

$$\mathbf{U}_n = \left[\mathbf{u}_{\hat{L}+1} \cdots\cdots \mathbf{u}_{2P}\right] \qquad (24)$$

Once $\mathbf{U}_n$ has been estimated, we can perform a search to find the path DOAs using one of the following two methods which are both based on the test of orthogonality of the candidate code/DOA to the subspace U_n:

3.1 Joint delay/DOA estimation:

This search uses knowledge of the pulse shaped code waveform[2] of the desired user, and hence, knowledge of $\mathbf{c}_{\mathbf{k}}(\tau)$, a ($P$x1) vector comprised of the chip-rate samples of this waveform starting at a specific delay τ. This vector is used in a search over all the possible

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[1] The noise subspace of $\mathbf{Y}_m^{\mathbf{T}}$ here is defined as the subspace spanned by the left singular vectors of $\mathbf{Y}_m^{\mathbf{T}}$ that are associated with the least significant ($2P$-K) singular values. For details see [7].

[2] Obviously, this waveform is the convolution of the desired user binary code with the pulse shape.

τ - values (typically within a period of few chips in quasi-synchronous systems) making use of the fact that any path code $\mathbf{c}'_{k,l}$ (i.e. over two antennas) should be of the form $\mathbf{Zf}$, where

$$\mathbf{Z} = \begin{bmatrix} \mathbf{c_k}(\tau) & \mathbf{0} \\ \mathbf{0} & \mathbf{c_k}(\tau) \end{bmatrix} \quad (25)$$

and $\mathbf{f}$ is the array response vector. Hence, this τ-search will simultaneously provide DOA's with the delays, as follows:

3.1.1- For $\tau = 0$ *to* $\tau_{\max}$, in steps (*step*<< chip period), perform the following two steps:

(i)Form the matrix $\mathbf{Z}$ as in (25).

(ii) Find $\mathbf{f}_0$, a two-element vector, as

$$\mathbf{f_0} = \arg\min_{\mathbf{f}} (\mathbf{f^H Z^H U}_n \mathbf{U}_n^{\mathbf{H}} \mathbf{Z f}) \quad (26)$$

subject to $\mathbf{f^H f} = 1$

$$r(\tau) = 1 / (\mathbf{f}_0{}^{\mathbf{H}} \mathbf{Z^H U}_n \mathbf{U}_n^{\mathbf{H}} \mathbf{Z f}_0) \quad (27)$$

3.1.2- Plot $r(\tau)$ as a function of τ, and take the $\hat{L}$ significant peaks of the plot, where the τ's corresponding to these peaks represent the estimated path delays. If we denote the $\mathbf{f_0}$-vector corresponding to the l^{th} significant peak of $r(\tau)$ by $\mathbf{f}_l$, then the DOA's of the desired user's paths can be found as

$$DOA(l) = \sin^{-1}[(1/\pi) * angle(\mathbf{f}_l(2) / \mathbf{f}_l(1))]$$

for $l = 1$ *to* L. (28)

3.2 DOA only estimation:

This search is done over θ without using the pulse shape information, by making use of the fact that any path code $\mathbf{c}'_{k,l}$ should be composed of two identical halves (upper and lower) except for the array response factor ρ multiplying the lower part. Hence any desired user path code should have the form of

$$\begin{bmatrix} \mathbf{B}_k \\ \rho \mathbf{B}_k \end{bmatrix} \mathbf{w} \quad (29)$$

Hence, using this representation for a candidate path code, we can do a search over θ as follows:

3.2.1- For $\theta = -\pi/3$ *to* $\pi/3$ (the view angle) do the following two steps:

(i) form the candidate code matrix $\mathbf{C_c}$ given by (29), $\rho = \exp(-j\pi\sin(\theta))$.

(ii) Test the candidate code ($\mathbf{C_c w}$) orthogonality to $\mathbf{U}_n$, using the measure $w(\theta)$ given by 1/[min. eigen value of $(\mathbf{C_c^H U}_n \mathbf{U}_n{}^{\mathbf{H}} \mathbf{C_c})$].

3.2.2-The estimated path DOA's of the desired user are found as the θ-values corresponding to the significant peaks of the plot of $w(\theta)$ (they should be less than or equal to $\hat{L}$).

4. SIMULATIONS

To test the performance of the proposed DOA estimation methods, we carried out the following experiments via computer simulations.

Exp.1: In this experiment, we assume a quasi-synchronous system with a process gain of 22 employing Gold codes. We consider the performance as a function of SNR. We assume here a raised cosine chip waveform (roll off factor of .5), and consider DOA estimation with 6 antennas. A scenario of 8-users is simulated with 3-paths /user with relative powers of (-6, -3 and 0) dB, and an SIR=-16 dB. DOA's of the desired user were (-17.2°, 4.6°, 11.4°) with delays of (.19,1.48, 2.13) chips. The size of the block used for estimation is 200-symbols. Fig.1 shows the rms error in DOA estimation versus SNR of the strongest path of the desired user. The figure generally shows a fair accuracy with the method 1 being more accurate than method 2. It also shows that accuracy of estimation is better for more spatially separated paths (estimation of the –17.2° is better than 4.6° and 11.4°), and also better for higher SNRs.

Exp.2: To show the near-far resistance of the proposed methods, we carried out an experiment similar to experiment 1 but with the following differences in the parameters of the desired user:(DOAs=-8.6°,8.5°, 17.1°.),(Delays= .23, .51, 2.13 chips),.Path relative powers (0,0,-3) dB. SNR of the strongest path of the desired user is fixed at (6 dB). Again, 200-symbols were used for estimation. Fig.2 shows the rms error in the DOA estimation versus SIR. The figure clearly shows that both of the proposed search methods are extremely near-far resistant.

For lack of space, we are not presenting here a comparison with the performance achieved via other methods. However, our experiments clearly show that the method proposed here has significantly better performance than [6].

5. CONCLUSIONS

In this paper, a new approach for blind multiuser DOA estimation in multipath DS/CDMA channels, is presented. The approach employs a subspace MUSIC-like technique, and develops two search methods; the first is a delay/DOA search, and the second is a DOA-only search. The pulse shape information is used only in the first method. The first method, generally, has higher accuracy than the second method,providing delays/DOAs simultaneously. Simulation results show that the two methods can provide DOA estimation of quite reasonable accuracy. Results also show that the method is very near-far resistant and hence can be employed in loosely power controlled scenarios. Hence, the proposed method can be very useful to base stations employing smart antennas and performing multiuser detection.

6. REFERENCES

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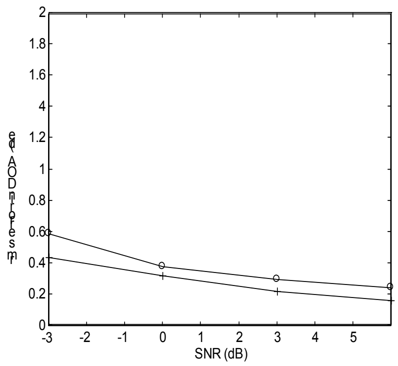

Fig.1a

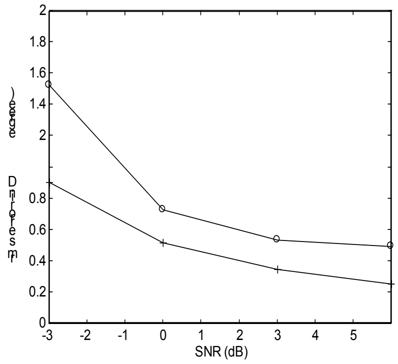

Fig.1b

Fig.1 RMS error in DOA estimates versus SNR(strongest path) of the desired user,(++: method-1, oo:method-2). 1a: DOA=-17.2°, 1b: DOA=4.6°,

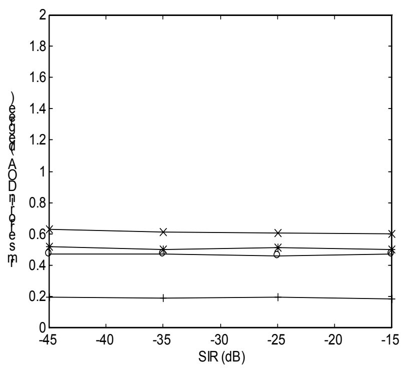

Fig.2- RMS error in DOA estimate, for the desired user, versus SIR, for exp.2. (method 1, ++:DOA=-8.6°, oo:DOA=8.5°) (method 2, **:DOA=-8.6°, xx: DOA=8.5°).