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Estimation of permeability of shaly sand from induced polarization relaxation time spectra

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ABSTRACT
The Singular Value Decomposition (SVD) method is used to invert induced polarization (IP) decay curve for the IP relaxation time spectrum. A nonnegative restriction is added during inversion for meaningful solution of the amplitude of the relaxation spectrum. The analysis of the experiment design, by comparing results in different range, shows that the appropriate number of relaxation arrangement points in data inversion lies with 32 to 64. The attenuation of noise in this range is better and resolution is good. IP relaxation spectrum indicates the pore size distribution that in turn indicates the permeability of shaly sand reservoir. Combining the average relaxation time (geometric average time) with the porosity can improve the accuracy of estimation of the permeability in comparison to that obtained using total porosity from core measurements.

INTRODUCTION

Permeability plays a crucial role in the evaluation of aquifers and hydrocarbon reservoirs. In general permeability is estimated from laboratory measurements of core samples. It may vary over several orders of magnitude from different plugs in the same core. For instance, drastic permeability reductions result from the growth of minute amounts of secondary clay minerals on quartz grains that further changes the geometry of the hydraulic capillaries. In general permeability is a function of the properties of the pore space like porosity as well as shape, size, spacing and orientation of the rock grains. Therefore permeability of fractured rocks representing insitu conditions can be obtained only if the exact spacing and orientation of rock fragments are duplicated in the laboratory that however is rarity in reality. Therefore, the possibility of accurate measurement of insitu permeability by using well logs is often mandatory.

Initially, the permeability was estimated by resistivity method. Conceptually, the resistivity method is based on the law of conservation of charges and Ohm's law and interestingly the hydrodynamics is also based on similar laws of conservation of mass and Darcy's law of laminar flow. Hence interrelationship between resistivity and permeability is expected to exist if medium is the same. Sri Niwas & Lima (2003) demonstrated some practical models for electrical and hydraulic effects and their linkage with some complimentary inputs that are fresh. Probably the theoretical link between resistivity and permeability can be found by using the "pore channel model " or "capillary tube model" derived by Carman (1956) for describing the streaming fluid flow in a porous rock. Hydrodynamic permeability and electrical conductivity are basically pore geometry controlled properties (Pfannkuch 1969). Therefore, if the medium is same the simplest relationship can be established (Lima & Sri Niwas 2000). However, the relationship contains the term of resistivity or conductivity of the clay grain, which is rather difficult to estimate.

Induced Polarization method plays a significant role in computing amount of clay present in a litho unit and its resistivity. Thus joint interpretation of resistivity and IP data can improve the computation of permeability.

Nuclear Magnetic Resonance (NMR) logging tool is also used for estimating the permeability. But it has small depth of investigation, besides low signal-to-noise ratio (Vinegar & Waxman 1982).

The estimation of permeability based on borehole measurements can be improved further if additional logs are interpreted in addition to porosity log. From these additional logs specific surface, clay content and bound water, irreducible water saturation, and cation exchange capacity can be estimated (Pape, Clauser & Iffland 1999). New developments, which promise a more direct access to values of the specific surface, are the nuclear magnetic resonance (NMR) method (Kenyon et al., 1995) and the interpretation of complete decay curves of induced polarization in time domain (Pape & Vogelsang 1996).

The induced polarization (IP) has several advantages over to the NMR logging. The IP has more depth of investigation and high signal-to-noise ratio. Since Induced Polarization phenomenon takes place when clay particles comes in contact with conducting fluid the IP log observes only water and not oil in the formation. Lima & Sri Niwas (2000) emphasized combined use of electrical resistivity and induced polarization measurement for computing hydraulic properties.

But as we know, the permeability estimation from time-domain IP has received little attention. In the time domain, the IP response is measured only along conducting pathways through the formation. Therefore, disconnected and dead-end pores, which do not contribute to permeability, are not ignored (Vinegar & Waxman, 1987). The detection of IP in the time domain is realized by measuring both the maximum voltage during the charging period of a direct-current flow, and the transient decay of electrical potential as a function of time, subsequent to the turn off of the current. According to IP theory, the decay curve is a superposition of signals coming from all pores within the measurement volume of the formation. The curve can be decomposed into a multi-exponential decay, and a relaxation spectrum can be estimated.

Maosong et al.,(2004) have described results of a study to estimate permeability from the IP measurement of core samples using the IP relaxation time spectra for the first time. Further, Nelson (1994) reviewed a number of porosity-permeability *(f-k)* relationships. For some homogeneous sandstone lithologies with a unique history of sedimentation and diagenesis, the data show quasi-linear relationships in *f-log (k)* plots. Maosong et al., (2004) proposed a new linear relationship in k and log (Txf^4) for shaly sands, where k is permeability in *md*, f is the porosity and T is the average IP relaxation time constant in milliseconds.

THEORY

Shaly sand contains randomly distributed sand zones and clay in the form of either laminar clay or dispersed clay. Since outer surface of clays are negatively charged the clay-rich zones having contact with formation water act as cation-selective membranes and restrict the flow of anions under the applied electric field leading to the formation of double layer of charges. After due polarization an electrochemical gradient is developed across the sand zone by the build up of electrolyte concentration at the edge of the clay-rich zone. Upon switching of the electric current, ions relax to the equilibrium states and the concentration gradient decreases that constitute the induced polarization decay curve.

For one pore, the IP decay curve follows the rule of a single exponential function with the decay time constant, $T = R^2 / D$, where D is the diffusion constant of electrolyte ions in the aqueous solution at formation temperature, and R is the length of the clay-free zone along the direction of the applied electric field (Jost 1952). But the natural rocks composed of pore groups with different sizes. The normalized IP decay signal $y(t)$ $(= V(t) / V(0))$ is a superposition of IP signal from each pores, where $V(t)$ is the IP decay signal at any time t and $V(0)$ at time $t=0$.

y(t) can be expressed as (Maosong et al., 2004)

$$y(t) = \frac{V(t)}{V(0)} = \sum_i f_i \cdot \exp(-\frac{t}{T_i}) \qquad (1)$$

where f_i is the fraction of the total pore volume in which the time constant equals to T_i.

The problem of representing an arbitrary function in terms of an exponential function consists of estimating $2n$ parameters, f_i and T_i. The problem of finding f_i and T_i is a non-linear inverse problem necessitates the fit of a set of exponentials to the decay curve. If the exponents T_i's in exponential expansion of $y(t)$ are known *a priori* or assumed to be known by some statistical criteria then the problem becomes to find only f_i's, which turns out to be a linear problem. This type of computational advantages have been adopted by Sri Niwas & Israil (1986, 1987) for computations of apparent resistivity over layered earth model.

The above problem can be cast as least-squares minimization problem with non-negativity constraint:

$$\psi = \|Mf - y\|^2 \rightarrow \min \qquad (2)$$

$$M_{ij} = \exp(-t_i / T_j) \qquad (3)$$

where $y = (y_1, y_2, \ldots, y_m)^t$ is the IP decay signal, $f = (f_1, f_2, \ldots, f_n)^t$ is the amplitude of the component with relaxation time constant T_j. Here T_j $(j = 1,2,\ldots,n)$ are relaxation time constants (which are *a priori* assumed), and superscript t stand for matrix transpose operation.

The curve of f_i versus T_i is named as induced polarization relaxation time spectrum. The clay-free relaxation distances are substantially the same as the pore size distribution. So the relaxation spectrum reflects the pore size distribution of shaly sand (Vinegar & Waxman 1987). Once the distribution of the pore size is known, the formation permeability in turn can be calculated.

Ordinary matrix inversion scheme can not be used for the above problem, as the condition number was very high. Therefore for obtaining the least square solution Singular Value Decomposition (SVD) can be used.

According to SVD theory, matrix M can be written as

$$M_{m\times n} = U_{m\times n} \cdot \Lambda_{n\times n} \cdot V^{t}_{n\times n} \tag{4}$$

where $U_{m\times n}$ is an orthogonal matrix whose columns are eigenvectors of $M\times M^t$, $V_{n\times n}$ is an orthogonal matrix whose columns are eigenvectors of $M^t\times M$, $\Lambda = |\mathrm{diag}(\lambda_j)|_{n\times n}$ is a non-negative diagonal matrix and λ_j's $(1\le j\le n)$ are the singular values of matrix M. By convention, the ordering of the singular vectors is determined by high-to-low sorting of singular values, with the highest singular value in the upper left index of matrix. Furthermore, $\lambda_j > 0$ for $(1\le k\le p)$, and $=0$ for $(p+1\le k\le n)$, where p is the rank of matrix M.

The resulting inverse solution can be determined as

$$\hat{f} = V \cdot \left[diag\left(\frac{1}{\lambda_1}, \frac{1}{\lambda_2}, \ldots \frac{1}{\lambda_p}, 0, \ldots 0 \right) \right] \cdot \left(U^t \cdot y \right) \tag{5}$$

A non-negative restriction on f_i's must be included in the inversion methodologies as the negative value of any of f is physically meaningless. Instead of adding non-negative restriction on at the time of matrix inversion, modulus or square of f_i's can be used. Experimental results show that the shape of the IP relaxation spectrum and the pore size distribution from mercury capillary pressure are very similar. And this indicates that the IP relaxation spectrum reflects the pore size distribution directly (Maosong et al., 2004).

NUMERICAL RESULTS

The present data obtained from website (http://www.icdp-online.de/sites/ktb/ welcome.html) of International Continental Scientific Drilling Program (ICDP). The data contains measures of IP decay curves of the KTB main borehole between 354 and 2000 *m* depth. One decay series contains the primary voltage and decay voltages after 0, 33, 40, 49, 60, 73, 88, 107, 130, 157 and 191 milliseconds. Derived porosity from IP data for each depth is also given for each 1 *m* depth is between 354-363 *m* shown in Table 1.

Construction of Matrix M

Matrix M is constructed whose elements are given by equation (3) $M_{ij} = \exp(-t_i/T_j)$, where T is the relaxation time (in ms) and t is the time (in ms).

Four matrices are constructed and for all matrices t varies between 0, 33, 40, 49, 60, 73, 88, 107, 130, 157, and 191 ms. A typical predefined relaxation time constant T series, which is named as T arrangement, is j points logarithmically selected from T_{min} to T_{max} in a uniform step. For the present data set, T_{min} = 0.01ms and T_{max} = 10,000 ms and the values of j are 8, 16, 32 and 64 in the present work.

SVD Analysis of the Matrix M

All four matrices were analyzed using Singular Value Decomposition (SVD). There are only 2 or 3 significant eigenvalues in all M matrices. It is observed that highest value of the first eigenvalue for the 64 points T arrangement is roughly 1.4 times as large as

Table 1. Measures of IP decay curves and porosity of the KTB main hole between 354 and 363 m depth.

Depth(m)	V(0)	V(33)	V(40)	V(49)	V(60)	V(73)	V(88)	V(107)	V(130)	V(157)	V(191)	Porosity (%)
354	732.17	48.25	39.65	28.95	28.07	23.27	19.39	18.22	14.79	12.55	8.98	0.15
355	767.26	46.78	39.16	29.19	28.33	24.25	19.87	17.84	15.33	13.02	8.94	0.15
356	797.65	46.22	39.14	30.55	29.49	26.81	22.1	19.6	16.39	14.03	9.94	0.15
357	818.66	45.26	38.8	31.1	29.51	27.4	22.66	20.32	16.99	14.41	10.55	0.15
358	872.17	40.95	35.52	29.59	27.42	26.18	21.43	19.16	16.6	13.67	10.26	0.15
359	908.92	38.21	33.07	28.73	26.36	25.57	21.52	19.87	16.83	13.74	10.98	0.41
360	914.44	37.4	32.53	28.8	26.19	24.86	21.75	20.91	17.73	14.46	11.71	0.24
361	897.91	38.92	34.18	30.5	27.55	25.59	22.45	21.5	17.73	14.39	11.1	0.24
362	864.69	44.51	39.16	34.52	31.1	28.69	25.05	24.13	19.12	16.15	12.02	0.8
363	815.8	46.37	40.6	35.2	31.46	29.38	25.71	24.59	19.15	16.01	11.8	0.24

for the 32 points T arrangement, roughly twice as large as for the 16 points T arrangement and roughly thrice as large as for the 8 points T arrangement. This has relevance to the noise sensitivity of various T arrangements.

Eigenvalue number and condition number ($\eta_i = \lambda_1/\lambda_i$) are shown in Table 2. To demonstrate the relative resolution of different T arrangements, condition numbers are plotted in Figure 1. Very large condition number points the ill-conditioned nature of the system that means that small error in data would results in large error in the solution. Both Table 2 and Figure 1 indicate that the resolution of 8 points T arrangement is not satisfactory because fifth eigenvalue is extremely small. Thus this arrangement should not be used. Whereas other 3 that is 16, 32 and 64 can very well be adopted. However, we have chosen 32 and 64 for the present work.

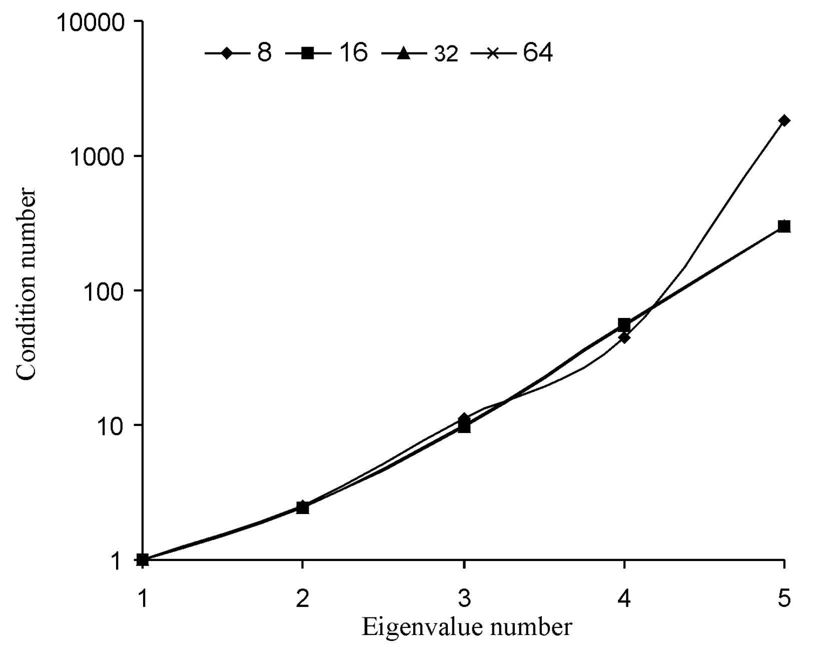

Figure 1. Plot of condition number of matrix M for various number of points (8, 16, 32, 64) of T arrangements.

Table 2. Eigenvalues and condition number of M matrices for various n points T arrangements (n = 8, 16, 32, 64).

	8 points T arrangement		**16 points T arrangement**		**32 points T arrangement**		**64 points T arrangement**	
i	λ_i	η_i	λ_i	η_i	λ_i	η_i	λ_i	η_i
1	5.1620	1	7.1295	1	9.9645	1	14.0082	1
2	2.0513	2.52	2.9039	2.46	4.1093	2.43	5.8127	2.41
3	0.4552	11.34	0.7128	10.00	1.0127	9.84	1.4348	9.76
4	0.1153	44.78	0.1265	56.34	0.1807	55.15	0.2564	54.63
5	0.0029	1806.17	0.0239	297.53	0.0336	296.40	0.0478	293.35
$\sum_i \lambda_i^{-1}$	361.4481		51.5225		36.6122		25.7810	

Inversion of Decay Curves- Singular Value Decomposition (SVD)

Decay curves are inverted in order to get the relaxation spectra. Maosong et al., (2004) suggested that minus terms in the solution of the amplitude of the relaxation spectrum are possible which has no physical meaning. A non-negative restriction, i.e., $f_j > 0$ (j = 1, 2 . . . n) must be added when we solve the model. Starting with a full matrix M, a tentative solution f^ is computed. If all components are positive or at least zero, the solution f^ is accepted. If one or more components are negative, the column of M corresponding to the most negative amplitude is eliminated, the corresponding component in f^ is set to zero, and the process is repeated with a reduced matrix M^1 to find a reduced solution $f\hat{}^1$. This calculation is done repeated until all components of *f* are positive or at least zero.

However, in doing so some useful information is lost affecting the resolution. We have adopted different approach using the square of the least square solution of the given problem to ward off negative value. Thus the induced polarization relaxation time spectrum is redefined as the curve of f_i^2 versus T_i instead of f_i versus T_i.

At the time of SVD inversion those eigenvalues are excluded for which the condition number was more than 300. This number was chosen keeping in mind the condition number vs eigenvalue number plot that invariably show the cut off eigenvalue having condition number approximately 300.

For the whole data set decay curves (Fig.2) are inverted and relaxation spectra are obtained for 32 and 64 points of T arrangements only. These computed spectra are given in Fig.3 (32 point arrangements) and Fig.4 (64 point arrangements).

Computation of Average Relaxation Time

Average relaxation time for geometric series of T arrangements is calculated by

$$T_g = \left(\prod_{i=1}^{n} T_i^{f_i^2} \right)^{1 / \sum_{i=1}^{n} f_i^2} \tag{6}$$

for the whole data set T_g.

Estimation of Permeability

According to the proposed model of Tong Maosong et al. (2004), the relationship in porosity and permeability is given by

$$K = c \times \left(T_g \times \phi^4 \right)^b \tag{7}$$

Since we don't have enough information about the permeability of the concerned area, it is impossible to correlate the permeability and porosity. Also we do not have any idea of the value of constants b and c, the average values of b and c provided in Tong Maosong et al. (2004) were used for the estimation of permeability and which are given as b = 1.57 and c = -9.6. Using these values of b and c the permeability is estimated for whole data set (i.e. for depth 354 to 2000 meter). A part of which is presented in Table 3.

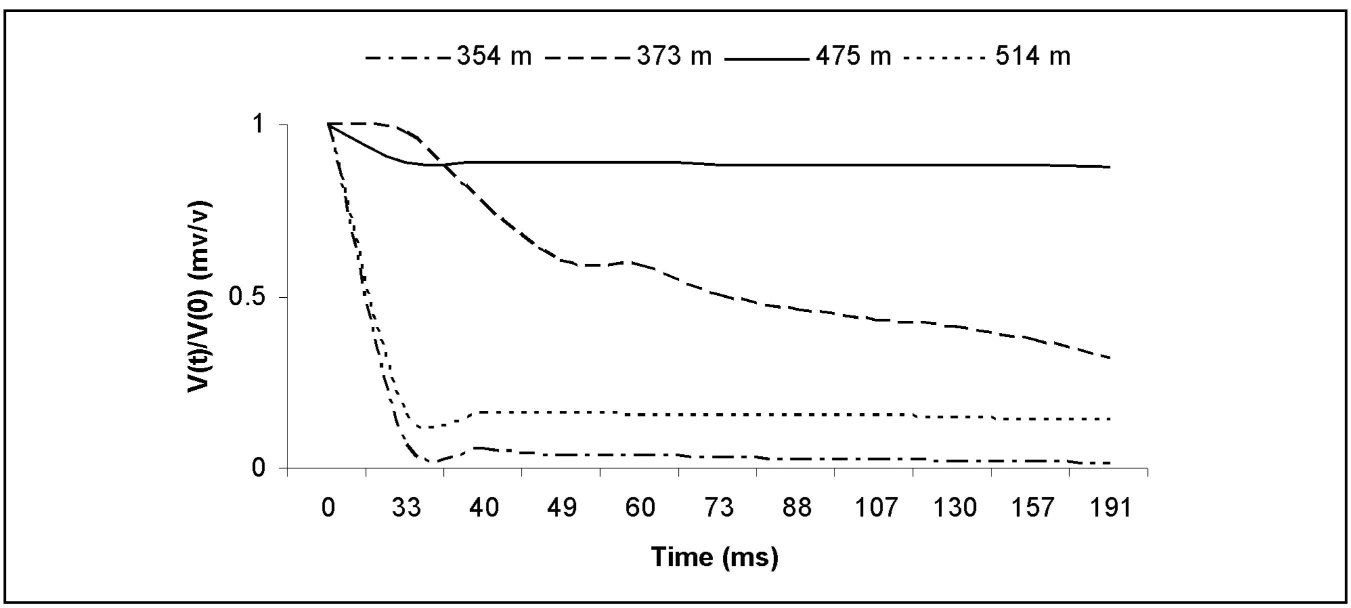

Figure 2. Analyzed 4 typical decay curves taken from ICDP data recorded in KTB main bore hole at different depths (354, 373, 475 and 514 m).

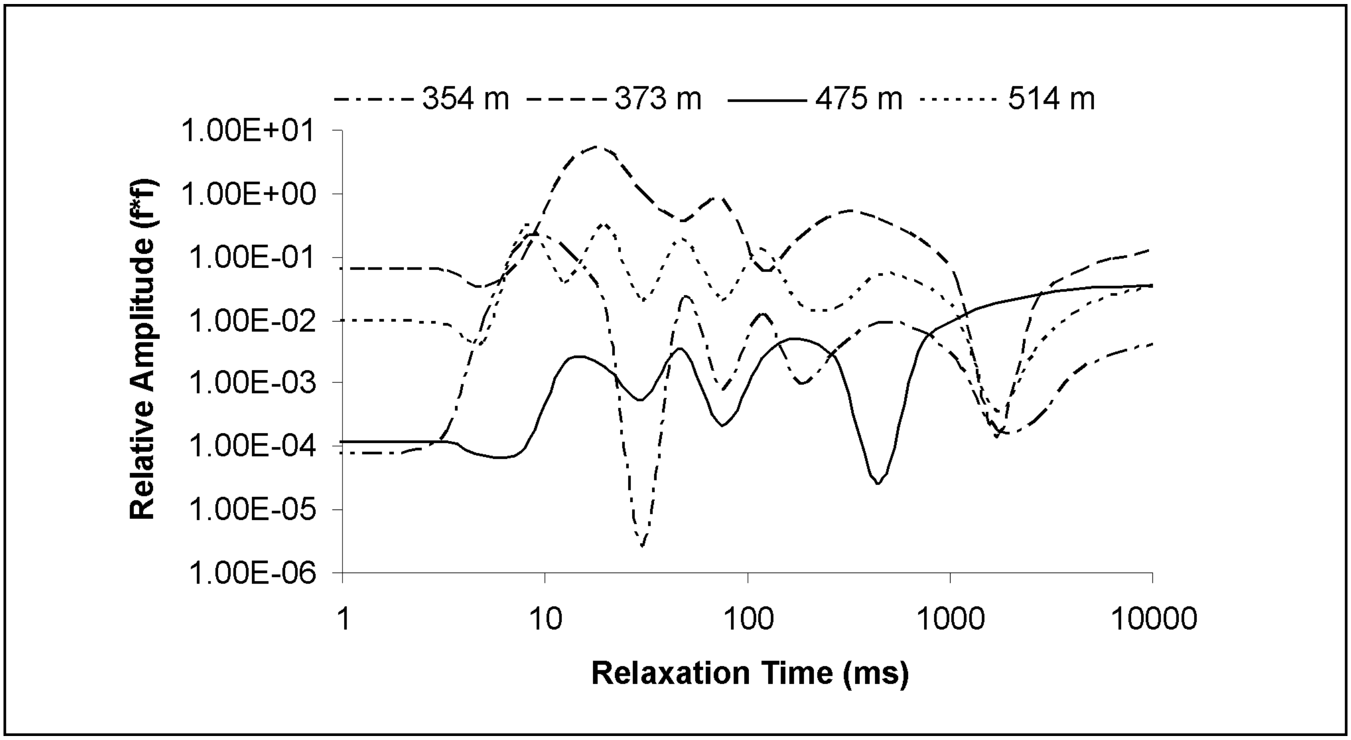

Figure 3. Relaxation Time spectra of data given in Figure 2 for different depth levels for 32 points of T arrangements.

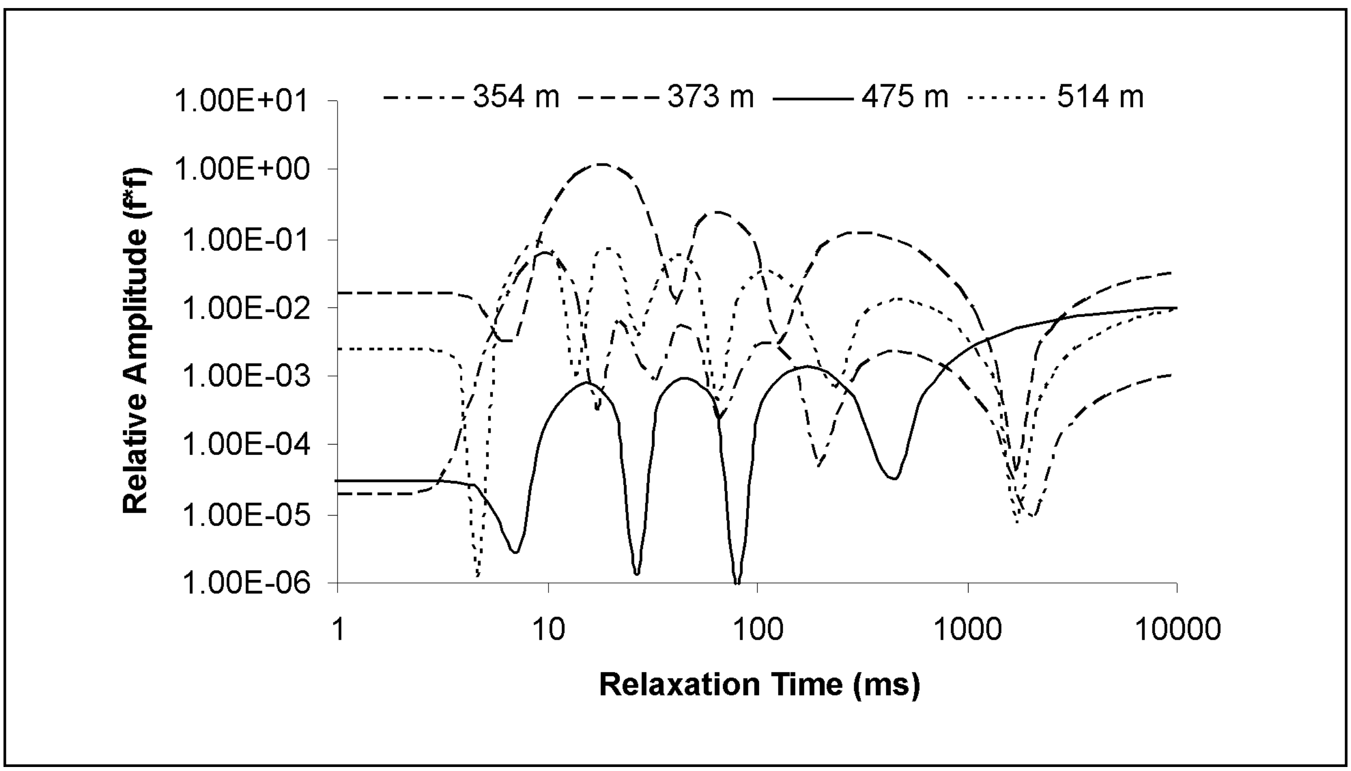

Figure 4. Relaxation Time spectra of data given in Figure 2 for different depth levels for 64 points of T arrangements.

Table 3. Average Geometric Relaxation Time (ms), Porosity (%) and Estimated Permeability (Darcy) for a set of data

Depth (m)	**T_g(ms)** (32points)	**T_g(ms)** (64points)	**Porosity (%)**	**Permeability (Darcy)** (32 points)	**Permeability (Darcy)** (64 points)
354	1.60E+01	1.56E+01	0.15	1.31E-16	1.3E-16
355	2.48E+01	2.47E+01	0.15	2.6E-16	2.59E-16
356	1.86E+01	1.86E+01	0.15	1.66E-16	1.65E-16
357	1.55E+01	1.54E+01	0.15	1.24E-16	1.23E-16
358	1.62E+01	1.61E+01	0.15	1.33E-16	1.32E-16
359	2.27E+01	2.34E+01	0.41	1.25E-13	1.31E-13
360	5.97E-01	6.21E-01	0.24	1.43E-17	1.52E-17
361	5.85E-01	6.09E-01	0.24	1.39E-17	1.48E-17
362	7.74E-01	8.02E-01	0.8	4.14E-14	4.37E-14
363	1.26E+00	1.30E+00	0.24	4.64E-17	4.84E-17

Thus IP decay curves are successfully inverted in order to get the IP relaxation time spectra and permeability is estimated.

CONCLUSIONS

The Singular Value Decomposition is used for the inversion of the Induced Polarization decay curves. The IP relaxation time spectra show a strong relationship of constant average relaxation time with the amount of clay present. The appropriate number of relaxation arrangement points in IP data inversion should be tried in the range of 32 to 64. It is estimated on the basis of trade off between noise and resolution. The IP relaxation time distribution indicates the pore size distribution. Further the permeability is estimated using the proposed permeability-porosity relationship.

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